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Kerr geodesics, the Penrose process and jet collimation by a black hole

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ABSTRACT

Aims. We re-examine the possibility that astrophysical jet collimation may arise from the geometry of rotating black holes and the presence of high-energy particles resulting from a Penrose process, without the help of magnetic fields.

Methods. Our analysis uses the Weyl coordinates, which are revealed better adapted to the desired shape of the jets. We numerically integrate the 2D-geodesics equations.

Results. We give a detailed study of these geodesics and give several numerical examples. Among them are a set of perfectly collimated geodesics with asymptotes $\rho = \rho_1$ parallel to the z-axis, with ρ_1 only depending on the ratios $\frac{Q}{E^2-1}$ and $\frac{a}{M}$, where a and M are the parameters of the Kerr black hole, E the particle energy and Q the Carter's constant.

Key words. black hole physics – acceleration of particules – relativistic processes

1. Introduction

It has long been speculated that a single mechanism might be at work in the production and collimation of various very energetic observed jets, such as those in gamma ray bursts (GRB, Piran et al. 2001; Sheth et al. 2003; Fargion 2003), and jets ejected from active galactic nuclei (AGN) (Sauty et al. 2002) and from microquasars (Mirabel & Rodriguez 1994, 1999). Here we limit ourselves to jets produced by a black hole (BH) type core. The most often invoked process is the Blandford-Znajek (Blandford & Znajek 1977) or some closely similar mechanism (e.g. Punsly & Coroniti 1990a,b; Punsly 2001) in the framework of magnetohydrodynamics, always requiring a magnetic field. However such mechanisms are limited to charged particles, and would be inefficient for neutral particles (neutrons, neutrinos and photons), which are currently the presumed antecedents of very thin and long duration GRB (Fargion 2003). Moreover, even for charged particles, some questions persist (see for instance the conclusion of Williams 2004). Finally, while the observations of synchrotron radiation prove the presence of magnetic fields, they do not prove that those fields alone cause the collimation: magnetic mechanisms may be only a part of a more unified mechanism for explaining the origin and collimation of powerful jets (see Livio 1999, p. 234, and Sect. 5), and, in particular, for collimation of jets from AGN to subparsec scales (see de Felice & Zanotti 2000).

Considering this background, it is worthwhile looking for other types of model to explain the origin and structure of jets. Other models based on a purely general relativistic origin for jets have been considered. A simple model was obtained by Opher et al. (1996) by assuming the centres of galaxies are described by a cylindrical rotating dust. That paper showed that confinement occurs in the radial motion of test particles while the particles are accelerated in the axial direction thus producing jets. Another relativistic model was put forward in Herrera & Santos (2007). This showed that the sign of the proper acceleration of test particles near the axis of symmetry of quasi-spherical objects and close to the horizon can change. Such an outward acceleration, that can be very big, might cause the production of jets.

However, these models show a powerful gravitational effect of repulsion only near the axis, and are built in the framework of axisymmetric stationary metrics which do not have an asymptotic behaviour compatible with possible far away observations. So we want to explore the more realistic rotating black hole, i.e. Kerr, metrics instead.

We thus address here the issue of whether it is possible, at least in principle (i.e. theoretically) to obtain a very energetic and perfectly collimated jet in a Kerr black hole spacetime without making use of magnetic fields. Other authors (see Bičák et al. 1993; Williams 1995, 2004; de Felice & Carlotto 1997, and references therein) have made related studies to which we refer below. Most such authors agree that the strong gravitational field generated by rotating BHs is essential to understanding the origin of jets, or more precisely that the jet originates from a Penrose-like process (Penrose 1969; Williams 2004) in the ergosphere of the BH; collimation may also arise from the gravitational field and that is the main topic in this paper.

Our work can therefore be considered as covering the whole class of models in which particles coming from the ergosphere form a jet collimated by the geometry. Although a complete model of an individual jet would require use of detailed models of particle interactions inside the ergosphere, such as that given by Williams (2004), we show that thin and very long and energetic jets, with some generic features, can be produced in this way. In particular the presence of a characteristic radius, of the size of the ergosphere, around which one would find the most energetic particles, might be observationally testable.

From a strictly general relativistic point of view, test particles in vacuum (here, a Kerr spacetime) follow geodesics; this applies to both charged and uncharged particles, although, of course, in an electrovacuum spacetime, such as Kerr-Newman, charged particles would follow accelerated trajectories, not geodesics. Thus, in Kerr fields, what produces an eventual collimation for test particles, or not, is the form of the resulting geodesics. Hence we discuss here the possibilities of forming an outgoing jet of collimated geodesics followed by particles arising from a Penrose-like process inside the ergosphere of a Kerr BH. We show that it is possible in principle to obtain such a jet from a purely gravitational model, but it would require the "Penrose process" to produce a suitable, and rather special, distribution of outgoing particles.

The model is based on the following considerations.

Most studies of geodesics, (e.g. Chandrasekhar 1983), employ generalized spherical, i.e. Boyer-Lindquist, coordinates. We transform to Weyl coordinates, which are generalized cylindrical coordinates, and are more appropriate, as we shall see, for interpreting the collimated jets.

We consider test particles moving in the axisymmetric stationary gravitational field produced by the Kerr spacetime, whose geodesic equations, as projected into a meridional plane, are known (Chandrasekhar 1983). Our study is restricted to massive test particles, moving on timelike geodesics, but of course massless test particles on null geodesics could be the subject of a similar study (Incidentally the compendium of Sharp 1979 shows that analytic studies of general timelike geodesics have been much less frequent than detailed studies of more restricted problems).

For particles outgoing from the ergosphere of the Kerr BH we examine their asymptotic behaviour. Among the geodesic particles incoming to the ergosphere, we discuss only the ones coming from infinity parallel to the equatorial plane, because these are in practice the particles stemming from the accretion disk. We show that only those with a small impact parameter are of high enough energy to provide energetic outgoing particles.

In the ergosphere, a Penrose-like process can occur. In the original Penrose process, an incoming particle decays into two parts inside the ergosphere. It could also decay into more than two parts, or undergo a collision with another particle in this region, or give rise to pair creation (e^-, e^+) from incident photons which would follow null geodesics. The different possible cases do not affect our considerations, and that is why we do not study them here, although the distribution function of outgoing particles would be required in a more detailed model of the type discussed, in particular to explain why only particles with low angular momentum and not diverging from the rotation axis are produced. For detailed studies see Williams (1995, 2004) and Piran & Shaham (1977). After a decay, one (or more) of the particles produced crosses the event horizon and irreversibly plunges into the BH, while a second particle arising from the decay can be ejected out of the ergosphere following a geodesic towards infinity. This outgoing particle could be ejected so that asymptotically it runs parallel to the axis of symmetry, but we do not discuss only such particles.

In our model there is no appeal to electromagnetic forces to explain the ejection or the collimation of jets, though the particles therein may themselves be charged. The gravitational field suffices, in the case of strong fields in general relativity, which is the case near the Kerr BH, provided the ergosphere produces particles of appropriate energy and initial velocity. The gravitomagnetic part of the gravitational field then provides the collimation. Hence, our model is, in this respect, simpler than the standard model of Blandford & Znajek (1977), and is in accordance with the analysis given in Williams (2004).

The paper starts with a study of Kerr geodesics in Weyl coordinates in Sect. 2; the next section studies the asymptotic behaviour of geodesics of outgoing particles with $L_z = 0$; Sect. 4 analyses incoming particles stemming from the accretion; a sample Penrose process and the plotting of geodesics are presented in Sect. 5; and finally we discuss in Sect. 6 the significance of our results for jets. In the conclusion, we succinctly summarize our main results and evoke some perspectives.

2. Kerr geodesics

We start from the projection in a meridional plane ϕ = constant of the Kerr geodesics in Boyer-Lindquist spherical coordinates $\bar{r}$, θ and ϕ. The metric is

$$\begin{aligned} ds^2 &= \left(\bar{r}^2 + a^2\cos^2\theta\right)\left(\frac{d\bar{r}^2}{\bar{r}^2 - 2M\bar{r} + a^2} + d\theta^2\right) \\ &+ \frac{\sin^2\theta}{(\bar{r}^2 + a^2\cos^2\theta)}\left(a dt - \left(\bar{r}^2 + a^2\right)d\varphi\right)^2 \\ &- \frac{(\bar{r}^2 - 2M\bar{r} + a^2)}{(\bar{r}^2 + a^2\cos^2\theta)}\left(dt - a\sin^2\theta d\varphi\right)^2, \end{aligned} \tag{1}$$

where M and Ma are, respectively, the mass and the angular momentum of the source, and we have taken units such that $c = 1 = G$ where G is Newton's constant of gravitation. The "radial" coordinate in Eq. (1) has been named $\bar{r}$ because it is more convenient for us to use the rescaled coordinate $r = \bar{r}/M$ (O'Neill 1995). The projected timelike geodesic equations are then

$$M^2\dot{r}^2 = \frac{(a_4r^4 + a_3r^3 + a_2r^2 + a_1r + a_0)}{\left[r^2 + \left(\frac{a}{M}\right)^2\cos^2\theta\right]^2}, \tag{2}$$

$$M^2\dot{\theta}^2 = \frac{b_4\cos^4\theta + b_2\cos^2\theta + b_0}{(1-\cos^2\theta)\left[r^2 + \left(\frac{a}{M}\right)^2\cos^2\theta\right]^2}, \tag{3}$$

with coefficients

$$a_0 = -\frac{a^2Q}{M^4}, \tag{4}$$

$$a_1 = \frac{2}{M^2}\left[(aE - L_z)^2 + Q\right], \tag{5}$$

$$a_2 = \frac{1}{M^2}\left[a^2(E^2-1) - L_z^2 - Q\right], \tag{6}$$

$$a_3 = 2, \tag{7}$$

$$a_4 = E^2 - 1, \tag{8}$$

and

$$b_0 = \frac{Q}{M^2}, \tag{9}$$

$$b_2 = \frac{1}{M^2}\left[a^2(E^2-1) - L_z^2 - Q\right] = a_2, \tag{10}$$

$$b_4 = -\left(\frac{a}{M}\right)^2(E^2-1); \tag{11}$$

where the dot stands for differentiation with respect to an affine parameter and E, L_z and Q are constants. Here Chandrasekhar's δ_1 has been set to 1, its value for timelike curves. Assuming that the affine parameter is proper time τ along the geodesics, then these equations implicitly assume a unit mass for the test particle[1], so that E and L_z have the usual significance of total energy and angular momentum about the z-axis, and Q is the corresponding Carter constant (which, as described in Hughson et al. 1972, for example, arises from a Killing tensor of the metric, while E and L_z arise from Killing vectors). With this understanding, E, L_z, and Q have the dimensions of Mass, Mass^2 and Mass^4 respectively, in geometrized units, while δ_1, though 1 numerically, has dimensions Mass^2, as do all the a_i and b_i. In this paper we consider only particles on unbound geodesics with $E \geq 1$ (for the conditions for existence of a turning point, giving bound geodesics, which are related to the parameter values for associated circular orbits, see Chandrasekhar 1983; Williams 1995, 2004).

The dimensionless Weyl cylindrical coordinates, in multiples of geometrical units of mass M, are given by

$$\rho = \left[(r-1)^2 - A\right]^{1/2} \sin\theta, \quad z = (r-1)\cos\theta, \tag{12}$$

where

$$A = 1 - \left(\frac{a}{M}\right)^2 . \tag{13}$$

From (12) we have the inverse transformation

$$r = \alpha + 1, \tag{14}$$

$$\sin\theta = \frac{\rho}{(\alpha^2 - A)^{1/2}}, \quad \cos\theta = \frac{z}{\alpha}, \tag{15}$$

with

$$\alpha = \frac{1}{2}\left(\left[\rho^2 + (z+\sqrt{A})^2\right]^{1/2} + \left[\rho^2 + (z-\sqrt{A})^2\right]^{1/2}\right). \tag{16}$$

Here we have assumed $A \geq 0$, and taken the root of the second degree equation obtained from (15) for the function $\alpha^2(\rho, z)$ that allows the extreme black hole limit $A = 0$. The other root, in this limit, is the constant $\alpha = 0$.

The Eq. (16) shows that in the (ρ, z) plane the curves of constant α (constant r) are ellipses with semi-major axis α and eccentricity $e = \sqrt{A}/\alpha$: for large α, these approximate circles. Note that $\rho = 0$ consists of the rotation axis $\theta = 0$ or π together with the ergosphere surface.

Now, with Eqs. (14) and (15) we can write the geodesics Eqs. (2) and (3) in terms of ρ and z coordinates, producing the following autonomous system of first order equations

$$M\dot{\rho} = \frac{\dfrac{P\alpha^3\rho}{\alpha^2 - A} + \dfrac{S(\alpha^2 - A)z}{\alpha\rho}}{(\alpha+1)^2\alpha^2 + \left(\dfrac{a}{M}\right)^2 z^2}, \tag{17}$$

$$M\dot{z} = (Pz - S)\alpha\left[(\alpha+1)^2\alpha^2 + \left(\frac{a}{M}\right)^2 z^2\right]^{-1}, \tag{18}$$

where

$$P = \epsilon_1\left[a_4(\alpha+1)^4 + a_3(\alpha+1)^3 + a_2(\alpha+1)^2 + a_1(\alpha+1) + a_0\right]^{1/2}, \tag{19}$$

$$S = \epsilon_1\epsilon_2\left(b_4 z^4 + b_2\alpha^2 z^2 + b_0\alpha^4\right)^{1/2}, \tag{20}$$

[1] An alternative interpretation is to assume that for a particle of mass m, the affine parameter τ/m has been used (Wilkins 1972; Williams 1995).

and $\epsilon_i = \pm 1$ for $i = 1, 2$: ϵ_1 indicates whether the geodesic is incoming or outgoing in r (i.e. the sign of $\dot{r}$), while $\epsilon_1\epsilon_2$ indicates whether θ is increasing or decreasing. Note we always mean the non-negative square roots to be taken.

The ratio between the first order differential Eqs. (17) and (18) yields the special characteristic equation of this system of equations

$$\frac{dz}{d\rho} = \frac{(|P|z - \epsilon_2|S|)(\alpha^2 - A)\alpha^2\rho}{|P|\alpha^4\rho^2 + \epsilon_2|S|(\alpha^2 - A)^2 z} . \tag{21}$$

We restrict our study to the quadrant $\rho > 0$ and $z > 0$ in the projected meridional plane (orbits in fact spiral round in ϕ in general: this information is contained in the conserved L_z). The results for the other three quadrants will follow by symmetry, although this symmetry does not imply that individual geodesics are symmetric with respect to the equatorial plane. From numerical solutions of the geodesics we obtained asymmetrical geodesics, confirming the analysis in Williams (1995, 2004). Geodesics can also cross the polar axis, which would be represented by a reflection from $\rho = 0$ back into the quadrant.

Geodesics going to or coming from the expected accretion disk would, if the disk were thin, go to or from values of ρ much larger than z. In this limit ($\rho \gg z$ and $\rho \gg \sqrt{A}$), we have

$$\alpha = \rho\left(1 + O\left(\rho^{-2}\right)\right), \tag{22}$$

$$|P| = \sqrt{a_4}\rho^2\left(1 + k/\rho + O\left(\rho^{-2}\right)\right), \tag{23}$$

$$\text{where } k = \frac{2E^2 - 1}{E^2 - 1}, \tag{24}$$

$$|S| = \sqrt{b_0}\rho^2(1 + O(\rho^{-2})), \tag{25}$$

and thus

$$\frac{dz}{d\rho} \approx \frac{z - \epsilon_2 z_1 + zk/\rho}{\rho(1 + k/\rho)} + O(\rho^{-3}), \tag{26}$$

where

$$z_1 = \left(\frac{b_0}{a_4}\right)^{1/2} = \frac{1}{M}\left(\frac{Q}{E^2 - 1}\right)^{1/2}, \tag{27}$$

and we have to assume $Q \geq 0$ to obtain a real z_1 (the form of Eq. (3) makes it obvious that geodesics with $Q < 0$ are bounded away from the equatorial plane $\cos\theta = 0$, though those with very small $|Q|$ could still satisfy $\rho \gg z$).

The truncated series development of $dz/d\rho$ now yields

$$\frac{dz}{d\rho} \approx \frac{1}{\rho}\left(z - \epsilon_2 z_1 + \frac{\epsilon_2 k z_1}{\rho}\right) + O\left(\rho^{-3}\right). \tag{28}$$

If $\epsilon_2 = -1$ the curve crosses the equatorial plane and gives similar asymptotic behaviour in the second quadrant, so we can take $\epsilon_2 = 1$.

A thicker accretion disk would absorb or release particles on geodesics with larger values of z/ρ, which might include particles with $Q < 0$.

Geodesics in an axial jet would have $z \gg \rho$. For this limit, we first observe that from Eq. (20) we have

$$S \approx \epsilon_2\left[(b_0 + b_2 + b_4)z^4 + (2b_0 + b_2)\rho^2 z^2\right]^{1/2} + O(z^{-1}), \tag{29}$$

where

$$b_0 + b_2 + b_4 = -\left(\frac{L_z}{M}\right)^2 \leq 0, \tag{30}$$

$$2b_0 + b_2 = \frac{1}{M^2}\left[a^2(E^2 - 1) - L_z^2 + Q\right]. \tag{31}$$

Hence in this limit S is well defined and real for indefinitely small ρ/z only for $L_z = 0$. The geodesics obeying this restriction, imposed after similar reasoning, were studied by Bičák et al. (1993), but in BL or Kerr-Schild coordinates. Here we re-examine these geodesics in the more revealing cylindrical coordinates.

Before doing so, we may note that in contrast to geodesics with $L_z \neq 0$, geodesics with $E^2 > 1$ and $L_z = 0$ may lie arbitrarily close to the polar axis (Carter 1968). For $L_z \neq 0$, the value of S^2 at the axis is $-z^2 L_z^2 < 0$ which is not allowed and thus there is some upper bound θ_0 on θ. The value of S^2 at $\theta = \pi/2$ is $b_0 \alpha^2$ so if $Q < 0$ there is also a lower bound θ_1 on θ.

3. Geodesics with $L_z = 0$

We shall discuss unbounded ($E^2 > 1$) outgoing geodesics. Corresponding incoming geodesics will follow the same curves in the opposite direction.

For $L_z = 0$, S^2 factorizes as

$$S^2 = (\alpha^2 - z^2)(Q\alpha^2 + a^2(E^2 - 1)z^2)/M^2. \tag{32}$$

Hence S can only be zero at the symmetry axis, where $\cos\theta = 1$, $\alpha = z$ and $\rho = 0$, or, if $Q < 0$, at some $z/\alpha = (|Q|/a^2[E^2 - 1])^{1/2} = \cos\theta_1$, say. Correspondingly $\dot{\theta} = 0$ only at the axis, at $\theta = \theta_1$ if $Q < 0$, and at $\alpha \to \infty$.

Thus for $L_z = 0$ and $Q < 0$, geodesics which initially have $\dot{\theta} < 0$ will become asymptotic to $\theta = \theta_1$. The angle may be narrow if

$$a^2(E^2 - 1) - |Q| \ll |Q|, \tag{33}$$

and then $\theta_1 \ll 1$. Such geodesics may provide a conical jet, as discussed later.

Our other polynomial, P^2, can be written as

$$P^2 = (E^2 - 1)\left(r^4 + \frac{a^2}{M^2}r^2 + \frac{2a^2}{M^2}r\right) \tag{34}$$
$$+2r^3 + 2\frac{a^2}{M^2}r - \frac{Q}{M^2}\left(r^2 - 2r + \frac{a^2}{M^2}\right).$$

From this form it easily follows that any unbound geodesic ($E^2 > 1$) with $L_z = 0$ has at most one turning point in r (i.e. value such that $\dot{r} = 0$) and this, if it exists, lies inside the horizon (and a fortiori inside the ergosphere, Stewart & Walker 1974). The argument is very simple. If $E^2 > 1$ and $Q \leq 0$, then P^2 is strictly positive for all $r > 0$. If $Q > 0$, P^2 is negative at $r = 0$ but positive at the outer black hole horizon (where $r^2 - 2r + a^2/M^2 = 0$), so its one zero lies inside the black hole. This implies that unbounded outgoing geodesics followed by particles with $L_z = 0$ must come from the ergosphere. Correspondingly, geodesics incoming from infinity with $L_z = 0$ will fall into the ergosphere.

Although there are no turning points of r, one can have turning points of ρ, if $\epsilon_2 = -1$. Such turning points are solutions of the equation

$$D(\rho, z) \equiv |P|\alpha^4\rho^2 - |S|(\alpha^2 - A)^2 z = 0 \tag{35}$$

where $D(\rho, z)$ is the denominator of (21) with $\epsilon_2 = -1$. At each of these turning points (ρ_2, z_2), $\mathrm{d}z/\mathrm{d}\rho \to \infty$, which means that the geodesics have a vertical tangent (parallel to the z-axis). Before reaching the turning point, these geodesics have $\mathrm{d}\rho/\mathrm{d}z > 0$, and, at any z, $\mathrm{d}z/\mathrm{d}\rho > \mathrm{d}z/\mathrm{d}\rho|_D$, where $\mathrm{d}z/\mathrm{d}\rho|_D$ is the slope of the curve (35), and afterwards they have $\mathrm{d}\rho/\mathrm{d}z < 0$, implying that they subsequently cross the axis. They will then cross the curve (35) again but from above in the (ρ, z) plane and hence with $\mathrm{d}z/\mathrm{d}\rho < \mathrm{d}z/\mathrm{d}\rho|_D$, and afterwards stay in the region outside (35).

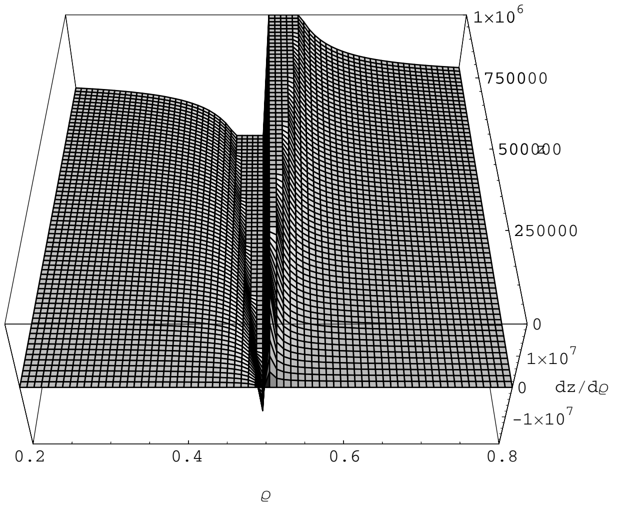

Fig. 1. Plot of the surface $\mathrm{d}z/\mathrm{d}\rho = f(\rho, z)$ given by Eq. (21) for an outgoing particle with the parameters $E = 10^4$, $L_z = 0$ and $Q = -9\times10^3$ for a black hole with parameters $M = 1$ and $a = 0.5$, in the case where $\epsilon_2 = -1$. The points where $\mathrm{d}z/\mathrm{d}\rho \to \pm\infty$ correspond to the asymptotes given by the Eq. (40), $\rho = \rho_1 = 0.49991$.

For outgoing geodesics outside (35) which reach points at large z and ρ ($\gg\sqrt{A}$), then unless the ratio of z to ρ is very large (the case which we discuss next) or very small, approximating Eq. (21) gives $\mathrm{d}z/\mathrm{d}\rho \approx z/\rho$, so all such geodesics approximate $\rho = Cz$ for suitable C, regardless of the sign of ϵ_2.

In the limit $z \gg \rho$ and $z \gg \sqrt{A}$,

$$\alpha = z(1 + O(z^{-2})), \tag{36}$$
$$|P| = \sqrt{a_4}\, z^2(1 + k/z + O(z^{-2})), \tag{37}$$
$$|S| = \sqrt{2b_0 + b_2}\, \rho z(1 + O(z^{-2})), \tag{38}$$

so the Eq. (21) can be approximated by

$$\frac{\mathrm{d}z}{\mathrm{d}\rho} = \frac{z(1 + k/z)}{\rho(1 + k/z) + \epsilon_2\rho_1} + O\left(\frac{1}{\rho z}\right), \tag{39}$$

where

$$\rho_1 = \left(\frac{2b_0 + b_2}{a_4}\right)^{1/2} = \rho_e\left[1 + \frac{Q}{a^2(E^2 - 1)}\right]^{1/2}, \tag{40}$$

and $\rho_e \equiv a/M$. Here ρ_1 is real if $a^2(E^2 - 1) + Q > 0$ but we see from Eq. (32) that for S to be real near the axis, this condition must be satisfied.

In Fig. 1 we show a plot of the values of $\mathrm{d}z/\mathrm{d}\rho$, using (21), for $\varepsilon_2 = -1$, with the parameters $M = 1$, $a = 1/2$, $E = 10^4$, $Q = -9 \times 10^3$. The only asymptotes are parallel to the z axis at $\rho = \rho_1$ as expected from Eq. (40).

We also plot in Fig. 2 a set of such outgoing geodesics obeying (21), for the same values of the parameters of the BH ($a = 1/2$, $M = 1$) and of the particle ($L_z = 0$, $Q = -2.2 \times 10^5$, $E = 2 \times 10^3$, so $\rho_1 = 0.441588$), but with different initial values of the position. The set of turning points of these geodesics is the curve defined by Eq. (35). For the rightmost of these geodesics, the numerical integration was also continued back towards the ergosphere as far as $\rho = 10^{-4}$, $z = 0.843407 < \sqrt{A} = 0.866025$.

To confirm the picture obtained from these numerical experiments, one can show, without assuming $z \gg \rho$, the existence of

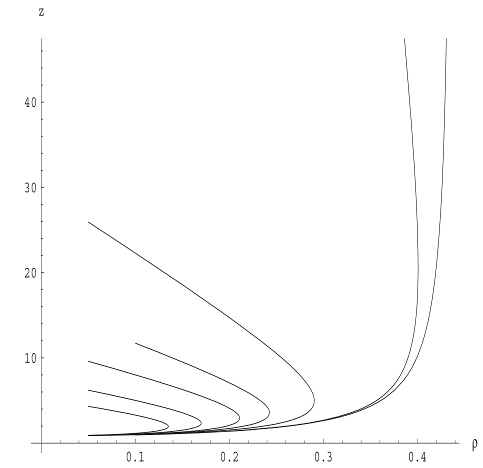

Fig. 2. Plots of geodesics obeying Eq. (21), showing the turning points. From left to right these curves start at $\rho_0 = 0.07$ and $z_0 = 0.98, 0.95, 0.93, 0.92, 0.91, 0.9$ and 0.89935501.

exactly one zero of D on any curve r = constant, $0 < \theta < \pi/2$, so that the conclusion that a geodesic has at most one turning point in ρ is not an artefact of the approximation at large z. The argument is as follows.

Along an r = constant curve, $|P|$ and α are constant, $\rho = \rho_0 \sin\theta$ and $z = \alpha\cos\theta$, where ρ_0 is a constant (related to r). Then

$$D = |P|\alpha^4\rho_0^2 \sin^2\theta - |S|(\alpha^2 - A)^2 \alpha\cos\theta \tag{41}$$

where, defining

$$F \equiv (b_0 + |b_4|\cos^2\theta)^{1/2} = [(Q + a^2(E^2 - 1)\cos^2\theta)]^{1/2}/M, \tag{42}$$

from Eq. (32) we have $|S| = \alpha^2 F \sin\theta$. For real S we need $F \geq 0$ and as θ decreases, F increases.

At $\theta = \pi/2$, $D > 0$, while at small θ, $D < 0$. Hence there is at least one zero of D. Let the largest one be at $\theta = \theta_0$ say. On the r = constant curve, we will then have

$$D = \alpha^3(\alpha^2 - A)^2 \frac{\sin\theta}{\sin\theta_0}(F_0 \sin\theta\cos\theta_0 - F\cos\theta\sin\theta_0). \tag{43}$$

Here we have used $D = 0$ at $\theta = \theta_0$ to substitute for the constant $|P|\alpha^4\rho_0^2\sin^2\theta_0$ in terms of α and θ_0. As θ decreases from θ_0, $\sin\theta$ decreases, $\cos\theta$ increases and F increases. Thus the combination $(F_0 \sin\theta\cos\theta_0 - F\cos\theta\sin\theta_0)$ becomes and stays negative for all $\theta < \theta_0$. Thus $D < 0$ for all $\theta < \theta_0$, though D approaches 0, due to the further factor $\sin\theta$, as $\theta \to 0$. This implies that the only points on r = constant such that $D = 0$ are at $\theta = 0$ and $\theta = \theta_0$.

For large z we see from Eq. (39) that the turning points lie approximately on a curve $\rho = \rho_1 z/(z + k)$ or $z = -k\rho/(\rho - \rho_1)$. Actually, the differential equation for large z, if we drop the $1/z^2$ terms, has an analytic solution

$$Ckz = k\rho + \rho_1 z \ln[z/(z + k)], \tag{44}$$

where C is a constant of integration, so

$$\rho = \rho_1 z/k \ln(1 + k/z) + Cz \to \rho_1 + Cz \ldots \tag{45}$$

as $z \to \infty$. Keeping the next order terms in $1/z$ would be inconsistent with the terms dropped during the derivation (Similarly, there is an analytic solution for the approximate Eq. (26) at large ρ which gives $z = z_1\rho\ln(1 + \rho/k)/k + c\rho$ with similar interpretation).

From Eq. (45), either (a) ρ/z is approximately constant or (b) $\rho \to \rho_1$. In case (a), we note that for consistency of the approximation $z \gg \rho$, C must be small, although the conclusion is the same as was reached above merely with the assumption that both z and ρ are $\gg \sqrt{A}$. In case (b), we have a limit-outgoing geodesic for which $\rho < \rho_1$ at all points and as $z \to \infty$, $\rho \to \rho_1$. This limit is obtained since the turning point for ρ has $z_2 \to \infty$ when $\rho_2 \to \rho_1$. We can see from Eq. (44) that the coordinate z_2 of this turning point tends to infinity like $z_3 = -k\rho_1/(\rho_2 - \rho_1)$. The geodesics asymptotic to ρ_1 would provide a perfectly collimated jet parallel to z.

One might think (and we initially thought) that there also existed geodesics eventually tending to the same asymptote but approaching it from the right in the (ρ, z) plane (for example, directly from the accretion disk, or coming from the ergosphere but with a turning point $\rho_2 > \rho_1$). However, such geodesics do not exist, since they require that $\mathrm{d}z/\mathrm{d}\rho < 0$ in the limit $z \gg \rho$ and for $\rho > \rho_1$, contradicting Eq. (39) which implies $\mathrm{d}z/\mathrm{d}\rho > 0$. This is entirely in agreement with the results of Stewart & Walker (1974).

The geodesics in $\rho > \rho_1$ may asymptote to any ratio z/ρ, from Eq. (39). Moreover, geodesics which do turn in ρ then cross the axis, cannot cross the curve $D = 0$ from below again, and so cross it from above and also asymptotically have some fixed ratio z/ρ.

For astrophysical applications, it may be important to write the results in the normal units of length and time. We have, from Eq. (17), that asymptotically for outgoing particles in $z \gg \rho$, $\dot\rho \to 0$, $\dot t \to E$, $\dot z > 0$ (for $-\epsilon_2 = 1 = \epsilon_1$) and Eq. (18) is given by

$$M\dot z \approx \sqrt{a_4}; \tag{46}$$

hence, restoring normal units of length and time and taking a particle of mass m, the asymptotic value v of the speed of outgoing particles is given by

$$mM\dot z \approx \left(\frac{E^2}{c^4} - m^2\right)^{1/2} = m\gamma\frac{v}{c}, \tag{47}$$

where we have used (8) and

$$\gamma = \left[1 - \left(\frac{v}{c}\right)^2\right]^{-1/2} = \frac{E}{mc^2}, \tag{48}$$

is the Lorentz factor. Hence, asymptotically, the speed of the particle is

$$v = c\frac{M\dot z}{\dot t} = c\left(1 - \frac{m^2c^4}{E^2}\right)^{1/2}, \tag{49}$$

which is ultrarelativistic if $E \gg \sqrt{mc^2}$. From Eq. (47) we have that asymptotically limit-outgoing particles have an uniform motion parallel to the z axis.

4. Incoming particles

We describe as "incoming particles" the particles, with parameters E', L'_z, and Q', coming into the ergosphere following unbound geodesics and having a turning point in z (i.e. such that $\dot z = 0$). Such turning points $\{\rho_4, z_4\}$ are defined as solutions of the equation

$$N_2(\rho_4, z_4) = 0 \tag{50}$$

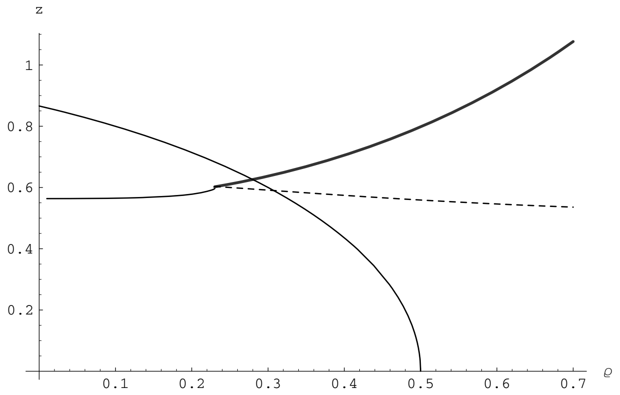

Fig. 3. Penrose process. Plots of the ingoing particle (dashed line) coming asymptotically, for $\rho \to \infty$, from $z_1 = 1.58116$, to the turning point ($\rho = 0.23$, $z = 0.59$) located inside the ergosphere, whose boundary is indicated, where it decays into an infalling particle and an outgoing particle (bold line) following a geodesic asymptotic to $\rho = \rho_1 = 1.64341$ when $z \to \infty$. The parameters of the black hole are $M = 1$ and $a = 0.5$. The parameters of the three geodesics are $E_{\rm in} = 200$, $E_{\rm out} = 202$, $E_{\rm fall} = -2$, $L_{z,\rm in} = L_{z,\rm fall} = -100$, $L_{z,\rm out} = 0$, $Q = Q' = 100\,000$.

where

$$N_2 = Pz - S \tag{51}$$

is the relevant factor in the numerator of the right side of Eq. (21). As remarked earlier we need only consider $\epsilon_2 = 1$. For large ρ these turning points are approximately at $z = z_1(1 - k/\rho)$, from Eq. (28). The set of these turning points $\{\rho_4, z_4\}$ forms a curve of the points of each geodesic with horizontal tangent (i.e. parallel to the ρ–axis). Each geodesic has a positive slope $\mathrm{d}z/\mathrm{d}\rho > 0$ for $\rho < \rho_4$, a negative one for $\rho > \rho_4$ and a maximum at the turning point. There exists a limit-incoming geodesic with its turning point at the infinite $\rho_4 \to \infty$ for $z_4 \to z_1$, i.e. with the asymptote $z = z_1$, when $\rho \to \infty$.

A test particle with parameters E', L'_z, and Q' coming from infinity (in practice from the accretion disk) parallel to $z = 0$ towards the axis of the black hole corresponds to a geodesic which, in the limit $\rho \to \infty$, has an asymptote defined by $z = z_1 =$ constant where z_1 is the impact parameter. Therefore it is a limit-incoming particle with $z < z_1$, $\dot{z} < 0$, $\dot{\rho} < 0$, $\epsilon_1 = \epsilon_2 = 1$. In the limit $\rho \gg z$, $\mathrm{d}z/\mathrm{d}\rho = 0$, so the tangent has to be parallel to the ρ axis and Eq. (27) produces

$$z_1 = \frac{1}{M}\left(\frac{Q'}{E'^2 - 1}\right)^{1/2}. \tag{52}$$

We have plotted in Fig. 3 (see Sect. 5) an example of a geodesic of an incoming particle. We see that, unlike ρ_1, z_1 does not depend upon the black hole parameter a. The incoming particles come from the accretion disk, which means that their energy E' is rarely very big, which means *rarely as big as a or M*, which characterize the black hole energy, but instead of the same order as 1 or $\sqrt{Q'}$. When $E' \to 1$ then $z_1 \to \infty$ if $Q' \neq 0$. However, if E' is very big compared to 1 and $\sqrt[4]{Q'}$, then $z_1 \to 0$.

For a given Q', the most energetic incoming particles are those with a small impact parameter z_1, near to zero. Hence only a thin slice of the accretion disk can participate with the greatest efficiency in producing Penrose processes leading to the most intense possible jet. The point where the ergosphere surface intersects the z axis is $z_{\rm e} = \sqrt{A}$. The value of $z_{\rm e}$, for the incoming particles, does not play a role like that of $\rho_{\rm e}$ for the outgoing particles (compare Eq. (40) to Eq. (52)).

5. Penrose process and plotting of geodesics

To make a jet using the geodesics discussed above, we would have to assume that incoming particles arrive in the ergosphere and undergo a Penrose process. As mentioned earlier, in its original version (Penrose 1969), each particle may be decomposed into two subparticles and one of them may cross the horizon and fall irreversibly into the BH, while the other is ejected to the exterior of the ergosphere; or the incoming particle may collide with another particle resulting in one plunging into the BH and the other being ejected to the exterior. The second case can correspond to a creation of particles, say e^- and e^+ from an incoming photon ($\delta = 0$) interacting with another inside the ergosphere. We do not present here all the possible cases, which are exhaustively studied, especially for AGN, in Williams (1995, 2004). There is also observational evidence for a close correlation between the disappearance of the unstable inner accretion disk and some subsequent ejections from microquasars such as GRS 1915+1105 (Mirabel & Rodriguez 1994, 1999), which from our point of view could correspond to the instability causing disk material to fall through the ergosphere and to then give rise to a burst of ejecta from Penrose-like processes. Here we are mainly interested in the outgoing particles which follow geodesics that tend asymptotically towards a parallel to the z axis, as described in the earlier Sect. 3. These events are closely dependent on the possibilities allowed by the conservation equations. In the case when the incoming particle splits into two (Rees et al. 1976), the conservation equations of the energy and angular momentum are

$$E_{\rm in} = E_{\rm out} + E_{\rm fall}, \tag{53}$$

$$L_{z,\rm in} = L_{z,\rm out} + L_{z,\rm fall}. \tag{54}$$

We know from Eq. (29) that for the outgoing particles we study $L_{z,\rm out} = 0$. The particles falling irreversibly into the BH have energy $E_{\rm fall}$ and angular momentum $L_{z,\rm fall}$. The energy and the angular momentum of a particle in the ergoregion can be negative which is the basis of the Penrose process. The particle falling into the black hole has negative energy, and hence the outgoing particle, leaving the ergosphere, has a bigger energy than the incoming particle,

$$E_{\rm out} = E_{\rm in} + |E_{\rm fall}|. \tag{55}$$

We have plotted numerically the geodesics for incoming, outgoing and falling particles with the following values for the parameters: $a/M = 1/2$, $E_{\rm in} = 200$, $E_{\rm out} = 202$, $E_{\rm fall} = -2$, $L_{z,\rm in} = L_{z,\rm fall} = -100$ and $Q = Q' = 10^5$. These values are chosen in such a way that the geodesics meet inside the ergoregion situated in the interior of the ergosphere

$$z^2 = \left\{1 - \rho_{\rm e}^2\left[1 - \left(\frac{\rho}{\rho_{\rm e}}\right)\right]\right\}\left[1 - \left(\frac{\rho}{\rho_{\rm e}}\right)\right], \tag{56}$$

and they produce for the asymptotes of the outgoing particles $\rho_1 = 1.64341$ and for the incoming $z_1 = 1.58116$. The curves are built with the initial conditions $z[1.55] = 20$ for outgoing particles and $z[0.25] = 0.60$ for incoming particles. Their intersection is situated at the point $\rho_i = 0.23$ and $z_i = 0.59$ inside the ergosphere which we can take as the initial condition for the falling particle and from where we trace the three curves (see Fig. 3).

The exhibition of these numerical solutions with an outgoing geodesic which leaves the ergosphere after the Penrose process and has vertical asymptote with the value ρ_1 precisely equal to Eq. (40) confirms that a model based on such geodesics is possible.

6. Implications for jet formation

We have shown that to obtain a jet of particles close to the rotation axis, it must be formed from particles with (almost) zero angular momentum, $L_z = 0$. If we consider only particles with $L_z = 0$, there is among them a subset which give a perfectly collimated jet, i.e. a set of geodesics exactly parallel to the axis: for each allowed value of Q they form a ring of radius

$$\rho_1 = \rho_e \left[1 + \frac{Q}{a^2(E^2-1)}\right]^{1/2}, \quad (57)$$

whwre $\rho_e = a/M$. Note that $\rho = \rho_e$ is the circumference of the ergosphere in the equatorial plane. For large E the set of all these geodesics will give a jet of radius of the order of ρ_e, with a density of particles dependent on the distribution in Q for given E, which remains perfectly collimated all the way to infinity.

We note that all other geodesics with $L_z = 0$ will spread out from the axis along lines $z = K\rho$. An astrophysical jet will of course be of only finite extent and not perfectly collimated, so it could include such geodesics for suitably large K, as well as geodesics with a small $L_z \neq 0$.

Thus forming a collimated jet of particles from a Penrose-like process, this jet having a narrow opening angle, for a rotating black hole without an electromagnetic field, depends on the initial distribution of particles leaving the ergosphere, or of some non-gravitational collimating force, even if we consider only particles with $L_z = 0$.

On the other hand, outgoing particles with small energies, namely of the order of their rest energy, $E \approx 1$, and $Q > 0$ have asymptotes parallel to the z axis with $\rho_1 \gg \rho_e$.

This predicted scale of the region of confined highly energetic particles might provide a test if the accretion disk parameters provided values for the BH mass and angular momentum, in a manner such as discussed in McClintock et al. (2006) and papers cited therein, and if the transverse linear scale of the jet near the BH could be measured (Particles of equally high energy may exist in $\rho > \rho_1$ but will spread out away from the axis).

Let us make a brief qualitative remark about the observability of the two species, (a) and (b), of geodesics outgoing from the ergosphere, studied in Sect. 3 (after Eq. (45)). As illustrated by the Fig. 2, for each fixed value of ρ_1 there is one (b)-geodesic only, which is the limit of many (one infinity of) (a)-geodesics when the turning point tends to the infinity ($z_2 \to \infty$, $\rho_2 \to \rho_1$). However, the (a)-type geodesics, though much more numerous than the (b)-type geodesics, are, directly or indirectly (i.e. by radiation, if charged), much more difficult to observe.

Indeed, contrary to the set of (b)-particles framing the jet in one direction (collimation along the poles), the (a)-particles ejected from the ergosphere along unbound geodesics at lower latitudes are dispersed into the whole 3D-space (4π steradians). The (a)-particles never produce a beam into one privileged direction but instead dilute in the whole space. Observed from the infinity in one line of sight (θ = constant, ϕ = constant), one single (a)-particle could directly be detected. While, from the infinity in the line of sight z ($\theta = 0$, $\forall\phi$), the observer will see one infinity (each point of the perimeter of the circle of radius ρ_1) of (b)-particles. The result is reinforced when we extend it to all the possible values of ρ_1. Encircling the foot of the (b)-jet, the (a)-particles frame a gerb, from the basis of which a possible indirect effect of isotropic radiation emission (from accelerated charged particles) could be observed, during the jet eruption.

Besides, by their dispersion, the pressure the (a)-particles locally exert on the ambient medium is much weaker than the pressure exerted by the numerous coherent (b)-particles of the jet (a narrow parallel beam is more incisive). The (a)-particles are probably more rapidly thermalised than the (b)-particles of the jet. So, one might expect that many particles ejected at lower latitudes never attain infinity (neither the height of the jet), and most of them feed the medium, framing a halo around the BH, falling inside again, or returning to the accretion disk.

We noted also that geodesics with $Q < 0$ can be asymptotic to lines with constant θ. These asymptotes allow us to define another type of jet which is bigger and less collimated than the previous one. It is interesting to remark that recent observations (Sheth et al. 2003; Sauty et al. 2002) suggest the existence of two different types of jets precisely of these sorts, i.e. narrowly and broadly collimated.

There exists an ensemble of geodesics that tend asymptotically to these conical characteristics. The unbounded geodesics have mainly been discussed, however, by using Boyer-Lindquist coordinates r and θ by the majority of authors. If we rewrite our results, using these coordinates, we may interpret our results and compare to those of other authors. However, as we show below, these coordinates are not as well-suited to the issues we have discussed.

Geodesics with $L_z \neq 0$ may reach low values of ρ/z, if b_2 is large enough, but must be bounded away from $|\cos\theta| = 1$ (i.e. $\theta = 0$ or $\theta = \pi$), since those values would imply $\dot{\theta}^2 < 0$, from Eq. (3), (cf. Chandrasekhar 1983, p. 348). In practice this means that a narrow jet along the axis must be composed of particles with very small L_z. Particles with non-zero L_z could only lie within a jet with bounded ρ for a limited distance, because large enough z would imply $\dot{\theta}^2 < 0$. If $Q \geq 0$, the orbits reverse the sign of $\dot{\theta}$ and reach the equatorial plane, and would thus be expected to be absorbed by the accretion disk. For $Q < 0$ they are confined to a band of values of θ given by the roots of $S^2 = 0$. These are the "vortical" trajectories of de Felice et al. (de Felice & Calvani 1972; de Felice & Curir 1992; de Felice & Carlotto 1997). Depending on the maximum opening angle θ, these may still hit, and presumably be absorbed by, a thick accretion disk (de Felice & Curir 1992). Such orbits can be adequately populated by Penrose-like processes (Williams 1995, 2004), and might undergo processes which reduce the opening angle (de Felice & Curir 1992; de Felice & Carlotto 1997). A jet composed of such particles would tend to be hollow and would have a larger radius ρ at large z than is obtained for orbits with $L_z = 0$, and hence be observationally distinguishable. The presence of these escaping trajectories spiralling round the polar axis can be associated with the gravitomagnetic effects due to the rotation of the hole, one of whose consequences is that even curves with $L_z = 0$ have a non-zero $\mathrm{d}\phi/\mathrm{d}t$ at finite distances.

Thus although an infinitely extended jet of bounded ρ radius would only contain particles with $L_z = 0$, which we would expect to be a set of measure zero among all particles ejected, we shall consider this as a good model even for real jets. In practice, interactions with other forces and objects, which would affect the jet both by gravitational and other forces, have to be taken into account once the jet is well away from the BH, and these influences might or might not improve the collimation. In (de Felice & Carlotto 1997), the authors discussed possible improved collimation for particles of low L_z using forces which have a timescale long compared with the dynamical timescale of the geodesics, and which act to move particles to new geodesics with changed parameters. It should be noted that if the object producing the jet is modelled as a rotating black hole, production of a collimated jet only arises naturally if the object throws out energetic particles with low L_z, since our discussion shows that other particles cannot join such a jet unless there is some other strong collimating influence away from the BH.

However previous authors have not pointed out the existence of asymptotes $\rho = \rho_1$, presumably because they are less obvious when using coordinates r and θ. In fact, considering $z \to \infty$, the expressions (12) and (15) produce $\cos\theta \approx 1 - (\rho_1^2/2z^2) + O(z^{-3})$, $\sin\theta \approx (\rho_1/z) + O(z^{-4})$ and $r \approx z + 1 + (\rho_1^2/2z^2) + O(z^{-3})$. With these expansions it is clear that in the limit $\theta = 0$ one would have to take the limit of $r\sin\theta$ to allow ρ_1 to be determined.

In the same vein, to find the values of asymptotes $z = z_1 \neq 0$ near the equatorial plane $\theta = \pi/2$ for the incoming particles (see Eq. (52)) one has to study $r\cos\theta$ if one uses the coordinates r and θ. In fact, one finds for the asymptotic expansion $\rho \to \infty$ the following expressions, $\cos\theta \approx (z_1/\rho) + O(\rho^{-3})$, $\sin\theta \approx 1 - (z_1^2/2\rho^2) + O(\rho^{-3})$ and $r \approx \rho + 1 + [z_1^2 + 1 - (a/M)^2]/2\rho + O(\rho^{-3})$.

7. Conclusion

Our main results are the following.

There are projections of geodesics all over the meridional planes. Among these geodesics there are some, with vertical asymptotes parallel to z which can form a perfectly collimated jet. There are, as well, geodesics with horizontal asymptotes parallel to the radial coordinate ρ, that can represent the paths of incoming particles leaving the accretion disk.

These two types of geodesics have intersection points that can be situated inside the ergosphere. At these points a Penrose process can take place, producing the ejection of particles along the axis with bigger energies than the energies of incoming particles close to the equatorial plane. The energies of outgoing particles are significantly larger than the ones of the incident particles for the asymptotically vertical geodesics near the scale a/M of the ergosphere diameter in the coordinate ρ, so such particles can show collimation around the surface of a tube of diameter $2a/M$ centred on the axis of symmetry. Such collimated outgoing particles have to have a zero orbital momentum $L_z = 0$, which implies, from the Penrose process, that the incoming particles have a negative orbital momentum, $L'_z < 0$. Thus the jet has to be fed from incoming particles with retrograde orbits in the accretion disk. There is evidence for the existence of substantial counterrotating parts of accretion disks (Koide et al. 2000; Thakar et al. 1997), and such counterrotations could explain the viscosity inducing the instabilities which trigger the falling of matter towards the ergosphere. It is now known (Mirabel & Rodriguez 1994, 1999; Mirabel 2006) that there is a close connection between instabilities in the accretion disk and the genesis of jets for quasars and microquasars.

The most energetic incoming particles are those near the equatorial plane. Hence the incoming particles which produce the most energetic outgoing particles by a Penrose process in the ergosphere, whose maximum size is $z_e = \sqrt{A}$, are those with angular momentum $L'_z < 0$ and a very small impact parameter z_1.

Also, the limiting diameter of the core of a perfectly collimated jet depends upon the size of the ergosphere. The effective thickness of this part of the jet in this case is of the order of $2\rho_e = 2a/M$.

Our idealised model is based on the well-behaved vacuum stationary exact solution of Einstein's equations with axial symmetry, namely the Kerr metrics, which does not take into account the ambient medium. Though this medium is very dilute, it plays a non-negligible role on the more complex global scenario for jets like progressive widening of the beam, advent of knots, lobes, etc. However, for the scenario that we are here concerned, namely the beginning of the jet (parsec scale for microquasars,while some hundred parsecs for AGN, depending on the BH mass), where it is strongly collimated, our approximation of test-particles along geodesics is relevant. Indeed, the observed jets stemming from active galactic nuclei ejected along the polar axis have ultrarelativistic speeds, typically $v_j = 0.99995$c. The ejected particles, forming the jets, are thermalized with temperatures of the order 10^5 K (Filloux 2009) producing a lateral force from the pressure gradient between the thermal energy of the particles in the outflow and the low density enveloping medium (Punsly 1999a,b). The internal particle trajectories to these jets expand laterally at the speed of sound, being of the order $v_s = 30$ km s^{-1} (Filloux 2009), asymptotically forming a conical shape with an opening angle of the order of the inverse Mach number $(v_s/v_j) = 10^{-4}$ radians. As we can see (Punsly 1999b, Appendix), the more realistic trajectories corresponding to such corrective terms represent only a small perturbation to the geodesics.

The model that we present to explain the formation and collimation of jets arises essentially from relativistic strong gravitational field phenomena without resort to electromagnetic phenomena. From this point of view the model could be interesting also for understanding observational evidence of neutral particles emitted from the inner jet itself. For example, the recent observations of Ultra High Energy Cosmic Rays (difficult to explain, implying neutral particles such as neutrinos, or H or Fe atoms, etc. Auger 2007a,b; Dermer et al. 2009; and HESS collab. 2009, and references therein) is a new challenge. To explain the Very High Energy of such neutral (massive) particles, especially neutrinos (Auger 2007a,b, which are able to travel freely over large distances), our model very naturally suggests that they could be directly coming from the collimated inner jet, which would privilege sources (BH) with rotational z-axis along the line of sight of the observation. Massless particles (photons) would be emitted by charged particles accelerated along the collimated inner jet (Dermer et al. 2009; HESS collab. 2009), which would privilege sources (BH) with rotational z-axis perpendicular to the line of sight of the observation.

Our model is sufficiently general to fit various types of observed jets, like GRB, jets ejected from AGN or from microquasars, whenever they are energetic enough to be explained by just a rotating black hole fed by an accretion disk in an axisymmetric configuration. The main drawback is the need to preferentially populate the geodesics which can form such collimated jets. Work is in progress on this question to determine a possible confrontation of the model with observations. Our preliminary studies led us to understand the fundamental role of the function $P(r)$ of the geodesics equations (See Eqs. (2) and (19)). As an example, in the special case where the equation $P(r) = 0$ has a real double root, there exist only two narrow ranges of ρ_1 values for large values of E. In this case, we can evaluate from the power, for example of radio loud extragalactic jets (Willott et al. 1999), or of microquasars jets (Fender et al. 2004), the particle density, the mean kinetic energy by particle, the mean velocity and the Lorentz factor of the jets. These results, since they require a long presentation, deserve a separate paper which is under preparation.

The existence of vacuum solutions of the Einstein equations of Kerr type but with a richer, not connected, topological configuration of the ergosphere (see Gariel et al. 2002, Figs. 7–10), allows us to propose the existence of double jets, because they are expected to come out from the ergosphere. These bipolar jets have been observed (see for instance Skinner et al. 1997; Sahai et al. 1998, Fig. 1; Fargion 2003, Fig. 2; and

Kwok et al. 1998) and could be naturally interpreted in a generalization of our model.

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