A&A 518, A3 (2010)
DOI: 10.1051/0004-6361/201014374

Astronomy & Astrophysics

Milne-Eddington inversion of the Fe I line pair at 630 nm *(Research Note)*

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Received 8 March 2010 / Accepted 26 May 2010

ABSTRACT

Context. The iron lines at 630.15 and 630.25 nm are often used to determine the physical conditions of the solar photosphere. A common approach is to invert them simultaneously under the Milne-Eddington approximation. The same thermodynamic parameters are employed for the two lines, except for their opacities, which are assumed to have a constant ratio.

Aims. We aim at investigating the validity of this assumption, since the two lines are not exactly the same.

Methods. We use magnetohydrodynamic simulations of the quiet Sun to examine the behavior of the ME thermodynamic parameters and their influence on the retrieval of vector magnetic fields and flow velocities.

Results. Our analysis shows that the two lines can be coupled and inverted simultaneously using the same thermodynamic parameters and a constant opacity ratio. The inversion of two lines is significantly more accurate than single-line inversions because of the larger number of observables.

Key words. radiative transfer – polarization – line: profiles – Sun: surface magnetism – Sun: photosphere – methods: data analysis

1. Introduction

Both solar physics and radiative transfer owe much to the Milne-Eddington (ME) approximation. In an incredibly simplistic description of spectral line formation, which provides useful hints for understanding the behavior of spectral lines that form in solar and stellar atmospheres. It also offers a key diagnostics to infer the physical conditions of the plasma. This is particularly true when magnetic fields are present. The first solution of the radiative transfer equation in the presence of a magnetic field was derived adopting the ME assumption that all physical quantities relevant to line formation are constant with depth (Unno 1956; Rachkovsky 1962a,b). Under these conditions, the solution is analytic and therefore, by simply varying the model parameters, one gains an understanding of the Stokes line profiles' behavior. Similarly, the analytic character of the ME solution allows perturbative analyses like those performed by Landi Degl'Innocenti & Landolfi (1983), who studied the influence of velocity gradients, or Orozco Suárez & Del Toro Iniesta (2007), who calculated the sensitivities of ME Stokes profiles to the various model parameters. Inversion codes of the radiative transfer equation are diagnostic tools that become simpler under the ME hypothesis. Since the pioneering work by Harvey et al. (1972) and Auer et al. (1977), and after improvements by Landolfi & Landi Degl'Innocenti (1982) and Landolfi et al. (1984), a number of ME inversion codes have been developed. These include the HAO code (Skumanich & Lites 1987; Lites & Skumanich 1990), MELANIE (Socas-Navarro 2001), HELIX (Lagg et al. 2004), MILOS (Orozco Suárez & Del Toro Iniesta 2007), and VFISV (Borrero et al. 2010).

Strictly speaking, the ME model is applicable to just one spectral line. The reason is that the so-called thermodynamic parameters of the model are meant to characterize the behavior of the specific line under consideration. The line-to-continuum opacity ratio, η_0, the Doppler width of the line, $\Delta\lambda_{\rm D}$, and the damping parameter, a, govern the shape of the Stokes profiles (i.e., they are the parameters of the Voigt and Faraday-Voigt functions). In turn, the source-fuction terms, S_0 and S_1, control the continuum level, the line depression, and the Stokes amplitudes. However, many investigations are based upon the simultaneous ME inversion of spectral line pairs, like the well-known Fe I doublet at 630 nm (e.g., Lites et al. 1993) or the Mg I *b* lines at 517.2 and 518.3 nm (Lites et al. 1988). These inversions are reported to use no extra free parameters as compared to single-line inversions, based on the similarities between the two lines belonging to the same multiplet: the opacities for each line are specified in the ratio of their respective oscillator strengths, while the Doppler widths and the damping parameters are assumed to be identical for the two lines. S_0 and S_1 are also the same for both lines because they lie very close in wavelength.

This strategy has been widely used for more than 20 years to analyze the Fe I 630 nm measurements taken with instruments such as the Advanced Stokes Polarimeter (Elmore et al. 1992) or the spectropolarimeter aboard the *Hinode* satellite (Lites et al. 2007; Kosugi et al. 2007; Tsuneta et al. 2008). The simultaneous inversion has been shown to provide better results than single-line inversions (e.g., Lites et al. 1994). The better performance is easy to understand: if the observables reproduced by the inversion are doubled, the results can be expected to be more accurate (at least by a factor $\sqrt{2}$). However, the two lines do not have exactly the same atomic parameters (see Table 1). For this

Table 1. Atomic data for the Fe I 630 nm lines.

λ_0 (nm)	χ (eV)	$\log gf$	Transition	α/a_0^2	σ	$g_{\rm eff}$
630.1501	3.654	−0.75	$5P_2$–$5D_2$	0.243	840.5	1.67
630.2494	3.686	−1.236	$5P_1$–$5D_0$	0.240	856.8	2.5

Notes. Shown are the central wavelength of the transition, λ_0, the excitation potential of the lower atomic level, χ, the multiplicity of the lower level times the oscillator strength, $\log gf$, the collisional broadening parameters, α and σ (in units of Bohr's radius a_0), and the effective Landé factor of the line, $g_{\rm eff}$. The $\log gf$ values have been derived from a fit to the solar spectrum using a two-component model of the quiet Sun (Borrero & Bellot Rubio 2002).

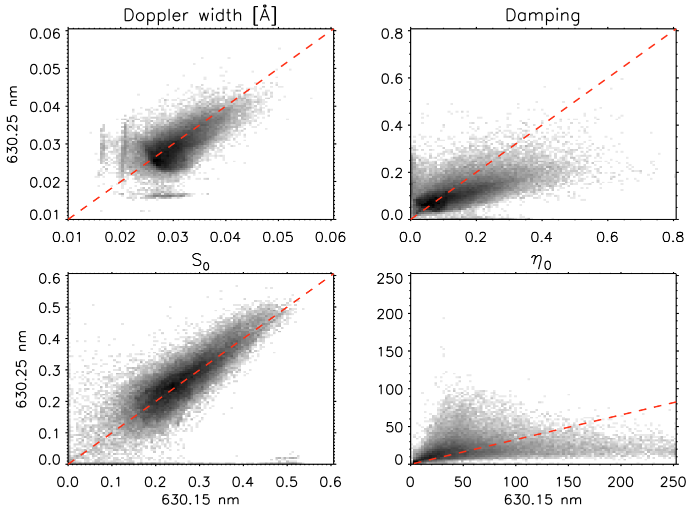

Fig. 1. $\Delta\lambda_{\rm D}$, a, S_0, and η_0 parameters from the inversion of the Fe I line at 630.25 nm vs. those obtained from the 630.15 nm line. The dashed lines represent one-to-one correspondences. In the bottom right panel the dashed line represents $\eta_{0,2}/\eta_{0,1}$ = 0.327. Both lines are inverted separately.

reason, they are formed at slightly different heights, where different physical conditions may exit (e.g. Martínez González et al. 2006). In view of these differences and of the large excursions of η_0, a, and $\Delta\lambda_{\rm D}$ in the real solar photosphere, both horizontally and vertically, one may wonder whether the simultaneous ME inversion of the two lines is valid and what the limitations are. General inversion codes not relying on the ME approximation perform exact line transfer calculations, so they are able to invert the two lines without inconsistencies. The purpose of the present Research Note is to check the ME case using the excellent test bench offered by modern magnetohydrodynamic (MHD) simulations.

2. Influence of the ME thermodynamic parameters

Trade-offs among the ME thermodynamic parameters have been reported and found not to be very important for the inference of vector magnetic fields and line-of-sight (LOS) velocities (e.g., Lites & Skumanich 1990; Westendorp Plaza et al. 1998). An explanation of this phenomenon has been given by Orozco Suárez & Del Toro Iniesta (2007). Therefore, the first question we should answer is whether or not the thermodynamic parameters of the lines need to be the same to obtain similar LOS velocities, $v_{\rm LOS}$, magnetic field strengths, B, inclinations, γ, and azimuths, φ, from their individual analysis.

To investigate this, we synthesized realistic Stokes profiles for the two Fe I lines at 630 nm using simulations performed with the MPS/University of Chicago Radiative MHD code. This code solves the MHD equations for compressible and partially ionized plasmas. Further information about the simulations can be found in Vögler (2003) and Vögler et al. (2005). The snapshot used here belongs to a mixed-polarity simulation run with an average magnetic field strength of $\langle B\rangle \simeq 140$ G at $\log \tau_{500} = -1$. We generated the Stokes spectra of the two lines from the MHD models using the SIR code (Ruiz Cobo & Del Toro Iniesta 1992), as explained by Orozco Suárez et al. (2010). We then inverted the two lines *separately* with the MILOS code. The inversion was carried out assuming a one-component model atmosphere (magnetic filling factor unity) and zero macroturbulent velocity. No noise was added to the Stokes profiles.

Figure 1 shows scatter plots of $\Delta\lambda_{\rm D}$, a, S_0, and η_0 as inferred from the inversion of the two lines. The results for S_1 are very similar to those for S_0, with less scatter. In this and other figures, the gray colors inform about the pixel density, with black meaning higher values. Over-plotted are dashed lines representing one-to-one correspondences, except for η_0 where the ratio $\eta_{0,2}$ / $\eta_{0,1}$ = 0.327 is indicated (1 and 2 stand for the 630.15 and the 630.25 lines, respectively; see below). The plots include all pixels in the simulation snapshot (288 × 288), independently of the polarization signal. Obviously, the thermodynamic parameters are far from being the same for the two lines. The scatter is large for $\Delta\lambda_{\rm D}$ and S_0, although they show a linear correlation with a slope close to unity. For η_0 and a the scatter is dramatic; differences of up to a factor 3 for a and of more than one order of magnitude for η_0 can be seen. Indeed, the η_0 values obtained for the 630.15 nm line span the full range of variation of the line-to-continuum opacity ratio in real solar atmospheres (see, e.g., Fig. 11 of Westendorp Plaza et al. 1998).

Do these discrepancies between the thermodynamic parameters affect the magnetic and velocity inferences? The answer to this question is negative, as can be seen in Fig. 2, where $v_{\rm LOS}$, B, γ, and φ are displayed as inferred from the individual inversion of the 630.15 and 630.25 nm lines. Despite the strong scatter in the thermodynamics, the results from both lines are remarkably similar. Therefore, we have to conclude that although the thermodynamic parameters of ME inversions may have little meaning, it is possible to establish approximate relations between the parameters of the two lines for use in simultaneous inversions, as we shall see in the next section.

3. Understanding the simultaneous inversion

Since iron is mostly ionized and local thermodynamic equilibrium conditions prevail in the solar photosphere, the dependence of the logarithmic derivative of η_0 on temperature (the most important quantity in line formation) is linear with the excitation potential χ and the ionization potential I, and quadratic with the temperature. $\Delta\lambda_{\rm D}$ is proportional to the square root of temperature and does not depend on the atomic parameters of the line. Assuming van der Waals broadening for the calculation of the damping coefficient, an intricate (but weak) dependence of a on χ, along with a proportionality with the 7/10-th power of temperature, is obtained (see, e.g., Gray 2008).

Neglecting the differences induced by the slightly different excitation potentials of both lines, the a and $\Delta\lambda_{\rm D}$ ratios are proportional to $\lambda_{0,2}/\lambda_{0,1}$. Thus, the damping and Doppler width can safely be assumed to be equal for both lines, which is also the

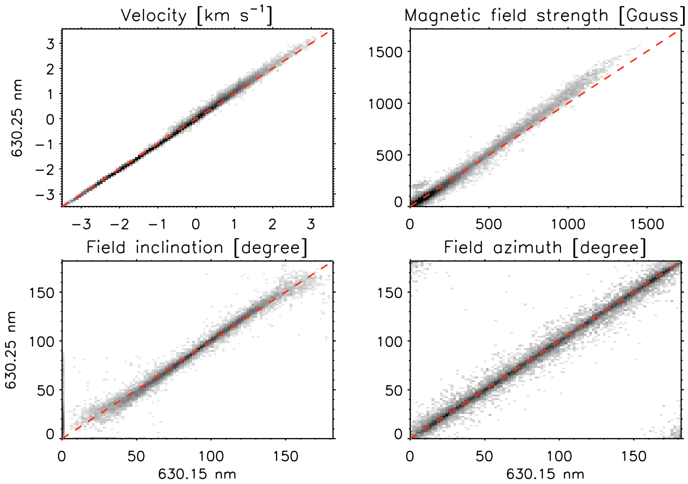

Fig. 2. Line-of-sight velocity, magnetic field strength, inclination, and azimuth from the inversion of the 630.25 nm line vs those from the inversion of the line at 630.15 nm. Both lines are inverted separately.

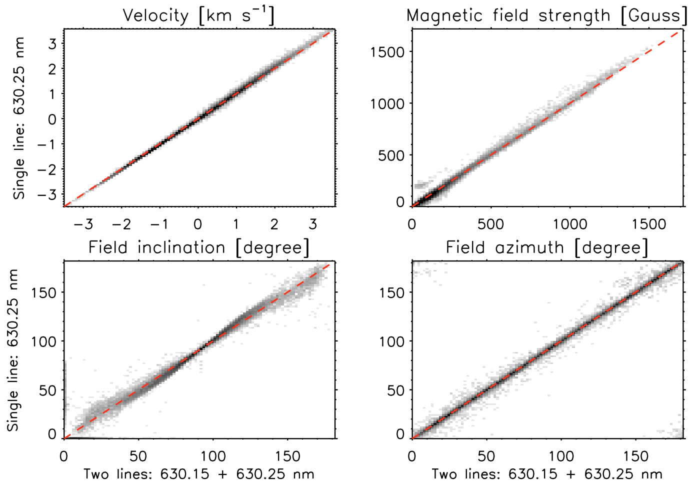

Fig. 4. Magnetic field strength, inclination, azimuth, and LOS velocity from the the inversion of the 630.25 nm line vs those from the simultaneous inversion of the two lines. The latter inversion was done assuming the same model atmosphere (total of nine free parameters) for the two lines, with their η_0 values coupled.

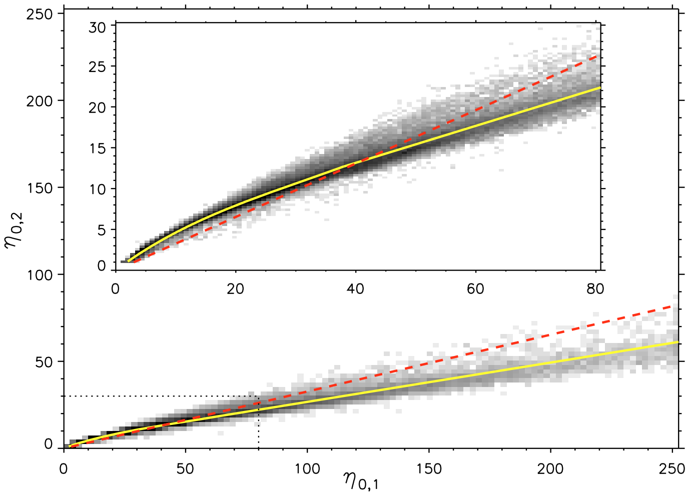

Fig. 3. η_0 values retrieved from the inversion of the Fe I lines at 630.15 and 630.25 nm. The dashed line represents the theoretical ratio $\eta_{0,2}/\eta_{0,1} = 0.327$. The solid line stands for a multi-polynomial fit. The inset zooms in the dotted box.

case for S_0 and S_1, because the Planck function should not present significant differences in such a short wavelength interval. η_0 is proportional to the square of the central wavelength of the line and to its gf value. Therefore, $\eta_{0,2}/\eta_{0,1} \approx (g_2\, f_2)/(g_1\, f_1)$, where g is the multiplicity of the lower level and f the oscillator strength. Note that this ratio is independent of the temperature or other thermodynamic quantities, so it should not change with height in the atmosphere even if the opacity varies by orders of magnitude. With the parameters of Table 1, the opacity ratio for the Fe I 630 nm lines turns out to be $\eta_{0,2} / \eta_{0,1} = 0.327$. This is the value implemented in MILOS. The HAO code and MELANIE use similar ratios[1].

[1] The opacity ratio depends basically upon f, which is known with limited precision and varies from one source to the next. This causes an uncertainty in the theoretical opacity ratio. For example, the laboratory measurements of Bard et al. 1991 give $\log gf = -0.718$ for Fe I 630.15 nm, rather than the −0.75 specified in Table 1. With this oscillator strength, the theoretical ratio would be 0.301 (closer to the values retrieved from the inversion, but only at the high η_0 end).

To check the validity of this estimate, we inverted the 630.25 nm line again but forcing S_0, S_1, $\Delta\lambda_\mathrm{D}$, and a to be equal to those obtained from the previous inversion of the 630.15 nm line. The remaining model parameters were allowed to vary freely. Figure 3 shows the η_0 values retrieved from the inversion. The dashed line represents the ratio $\eta_{0,2} / \eta_{0,1} = 0.327$. The solid line corresponds to a multi-polynomial fit $y = a[1]x^3 + a[2]x^2 + a[3]x$ with $a = [1 \times 10^{-4}, -9.3 \times 10^{-3}, 0.5]$ for $\eta_{0,1} \leqslant 38$ and $y = b[1]x + b[2]$ with $b = [0.23, 4.1]$ for $\eta_{0,1} > 38$. Note that the theoretical ratio provides a fair description of the relationship between the two η_0 values in the range where most of the points are located (to stress the differences, the inset zooms in on the boxed area). Since the exact values of the thermodynamical parameters are not very important for the determination of the magnetic field vector and the LOS velocity, we conclude that it is safe to use a constant opacity ratio to invert the two lines simultaneously without increasing the number of free parameters.

The final consistency proof is shown in Fig. 4, where the parameters obtained from the inversion of Fe I 630.25 nm are plotted against those coming from the simultaneous inversion of the two lines as coupled through their theoretical η_0 ratio. The scatter is very small for B and $\mathrm{v_{LOS}}$ and somewhat larger for γ and φ, but still much smaller than that of Fig. 2. This suggests that the accuracy of analyses based on one single line (e.g., Bommier et al. 2009) could be improved by adding the other line.

Acknowledgements. We thank an anonymous referee for raising the issue investigated in this paper. Our work has been supported by the Spanish MICINN through projects AYA2009-14105-C06-06 and PCI2006-A7-0624, by Junta de Andalucía through project P07-TEP-2687 (including European FEDER funds), and by the Japan Society for the Promotion of Science.

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