A&A 516, A22 (2010)
DOI: 10.1051/0004-6361/200912629

Astronomy & Astrophysics

On the observational bias of the Trojan swarms

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Received 4 June 2009 /Accepted 30 July 2009

ABSTRACT

Aims. I investigate whether the Trojan swarms are observationally biased and place a completion limit on their absolute magnitude.
Methods. Observations including Trojans from a number of observation sites are cross-checked with orbital integration of known Trojans and a set of fictitious Trojan orbits.
Results. The completion limit for the Trojans swarms can be set to be $H = 11.5$ mag. The L_5 swarm is 71% of the L_4 size down to this limit. It is not likely that any existing set of orbital elements can have escaped detections. However, parts of the orbital element space, especially in the inclination, are biased for Trojans fainter than the completion limit. In the absolute magnitude interval 11.5−13 mag, 65% of new objects should have inclinations 15−40°, while this inclination interval currently contains 49% of Trojans in the complete interval 9.5−11.5 mag. Trojans fainter than an absolute magnitude of 13 mag are also clearly biased in the ascending node at values coinciding with the Milky Way.

Key words. minor planets, asteroids: general – planets and satellites: individual: Jupiter

1. Introduction

For a long time after the first Trojan discovery in 1906 (Wolf 1906; Berberich 1906), Trojans were discovered sporadically. In 1960, the plates of the Palomar-Leiden survey (PLS) (Van Houten et al. 1970b) were acquired and although it was not a Trojan survey, 19 Trojan candidates were found. Four uncertain candidates among these Trojans were later confirmed and an additional five Trojans found (Van Houten et al. 1984). Inspired by the detection of the Trojans in the PLS, two fields towards the L_4 swarm were observed (Van Houten et al. 1970a). A resulting completion limit of absolute magnitude $H \approx 8.5$ mag for L_4 was suggested. Furthermore, the number of L_4 Trojans was estimated to be about 700 to $H \approx 13.3$ mag. Shoemaker et al. (1989) adopted a completion limit of $H \approx 8.75$ mag for the L_4 based on observations that extended to the late 1980s. There were indications that the previous size estimation might be too low and a new estimate at $H \approx 13$ mag was made, which equalled 1000 ± 200. Van Houten-Groeneveld et al. (1991) analyzed observations of L_4 completed in 1973 and got 626 L_4 Trojans to $H \approx 12.8$ mag. Furthermore, they lowered the Shoemaker et al. (1989) estimate, which had been based on an erroneously made correction, by about 7%, but they did not consider the completion limit. The Uppsala-DLR Trojan survey (Lagerkvist et al. 2002) used their L_4 observations from 1996 to estimate the number of Trojans brighter than $H = 13$ mag in that swarm to be 1100. A completion limit of $H \approx 10.5$ mag was used, based on the discovery statistics obtained between 1999 to 2001. In the search for Trans-Neptunian objects, Jewitt et al. (2000) analyzed coincidental Trojan observations and deduced that there should be between 1700 and 5700 L_4 Trojans to $H \approx 13$ mag. However, they observed an area between L_4 and L_3, where the Trojan distribution is more stretched out than between L_4 and Jupiter. In doing so, they overestimated the area density in the other directions and hence the population. Compensating for this and their number reduces to about 60% (Lagerkvist et al. 2002). At the 2 km size limit ($H \approx 17.6$ mag), Yoshida & Nakamura (2005) estimated about 4 times as many Trojans as Jewitt et al. (2000) in a similar type of survey. However, apart from making the same type of size overestimation as Jewitt et al. (2000), Yoshida & Nakamura (2005) also misplaced the L_4 point since the 60° longitude of the Lagrangian point applies to the heliocentric system, while Yoshida & Nakamura (2005) used a geocentric system.

The L_5 side was far less well examined than L_4. Degewij & Van Houten (1979) inferred a preliminary result of 3.5 as many Trojans in L_4 as in L_5. However, this was found to be incorrect and revised to be 2.0 (Van Houten-Groeneveld et al. 1991). The half size was also found for faint Trojans (Shoemaker et al. 1989), while the brighter Trojans L_4 and L_5 appeared to be about equal in number. They suggested that down to $H \approx 13$ mag, L_5 should be about 75% as numerous as L_4. Shoemaker & Shoemaker (1990) suggested that the number of L_5 Trojans should be about 100 to $H \approx 11$ mag, corresponding to about 700 to $H \approx 13$ mag or 74% of the L_4. Furthermore, they placed a completion limit on the L_5 to $H \approx 9$ mag.

Considering Trojans to a limit of $H = 11$ mag, there are now 116 in L_5 and 155 in L_4, which implies that the L_5 swarm is 75% of the L_4. Down to $H = 13$ mag, there are so far 632 Trojans in L_5 and 827 in L_4. Lagerkvist et al. (2002) verified their results derived from observations by using the number of known Trojans at $H = 10$ mag and the slope of 0.4 to infer the presence of about 1100 Trojans in L_4. The current slope between $H = 10$ mag and $H = 11$ mag is slightly lower (0.34), but using this slope from $H = 11$ mag would infer about 742 L_4 Trojans to $H = 13$ mag, which is lower than the known number. The slope of 0.4, counted from $H = 11$ mag, infers 978 L_4 Trojans to $H = 13$ mag.

All of these Trojan surveys have two important aspects in common: they are limited in time and only a fraction of the swarm is covered. This requires quite large correction factors. A concern is how much the numbers must be corrected for the inclination and wether the distribution is the same for high inclination Trojans as for low inclination Trojans.

A way to evaluate the bias problem is to use the extensive amount of observational material coming from near-earth-object (NEO) searches. Although much observation effort has been concentrated on the ecliptic, the length of time these surveys have been active will compensate for the smaller observed space. As a consequence of their dynamics, every Trojan must pass the ecliptic twice during their annual period of 10.4–13.5 yr. Furthermore, they must also pass the perihelion once. Many of the NEO surveys have now been operating for about 10 yr, thus covering about one annual Trojan period.

Trojans have been observed for slightly more than a hundred years. However, 96% of all Trojan observations have been made after 1 Jan. 1995 (until 1 Jan. 2008) and 85% of these are from six observatory sites specially selected for this work. The remaining observations have been obtained by 347 other sites, but the 10 largest contributors of data represent 90% of the observations.

Although no longer the site that contribute the most Trojan observations, the two sites with dedicated surveys are still among the top ten contributing observing sites. Palomar (code 675) is the site of the PLS, three follow-up surveys T-1, T-2, and T-3 (Marsden 1991, 1989, 1987), and additional observations (Shoemaker et al. 1989; Shoemaker & Shoemaker 1990). The second site is La Silla (code 809), where ESO is located and the Uppsala-DLR Trojan survey was conducted in 1996, 1997, and 1998 (Lagerkvist et al. 2000, 2002).

The remaining text of this work is organized in the following way. Section 2 describes the data sets and my analysis methods. My result follows in Sect. 3 and discussions are presented in Sect. 4. Conclusions can be found in Sect. 5.

2. Method

A time span from Julian day 2 449 718 (31 Dec. 1994 12:00 UT) to 2 454 467 (1 Jan. 2008 12:00 UT) was selected for this investigation. There are two reasons for this choice. First, several large asteroids surveys have been active during the past decade. These surveys have produced many observations in a general search for NEOs, thus they are not specifically targeted at the Trojan swarms. Second, the time span of 13 years is about the same as the longest orbital period for a Jupiter Trojan, thus all Trojan obits must have passed the ecliptic at least twice and have been close to perihelion at least once during this interval.

The data that have been processed are of three types: observations, Trojan orbit integrations, and integration of fictitious Trojan orbits. The observations are of two types: non-Trojan observations used for detection limits and sky coverage. The Trojan observations are used together with the integration of the known Trojans to confirm how good the surveys are at finding Trojans and keep a track of errors introduced by the investigation method. The integrated Trojans can also be used to adjust the magnitude zeropoint between the surveys, which affects the absolute magnitude detection limits for unknown Trojans. The non-Trojan observations and those of the fictitious Trojans infers whether any specific part of the orbital element space is not mapped e.g. high-inclination Trojans, and how bright an absolute magnitude an unknown Trojan might have and yet still be missed.

2.1. Known Trojans

The Trojan sample was taken from the Minor-Planet-Center's (MPC's) Trojan list[1], which contained orbits for 2295 objects in early January 2008. The orbital elements used were not extracted from this page but form the orbit database at MPC. There was no removal of any orbits from this sample, although it contains e.g., short arc orbits, objects not observed recently and even possibly non-Trojans (Karlsson 2004). The temporarily designated Trojans were compared against all designations and possible numbering so as not to mismatch any objects.

The orbit integration was performed with a RADAU15 integrator (Everhart 1985) with a data dump step of 1 day at 0 UT. Orbital data for the 8 major planets was taken from Larsen & Holdaway (2001) and the effect of the larger moons of Jupiter, Saturn, and Neptune was included in the respective planet (Seidelmann 1992).

2.2. Fictitious Trojans

The fictitious Trojans was analyzed in two steps. Initially, a set of 15 000 objects were created with random angles in ascending node (Ω), perihelion argument (ω), and mean anomaly (M) with the requirement that the mean longitude with respect to Jupiter was less than 170°. Inclination (i) was randomly selected up to 80° and eccentricity (e) up to 0.5. The semimajor axis (a) was selected between 4.92 AU and 5.48 AU but with the requirement that the object dynamically should be in (or close to) the 1:1 resonance region when the mean longitude (λ) was taken into consideration. The resulting distribution along the semimajor axis was shaped as a triangle peaked at 5.2 AU but with two other corners cut because of the initial limit. There are five objects outside this limit in the true Trojan sample. The two most extreme cases 2002 AO_{148} and 2003 FH_7 do not have Trojan orbits, while the other three 1997 TW_2, 2002 ES_{77}, and 2006 DS_{70} might be in horseshoe orbits. A Horseshoe orbit is a natural transition between a tadpole orbit and non-Trojan space but temporarily captured objects can also have this type of orbit for some time (Karlsson 2004). However, the number of Trojans in horseshoe orbits compared to the tadpole population should be small because of the relatively short transition period. The sample thus includes the know swarms. The RMVS3 integrator in the SWIFT package (Levison & Duncan 1994) was used to integrate the fictitious Trojans, and objects still in the Jupiter Trojan area after 200 000 yr were sorted to form a second sample.

This second sample contained 1027 of the originally 15 000 objects. Of these, 539 were in the L_4 group and 488 in L_5. Only one orbit had $a < 5$ AU and none had $a > 5.45$ AU. One orbit remained with $e > 0.4$ and two with $i > 60°$. As expected, the other three orbital elements still occupied the originally selected parameter space. Although the remaining set had distributions that were more similar to the true sample, some differences remained. The fictitious sample was randomly distributed in the (Ω, ω, M)-space, while for the real Trojans, M is generally in phase with Jupiter, and Ω has separate peaks for the two swarms, so that the perihelion longitude $\varpi = \Omega + \omega$ is shifted 60° from Jupiter's value (see Sect. 3.3 for comments related to these peaks). The inclination distribution among the true Trojans has a peak around 10° and falls to nearly zero at 40°. The fictitious sample looked similar but was almost flat at 10–30° (with a small dip at 20°) and then fell off towards 60°. This implies

[1] http://www.cfa.harvard.edu/iau/lists/JupiterTrojans.html

Table 1. Observation statistics for the selected observatories from 1995 to 2008.

Site	Observations[a]			Trojans		Area coverage[b]			Grid size
Code	Total	Trojan	Trojan part [%]	Observed[c]	Discovered	Total	Trojan	Trojan part [%]	Sq. deg.
644	3 738 472	24 977	0.67	1618	141	373 240	7617	2.0	0.96
691[d]	937 706	7875	0.84	1056	224	15 378	221	1.4	0.08
691	3 586 484	17 325	0.48	1640	304	106 741	4117	3.9	0.78
699	5 171 452	35 695	0.69	1304	62	475 990	11 121	2.3	1.24
703	4 769 603	15 470	0.32	1053	27	480 719	7375	1.5	2.02
704	22 328 359	133 653	0.60	1634	577	721 336	7386	1.0	0.25
G96	3 393 690	13 868	0.41	1215	154	50 814	933	1.8	0.28
Rest	...	43 356	...	1804	414	...	...	...	...

Notes. [(a)] The number of reported positions of which each object usually have three or more at every occasion.
[(b)] The number is based on observations and on the assumed grid-sizes of each site and cumulatively counted for each night. In comparison, the half sky is about 20 626 square degrees.
[(c)] The number refer to unique objects.
[(d)] Old instrument configuration.

that the fictitious sample to a greater extent than the real sample, resided outside the ecliptic stripe that most surveys focus on. The distribution of eccentricity is also similar between the real and the fictitious samples but, as for the inclination, the decrease towards higher values from a peak around 0.05, is faster for the real sample. The consequence is that the general perihelion distance is smaller for the fictitious set. The difference corresponds to about 1 mag at the same detection limit for these *excess* eccentricity objects. The semimajor axis distribution had an excess of values at the outskirts of the 1:1 resonance space for the fictitious Trojans. This caused one group to be more easily detected and one more difficult to detect compared to the true sample. However, the effect of the semimajor axis on the brightness is smaller than the effect of the eccentricity.

As a consequence of the distribution in M of the real Trojans, they generally reach perihelion within a close time span, which affects the timing of discovery of Trojans to within about 1 mag from the detection limits of the surveys. Since the semimajor axis changes periodically around the semimajor axis of Jupiter, the mean anomaly oscillates with an amplitude of about 50° with respect to the mean anomaly of Jupiter. The last time Jupiter passed perihelion was in 1999.

The planetary system was in both integration sets taken from DE405/LE405 (Standish 1998). The masses of the planets (Mars-Neptune in the first, and Earth+Moon-Neptune in the second integration) were taken from Standish (1995). The second set was integrated for the main time period (1995 to 2008) with a output data step of 1 day at 0 UT.

2.3. Observation data

From the two MPC data files containing numbered and unnumbered asteroids respectively, the asteroid number/designations were compared with the list of 2295 Trojans. On the basis of their detections of Trojans and overall sky coverage, six observatory codes were selected. Some basic statistics for these observatory sites are shown in Table 1. The sites are Palomar Mountain, NEAT (644), Steward Observatory, Kitt Peak-Spacewatch (691), Lowell Observatory-LONEOS (699), Catalina Sky Survey (703), Lincoln Laboratory, LINEAR (704), and Mt. Lemmon Survey (G96). The site 691 is divided into two because of an instrument upgrade, counted from Julian day 2 452 570 (22 Jan. 2002). In addition, a general Trojan have been observed 133 times, by seven different sites. The most observed Trojan is (2357) Phereclos with 1010 observations, and it also has the most observations performed by the same site (704), 499.

2.4. Working procedure

For each day number (i.e., from 12:00 UT to 12:00 UT the next day) and for each site, all observations were placed on a sky-grid with a grid-size of about half of the CCD field used at the site. The grid-areas containing observations was compared with that of each object in the fictitious Trojan set. The number of observations for that night, observatory code, and the number of the fictitious objects was recorded along with the magnitude of the faintest observation. Based on this faintest magnitude, an absolute magnitude was calculated (Bowell et al. 1989) for the grid-area of the fictitious object as well as an absolute magnitude for a worst case scenario, where the Earth-object distance was extended so that the Sun-object distance corresponded to the largest aphelion distance present in the fictitious sample at that time.

The same sky-grid fields were also checked against the real integrated Trojans. Three types of matching were recorded. First, when the same Trojan from both the integrated set and the observation set was in the same grid-area, it was considered a match. The number of observations in that area, the observatory code, and the object in question was recorded. The faintest magnitude in the area were all recorded along with the magnitude of the Trojan observation (which in many cases was the same object) and the theoretical magnitude (Bowell et al. 1989). Second, if the integrated Trojan and the observation were not in the same grid-area, then the distance the Trojan observation was away from the integrated Trojan (measured in grid-areas) was also recorded and flagged as a near miss. For some points on the mismatching, see Sect. 4. Third, if there was no Trojan observation at all, although the integrated Trojan was in a grid-area with positions of other asteroids, then it was flagged as a miss.

In a true situation, one detection does not constitute an orbit. To simulate a true discovery routine for the fictitious Trojans, I adopted the following discovery criteria based on information from MPC[2]. For observations from the same site, there had to be two different nights with intra-night time spans of at least 30 min, where the time span between the two nights should be less than or equal to 7 days. If two or more observatories had detections during the same night, then intra-night time span of

[2] http://www.cfa.harvard.edu/iau/info/Astrometry.html

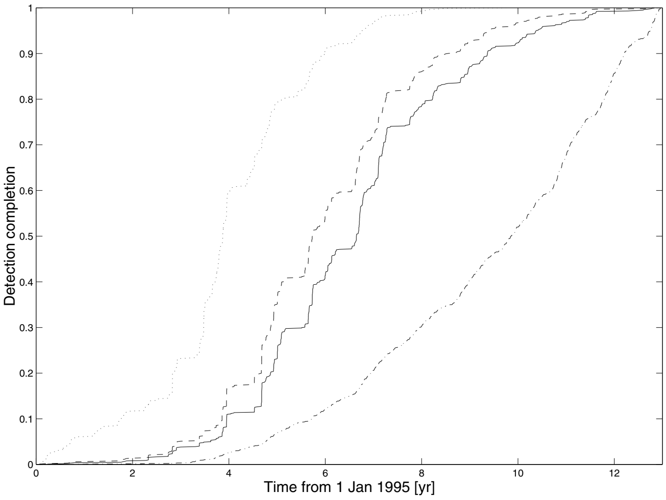

Fig. 1. Detection of fictitious Trojans. The figure show how analyses of real observed fields detected the 1027 fictitious Trojans. The time is counted in Julian years from 1 Jan. 1995. The dotted line show the cumulated number of the first time a fictitious Trojan is in an observed grid-area. The dashed line show the identifications when the detection simulation (see Sect. 2.4) is used. The solid line is the same as the dashed but with the additional requirement that the identification produces the faintest absolute magnitude possible for that object. The dash-dotted line shows the cumulative number of observed grid-areas on a night and site basis.

15 min for the different sites was allowed. Grid-areas with only one real position were considered non-linkable. In the following text, *detection* refers to when the observation and the integrated object is in the same grid-area, and *identification* or *finding* refers to the success of the detection simulation software in linking detections of the same object to each other. The simulated detection of Trojans is shown in Fig. 1.

3. Results

As the completion level is being determined, two main questions emerge. Are all possible orbits covered? What is the limiting magnitude? In this case, the first question is difficult since the true areas observed are not recorded (I described in Sect. 2 how I resolved this problem). The second question poses a problem because the observations are from different sites and taken during a long time period. Since each site contributes with many observations, these can be compared and different ways of deriving the limiting magnitude can be used to constrain it.

3.1. Orbit coverage

The detection and identification of the Trojans in the fictitious sample is shown in Fig. 1. The dotted line records in a cumulative way the first time an object came into an observed grid-area. Every object except one (see the end of this subsection) is in this case within an observed grid-area before nine years have past, which shows that the Trojan area is well covered with observations. If objects were evenly distributed in circular orbits, half of them would pass the ecliptic in a quarter of the orbital period (about three years). In Fig. 1, the half-way mark is passed after less than four years. The dashed-dotted line show the normalized cumulative area covered during the time period for every night and observatory. From the beginning, Trojans are already easily found. Although the first three years only represent 0.3% of the total coverage, 5% of the fictitious Trojans were found. When the coverage rate goes up around the beginning of 1998, the identification rate (dashed) follows for some five years. The delay in achieving the same completion level as for the case of just being detected, is 1−2 yr after 1998 compared to about 3 yr before. In Fig. 1 there appears to be a shift of another year to obtain the faintest magnitude possible (solid line). This is given by the time it takes to pass through the ecliptic area and the Trojan is closer to perihelion the *next* year. The median magnitude improvement between the dashed and solid case is 1.42 mag and 53% of the objects are affected. A closer look on an object-to-object basis indicate that much of the *improvement* (64%) occured already within the same year. This is because of the several observations of the same object, and for brighter objects it is more likely that one of the subsequent identifications has a fainter limiting magnitude than the original one. Since these last two lines follow each other quite well, it is possible to conclude that there have been no major changes in the limiting magnitude capability at the different sites during the analyzing time period, at least not for the dominating site.

The fall off of detections/identifications rate for the last 25% of the objects is not due to lack of observations, but fewer objects being left undetected. This cis evident from the dashed-dotted line, which shows a still faster growing curve of area coverage. As expected, the shift in time to find the last elusive objects is considerably longer than when there were still plenty of undetected bodies. The identification routine only missed five objects, not counting the object that was not even detected.

Only grid-areas containing positions with a magnitude were used in Fig. 1. However, detections without magnitudes represent only 3.0% of all object detections and none of the missed objects are affected by this removal of grid-areas.

Comparing the integrated Trojans with the observations, no site appears to be very efficient in detecting Trojans. Overall, they detect only about half of the possible objects. The situation improves slightly for the brighter Trojans; twice as many of the known objects are detected as missed. However, this should not affect the detection efficiency in Fig. 1 for Trojan observations, if the effect is purely due to the magnitude since the inefficiency is already determined by the observations. On the other hand, the inefficiency might be caused by the slow sky movement of the Trojans, where mainbelt observations, which are generally used to measure the limiting magnitude, would not be as affected. Assuming that 20 detection nights are needed for identification, based on the fictitious Trojans missed, then 16% of the identifications might be affected (the mean number of detection nights being 100). However, 47 of the fictitious Trojans have fewer than 20 detections and are still identified. This could reduce the affected number to about 1.6% or 16 objects. In the end, no more than 2% of the objects should remain undetected during a period of 13 yr, not taking the magnitude into consideration.

Considering at the orbital elements for the last 10% and 2% of unidentified objects (considering first, dashed, and identification cases) infers no preferred area in element space except for the inclination, the combination $\omega + M$, and the ascending node. In Fig. 2, these elements (top panels) are compared with the true sample (bottom panels).

As expected, it is easiest for medium to high inclination objects to avoid detection because of the concentration of observations near the ecliptic. However, there appears to be no direct connection between the inclination and how difficult it is to find the object, because the 2% group still occupies the same range as in the 10% cases. A reason for this is that although most of the sites used here are capable of finding almost

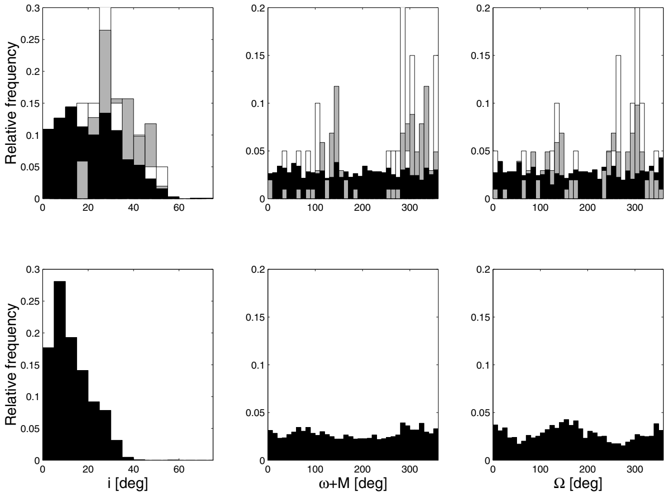

Fig. 2. Orbital elements affected by sky coverage. In *the top panels*, the distribution of the initial sample is shown in black, the last 10% of identifications in grey, and the last 2% of identifications in white. The shortest bar is in front of the other and the longest bar is furthest behind. All elements were taken at the epoch JD 2 453 005.5 (1 Jan. 2004). The *top left panel* show the relative distribution of inclinations for the fictitious Trojans and the *top middle panel* show $\omega + M$ for the same objects, and *the top right* shows the ascending node. The *bottom panels* show the real Trojan sample in the same way.

every object themselves, their search pattern is not correlated with each other. Objects that pass through the ecliptic undetected have some 3–6 years of safe time, which is more than the time between when the two groups considered here start being recorded. For inclinations below 15°, there is no such safe time and the observations intensity is so high and the coverage so good, that no orbit set can remain undetected for long. Starting from zero, the inclination completion pace is about $4^\circ\ \mathrm{yr}^{-1}$ after year 6. However, the medium-inclined Trojans appear more difficult to find than those inclined the most. This is partly because there are more objects available for detection in the medium-inclined interval than the high-inclined interval.

In the middle panels, the mean anomaly was assumed to be that for the epoch JD 2 453 005.5 (1 Jan. 2004). This is slightly after the breaking point for the 10% limit. There are concentrations of objects at roughly 300° and 140°. The ascending node also exhibits peaks at similar angles. Three main causes are: the location of the Milky Way, the amount of observations, and the location of the surveys. The ecliptic crosses the Milky Way at about $\Omega = 280^\circ$ (towards the galactic center) and $\Omega = 80^\circ$ (away from the Galactic center). The observation sites observe mostly the northern sky, so objects with $180^\circ < \omega + M < 360^\circ$ are primarily in the southern sky (depending also on Ω), shielded by the Earth. The epoch of elements corresponds to year 9 in Fig. 1, and more than about four years before that, the observation rate was considerably lower. Since the mean anomaly increases by about $30^\circ\ \mathrm{yr}^{-1}$ for the Trojans, four years corresponds to a change of 120°. This means that the Trojans at the peak around $\omega + M = 300^\circ$ in the top middle panel were in the southern sky during the more intensely observed period. The high inclination Trojans avoid being observed by spending a lot of time away from the ecliptic, where observations are concentrated. However, there is no connection between the inclination and when identification takes place. Even when an object is in a well observed area, there is still a random-like process. All areas cannot be observed at once and a Trojan generally crosses a grid-area within a few days. It is more a matter of having a sufficiently high inclination. The peaks in Ω do not really coincide with the Milky Way, except around $\Omega = 160^\circ$, which have a descending node at $\Omega = 80^\circ$. The location of the other Milky Way crossing is more or less empty for objects that are found late. The peaks at $\Omega = 300^\circ$ and $\Omega = 120^\circ$ are basically opposite to each other but do not coincide exactly with the Milky Way. Note that the middle and left panels are connected by the location of the two Trojan swarms, $\lambda = \Omega + \omega + M = 100^\circ$ (L_5) and $\lambda = 210^\circ$ (L_4) at this epoch.

There are objects from both swarms in the 10% group in the $\omega + M = 300^\circ$ peak, but only L_4 objects in the 2% sample. This is because these last L_4 Trojans generally followed the Milky Way path, while the L_5 Trojans passed through a high galactic latitude area. The 50–150° peak in the top middle panel also contains Trojans from both groups in the 10% sample, while only L_4 objects remain in the 2% group. The reason for this is that the L_5 part emerged from the Milky Way at high ecliptic latitude and moved towards low ecliptic latitudes at high galactic latitudes where they were easily found. The L_4 group was initially at high latitude with respect to both the ecliptic and to the Milky Way. The group was then gradually discovered as it reached the ecliptic. The reason that the objects from $\omega + M < 100^\circ$ survived into the 2% group is time, since they simply reached the ecliptic last. They also had some shielding from the Milky Way when they approached the ecliptic. The only L_5 Trojan in the 2% group remained undetected by crossing the ecliptic before entering the Milky Way (away from the galactic center) on a north-bound trajectory. However, since no other object from the 10% group occupies that part of the $(\omega + M, \Omega)$-space, it appears like a coincidental identification escape from its previous descending node crossing, which took place about six years before (in roughly year 3 in Fig. 1) at high Galactic latitudes.

When considering the Trojans who avoid identification the longest, it is clear that the two swarms do not behave in the same way. The ratio L_5/L_4 was initially 0.91. In the 10% group, it decreased to 0.67, and in the 2% group, it decreased to 0.05. The reason for this is the location of the swarms in the sky, the short period of time of intense observation, and the smaller coverage of the southern sky. The L_5 swarm were in general in more favorable positions for observations. Looking at the real discovery statistics, 1028 Trojans have been found in L_4 and only 875 in L_5 since 1995. However, this includes the Trojans detected by the Uppsala-DLR survey (Lagerkvist et al. 2002). Bypassing these years by counting from 1999, the L_5 contains most discoveries with 817, compared to 734 for L_4.

When comparing the fictitious objects to the real sample in the bottom panels of Fig. 2, it can be seen that there are no dips in the $\omega + M$ distribution at the above mentioned places, but the distribution instead has small peaks. There are dips in the ascending node distribution roughly where the Milky Way crosses the ecliptic, but these are much broader than the shielding capability of the Milky Way (see Sect. 3.2). The decrease in the inclination occurs before the inclination effects in the panel above, and note that all objects in the top panel were found before the end of the investigated time period. The lack of true high inclination Trojans is thus not caused by observation bias but is a reflection of the true lack of high inclination Trojans, unless the inclination is correlated with size and the high inclination Trojans are faint. There is also a bias effect depending on size, inclination, and observations beyond the absolute magnitude completion limit (see Sect. 3.3).

The Trojans that were not identified at all differs a bit from those that were identified at a late stage. The $\omega + M$ are between 140° and 260°, where the top middle panel of Fig. 2 show a lack of the late discoveries. They also avoid the peaks in Ω, but are close to the nodes of the Milky Way. In the inclination, there are two distinct groups, one with medium-high inclinations from 25° to 40° and one with an extremely high inclination of about 70°. This separation and the separation into the ascending node is found to reflect the Trojan group to which they belong to. Those from the L_4 group (four of six objects) have medium-high inclination and an ascending node close to the Milky Way away from the Galactic center. The L_4 Trojans avoided detection partly by shielding and partly by coincidence. At the end of the time span, these objects reside in the southern sky. Their last crossing of the ecliptic occurred slightly away from the Milky Way and their non-detection here must coincidental. The ascending node crossing before that was both partly shielded by the Milky Way and occurred at a time when the observation intensity was lower. The time between the crossings were spent away from the ecliptic. The L_5 Trojans have orbits that follow the Milky Way in the north sky, and their high inclination also cause them to pass the ecliptic quite rapidly. However, although none of these objects was selected by the simulated identification routine, only one managed to avoid all the observed grid-areas.

The Trojan missed has one of the L_5 orbits with a high inclination ($\approx$74°). The first ecliptic transit took place in December 1998 at an elongation of 24°. Furthermore, it was close to the Galactic plane in the direction of the Galactic center and travelled from the southern sky to the north. However, the detection conditions could not have been much worse. After the transit of the ecliptic, the object remained close to the Galactic plane when the elongation became large enough for observations to resume. It continued to orbit close to the ecliptic for a year and a half, but at that time had reached a high ecliptic latitude (>50°). The second ecliptic transit occurred in June 2001 in a way very similar to the first time, close to the Galactic plane and small elongations. Thus there were four important reasons for the non-detection of this fictitious Trojan: first, an orbit close to the Galactic plane; second, the ecliptic transit took place at small elongation; third, a high inclination kept the orbit away from the ecliptic and ensured a rapid passage of the ecliptic; fourth, both the eccentricity and orientation of the orbit ensured that it spent less time in the northern than in the southern sky.

3.2. Magnitude limit

The observations were performed from different sites and to be able to compare, I used the theoretical magnitude (Bowell et al. 1989) as a reference. There are many reasons why this magnitude does not correlate with the observed magnitude. The theoretical magnitude is by itself not a perfect representation of the true mean conditions, but the differences also depends on the shape and rotation of the object, and different telescopes use filters that can naturally differ from each other.

The differences between observed Trojans magnitudes and their integrated orbits with corresponding absolute magnitudes (G-parameter assumed 0.15) are shown in Fig. 3 and the values for the lines can be found in Table 2 (Cols. 4 and 5). The number of events that these values are based on is shown in Col. 2 of the same table. The differences are generally small with site G96 being nearly zero. The site 699 is the only with a negative compensation, that is the theoretical magnitude is fainter than the observed, and site 704 exhibits the largest difference. The dotted curve is a corresponding Gaussian curve for comparison. From this it can be seen that the high tail of the distributions is usually more stretched than the low end. If the theoretical magnitude is plotted against the observed magnitude, it becomes clear that the low end closely follows a normal distribution for all magnitudes, while the high tail does not. The fainter the theoretical magnitude, the more the observed magnitude becomes fainter and deviates. There also appears to be a general breaking point around observed magnitude 21, where the deviation becomes larger. Without further analysis, I can only speculate that assuming the theoretical magnitude is generally correct, the faint end deviation might originate form the stellar reduction catalog and the noise estimation of the observation. The high tail could perhaps be cause by non-photometric weather conditions. The compensation term in Table 2, Col. 4, was applied to all numbers in that table and Fig. 4.

The absolute magnitudes of the fictitious Trojans detected at optimal viewing conditions (large dots) are compared with the real sample (small dots) in Fig. 4. The fictitious Trojan discoveries are concentrated between 2000 and 2004, nearly 2/3 of the Trojans being found during this period. This reflect the wider coverage compared to prior to 1998. The decreasing detection rate after 2002 indicates that there has been little change to the principal observation sites or sky coverage of the Trojan groups. The lines correspond to the median values for the detections from the different sites (see Table 2, Col. 10).

The effect of the two Trojan clouds can be seen as concentrations in both the real and fictitious samples and Fig. 4. Since the Earth has a shorter period than Jupiter, the L_5 swarm is first encountered after the large gaps, Jupiter is passed during the small gap, and then the second group consits mostly of L_4 Trojans. These gaps also manifest themselves in Fig. 1 as plateaux. The unevenness within such a pair has three basic reasons: first, the passing of the Galactic plane by the swarms at different times; second, location of the swarm in the south or north sky; and third, observations during local summer (in practice, a combination with the second reason) or winter, which affects the length of the night.

If the limiting magnitude in Table 2, Col. 6, is compensated for by the distance, then there is a remaining difference of 1.5 to 2 mag to the absolute magnitudes given in Table 2, Col. 10. The difference between Cols. 10 and 12 in a similar way is about 1.2 mag. This difference is caused by the observational demands of the identification of a new object (observations from more than one night) compared to just detecting an object. The median magnitude of real detections (Col. 11) is essentially in-between these numbers. In the real discoveries, a contributing effect is also included of observations from other sites. However, since this contribution is relatively small it suggests that the identification routine is a bit restrictive. The limiting magnitude numbers might have to be compensated to be about 1 mag fainter.

Considering the orbital elements of the 100 fictitious objects with the brightest and faintest absolute magnitude, respectively, there are some small differences. The brighter objects have the same distribution as the fainter objects, but have on average larger semimajor axes (about by 0.1 AU). There is also an autocorrelation in the mean anomaly between the bright and the faint objects. The bright objects have one perihelion passage (when L_4 was in the southern sky) during the time span. The faint objects generally have two perihelion passages during the time span. The true number of grid-area detections is about similar for the two groups. Thus, the conclusion is that the combination provides the bright objects with slightly less favorable geometrical conditions that cause their absolute magnitude to not be pushed

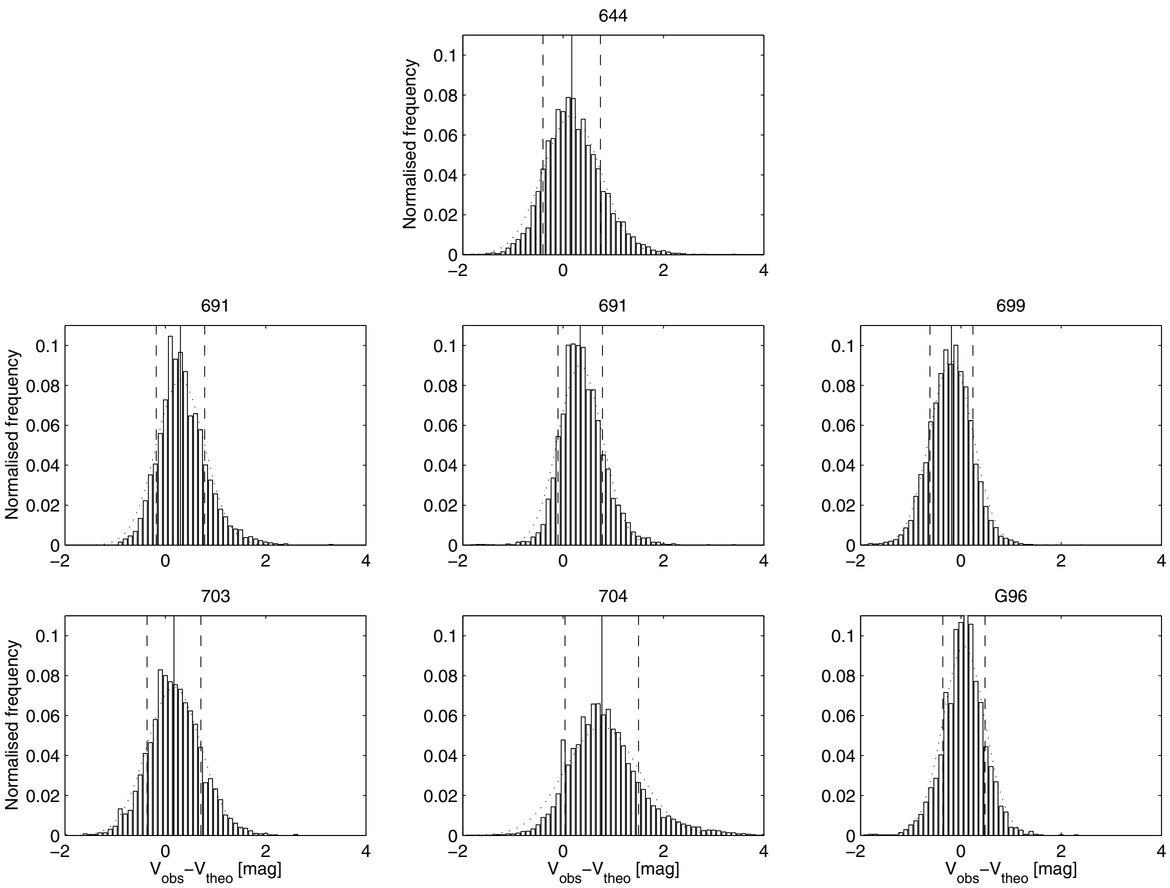

Fig. 3. Distribution of differences between observed and theoretical magnitudes for the different sites. The solid vertical and dashed lines show the location of the median and standard deviation values, respectively. The number can be found in Table 2 (Cols. 4 and 5). The dotted curves represent the corresponding Gaussians. The left middle panel is for the old settings of site 691, while the center panel is the current configuration.

Table 2. Photometry results for the observatories.

Site Code	Events[a] [pcs.]	Grid miss[b] [%]	Δmag [mag]	σ [mag]	limit[c] [mag]	limit[d] [mag]	Δlimi[e] [mag]	Found[f] [pcs.]	H^g [mag]	H^h [mag]	H^i [mag]	H^j [mag]
644	8274	5.6	0.17	0.57	19.75	19.43	0.5	186	12.1	12.9	13.5	12.4
691[k]	2611	11.1	0.30	0.48	20.30	20.10	0.8	35	12.4	13.4	13.5	12.9
691	5333	5.2	0.34	0.44	21.07	20.96	0.1	24	13.4	13.5	14.3	13.1
699	9030	2.9	-0.19	0.43	19.14	18.99	0.3	5	12.1	12.3	12.8	11.8
703	3896	3.7	0.17	0.54	19.79	19.53	0.4	73	11.7	12.4	13.5	12.4
704	28 456	4.6	0.77	0.73	18.69	18.63	0.3	694	11.3	12.3	12.6	11.3
G96	3423	6.4	0.06	0.42	21.19	20.94	0.3	5	12.7	13.5	14.1	13.8

Notes. [(a)] The cumulative number of matched Trojans with magnitude on a night to night basis.
[(b)] The number of the integrated real Trojans which did not end up in the same grid-area as the observations.
[(c)] Median of the faintest object in fields containing real Trojans.
[(d)] Median of the faintest object in all fictitious Trojan containing fields.
[(e)] Difference in using all fields with fictitious Trojans and those fields which contained 10 observations or more.
[(f)] Number of simulated discoveries.
[(g)] Median magnitude of simulated identifications.
[(h)] Median magnitude of real discoveries.
[(i)] Median magnitude of the deepest magnitude for identified objects.
[(j)] Magnitude based on Col. 6 and a combined median distance and phase angle compensation (−7.36 mag).
[(k)] Old instrument configuration.

towards fainter values. However, the variations between the two groups are smaller than the intra-group variations.

When the situation of first identification is compared on an object-to-object basis for optimal conditions, then the magnitude difference is about 1.3 mag within the same year. This is a combination of an early detection by one site and an improvement from another site with superior capabilities, and an early detection and improvement of observation conditions closer to opposition. The improvement from the next apparition is 1.5 mag, which also reflects objects approaching perihelion with

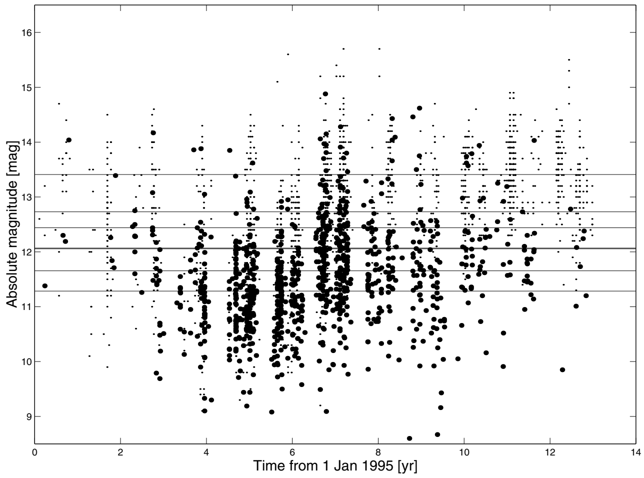

Fig. 4. Simulated and real Trojan discoveries. The absolute magnitudes of the fictitious Trojans after linking detections, marked with big dots, and real Trojans shown as small dots. The lines show the median magnitude (Table 2, Col. 10) for the fictitious identifications at the different sites.

respect to the previous year. For the fictitious Trojans, the overall discovery magnitude is 11.5 ± 0.9. This is based on a median value. From Sect. 3.1, I estimated that halving the number of detections would still pselect 98% of the objects and no orbit type would be missed. The limiting magnitude should therefore be confident to a level of 98%. The average difference in absolute magnitude between the correct detection condition and the worst case scenario is 2.3 ± 0.5 mag. This effectively excludes any chance of undetected Trojans being brighter than H = 9.2 mag.

Since the detection capabilities can easily reach H = 11−12 mag and there has been no detection of Trojans brighter than H = 11 mag after 2002, this is another indication that the Trojan sample should be complete to this magnitude. Even in the worst case scenario (which in itself is unlikely), only 1.4% of the detections had an absolute magnitude brighter than the brightest Trojan (H = 7.49). It is therefore in practice possible to exclude the possibility that the brightest Trojan could have been undetected, i.e. (624) Hektor, is the brightest Trojan.

3.3. Bias beyond the completion limit

In general, the surveys detected a fictitious Trojan every 20 square degrees and each of the four surveys with the largest area coverage could detect more than 90% of the objects. Furthermore, those objects that were not detected could be found above 20−30° in inclination in a random-like manner but with a slight general gradient. Surveys with smaller areal coverage could only find a complete sample of below 5−10° in inclination, and high inclination detections were rare. Since these smaller surveys are also those with the faintest limiting magnitude, it will affect the inclination-absolute magnitude dependence, where the fainter the object, the higher the inclination bias.

There is a correlation between the inclination and the number of observations in the fictitious sample, where low inclination Trojans are more frequently observed. A similar trend can be found in the real sample with H = 9.5−11.5 mag, although with a cut off at about i = 40°. This could be an indication of observational bias, where high inclination Trojans are missed due to less sky covarage. However, when the inclination distribution for a few observations and plenty of observations is compared, there are no clear differences at all. The increase in observations is not generally inclination dependent. On the other hand, when the absolute magnitude is also considered, then there is a clear difference in both the inclination and the number of observations. For the bright sample (H = 9.5−11.5 mag), Trojans with i < 10° have generally been observed 1.5 as many times as the rest. For faint Trojans (H >= 13 mag), the difference increases to a factor 2. If an even lower inclination is used i < 5°, the factor is 2.75. There is also a correlation between the absolute magnitude and the number of observations. The combined effect is that for the same number of observations, the observed absolute magnitude decreases by about 2 mag for an inclination increase of 30°. However, the spread in both inclination and absolute magnitude for the same number of observations is much larger than the effect itself, thus the centre of the known sample is well mixed. Although the number of observations does not affect the inclination directly, it does so via the absolute magnitude. Medium-inclination Trojans are under-represented at inclinations 10−35° for absolute magnitudes in the nearly complete interval H = 11.5−13 mag. Assuming that the true number of Trojans to H = 13 mag is about 1700 and has the same distribution as H = 9.5−11.5 mag, then about 65% of new discoveries should have i = 15−40°, and 34%, i = 0−15°. The interval i = 0−5° should only lack 3% of its Trojans. Trojans with inclinations above 40% should also be almost complete, lacking only 1% of the true number. The situation is similar for H >= 13 mag but with even more middle-inclination Trojans missing.

If the ascending node for the real Trojans is divided into three magnitude intervals, i.e. a complete sample 11.5 > H > 9.5, a near-complete sample 13 > H > 11.5, and a faint sample H > 13, then there is a clear difference. While the complete sample is randomly distributed, the faint sample has two peaks around Ω = 160° and Ω = 350°, and two troughs around Ω = 60° and Ω = 270°. The troughs coincide well with a passing through the Milky Way near the ecliptic, and both the troughs and peaks have a peak deviation of σ > 2.5 from a random sample. The near-complete sample has similar peaks and troughs but smaller and shifted downwards by some 30°. It is the combined effect from the two incomplete groups that is responsible for the broad wavy feature in the lower right middle panel of Fig. 2.

The concentration of the mean anomaly around the value of Jupiter is not dependent on magnitude and small dependencies between (H, M, e) and (H, M, a) are caused by changes in observed magnitude depths of surveys in the past few years. However, there might be both a difference in absolute magnitude and how the semimajor axis is distributed. Among the faint discoveries, there have been several Trojans with 4.9 < a < 5.05 AU. None of these Trojans or earlier discoveries are brighter than H = 13 mag in this area.

It is no surprise to find smaller Trojans in this area. While the Trojan swarms slowly dissipate (see e.g., Levison et al. 1997; Robutel & Gabern 2006) among all sizes, fragment from collisions can have orbital elements that differ significantly from the parent bodies and the smaller fragments are generally affected the most (Marzari et al. 1995). The lower semimajor axis limit for tadpole orbits is around a = 4.95 AU, while horseshoe orbits exists from this limit down to at least a = 4.8 AU.

4. Discussion

How does the true number of Trojans compare to previous predictions? Considering only the L_4 for which there have been several investigations, then there are 267 Trojans listed with $H \leq 11.5$ mag. The result of Van Houten et al. (1970a)

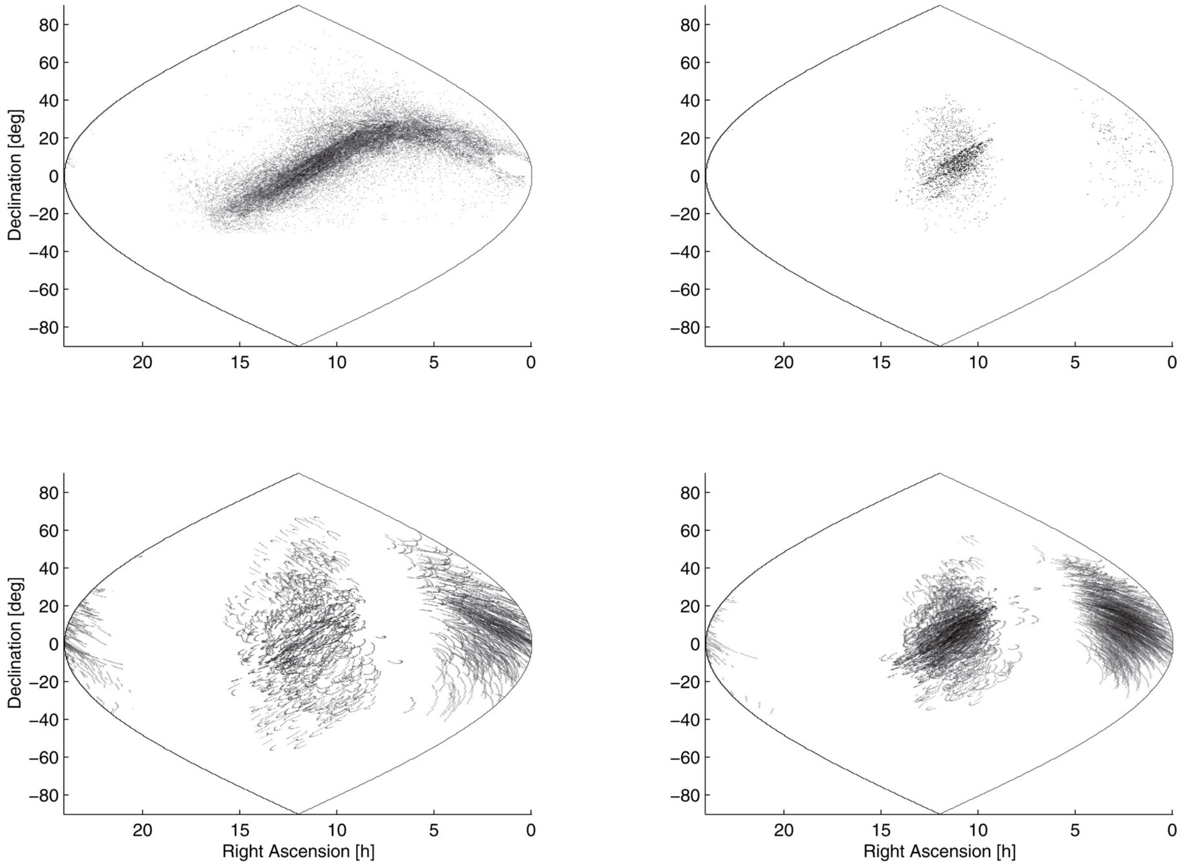

Fig. 5. Example of sky coverage. The four panels show a cumulated coverage example for the site 704 of the sky during 120 days centered on 6 Mar. 2002, when L_4 was in opposition. *The top left panel* show grid-areas for all observations. In the *top right panel*, only the grid-areas containing Trojans are shown. The fictitious Trojans are shown in the bottom left panel and the integrated real Trojans in *bottom right panel*.

is difficult to reproduce because they used a different absolute magnitude system and a plate/filter combination that corresponds approximately to a *B*-filter but it should correspond to 170 Trojans using $g = B(1,4) = H + 1.14$ mag and a single distribution, disregarding the knee at $H = 9.5$ mag. In Shoemaker et al. (1989), Eq. (3) with $B(1,0) = H + 0.77$ mag (typical for a $(B - V)$ color of a Trojan) would give 178 Trojans (223 if $B(1,0) = H + 1$ mag Marsden 1985). From Eqs. (3) and (10) in Jewitt et al. (2000), $V(1,1,0) = H + 0.36$ mag would imply between 350 and 1700 Trojans. Lagerkvist et al. (2002) do not have a useful equation but scaling down their 1122 objects from $H = 13$ mag to $H = 11.5$ mag with a slope of 0.4 would give 282 Trojans.

The only way for a bright Trojan to avoid detection is to have an orbit coinciding with the Milky Way. However, not even an orbit such as that is likely to go undetected for long since the aberration caused by Earth's annual movement is not negligible for a Trojan and from time to time will force the positions of such a Trojan orbit to be outside the Galactic plane. Furthermore, the analysis in this paper is only based on reported observations. The large surveys, especially LINEAR, in reality cover a much larger part of the sky. In Fig. 5, (top left panel) observations are found between about $-30°$ and $+70°$ in declination. The whole area between these declinations is observed but the area with observable objects is just a minor portion. This means that this site alone should be able to place a limit on new bright objects, which should not be brighter than $H = 11.3$ mag. Another point to into consider is that about 15% of the Trojan observational material has not been used here. A significant part of these observations were performed from sites capable of observing fainter objects than the majority of the selected sites in Table 2.

Although the majority of Trojan observations are made from the northern sky, the southern sky is not without its observing sites. For instance, there are four sites with observations of more than a hundred different Trojans each, i.e., Siding Spring survey (code E12) with 507, ESO La Silla (code 809) with 502, Siding-Spring-DSS (code 260) with 205, and Cerro Tololo Observatory (code 807) with 133. Both E12 and 809 are among the top ten observing sites and both 807 and 809 have Trojan observations fainter than 23 mag. Thus, Earth's shielding capability of Trojans in the southern sky, which is significant in this work, should be less important. However, the Trojan observations from the southern sky represents in total only 4.9% of all observations.

The absolute magnitude is not a true constant for an asteroid because of e.g., shape, spin, and surface composition. From apparition to apparition, the changes on absolute magnitude can be in the order of 0.1−0.2 mag (Lagerkvist & Magnusson 1990). The taxonomic type also affects the result: I assumed a *G*-parameter of 0.15, which is probably close to that of D-type asteroids. However, among the Trojans, P-type and C-type are also present. Harris (1989) derived $G = 0.086$ for a common group of low albedo asteroids, but without any values for D-type asteroids. Lagerkvist & Magnusson (1990) inferred a value of $G = 0.04$ for C-type asteroids, which would make the calculated absolute magnitudes brighter if used. The effect is stronger with increasing phase angle. Fortunately, Trojans always have small phase angles, at most some 20°, which limits the effect to less than 0.2 mag.

Although filter information is generally included in the observational data, I did not made any attempt to correct for this, other than by means of the corrections from Fig. 3. A reason for this is that the filter used at the telescope might differ from the wavelength band used in the stellar catalogs for the magnitude determination. The filter information is furthermore not always included in the observation. However, the different survey sites probably use the same telescope setting and reduction procedure all the time, so the correlation with the theoretical magnitude should be fairly good. Asteroid observations today are mostly performed in the V or R broad bands and occasionally in I. The difference in magnitude for a typical D-type Trojan with a spectral slope of around 10% kÅ^{-1} is about 0.46 mag in both $(V-R)$ and $(R-I)$ (Karlsson et al. 2008).

Since there is no actual size information in the top panels of Fig. 2, these figures can be compared with bright real Trojans. If the absolute magnitude is divided into three groups, i.e., $H > 14$ mag, $H = 12{-}14$ mag and $H \leq 12$ mag, then there is no large difference in inclination for the faintest group when comparing Trojans found after the beginning of 2003 to all Trojans in the group, although a slight increase in median inclination is evident from 9.5° to 10.7°. The same situation can be found in the middle group, a slight increase in median inclination from 9.7° to 11.8°. Inclinations of lower than 2° can still be found in both groups. For the bright group, the situation clearly differs. For all Trojans in the group, the minimum inclination is below 1° and the median inclination is 14.8°, but during the 2003−2008 period the median inclination is 23° and the lowest inclination found is 13°. This corresponds well to the top left panel of Fig. 2, where all orbits with inclinations below 15° were found before the end of 2003. Since the confidence level of 98% is based on high-inclination Trojans, any possible remaining Trojan brighter than $H = 11.5$ mag should have an inclination of above 15°.

Lagerkvist et al. (2002) found that the inclination distribution differs between L_4 and L_5. They studied the entire sample known at that time. The complete sample with $H \leq 11.5$ mag can be divided into two groups for both swarms at $H = 9.5$ mag and $H = 9.3$ mag for L_4 and L_5, respectively. The inclination distribution for the fainter L_4 group has more objects at $i < 15°$, while the corresponding L_5 swarm has more objects at $i > 15°$. The distribution for the fainter groups continues into the incomplete sample. The situation is quite different for the bright groups where L_5 dominates in a peak at 5−10° and L_4 around 20°. Is this the effect of small number statistics (24 Trojans in L_5 and 41 Trojans in L_4), or is it a remnant from the formation of the Trojan groups, or has this group evolved collisionally or dynamically? All of these *excess* Trojans in the peaks can be found around $H = 9$ mag. Is it possible that some of these are fragments of a larger body? For instance, two or three of the L_5 peak objects may constitute a body similar in size to (1143) Odysseus in L_4. At the $i = 20°$ peak, combining some six objects with $H = 8.9$ mag would produce an object around $H = 7.4$ mag, similar in size to (624) Hektor. There might be a statistic deficiency corresponding to a pair of larger L_4 Trojans. The five brightest L_5 Trojans differ in terms of absolute magnitude by less than 0.5 mag, while the five brightest L_4 Trojans occupy a 1 mag interval. Among these objects, there are those that may belong to the same collisional family[3]. However, spectral slopes for these Trojans can differ significantly (Jewitt & Luu 1990; Fornasier et al. 2007; Roig et al. 2008; Melita et al. 2008). There can on the other hand be several causes of these differences that do not affect membership of the same family. These brightest Trojans must be studied further in order to correctly interpret their context.

In Fig. 4, there is a clear difference between the real and the fictitious Trojans during the last three years of the time span. This is probably due to the identification program, which might be too restrictive. The use of the faintest detection magnitude for each object produces a picture more similar to that of the real Trojans, but with a shift of 2 mag towards fainter values. The bulk of new real Trojans originate in the slightly smaller, but deeper surveys of sites 691 and G96, which might not yield enough detections for the identification software. It should also be mentioned that a significant part of the latest discoveries are made by other sites, which also produces additional faint observations. The simulated dots in Fig. 4 corresponds very well to the real discoveries made by site 704 when shifted about 0.5 mag towards fainter values.

The detection simulation is also affected by the limiting magnitude value. Because some of the non-detections were those with deepest magnitude. Since there had to be at least two real detections from an observatory each night, the brighter magnitudes were used for the detection limit. The reason is that the brightest magnitude should have been observable in the same area and time as the faint observations but not vice versa.

It is impossible to be sure that two objects close to each other (e.g., an observation and an integrated object) should be in the same field since there is no information about the central point of the field in which a particular observation was taken. To compensate for this, I assumed that the sky-grid for each observatory corresponded to a quarter area (half side) of the CCD field for the telescope used. If the field were taken in a random way, then there is about a 67% chance that an integrated Trojan in the same grid-area as an observation is within the real CCD field. However, survey fields are usually not placed in a random, but systematic way. The consequence is that the probability depends on how the sky-grid and the field are placed with respect to each other. In any case, the chance should not be less than 25% that the two points in the same grid-area should also be in the same CCD field. If there is a perfect alignment, then the chance could be 100%. In Table 2, Col. 3, the general matching is 95%.

Another source of mismatch comes from the general sky motion of the objects. For Trojans, this is less than 10′ day^{-1} (Lagerkvist et al. 2000) close to opposition. All integrated data is given at midnight UT, so the position mismatch between integrations of an observation should be at most on the order of 5′ day^{-1} due to the time resolution. For the smallest sky-grid, this could affect up to 29% of the positions. For the largest sky-grid it is less than 16%. Sites located at more than about 90° east or west longitude will have observations taken during the local night divided into two nights when analysed as in Sect. 2.4. If the site is between 90° and 180° west, then observations from the previous local night will be recorded (the more, the further west), i.e., the right ascension will be higher than for the integrated data because Trojans moves towards lower right ascension close to opposition. Missed observation from the same night at the same sites will generally have lower right ascension than the integrated Trojans because of the time delay of the local midnight. In declination, the difference between integrated objects of higher or lower declination than the observation is much smaller.

There is a clear correlation between the number of missed detections where the object was fainter then the background and the magnitude of the object, and likewise for the number of detections. If the number of missed detections is normalized to the number of detections, then unity is reached at $H = 11.5{-}11.7$ mag. In the case where the missed object had a

[3] See http://www.daf.on.br/froig/\discretionary-petra/

brighter magnitude than the limit for that grid-area, the number correlates mostly with the number of detections. Trojans with uncertain orbits do not have more missed detections than well observed Trojans. However, they did have, as expected, very few detections and were dominant at absolute magnitudes fainter than 13.9, while almost absent for $H < 12$. The orbit uncertainty for some of these Trojans can easily be more than a sky-grid frame away in reality with respect to the integrated orbit. In the sample, more than a hundred Trojans had been discovered more than a year ago with observational arcs less than two weeks.

The orbit integration can itself give rise to position errors caused by the selection of the planetary system. However, the difference between the observed positions and the integrated positions are on the order of $1''$, which is in general compatible with the uncertainty of the observations. None of the abovementioned effects influenced my results.

5. Conclusions

From my simulation of a fictitious Trojan group compared to areas in the sky with real observations, and integrations of real Trojan orbits, I draw these main conclusions.

- No possible set of orbital elements in the Jupiter Trojan swarms can have been missed. The completion limit in absolute magnitude can therefore be set by the general detection limit of the NEO surveys, which are the main providers of asteroid observations.
- The only dynamical way for a Trojan to hide is in the Milky Way while traveling in the northern sky. Among the Trojans faint enough not to be detected by most surveys, their inclinations and ascending nodes are still biased.
- The inclination distribution is biased for absolute magnitudes fainter than 11.5 mag. The effect is strongest for the range 15−40°. Trojans fainter than $H = 13$ mag are also biased in terms of the ascending node at angles coinciding with the Milky Way.
- A general completion limit in the absolute magnitude can be set to be $H = 11.5$ mag. For Trojans with $i < 15°$, this limit can be extended another 0.5 mag.
- There appears to be a general cut off at about $i = 40°$ for the Trojan swarms.
- The L_5 swarm is smaller than L_4 to the completion limit, being about 71% of the L_4 size.
- Both Trojans swarms have a knee in the absolute magnitude distribution. The shift in distribution takes place at $H = 9.5$ mag for L_4, and $H = 9.3$ mag for L_5.
- No Trojan larger than (624) Hektor can exist.

Acknowledgements. I thank everyone who performs astrometric observations of minor planets and the staff at the MPC, who continuously keep track of and store the data.

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