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Relativistic satellite astrometry at μarcsec precision and the measurement of the stellar aberration *(Research Note)*

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ABSTRACT

Analysis and interpretation of the μarcsec precision astrometric data shortly to be provided by the ESA-GAIA satellite has prompted the development of a fully general-Relativistic Astrometric MODel (RAMOD), which is able to account for all the general-relativistic corrections up to the required order of precision $O(c^{-3})$. We obtain the covariant expression for the general relativistic aberration effect and provide the analytical module for determining the stellar aberration from the GAIA observables (the direction cosines of the incoming light-ray trajectory) at this same $O(c^{-3})$ order of precision.

Key words. gravitation – relativity – methods: analytical – astrometry – reference systems

1. Introduction

Stellar aberration is a well-known astronomical effect, discovered by Bradley (1728); it consists of the variation of the apparent line of sight caused by the motion of the observer, and is a consequence of the finiteness of the velocity of light, first measured by Römer (1676). As is well known, in the context of special relativity the aberration formula can be obtained from Lorentz-transforming the coordinate components of the velocity of light, and expressing these components in terms of the angles measured by two different observers between the line of sight of the star and the apex of their motion. The case of uniformly accelerated observers has been recently dealt with in Beig & Heinzle (2008). In the context of general relativity, a *covariant* formula for the stellar aberration can be directly derived from the general-Relativistic Astrometric MODel (RAMOD)[1], which was conceived and developed in de Felice et al. (1998, 2001), Bini & de Felice (2003), Bini et al. (2003), de Felice et al. (2004, 2006), de Felice & Preti (2006, 2008), Crosta & Vecchiato (2010), to reconstruct the trajectory of a light ray from the event of observation back to the star position in terms of the observational data represented by the direction cosines of the incoming light ray with respect to a given spatial frame. These data, which will soon be provided by the GAIA satellite (Turon et al. 2005), will allow us to obtain a map of the celestial sphere with a μarcsec accuracy in the measurements of angles. This accuracy is assured by retaining terms of the order of c^{-3} – namely, $O(3)$ – in the general-relativistic astrometric model. The problem of fixing the boundary conditions for the inverse ray-tracing problem was analytically discussed and fully solved in a series of works (Bini & de Felice 2003; Bini et al. 2003; de Felice & Preti 2006, 2008); in this paper we derive the covariant formula for the general relativistic aberration effect from the basic equation of the boundary conditions and provide a full analytical model for the direct determination of the stellar aberration angle in terms of the GAIA observables.

[1] For other approaches see, e.g., Bolos 2006 and Teyssandier & Le Poncin-Lafitte 2006).

Notation and conventions: metric signature is +2; Greek (spacetime) indices run from 0 to 3, while Latin (space) indices run from 1 to 3; hatted indices identify tetrad quantities; expressions like $B^\alpha C_\alpha$ for any B^α and C_α indicate scalar products relative to the given metric.

2. The aberration formula

Let our Galaxy be contained in a coordinate neighbourhood of a spacetime manifold, which we assume to be generated by the solar system alone. Let $\{x^\alpha\}$ be a coordinate system centred at the baricentre of the solar system; the spatial coordinates $\{x^i\}$ span spacelike hypersurfaces, which we require to exist at $t = \text{const.}$, t being a time coordinate. On each of these hypersurfaces, the spatial coordinate axes are chosen to identify a kinematically nonrotating frame, termed barycentric celestial reference system (BCRS). This coordinate system generates a field of coordinate bases for the tangent spaces on the manifold; all tensor components referred to in this paper are relative to those bases. The background metric is a solution of Einstein's equations with the solar system as the only source; its form is therefore dictated by the weak-field, slow-motion approximation, with metric components reading

$$
\begin{aligned}
g_{00} &= -1 + h_{00} + O(4), \quad g_{0i} = h_{0i} + O(5),\\
g_{ij} &= \delta_{ij} + h_{ij} + O(4),
\end{aligned}
$$

where, to the lowest order, the metric perturbations are $h_{00} \sim O(2)$, $h_{0i} \sim O(3)$, $h_{ij} = h_{00}\delta_{ij} \sim O(2)$. This choice was recommended by the IAU (2000).

Since the following discussion will deal primarily with the measurement of angles, it is essential to identify the observers and the frames of reference chosen to perform these measurements. We define

- The *locally baricentric observers*. They are a family of observers who are static with respect to the spatial axes of the BCRS, and are described at each spacetime point by a four-vector $\boldsymbol{u}$ with coordinate components $u^{\alpha} = A(x)\delta^{\alpha}_{0}$, where $A(x) = (-g_{00})^{-1/2}$ is a scalar function, that assures unitarity of $\boldsymbol{u}$ with respect to the given metric: $u^{\alpha}u_{\alpha} = -1$. A tetrad frame adapted to a locally baricentric observer is termed "Sun-locked" if one of the vectors of its spatial triad is identified as stably pointing to the geometrical centre of the Sun. Below, the spatial triad of the tetrad frame adapted to the BCRS observer will be indicated with $\boldsymbol{\lambda}_{\hat{a}}$.

- The *boosted observer*. This observer is at rest with respect to the centre of mass of the satellite and carries a triad, which is obtained from the local baricentric Sun-locked frame by boosting $\boldsymbol{\lambda}_{\hat{a}}$ to the rest space of the satellite. The boosted observer is described by a unitary four-vector field $\boldsymbol{u}'$ with coordinate components $u'^{\alpha} = A'(x)(\delta^{\alpha}_{0} + \beta^{j}\delta^{\alpha}_{j})$, where $\beta^{j} = \mathrm{d}x^{j}/\mathrm{d}t$ and $A'(x) = (-g_{00} - 2g_{0i}\beta^{i} - g_{ij}\beta^{i}\beta^{j})^{-1/2}$ is a scalar function, that assures unitarity of $\boldsymbol{u}'$ with respect to the given metric: $u'^{\alpha}u'_{\alpha} = -1$. The vector $\boldsymbol{u}'$ is tangent to the trajectory of the centre of mass of the satellite. The spatial triad of the tetrad frame adapted to this observer will below be indicated with $\boldsymbol{\lambda}_{\hat{a}(bs)}$.

- The *satellite observer*. This observer is at rest with respect to the centre of mass of the satellite, but is also at rest with respect to the spatial rotations as established by the satellite attitude (Turon et al. 2005). This observer is characterized by the same unitary four-vector field $\boldsymbol{u}'$, tangent to the trajectory of the centre of mass of the satellite considered above, but it carries a spatial triad that rotates with respect to $\boldsymbol{\lambda}_{\hat{a}(bs)}$, as stated. The spatial triad of the tetrad frame adapted to this observer will be indicated below with $\boldsymbol{E}_{\hat{a}}$. Note that $\boldsymbol{E}_{\hat{a}}$ both translates and rotates with respect to $\boldsymbol{\lambda}_{\hat{a}}$.

The unitarity of the vector fields $\boldsymbol{u}$ and $\boldsymbol{u}'$ assures that the parameter along their integral curves has the meaning of physical time for the corresponding observer. The raw data provided by GAIA will be the measurements made in the reference frame of the satellite observer, namely with respect to a tetrad $\boldsymbol{E}_{\hat{\alpha}}$ so that $\boldsymbol{E}_{\hat{0}} \equiv \boldsymbol{u}'$, while the vectors of the spatial triad $\boldsymbol{E}_{\hat{a}}$ are fixed according to the technical configuration of the mission.

The instantaneous physical ("spatial") velocity of the satellite observer $\boldsymbol{u}'$ with respect to the local barycentric observer $\boldsymbol{u}$ is given by the modulus of the four-vector $\boldsymbol{\nu}(\boldsymbol{u}', \boldsymbol{u})$ the components of which are given by

$$\nu(\boldsymbol{u}', \boldsymbol{u})^{\alpha} = -\frac{P(u)^{\alpha}{}_{\beta}u'^{\beta}}{u'_{\rho}u^{\rho}} = \frac{1}{\gamma}u'^{\alpha} - u^{\alpha}, \tag{1}$$

where the tensor $P(u)_{\alpha\beta} = g_{\alpha\beta} + u_{\alpha}u_{\beta}$ is the projection operator on the rest space of the local baricentric observer, and $\gamma = -u'_{\rho}u^{\rho}$ is the relative Lorentz factor between $\boldsymbol{u}$ and $\boldsymbol{u}'$ (de Felice & Clarke 1990). Similarly, if we indicate with $\boldsymbol{k}$ the tangent field to the geodesic trajectory of a light signal emitted by a given star, we can define the *line of sight* of the local baricentric observer $\boldsymbol{u}$ at the point of observation as the spatial four-vector $\bar{\boldsymbol{\ell}}_{(0)}$, the components of which are given by

$$\bar{\ell}^{\alpha}_{(0)} \equiv -\frac{P(u)^{\alpha}{}_{\beta}k^{\beta}}{k_{\rho}u^{\rho}} = -\frac{k^{\alpha}}{k_{\rho}u^{\rho}} - u^{\alpha}; \tag{2}$$

note that

$$\bar{\ell}^{\alpha}_{(0)}u_{\alpha} = 0, \qquad \bar{\ell}^{\alpha}_{(0)}\bar{\ell}_{(0)\alpha} = 1. \tag{3}$$

The astrometric observables are represented by the direction cosines between the $\hat{a}$th spatial direction fixed by the satellite attitude vector $\boldsymbol{E}_{\hat{a}}$ and the direction of the light ray in the rest frame of the observer $\boldsymbol{u}'$; these observables are given by

$$\cos\psi_{\hat{a}} \equiv \cos\psi_{(\boldsymbol{u}', \boldsymbol{E}_{\hat{a}}, \bar{\ell}_{(0)})} = \frac{P(u')_{\alpha\beta}(\bar{\ell}^{\alpha}_{(0)} + u^{\alpha})E^{\beta}_{\hat{a}}}{|u'_{\rho}(\bar{\ell}^{\rho}_{(0)} + u^{\rho})|} \tag{4}$$

(de Felice & Clarke 1990; Brumberg 1991), where the tensor $P(u')_{\alpha\beta}$ is the projection operator on the rest space of the satellite. In order to calculate the aberration effect due to the translational motion of the satellite with respect to the local BCRS, we need two angles:

- The angle $\tilde{\psi}_{\hat{a}}$, which the local baricentric observer would measure between the direction of $\boldsymbol{k}$ and the corresponding $\hat{a}$th direction of the triad $\boldsymbol{\lambda}_{\hat{a}}$:

$$\cos\tilde{\psi}_{\hat{a}} \equiv \cos\tilde{\psi}_{(u, \lambda_{\hat{a}}, \bar{\ell}_{(0)})} = \frac{P(u)_{\alpha\beta}(\bar{\ell}^{\alpha}_{(0)} + u^{\alpha})\lambda^{\beta}_{\hat{a}}}{|u_{\rho}(\bar{\ell}^{\rho}_{(0)} + u^{\rho})|} \cdot \tag{5}$$

- The angle $\tilde{\psi}_{\hat{a}(bs)}$, which the boosted observer would measure between the direction of $\boldsymbol{k}$ and the corresponding $\hat{a}$th direction of the triad $\boldsymbol{\lambda}_{\hat{a}(bs)}$:

$$\cos\tilde{\psi}_{\hat{a}(bs)} \equiv \cos\tilde{\psi}_{(\boldsymbol{u}', \lambda_{\hat{a}(bs)}, \bar{\ell}_{(0)})} = \frac{P(u')_{\alpha\beta}(\bar{\ell}^{\alpha}_{(0)} + u^{\alpha})\lambda^{\beta}_{\hat{a}(bs)}}{|u'_{\rho}(\bar{\ell}^{\rho}_{(0)} + u^{\rho})|}. \tag{6}$$

Recalling Eq. (3), from Eq. (5) we easily find that

$$\cos\tilde{\psi}_{\hat{a}} = \bar{\ell}_{(0)\beta}\lambda^{\beta}_{\hat{a}} \cdot \tag{7}$$

As far as Eq. (6) is regarded, first we note that

$$u'_{\alpha}u'_{\beta}(\bar{\ell}^{\alpha}_{(0)} + u^{\alpha}) = \gamma(\nu_{\alpha}\bar{\ell}^{\alpha}_{(0)} - 1)u'_{\beta},$$

as it follows from Eqs. (1) and (3); then we observe that

$$u'_{\alpha}u'_{\beta}(\bar{\ell}^{\alpha}_{(0)} + u^{\alpha})\lambda^{\beta}_{\hat{a}(bs)} = \gamma(\nu_{\alpha}\bar{\ell}^{\alpha}_{(0)} - 1)u'_{\beta}\lambda^{\beta}_{\hat{a}(bs)} = 0,$$

and that

$$u'_{\rho}(\bar{\ell}^{\rho}_{(0)} + u^{\rho}) = \gamma(\nu_{\rho}\bar{\ell}^{\rho}_{(0)} - 1) < 0.$$

Hence we finally obtain from Eq. (6)

$$\cos\tilde{\psi}_{\hat{a}(bs)} = \frac{(\bar{\ell}_{(0)\beta} - \nu_{\beta})\lambda^{\beta}_{\hat{a}(bs)}}{\gamma(1 - \nu_{\rho}\bar{\ell}^{\rho}_{(0)})} \cdot \tag{8}$$

This expression can be rewritten as a function of $\cos\tilde{\psi}_{\hat{a}}$ as given by Eq. (7), thus taking a more suggestive aspect, as we are going to see. To begin with, we note that the triad associated with the boosted observer, determined in Bini et al. (2003), can be rewritten in the following convenient form

$$\lambda^{\alpha}_{\hat{a}(bs)} = \lambda^{\alpha}_{\hat{a}} + \Lambda^{\alpha}_{\hat{a}} \qquad \alpha \in \{t, x, y, z\}, \quad \hat{a} \in \{\hat{1}, \hat{2}, \hat{3}\}, \tag{9}$$

where

$$\lambda^t_{\hat 1} = (h_{01}\cos\phi + h_{02}\sin\phi)\cos\theta + h_{03}\sin\theta, \tag{10}$$

$$\lambda^x_{\hat 1} = \left(1-\frac{h_{00}}{2}\right)\cos\theta\cos\phi, \tag{11}$$

$$\lambda^y_{\hat 1} = \left(1-\frac{h_{00}}{2}\right)\cos\theta\sin\phi, \quad \lambda^z_{\hat 1} = \left(1-\frac{h_{00}}{2}\right)\sin\theta; \tag{12}$$

$$\lambda^t_{\hat 2} = -h_{01}\sin\phi + h_{02}\cos\phi, \quad \lambda^x_{\hat 2} = -\left(1-\frac{h_{00}}{2}\right)\sin\phi, \tag{13}$$

$$\lambda^y_{\hat 2} = \left(1-\frac{h_{00}}{2}\right)\cos\phi, \qquad \lambda^z_{\hat 2} = 0;$$

$$\lambda^t_{\hat 3} = -(h_{01}\cos\phi + h_{02}\sin\phi)\sin\theta + h_{03}\cos\theta, \tag{14}$$

$$\lambda^x_{\hat 3} = -\left(1-\frac{h_{00}}{2}\right)\sin\theta\cos\phi, \tag{15}$$

$$\lambda^y_{\hat 3} = -\left(1-\frac{h_{00}}{2}\right)\sin\theta\sin\phi, \quad \lambda^z_{\hat 3} = \left(1-\frac{h_{00}}{2}\right)\cos\theta \tag{16}$$

is the local BCRS Sun-locked triad, and the boost is provided by

$$\Lambda^\alpha_{\hat a} = \xi^\alpha \zeta_{\hat a} \qquad \alpha\in\{t,x,y,z\}, \quad \hat a \in \{\hat 1, \hat 2, \hat 3\}, \tag{17}$$

where

$$\xi^t = 1 + \frac{\beta^2}{2} + \frac{3h_{00}}{2}, \qquad \xi^i = \frac{\beta^i}{2}, \qquad i\in\{x,y,z\} \tag{18}$$

and

$$\zeta_{\hat 1} = \beta^x\cos\theta\cos\phi + \beta^y\cos\theta\sin\phi + \beta^z\sin\theta, \tag{19}$$

$$\zeta_{\hat 2} = -\beta^x\sin\phi + \beta^y\cos\phi, \tag{20}$$

$$\zeta_{\hat 3} = -(\beta^x\cos\phi + \beta^y\sin\phi)\sin\theta + \beta^z\cos\theta. \tag{21}$$

In the above expressions the β^i are the coordinate components of the three-velocity of the satellite with respect to the local BCRS, and $\beta^2 \equiv \delta_{ij}\beta^i\beta^j$. The angles θ and ϕ are given by

$$\theta = \tan^{-1}\frac{z_\odot - z_0}{\sqrt{(x_\odot - x_0)^2 + (y_\odot - y_0)^2}} \tag{22}$$

$$\phi = \tan^{-1}\frac{y_\odot - y_0}{x_\odot - x_0}, \tag{23}$$

where $\{x_\odot, y_\odot, z_\odot\}$ and $\{x_0, y_0, z_0\}$ are the BCRS coordinates of the Sun and of the satellite, respectively.

Using Eq. (9), and recalling Eq. (7), we now see that Eq. (8) can be rewritten in the following form

$$\cos\tilde\psi_{\hat a(bs)} = \frac{\cos\tilde\psi_{\hat a} + (\bar\ell_{(0)\alpha}\Lambda^\alpha_{\hat a} - \nu_\alpha\lambda^\alpha_{\hat a(bs)})}{\gamma(1-\nu_\rho\bar\ell^\rho_{(0)})}, \tag{24}$$

which, clearly reminiscent of the form characterising the well-known special relativistic formula for the aberration effect, provides the generalization of the latter to the curved spacetime case, with arbitrary direction of the relative motion of the two observers $\boldsymbol{u}$ and $\boldsymbol{u}'$.

The consistency of Eq. (24) can be checked by deducing from it, as a special case, the well-known text-book special relativistic expression for the aberration effect. To this end, we have to neglect all the gravitational potentials and consider the simple case of the two observers $\boldsymbol{u}'$ and $\boldsymbol{u}$ being in inertial relative motion along the coordinate x axis. The general expression of a boosted vector – a vector of the triad in our case – reads

$$\lambda^\alpha_{\hat a(bs)} = P(u')^\alpha{}_\sigma\left(\lambda^\sigma_{\hat a} - \frac{\gamma}{\gamma+1}\nu^\sigma\nu_\rho\lambda^\rho_{\hat a}\right)$$

(Jantzen et al. 1992); hence, setting $u'^\alpha = \gamma(\delta^\alpha_0 + \beta\delta^\alpha_x)$, where $\beta = \beta^x = $ const., $\beta^y = 0 = \beta^z$, which implies from (1) that $\nu^0 = 0$ and $\nu^i = \beta\delta^i_x$, hence that $\nu_\rho\lambda^\rho_{\hat a} = \beta\lambda^x_{\hat a}$, we get

$$\lambda^\alpha_{\hat a(bs)} = \lambda^\alpha_{\hat a} + \beta\gamma\left[u^\alpha + \frac{\gamma}{\gamma+1}\nu^\alpha\right]\lambda^x_{\hat a}. \tag{25}$$

From Eq. (25), recalling Eqs. (7) and (9), and observing that $\nu_\rho\bar\ell^\rho_{(0)} = \beta\cos\tilde\psi_x$, where $\tilde\psi_x$ is the angle the local line-of-sight $\bar\ell_{(0)}$ forms with the coordinate x-direction as seen by the local BCRS observer, the numerator N and the denominator D of Eq. (24) in the simple case under scrutiny become

$$N = \cos\tilde\psi_{\hat a} + \beta\gamma\left(\frac{\beta\gamma}{\gamma+1}\cos\tilde\psi_x - 1\right)\lambda^x_{\hat a},$$

$$D = \gamma(1-\beta\cos\tilde\psi_x).$$

From these espressions, requiring the vectors of the triad $\{\lambda_{\hat a}\}$ to coincide with the local coordinate directions – namely $\lambda^i_{\hat a} = \delta^i_{\hat a}$, $\hat a \in \{x, y, z\}$ – and then considering in Eq. (24) the case $\hat a = x$ (namely, the direction of the relative motion between the two inertial observers we now consider), we finally recover the well-known special relativistic aberration formula

$$\cos\tilde\psi_{x(bs)} = \frac{\cos\tilde\psi_x - \beta}{1-\beta\cos\tilde\psi_x},$$

valid at all orders of β.

3. The aberration angle

Since aberration is an effect caused by the state of motion of the observer with respect to the star, it arises at the event of observation and is therefore accounted for – although implicitly – in the process of fixing the boundary conditions. For RAMOD, aberration is a correction of the observed line of sight due to the motion of the satellite with respect to the local baricentric observer. The angular correction ("aberration angle") due to translational motion is given by $\Delta\tilde\psi_{\hat a} = \tilde\psi_{\hat a(bs)} - \tilde\psi_{\hat a}$, namely

$$\Delta\tilde\psi_{\hat a} = \arcsin\left(\sqrt{1-\cos^2\tilde\psi_{\hat a(bs)}}\cos\tilde\psi_{\hat a} - \sqrt{1-\cos^2\tilde\psi_{\hat a}}\cos\tilde\psi_{\hat a(bs)}\right). \tag{26}$$

In order to obtain the aberration angle directly from the satellite observables, we need to reexpress the director cosines $\cos\tilde\psi_{\hat a(bs)}$ appearing in Eq. (26) in terms of the actually observed data, namely the director cosines $\cos\psi_{\hat a}$ defined in Eq. (4). Resting on the results of Bini et al. (2003), we can derive the link between the non-rotating boosted triad $\boldsymbol{\lambda}_{\hat a(bs)}$ and the triad $\boldsymbol{E}_{\hat a}$ adapted to the satellite observer, namely

$$\lambda_{\hat a(bs)} = M_{\hat a}{}^{\hat b}\boldsymbol{E}_{\hat b}, \tag{27}$$

where the matrix elements $M_{\hat a}{}^{\hat b}$ explicitly read

$$M_{\hat 1}{}^{\hat 1} = \cos\alpha, \tag{28}$$

$$M_{\hat 1}{}^{\hat 2} = -\sin\omega_r t\,\sin\alpha, \tag{29}$$

$$M_{\hat 1}{}^{\hat 3} = -\cos\omega_r t\,\sin\alpha, \tag{30}$$

$$M_{\hat 2}{}^{\hat 1} = -\sin\omega_p t\,\sin\alpha, \tag{31}$$

$$M_{\hat 2}{}^{\hat 2} = \cos\omega_p t\,\cos\omega_r t - \sin\omega_p t\,\sin\omega_r t\,\cos\alpha, \tag{32}$$

$$M_{\hat{2}}{}^{\hat{3}} = -\cos\omega_p t\, \sin\omega_r t - \sin\omega_p t\, \cos\omega_r t\, \cos\alpha, \tag{33}$$

$$M_{\hat{3}}{}^{\hat{1}} = \cos\omega_p t\, \sin\alpha, \tag{34}$$

$$M_{\hat{3}}{}^{\hat{2}} = \sin\omega_p t\, \cos\omega_r t + \cos\omega_p t\, \sin\omega_r t\, \cos\alpha, \tag{35}$$

$$M_{\hat{3}}{}^{\hat{3}} = -\sin\omega_p t\, \sin\omega_r t + \cos\omega_p t\, \cos\omega_r t\, \cos\alpha, \tag{36}$$

in terms of the attitude parameters $\{\alpha, \omega_r, \omega_p\}$ of the satellite. From Eqs. (8) and (27) it therefore follows that

$$\cos\tilde{\psi}_{\hat{a}(bs)} = M_{\hat{a}}{}^{\hat{b}} \cos\psi_{\hat{b}}, \tag{37}$$

in terms of the GAIA observables, defined in Eq. (4). Hence, using Eqs. (7) and (37) with Eq. (26) we can write

$$\Delta\tilde{\psi}_{\hat{a}} = \arcsin\left(\sqrt{1-\left(M_{\hat{a}}{}^{\hat{b}}\cos\psi_{\hat{b}}\right)^2}\,\bar{\ell}_{(0)\beta}\lambda_{\hat{a}}^{\beta} - \sqrt{1-\left(\bar{\ell}_{(0)\beta}\lambda_{\hat{a}}^{\beta}\right)^2}\, M_{\hat{a}}{}^{\hat{b}}\cos\psi_{\hat{b}} \right), \tag{38}$$

where all the quantities appearing in the rhs are known. In fact, the $\cos\psi_{\hat{b}}$ are the observables, the $\bar{\ell}_{(0)\beta}$ are deducible from the observables themselves as shown in de Felice & Preti (2006), the $\lambda_{\hat{a}}^{\beta}$ of Eqs. (10)–(16) are fixed by the metric and the BCRS coordinates of the Sun and of the satellite (Eq. (22)), and the components $M_{\hat{a}}{}^{\hat{b}}$ of Eqs. (28)–(36) are fixed by the attitude parameters of the satellite. Equation (38) therefore provides the analytical tool for computing the stellar aberration angle directly from the GAIA observational data.

4. Conclusions

We showed how the stellar aberration issue can be tackled from the general relativistic point of view, resting on the theoretical framework provided by the Relativistic Astrometric MODel (RAMOD), which was conceived and developed to provide an all-inclusive general relativistic analysis of the inverse ray-tracing problem at the $O(3)$ order of precision, which is the order required for a consistent dealing with the high-precision astrometric data, that will soon be provided by the ESA-GAIA mission. The results presented here can be employed as an independent module that can be attached to the main body of RAMOD, enabling us to single out and quantify the stellar aberration effect directly from the GAIA observational data.

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