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Observational studies of Cepheid amplitudes

I. Period-amplitude relationships for Galactic Cepheids and interrelation of amplitudes⋆

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ABSTRACT

Context. The dependence of amplitude on the pulsation period differs from other Cepheid-related relationships.

Aims. We attempt to revise the period-amplitude (P-A) relationship of Galactic Cepheids based on multi-colour photometric and radial velocity data. Reliable P-A graphs for Galactic Cepheids constructed for the U, B, V, R_C, and I_C photometric bands and pulsational radial velocity variations facilitate investigations of previously poorly studied interrelations between observable amplitudes. The effects of both binarity and metallicity on the observed amplitude, and the dichotomy between short- and long-period Cepheids can both be studied.

Methods. A homogeneous data set was created that contains basic physical and phenomenological properties of 369 Galactic Cepheids. Pulsation periods were revised and amplitudes were determined by the Fourier method. P-A graphs were constructed and an upper envelope to the data points was determined in each graph. Correlations between various amplitudes and amplitude-related parameters were searched for, using Cepheids without known companions.

Results. Large amplitude Cepheids with companions exhibit smaller photometric amplitudes on average than solitary ones, as expected, while s-Cepheids pulsate with an arbitrary (although small) amplitude. The ratio of the observed radial velocity to blue photometric amplitudes, $A_{V_{\rm RAD}}/A_B$, is not as good an indicator of the pulsation mode as predicted theoretically. This may be caused by an incorrect mode assignment to a number of small amplitude Cepheids, which are not necessarily first overtone pulsators. The dependence of the pulsation amplitudes on wavelength is used to identify duplicity of Cepheids. More than twenty stars previously classified as solitary Cepheids are now suspected to have a companion. The ratio of photometric amplitudes observed in various bands confirms the existence of a dichotomy among normal amplitude Cepheids. The limiting period separating short- and long-period Cepheids is 10.47 days.

Conclusions. Interdependences of pulsational amplitudes, the period dependence of the amplitude parameters, and the dichotomy have to be taken into account as constraints in modelling the structure and pulsation of Cepheids. Studies of the P-L relationship must comply with the break at $10^{\rm d}.47$ instead of the currently used "convenient" value of 10 days.

Key words. Cepheids – stars: fundamental parameters – astronomical data bases: miscellaneous

1. Introduction

Cepheid variables are considered to be among the most important stars to both astrophysics and establishment of the cosmic distance scale. Their pulsation period, P, eigenperiod of free radial oscillation (or its overtone) developing in the star, depends on the average density, ρ, of the star, according to the well known formula $P\sqrt{\rho} = Q$, where Q is practically constant for a given type of pulsator (neglecting the slight dependence on stellar mass). The existence of the period–luminosity (P-L) relationship of Cepheids is implied by this formula and because this pulsation is maintained in a narrow, nearly vertical region (referred to as the instability strip) in the Hertzsprung-Russell diagram.

The pure radial pulsation gives rise to a number of other relationships for Cepheids, e.g., between the period and radius, period and colour, and period and age. These relationships are usually expressed as a function of the decimal logarithm of the period. This representation is reasonable because in such a way most relationships are linear. There exists, however, an obvious exception: the dependence of the *pulsational amplitude* on log P is neither linear, nor single valued, i.e., a wide range of amplitudes is possible at a given pulsation period. Even the range of the pulsation amplitudes is a complicated function of the period. The peak-to-peak amplitude of variations during a complete cycle of the pulsation, a characteristic property of a Cepheid, provides important information about the energy of the pulsation, and the pattern of the period-amplitude (P-A) graph plotted for Cepheids is specific to the host galaxy.

Several features of the period dependence of pulsation amplitudes can be explained qualitatively by the physical properties of Cepheids. Longer period Cepheids are more luminous and have a lower surface gravity, therefore they pulsate in general with a larger amplitude. However, longer period Cepheids are of lower effective temperature. The longer the period, the deeper the relevant hydrogen and helium partial ionization zones responsible for driving radial pulsation. At a certain depth,

⋆ Table 1 is only available in electronic form at the CDS via anonymous ftp to cdsarc.u-strasbg.fr (130.79.128.5) or via http://cdsweb.u-strasbg.fr/cgi-bin/qcat?J/A+A/504/959

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convection occurs that dampens the ordered pulsational motion. As a result, the longest period Cepheids pulsate with a smaller amplitude than their shorter period (say 15–30 day) counterparts.

Previous empirical studies have shown that the photometric amplitude is a function of the position of the Cepheid within the instability region: amplitudes are largest near the blue side of the strip, and become gradually smaller toward the red edge (Sandage et al. 2004; Turner et al. 2006a, and references therein). Theoretical calculations modelling the pulsation of Cepheids have been unable to fully reproduce the *P*-*A* diagram delineated by observational data. Model calculations by Szabó et al. (2007), however, confirm theoretically that the spread in photometric amplitude at a given pulsation period is partly caused by differences in the location of the datapoints within the instability strip (see their Fig. 7). The peak-to-peak radial velocity amplitudes, however, have not been studied since Joy (1937).

Cepheid-related relationships are recalibrated from time to time but the strange *P*-*A* relationship is an exception. Ample photometric data obtained in the past few decades are of higher precision than samples used for similar studies 3–4 decades ago; they have facilitating the revision of the dependence of amplitude on the pulsation period and other properties, which is the specific purpose of this paper. Here we study the effect of binarity on the amplitude of pulsation, while the effect of metallicity will be investigated in Paper II.

A newly compiled homogeneous database containing physical and phenomenological properties of Galactic Cepheids is described in Sect. 2. The *P*-*A* relationships based on amplitudes in 5 photometric bands and radial velocity data are discussed in Sect. 3. Section 4 deals with newly introduced amplitude parameters and a discussion of relationships between these amplitudes. The conclusions are drawn in Sect. 5.

2. Data sample

2.1. Content of the database

We collected observational data of 369 Galactic classical Cepheids. From these data, we determined amplitudes of each Cepheid and derived some parameters characterizing the amplitudes. Cepheids with varying pulsational amplitudes were excluded from this study. In addition to more than 20 double-mode radial pulsators, we omitted the star V473 Lyrae, whose pulsation exhibits a modulation of a cycle length as long as 1258 days (Cabanela 1991), and Polaris, whose extremely low and changing amplitudes (Turner et al. 2005) would result in too large uncertainties when forming amplitude ratios. Two Galactic beat Cepheids, however, were used for checking some of our results (see Sect. 4). This sample of Galactic classical Cepheids is otherwise complete to the limit of 10^{m} average brightness in V. Some Cepheids of about 10–11th magnitude in V band were not included because of a lack of photometric data, although fainter Cepheids with known spectroscopic [Fe/H] values and/or reliable radial velocity phase curves do occur in the database.

This data base, published in Table 1, available at the CDS, contains the following information:

Col. 1: name of the Cepheid;
Cols. 2, 3: Galactic longitude and latitude (taken from the *SIMBAD* data base);
Col. 4: pulsation period (in days);
Col. 5: mean apparent brightness in V band;
Cols. 6–10: peak-to-peak amplitudes in U, B, V, R_{C}, and I_{C} bands, respectively;
Col. 11: peak-to-peak amplitude of the radial velocity variations (corrected for the effect of orbital motion in the case of known spectroscopic binaries);
Col. 12: ratio of radial velocity to photometric B amplitudes, q (see Sect. 4.2);
Cols. 13–14: the m parameter (to be defined in Sect. 4.3.1) characteristic of the wavelength dependence of the photometric amplitude and its uncertainty;
Cols. 15–16: the k parameter (to be defined in Sect. 4.3.2) characteristic of the wavelength dependence of the photometric amplitude and its uncertainty;
Col. 17: iron abundance, [Fe/H];
Col. 18: binarity status: 0: no known companion, 1: known binary (or more than one known companion);
Col. 19: mode of pulsation: 0: fundamental mode, 1: first overtone.

Although our data base involves a smaller number of Cepheids than previous ones compiled by Fernie et al. (1995, referred to as the DDO database), Szabados (1997), and Berdnikov et al. (2000), it is homogeneous and contains more information about the stars including [Fe/H] values, information about binarity and pulsational mode. In the case of several Cepheids, some fields have remained blank in Table 1, because of the large uncertainty in a given quantity (radial velocity amplitudes based on early data and photometric amplitudes in the U band) derived from existing observations.

2.2. Source and determination of the tabular data

2.2.1. Pulsation period

Since the pulsation period of Cepheids is affected by changes partly due to stellar evolution, especially in the case of periods longer than 10^{d} (i.e. luminous, therefore rapidly evolving Cepheids), special care was taken to use the true period for the epoch of photometric data from which amplitudes were determined (in general, these period values were effective in the 1990s). Periods were deduced by a Fourier-type periodogram analysis (see Sect. 2.2.2) and rounded to 3 decimal figures in Table 1.

The pulsation period of more than 70 Cepheids in our sample differs from the value given in the GCVS (Samus et al. 2004) to the third decimal place. Evolutionary or other secular period changes result in differences smaller than two thousandth parts of the period with respect to the catalogued value decades ago. However, the period listed in the GCVS deviates considerably from the true value for CU Ori ($1\overset{\mathrm{d}}{.}864$ instead of $2\overset{\mathrm{d}}{.}160$ given in the GCVS), V510 Mon ($7\overset{\mathrm{d}}{.}457$, instead of $7\overset{\mathrm{d}}{.}307$), and CI Per ($3\overset{\mathrm{d}}{.}297$ instead of $3\overset{\mathrm{d}}{.}378$). In these cases, periods given by the GCVS were determined from data covering one or two seasons. We involved all photometric data in the period analysis when determining the true value.

We did not convert the true period of first overtone Cepheids into the corresponding fundamental mode period, in contrast to the approach of Berdnikov et al. (2000). The fundamental period that corresponds to the first overtone periodicity can be calculated using the conversion formula

$$P_1/P_0 = -0.0143 \times \log P_0 - 0.0265 \times [\mathrm{Fe/H}] + 0.7101 \qquad (1)$$

as derived by Sziládi et al. (2007).

2.2.2. Pulsation amplitudes

In some cases, widely differing amplitudes are listed for the same Cepheid in various sources. To avoid using erroneous data, we redetermined the amplitudes from the original observational data. If Berdnikov (2008) and his coworkers had not observed the given Cepheid or if the unfavourable phase coverage resulted in an unacceptable value, other photometric series, obtained mainly by Coulson & Caldwell (1985), Coulson et al. (1985), Gieren (1981, 1985), and Moffett & Barnes (1984) were analysed. Amplitudes taken from Szabados (1997), also derived by Fourier decomposition, are listed for up to two decimal places in Table 1.

Most radial velocity amplitudes were determined from data published following the last update of the tables in the DDO database, the main sources of which were Barnes et al. (2005), Bersier (2002), Bersier et al. (1994), Gorynya et al. (1992, 1996), Groenewegen (2008), Imbert (1999), Kienzle et al. (1999), Kiss (1998), Petterson et al. (2005), Pont et al. (1994), and Derekas (personal communication). More recent data infer a scatter of about 1–2 per cent in the radial velocity phase curve. Earlier radial velocity data obtained by Joy (1937), Stibbs (1955), Feast (1967), and Lloyd Evans (1968, 1980) were taken into account if more recent data were not available for a given Cepheid.

The amplitudes were determined by decomposing the phase curves into Fourier terms using the program package MuFrAn (Kolláth 1990). In the Fourier decomposition, the observed time series was fitted by the sum of sinusoidal terms with frequencies corresponding to the observed pulsation period and its harmonics. In the case of monoperiodic variable stars, the instantaneous brightness value can be written as

$$m(t) = A_0 + \sum_{i=1}^{n} A_i \cos[i\omega(t - t_0) + \phi_i], \quad (2)$$

where t is time counted from an arbitrary t_0 moment, and the coefficients A_i and ϕ_i represent the amplitude and the phase of the corresponding term in the Fourier expansion, respectively, while $\omega = 2\pi/P$, where P is the observed pulsation period.

The shape of the light curve can be described quantitatively by properly defined parameters based on Fourier coefficients. The most useful set of parameters was proposed by Simon & Lee (1981). Following their suggestion and notation, the amplitude ratios, $R_{ij} = A_i/A_j$, and the $\phi_{ij} = j\phi_i - i\phi_j$ phase differences are commonly investigated. In spite of more recent interest in the R_{ij} and ϕ_{ij} Fourier parameters, here we study only the peak-to-peak amplitudes. The behaviour of the R_{ij} and ϕ_{ij} parameters will be studied in a later paper.

When decomposing photometric and radial velocity phase curves, amplitudes were derived from the fundamental period and its first four harmonics but for Cepheids with a complicated light curve shape the fit was extended to two more harmonics. Possible systematic differences between the amplitudes obtained from fits involving different numbers of harmonics were also studied. In the case of well covered phase curves, differences between 5- and 7-harmonic fits turned out to be insignificant. The goodness of the fit is very sensitive to the deviations from the true period. This is why special care was taken to use the value of the period valid for the epoch of observations analysed (see Sect. 2.2.1).

We decided to use data for the R and I bands of the Kron-Cousins system. The linear transformation formulae between the amplitudes in the Johnson and Kron-Cousins systems, based on Cepheids observed in both systems (93 stars in R and 91 stars in I bands), are as follows:

$$A_{R_C} = 1.157(\pm 0.008) \times A_{R_J} \quad (3)$$
$$A_{I_C} = 1.175(\pm 0.012) \times A_{I_J} \quad (4)$$

where A_{R_C} and A_{I_C} are amplitudes in the Kron-Cousins system, and A_{R_J} and A_{I_J} are in the Johnson system.

For Cepheids belonging to spectroscopic binary systems, the amplitude of the radial velocity variations of pulsational origin was determined by removing the orbital effect from the radial velocity data based on the orbital elements available in the online database[1] of binary Cepheids (Szabados 2003b).

2.2.3. Iron abundance

We characterize metallicity in terms of [Fe/H] values. Conventionally, [Fe/H] = log(Fe/H) − log(Fe/H)$_\odot$, is the logarithmic *iron abundance* relative to the Sun (where Fe/H is the ratio of the number of iron atoms to the number of hydrogen atoms in a volume unit of the stellar atmosphere). The sources of [Fe/H] data are: Giridhar (1983), Fry & Carney (1997), Groenewegen et al. (2004), Andrievsky et al. (2002a,b,c), Luck et al. (2003), Andrievsky et al. (2004, 2005), Kovtyukh et al. (2005a,b), Romaniello et al. (2005), Mottini (2006), Yong et al. (2006), and Lemasle et al. (2007).

There are 187 Galactic Cepheids in our catalogue with known spectroscopic [Fe/H] values. Quite a few bright southern Cepheids, some of which are binary systems, (e.g., AX Cir, V636 Sco) were neglected spectroscopically.

Various authors accept different solar chemical compositions. To homogenize the scale of [Fe/H] values, data were shifted to a common solar metallicity, log[n(Fe)] = 7.45 on a scale where log[n(H)] = 12 (Grevesse et al. 2007). Most [Fe/H] data have been taken from Andrievsky and his collaborators' papers (Andrievsky et al. 2002a,b,c; Luck et al. 2003; Andrievsky et al. 2004, 2005; Kovtyukh et al. 2005a,b). Their scale was shifted by 0.05 because they used an earlier [Fe/H] value for the solar chemical composition (Grevesse et al. 1996). The [Fe/H] values obtained by others were transformed to this modified Andrievsky scale based on common Cepheids in the respective projects, Fry & Carney (1997), Lemasle et al. (2007), Mottini (2006), and Romaniello et al. (2005). The transformation of [Fe/H] values obtained by Yong et al. (2006) to the common scale was taken from Luck et al. (2006). The transformation equations are as follows:

$$[\mathrm{Fe/H}]_{\mathrm{And.}} = 0.831(\pm 0.233) \times [\mathrm{Fe/H}]_{\mathrm{Fry}} + 0.053(\pm 0.032), \quad (5)$$
$$[\mathrm{Fe/H}]_{\mathrm{And.}} = 0.838(\pm 0.196) \times [\mathrm{Fe/H}]_{\mathrm{Lem.}} + 0.050(\pm 0.030), \quad (6)$$
$$[\mathrm{Fe/H}]_{\mathrm{And.}} = 0.627(\pm 0.132) \times [\mathrm{Fe/H}]_{\mathrm{Mot.}} + 0.013(\pm 0.014), \quad (7)$$
$$[\mathrm{Fe/H}]_{\mathrm{And.}} = 1.254(\pm 0.291) \times [\mathrm{Fe/H}]_{\mathrm{Rom.}} + 0.076(\pm 0.039), \quad (8)$$

and

$$[\mathrm{Fe/H}]_{\mathrm{And.}} = 0.965(\pm 0.106) \times [\mathrm{Fe/H}]_{\mathrm{Yong}} + 0.175(\pm 0.130). \quad (9)$$

These transformed [Fe/H] values were used only if no data were available from the databases of Andrievsky and his collaborators, to keep the data sample as homogeneous as possible. If Andrievsky et al. published more than one [Fe/H] value for the same Cepheid, priority was given to the most recent value. The metallicity data of Giridhar (1983) could not be transformed because of insufficient common stars.

[1] `http://www.konkoly.hu/CEP/intro.html`

2.2.4. Binarity status

The presence of a companion may affect the photometric amplitudes since an additional constant source of light always reduces the observable amplitude of the brightness variation. The amount of amplitude decrease is a function of the temperature and brightness differences between the Cepheid and its companion(s), and depends also on the photometric band considered. Physical and optical (i.e., line-of-sight) companions are identical in this respect.

The situation is different for radial velocity variations. If a Cepheid belongs to a spectroscopic binary system, orbital and pulsational radial velocity changes are superimposed on each other. The observable radial velocity amplitude is, therefore, larger than the amplitude caused by the pulsational motion alone. If the spectroscopic orbit of the Cepheid is known, the amplitude of pulsational radial velocity variations can be determined by removing the orbital effect from the observed changes in the radial velocity.

Binarity is an important factor when studying the observable amplitudes of Cepheids because more than 50% of Galactic Cepheids belong to binary or multiple systems (Szabados 2003b). A number of Cepheids may have undetected companions because of a selection effect preventing the discovery of duplicity in fainter Cepheids (Szabados 2003c).

Binarity status assigned to individual Cepheids in Table 1 is based on the online database of binary Cepheids[2], also giving references on star-by-star basis.

2.2.5. Pulsation mode

Although the excited mode of the pulsation is a fundamental property of individual Cepheids, there are no infallible methods for its determination. Cepheids in the Magellanic Clouds demonstrate that a separate *P*-*L* relation exists for each pulsation mode (see e.g., Udalski et al. 1999b). Cepheids pulsating in the first overtone are more luminous than fundamental mode oscillators of the same pulsation period, and monoperiodic second overtone Cepheids are even more luminous. Identification of the pulsation mode is necessary to determine the luminosity of a given Galactic Cepheid but contradictory results are often found in the literature.

Phenomenologically, monoperiodic Cepheids can be divided into two groups. The majority of Cepheids have a large amplitude (larger than $0\overset{\mathrm{m}}{.}5$ in the Johnson *V* photometric band) and an asymmetric light curve described by the well known Hertzsprung progression (Hertzsprung 1926). Members of the other group, containing Cepheids of low amplitude (smaller than $0\overset{\mathrm{m}}{.}5$ in Johnson *V* band), are often referred to as s-Cepheids because their light curves are *s*inusoidal, *s*ymmetric, and of *s*mall amplitude.

For another type of radially pulsating stars, the RR Lyrae variables, the pulsation mode can be inferred from the shape and amplitude of the light curve. Variables of RRab subtype (asymmetric light curve of large amplitude) are fundamental mode pulsators, while the small amplitude RR Lyraes with sinusoidal light curves (RRc subtype) pulsate in the first overtone (Castellani et al. 2003, and references therein).

Based on the study of Cepheids in the Large Magellanic Cloud (LMC), Connolly (1980) suggested that small amplitude Cepheids with sinusoidal light curves are first overtone pulsators. Later on, this statement was generalized and, by an analogy with the RRab–RRc dichotomy, it was assumed that s-Cepheids in our Galaxy are also overtone pulsators. Editors of the General Catalogue of Variable Stars (Kholopov 1985) avoid firm statement in this respect: they mention that DCEPS stars (the GCVS type of s-Cepheids) are *possibly* first overtone pulsators and/or cross the instability strip for the first time after evolving off the main sequence.

Nowadays the mode determination is usually based on Fourier decomposition of the light variations. Antonello et al. (1990) found two suitable criteria that can be applied for discriminating s-Cepheids from their normal amplitude siblings. In the R_{21} versus period diagram s-Cepheids form a lower sequence ($R_{21} < 0.2$) below the normal amplitude Cepheids, while in the ϕ_{31} versus period diagram, they form an upper sequence ($\phi_{31} > 3$) above the locus of normal amplitude Cepheids.

This quantitative procedure is usually followed by a step that is unjustified for Cepheids, the assumption that DCEPS stars and Cepheids pulsating in the first overtone mutually correspond to each other. Szabó et al. (2007) discussed how low pulsational amplitudes do not necessarily relate to oscillations in an overtone. Their nonlinear pulsation models indicate that fundamental mode Cepheids of periods longer than 10 days have small amplitude oscillations near both edges of the instability strip. Cepheids in the Magellanic Clouds whose pulsation mode can be identified from the colour-magnitude diagram clearly demonstrate that there are s-Cepheids that oscillate in the fundamental mode and large amplitude Cepheids that exhibit first overtone pulsation (Udalski et al. 1999a,b).

In Table 1, the pulsation modes are taken mostly from the extensive and homogeneous list compiled by Groenewegen & Oudmaijer (2000).

3. Period-amplitude relationships

In his exhaustive paper on the *P*-*A* relationship of Galactic Cepheids, Efremov (1968) described how the period-dependent maximum amplitude is manifested in two local maxima at both $\log P = 0.73$ and 1.4, while the largest possible amplitude drops at $\log P = 0.96$; this drop is the consequence of a resonance between the fundamental eigenmode and its second overtone (Buchler et al. 1990). Long-period Cepheids tend to pulsate with larger amplitude than short period ones, but there is a range of possible amplitudes at each period. A lower boundary appears for normal amplitude Cepheids at $A_B = 0.7$ mag. There is a separate group of small amplitude Cepheids, identical to the s-Cepheids, whose amplitude in the *B* band is smaller than $0\overset{\mathrm{m}}{.}6$. Although Efremov (1968) considered $0\overset{\mathrm{m}}{.}4$ to be the lower limit to the amplitudes of s-Cepheids in the *B* band, discovery of even lower amplitude s-Cepheids (e.g., V636 Cas, BG Cru, V1334 Cyg, V1726 Cyg, V440 Per) demonstrated that there is no lower amplitude limit for the pulsation of such Cepheids.

The new *P*-*A* graphs based on the data listed in Table 1 are shown in Figs. 1a-f for the *U*, *B*, *V*, R_C, and I_C band and radial velocity amplitudes, respectively. In this figure, circles denote short period ($\log P < 1.02$) Cepheids pulsating in the fundamental mode, squares refer to their long period counterparts ($\log P > 1.02$), triangles correspond to first overtone Cepheids, while × symbols are used to represent ambiguous pulsation mode. Empty symbols refer to Cepheids with known companion(s), filled symbols mean that there is no evidence of a companion.

The division between short- and long-period Cepheids is defined to be at $\log P = 1.02$, instead of $\log P = 0.96$ (Efremov 1968), or to be the pulsation period of 10 days adopted

[2] http://www.konkoly.hu/CEP/intro.html

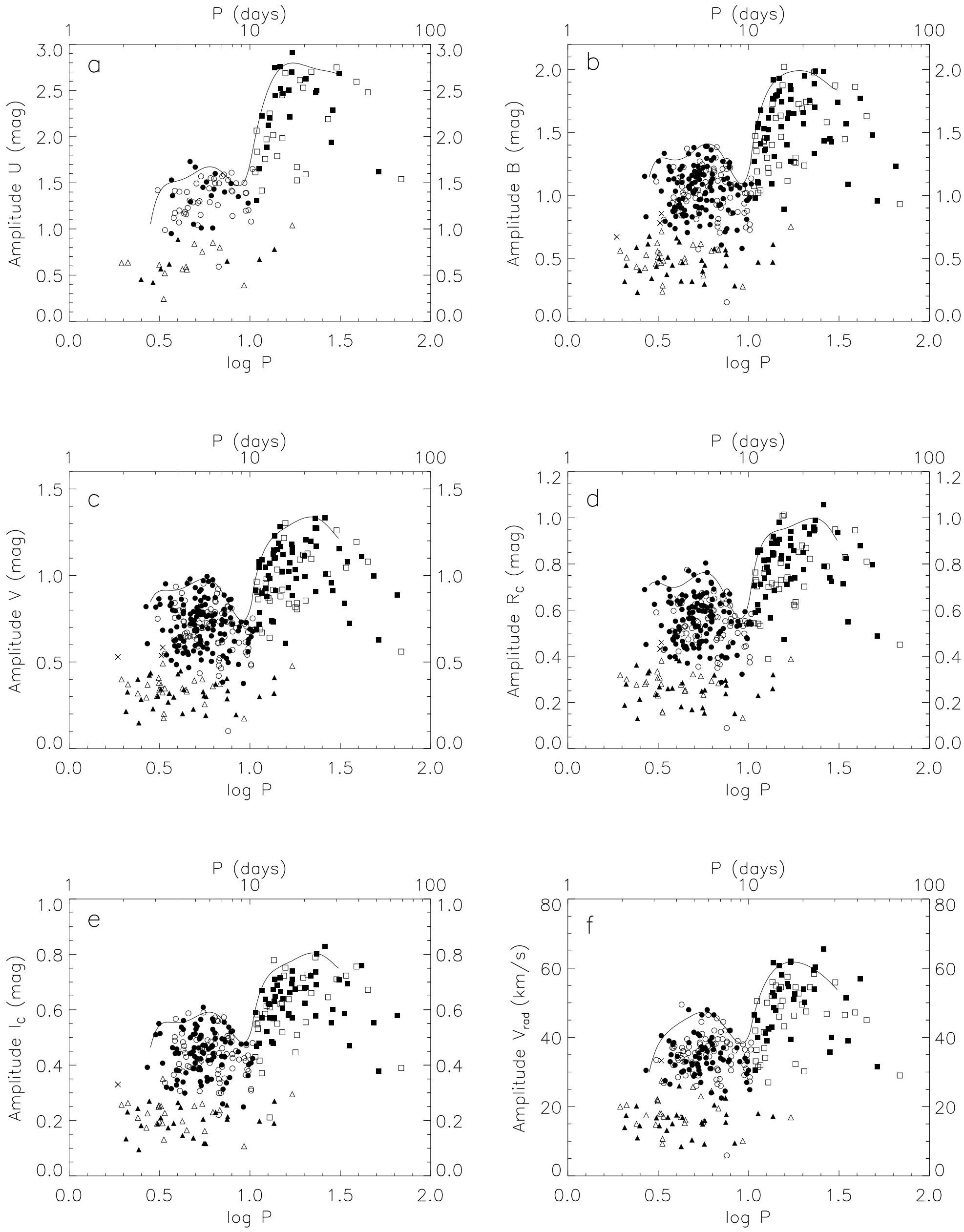

Fig. 1. Period-amplitude diagrams. Amplitudes in U, B, V, $R_{\rm C}$, and $I_{\rm C}$ photometric bands (panels **a**–**e**, respectively), and of radial velocity variations (panel **f**) are plotted. Circles and squares refer to fundamental mode Cepheids pulsating with short ($\log P < 1.02$) and long period ($\log P > 1.02$), respectively, while triangles represent Cepheids pulsating in the first overtone. Filled symbols are used for Cepheids without known companion, empty symbols represent Cepheids belonging to binary (or multiple) systems. × symbols are used if the pulsation mode of the star is ambiguous. The upper envelope (for its construction see the text) is also shown in each plot.

in studies dealing with the break in the *P*-*L* relationship (e.g. Sandage et al. 2009). This choice of a longer period limit (which corresponds to $P = 10^{\rm d}.47$) is supported by Fig. 2. In this diagram, the product of the pulsation period and the radial velocity amplitude is plotted against $\log P$. This product has no direct physical meaning but, due to its dimensions, it is related to the variation in the stellar radius during a pulsational cycle. The different behaviour of short- and long-period Cepheids (and the third group, namely the s-Cepheids) is obvious. The intersection of the two linear sections fitted to the data representing fundamental mode Cepheids indicates that the break occurs at $\log P = 1.02$.

To determine the upper envelope to our *P*-*A* plots, we followed the statistically sound method applied by Eichendorf & Reinhardt (1977) with some modifications. We divided the range of $\log P$ into equal intervals. Each interval has a width of 0.05 in $\log P$. The problem of individual bins containing different numbers of Cepheids can be mitigated by weighting the envelope-points by the number of stars in the given bin.

The largest observed amplitude in each interval was taken as a preliminary envelope point. The corresponding period for each envelope point was calculated to be the mean $\log P$ of Cepheids in the given interval. Unlike Eichendorf & Reinhardt's (1977) procedure, we divided the envelope into two parts and the fits were determined separately. The two parts cover $0.4 < \log P < 1.02$ and $1.02 < \log P < 1.5$ intervals, representing short- and long-period Cepheids, respectively. Both the decrease in the pulsation amplitude near $\log P = 1.0$, and the different behaviour of

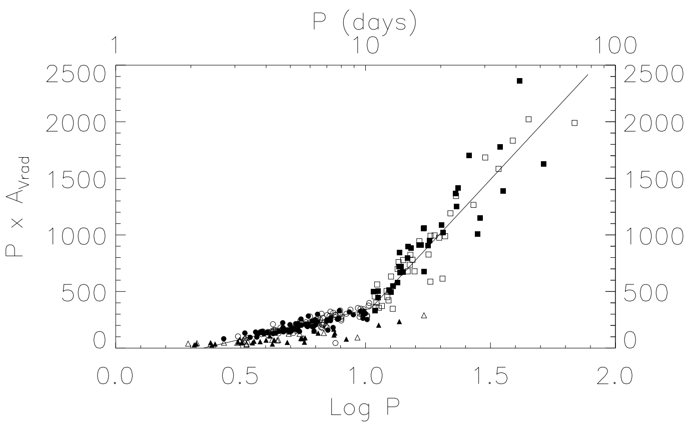

Fig. 2. Different behaviour of short- and long-period Cepheids. Meaning of the symbols is the same as for Fig. 1. The best-fit linear sections intersect at $\log P = 1.02$. s-Cepheids were not involved in the fitting procedure.

short- and long-period Cepheids justify this division. Outside the $0.4 < \log P < 1.5$ interval, the sample contains an insufficient number of Cepheids.

The most realistic upper envelopes were obtained by a least squares fit to the preliminary envelope points for each part in the form of a fifth order polynomial

$$\text{Envelope} = c_0 + \sum_{i=1}^{5} c_i \times (\log P)^i. \qquad (10)$$

Coefficients describing the upper envelopes and their errors for the amplitudes in U, B, V, R_C, I_C bands and for radial velocity amplitudes are listed in Table 2. These upper envelopes represent the largest possible pulsation amplitude at a given period. A few datapoints above the upper envelope in Fig. 1 indicate the uncertainties in the envelope curves. Nevertheless, the fitting could not have been constrained such that the envelope passed through the points of the largest true amplitudes.

All individual graphs in Fig. 1a–f exhibit similar patterns, as far as the shape of the upper envelope and the dichotomy between normal and small amplitude Cepheids are concerned, independently of the wavelength of the photometric band and even for the amplitude of the V_{RAD} variations. The upper envelopes define the largest pulsational amplitudes at $\log P = 0.76$ (in accordance with the value given by Efremov 1968) and at $\log P = 1.30$ (differing from Efremov's corresponding value of 1.4). The minimum amplitude at intermediate periods predicted by the envelope curves occurs at $\log P = 0.96$ in perfect agreement with the value given by Efremov. Nevertheless, we divide the normal amplitude Cepheids into two groups at the period limit of $\log P = 1.02$, in accordance with the dichotomy pointed out earlier in this Section.

Fundamental mode Cepheids belonging to binary systems tend to have smaller photometric amplitudes than their solitary counterparts, but this effect is not discernible in the case of s-Cepheids. Because the majority of companions to Cepheids are blue stars, the brightness difference between the Cepheid and its companion usually decreases towards shorter wavelengths, thus the observable amplitudes in U and B bands are lower than for the V and R bands. Numerical data listed in Table 3, i.e., the average amplitudes for each mode of oscillation and each photometric band studied, and for radial velocity variations, separately for binary and solitary Cepheids, support these statements. The average A_U of Cepheids with companions is about 85% of the corresponding value for solitary Cepheids, and this ratio decreases to between 0.92 and 0.94 for amplitudes in other photometric bands. In contrast, observed pulsational radial velocity amplitudes should not depend on the duplicity status of Cepheids, and this is confirmed by the observed values: the average pulsational radial velocity amplitude of Cepheids with companions differs by only 4% from the corresponding amplitude of solitary Cepheids.

The absence of the photometric effect of companions on the amplitude of first overtone Cepheids is somewhat surprising. Although their sample is smaller, the general behaviour cannot be doubted: binaries pulsate with a larger (by a factor of about 1.1) amplitude than overtone Cepheids without known companions (see Table 3). This behaviour implies that first overtone Cepheids can oscillate with any amplitude smaller than the largest possible value, while Cepheids pulsating in the fundamental mode prefer oscillating with a large amplitude, even though their physical properties do not place them in the middle of the instability strip. A remarkable exception is V440 Persei, which is classified as a fundamental mode Cepheid in spite of its extremely low amplitude (Szabó et al. 2007).

Because companion stars leave the amplitude of radial velocity variations unchanged (having removed the orbital effect), the *P*-*A* diagram constructed for radial velocity data is expected to show a relatively smaller spread than the period – photometric amplitude graph in Fig. 1. In contrast to this expectation, the pattern of the $A_{V_{RAD}}$ versus $\log P$ graph for Galactic Cepheids is largely similar to that of photometric amplitude versus $\log P$ diagrams without a noticeable decrease in the ratio of minimum and maximum amplitudes at a given pulsation period (and treating s-Cepheids separately). This feature can be explained, at least in part, by the lower precision of the radial velocity data and possibly by unrecognised spectroscopic companions. In this latter case, the observable amplitude is larger than the pulsational amplitude. The excess is caused by the contribution from the projected orbital motion superimposed on the amplitude of pulsational origin. An additional cause of the wide range of observed radial velocity amplitudes is the effect of the atmospheric metal content to be discussed in Paper II.

4. Discussion

The observable amplitude of a Cepheid may depend on: the effective temperature of the star, as well as its luminosity (Sandage et al. 2004), which are both related to the position of the star within the instability strip; the atmospheric metallicity (by means of effect on the energy balance of the pulsation); helium content; and the presence of companions. Investigations of the effects of the temperature, luminosity (which correlates with surface gravity in the case of Cepheids), and chemical composition on the oscillation amplitude were beyond the scope of this paper. Here we have concentrated on relations between various amplitudes, including their period dependence. Interrelations of various amplitudes facilitate the identification of binary stars among Cepheids. In view of the photometric effects of companions, known binaries have to be excluded when studying the intrinsic pulsational behaviour of the amplitudes.

Binarity is usually identified by means of spectroscopy. Owing to the regularity of the pulsation, there are also photometric methods for detecting the duplicity of Cepheids. Because of the finite range of amplitudes of solitary Cepheids, small or moderate amplitudes do not necessarily hint at the presence of a companion. Properly selected combinations of photometric amplitudes in different colours, however, can be suitable duplicity

Table 2. Coefficients of the envelope curves (with the formal errors given in parentheses below the respective coefficient).

	c_0	c_1	c_2	c_3	c_4	c_5	c_0	c_1	c_2	c_3	c_4	c_5
	$\log P < 1.02$						$\log P > 1.02$					
U	−126.5	936.3	−2712.3	3882.8	−2742.4	763.8	241.7	−1209.8	2267.4	−2024.2	873.9	−147.3
	(75.6)	(525.7)	(1439.9)	(1941.5)	(1290.6)	(338.7)	(1538.1)	(6254.0)	(10110.7)	(8132.8)	(3251.2)	(517.7)
B	−95.4	726.8	−2151.1	3128.6	−2233.6	625.9	−979.8	3783.1	−5836.3	4503.0	−1736.4	267.5
	(26.2)	(189.8)	(539.1)	(752.3)	(516.3)	(139.5)	(2003.3)	(8383.5)	(13983.4)	(11612.6)	(4796.2)	(789.7)
V	−46.7	364.5	−1099.0	1629.0	−1184.9	337.9	−885.1	3461.5	−5396.1	4194.3	−1624.5	250.7
	(12.3)	(91.2)	(265.1)	(378.6)	(265.7)	(73.3)	(1594.3)	(6656.8)	(11072.6)	(9166.6)	(3778.5)	(620.2)
R_C	−37.8	302.2	−928.7	1396.9	−1027.7	295.7	−750.6	2902.9	−4466.9	3420.9	−1303.2	197.5
	(12.8)	(94.2)	(272.7)	(386.7)	(269.2)	(73.6)	(505.6)	(2050.6)	(3308.8)	(2656.5)	(1060.7)	(168.5)
I_C	−37.6	282.2	−822.3	1179.8	−832.7	231.1	−579.8	2266.9	−3531.1	2741.5	−1060.3	163.4
	(9.0)	(65.9)	(188.2)	(264.3)	(182.4)	(49.5)	(505.5)	(2075.3)	(3390.8)	(2756.8)	(1114.6)	(179.2)
V_{RAD}	−2074	15367	−44551	64191	−45746	12855	−11217	41017	−59974	44053	−16236	2397
	(2083)	(14664)	(40690)	(55672)	(37584)	(10010)	(41150)	(167648)	(271723)	(219110)	(87922)	(14035)

Table 3. Average pulsational amplitudes.

Mode &	A_U	σ_{A_U}	n_U	A_B	σ_{A_B}	n_B	A_V	σ_{A_V}	n_V	A_{R_C}	σ_{A_R}	n_R	A_{I_C}	σ_{A_I}	n_I	$A_{V_{RAD}}$	$\sigma_{A_{V_{RAD}}}$	$n_{V_{RAD}}$
Binarity	(m)			(m)			(m)			(m)			(m)			(km s^{-1})		
Fundamental																		
Solitary	1.882	0.587	40	1.193	0.320	175	0.810	0.205	189	0.635	0.161	176	0.507	0.120	141	39.56	10.19	101
Binary	1.585	0.476	67	1.155	0.308	117	0.772	0.196	120	0.606	0.155	116	0.483	0.122	109	38.01	8.38	118
First overtone																		
Solitary	0.631	0.155	8	0.462	0.120	30	0.314	0.081	30	0.257	0.070	28	0.197	0.051	24	15.76	4.07	26
Binary	0.644	0.202	14	0.500	0.124	25	0.342	0.078	25	0.275	0.063	24	0.221	0.054	23	18.24	4.68	24

indicators. Photometric duplicity tests are based on the amplitudes in various bands and colour indices, and their phase relations (see the summary in Szabados 2003a).

4.1. Ratio of amplitudes

Amplitudes observed in two photometric bands are normally tightly correlated. During the first detailed study of this correlation, van Genderen (1974) noted that the average ratio of A_V/A_B differs for the largest amplitude Cepheids from those pulsating with an amplitude smaller than $1\overset{\mathrm{m}}{.}5$ in the B band. He obtained $A_V = 0.64A_B$ for $A_B > 1\overset{\mathrm{m}}{.}5$, while the complete A_V versus A_B plot could be approximated by a line of slope 0.67. Because only long-period Cepheids can pulsate with $A_B > 1\overset{\mathrm{m}}{.}5$, this dichotomy means a difference between the pulsations of short- and long-period Cepheids.

In the case of classical Cepheids, photometric amplitudes decrease with increasing wavelength: $A_{\lambda_1}/A_{\lambda_2} > 1$ ($\lambda_1 < \lambda_2$). Freedman (1988) determined the average ratios to be $A_B : A_V : A_{R_J} : A_{I_J} = 1.00 : 0.67 : 0.44 : 0.34$ from photoelectric observations of 20 classical Cepheids published by Wisniewski & Johnson (1968). However, 16 Cepheids in that sample belong to binary systems, which obviously falsifies the derived amplitude ratios.

When considering A_U and A_B amplitudes, there are exceptions: the A_U/A_B ratio is about unity for V495 Cyg, V1334 Cyg (a known spectroscopic binary), and V950 Sco, indicating a bright blue companion to these Cepheids. For V950 Sco, this is the first evidence of binarity, while for V495 Cyg this ratio is a further piece of evidence in addition to the large q value mentioned in Sect. 4.2.

The A_V/A_B ratio of V495 Cyg (0.737) also indicates a blue companion, so does the large A_V/A_B value (0.722) of UZ Cas. There are a number of Cepheids whose hot companion has been revealed by ultraviolet spectroscopy (see the online database on Cepheids in binaries and its description by Szabados 2003b). The excessive A_V/A_B ratios of these Cepheids, e.g., RW Cam (0.751), KN Cen (0.723), SU Cyg (0.767), S Mus (0.728), and AW Per (0.714) also confirm the diagnostic value of this amplitude ratio in detecting blue companions.

Figure 3 is a collection of amplitude-amplitude dependences. Panels (a) through to (e) show the A_U, A_B, A_R, A_I, and $A_{V_{RAD}}$ values as a function of A_V, respectively. Because companion stars exert a wavelength dependent influence on the photometric amplitudes, only Cepheids classified as 'solitary' have been taken into account in constructing Fig. 3.

Remarkably, this cleaned sample shows a dichotomic behaviour: long-period Cepheids ($\log P > 1.02$; filled squares) are characterized by a different slope in the A_λ versus A_V relationship than Cepheids pulsating with short periods ($\log P < 1.02$; filled circles). This behaviour is in accordance with the finding of van Genderen (1974) but now the dichotomy in amplitude ratios is generalized to other photometric bands.

The slopes of the linear fits (forced to cross the origin in each panel of Fig. 3) are summarized in Table 4, which also includes the slopes of other relationships not shown in Fig. 3. Column 6 of Table 4 lists the ratio of the slopes obtained for long period over the slope for short period pulsators. Marked as triangles in Fig. 3, s-Cepheids, behave in a similar way to short-period Cepheids.

The ratio of the slopes itself is also wavelength dependent: the larger the wavelength difference of the photometric bands involved, the larger is the difference in the slopes for the long- and short-period Cepheids.

We checked by statistical tests whether fits to short- and long-period Cepheids really have different slopes in the A_λ versus A_V diagram, or a fit with a single straight line or a parabola is more appropriate. The Student's t-test shows

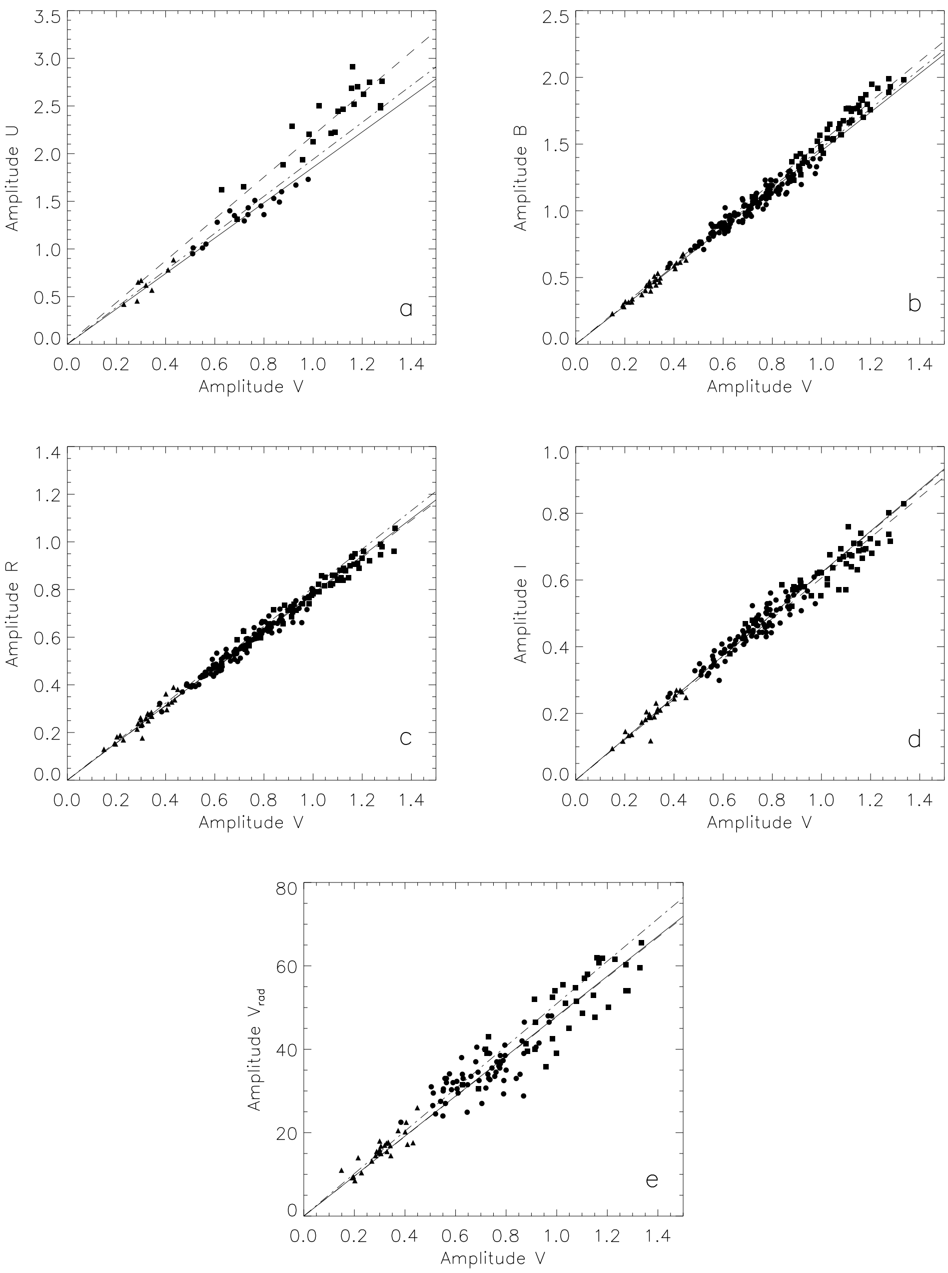

Fig. 3. Dependence of A_U, A_B, A_R, A_I, and $A_{V_{\rm RAD}}$ on A_V. Circles denote short period ($\log P < 1.02$) Cepheids pulsating in the fundamental mode, squares refer to their long-period counterparts ($\log P > 1.02$), triangles represent overtone pulsators. Linear least squares fits are also shown: a dashed line for long-period Cepheids, a solid line for short-period Cepheids, and a dash-dotted line for Cepheids pulsating in the first overtone. These plots only involve Cepheids without known companions.

Table 4. Slopes of the linear relationships between various amplitudes (omitting the known binaries). "Ratio" refers to the rate of slopes in the sense value for long- over value for short-period Cepheids.

Amplitudes involved	Slope	$\sigma_{\rm slope}$	N	Slope	$\sigma_{\rm slope}$	N	Ratio	Slope	$\sigma_{\rm slope}$	N
	Fundamental mode							1st overtone		
	$\log P < 1.02$			$\log P > 1.02$						
$A_U - A_V$	1.856	0.027	18	2.192	0.037	22	*1.182*	1.940	0.045	8
$A_B - A_V$	1.449	0.007	119	1.515	0.008	54	*1.046*	1.471	0.007	30
$A_R - A_V$	0.784	0.003	118	0.779	0.004	56	*0.994*	0.808	0.005	28
$A_I - A_V$	0.622	0.004	90	0.606	0.005	49	*0.974*	0.620	0.003	24
$A_{V_{\rm RAD}} - A_V$	47.93	0.73	64	47.76	0.86	35	*0.996*	50.93	0.90	26
$A_R - A_B$	0.540	0.002	111	0.513	0.004	51	0.950	0.548	0.008	28
$A_I - A_B$	0.424	0.003	82	0.398	0.004	46	0.939	0.423	0.007	24
$A_{V_{\rm RAD}} - A_B$	32.81	0.50	63	31.12	0.59	33	0.948	34.62	0.92	26

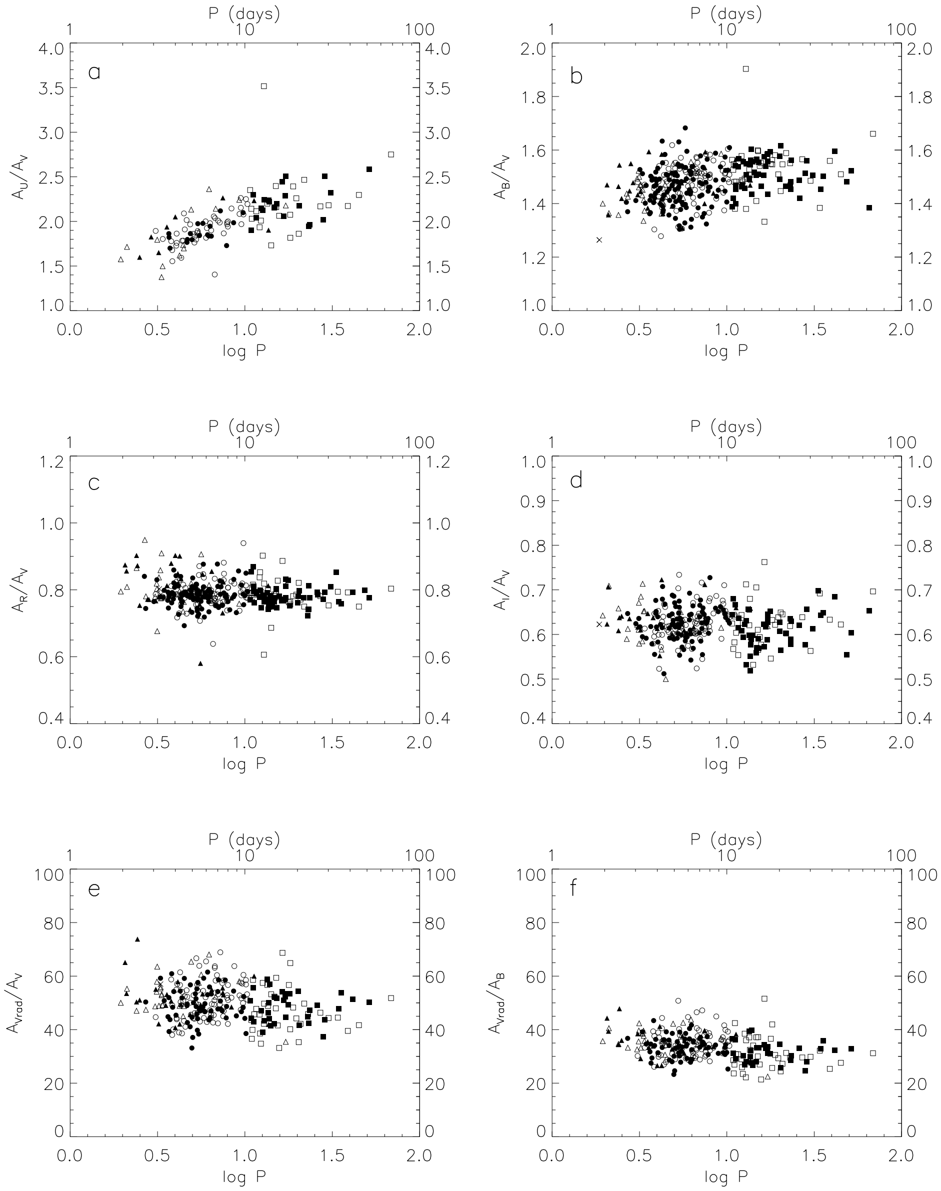

Fig. 4. Ratio of various amplitudes as a function of the pulsation period. The meaning of the symbols is the same as for Fig. 1. The effect of binarity on these amplitude ratios is discussed in the text. The strange outlier in the top of the two uppermost panels corresponds to SU Cru.

whether the means of the two subsamples (short and long period stars) are different. This test was used assuming that the two distributions have the same variance (to be checked with an F-test). If the points correspond to the fit, the means of the subsamples should be the same (null hypothesis).

In our case, the variances were the same with a confidence range of 95%, so we could use the Student's two-sample t-test. Neither in the case of the single straight line, nor of the parabola was the null hypothesis acceptable with a confidence range of 95%, so the means of the subsamples are not the same, and a single linear fit is not consistent with the points.

An extrinsic cause of the different A_B/A_V ratios of Cepheids with short and long pulsation periods might be that the effective wavelength, $\lambda_{\rm eff}$, of the photometric bands depends on the effective temperature, $T_{\rm eff}$, of the observed star: $\lambda_{\rm eff}$ shifts to longer values for cooler stars, which, in turn, causes a stronger decrease in A_B than A_V towards lower $T_{\rm eff}$, i.e., longer period Cepheids. This effect would result in a continuously decreasing A_B/A_V ratio toward longer pulsation periods. The observed effect is, however, just the opposite: the slope of the fitted line is larger for the sample with $\log P > 1.02$ than for the group of short-period Cepheids. The behaviour of the longest period Cepheids also implies that the dichotomy in the amplitude ratio has nothing to do with the convolution of filter transmission with the spectral energy distribution. Both II Car and GY Sge (disregarding long-period Cepheids with companions) are located close to the locus of much shorter period (11–14 days) Cepheids of intermediate amplitudes in the A_B versus A_V diagram, instead of deviating upwards.

It is noteworthy that Coulson & Caldwell (1989) found a linear relationship between A_λ/A_V and $\log P$ that is valid for the U, B, and I photometric bands. Amplitude ratios derived from our database, plotted in Fig. 4, however, indicate that the linear fit is a rough approximation that also contradicts the dichotomic behaviour evident in Fig. 3.

A common feature seen in each panel of Fig. 4 is the considerable scatter in the data points. A part of the scatter can be explained by the observed metallicity dependence of the amplitude ratios that will be discussed in Paper II. Binary companions may also have an adverse effect on various ratios of photometric amplitudes depending on the temperature difference between the Cepheid and its companion. It is noteworthy that a majority of points strongly deviating upwards or downwards correspond to binaries (open symbols in Fig. 4). Some deviating points representing solitary Cepheids may indicate binarity. Spectroscopy and multicolour photometry of these variables (FM Car, BP Cas, V459 Cyg, V924 Cyg, UY Per, V773 Sgr – possible blue companion; CY Car, AY Cen, GI Cyg – possible red companion) are recommended.

Large deviations from the typical values of the A_U/A_V and A_B/A_V amplitude ratios of SU Crucis are unique among Cepheids and cannot be explained by any reasonable companion star, although Coulson & Caldwell (1989) and Laney & Stobie (1993) hypothesized an extremely red companion.

4.2. The $A_{V_{\rm RAD}}/A_B$ amplitude ratio

The ratio of the amplitudes of the radial velocity to photometric variations is an indicator of the pulsation mode, based on purely observational data. The theoretical background behind why this ratio is suitable for differentiating between the modes of pulsation is laid by Balona & Stobie (1979). Based on their linear pulsational model, one expects a 1/0.7 = 1.43 times larger value of amplitude ratio for the first overtone pulsation compared with the fundamental mode oscillation (in a given photometric band) because the period ratio of the two excited modes is about 0.7 (see Eq. (1)).

The opportunity for determining the pulsation mode from the ratio of radial velocity to photometric amplitudes stimulated an in-depth study of the $A_{V_{\rm RAD}}/A_B$ amplitude ratio. The A_B amplitude was chosen because its value is known for most Cepheids and it is larger than in other bands (except U, but A_U values are available for a much smaller sample), therefore relative errors are smaller. In what follows, the value of the $A_{V_{\rm RAD}}/A_B$ amplitude ratio is referred to as q.

Cepheids in the Magellanic Clouds are excellent test objects for checking the validity of theoretically predicted q because their pulsation mode can be identified even in the case of single-mode oscillation. From observational data of 29 fundamental mode (F) and 9 first overtone (1OT) Magellanic Cepheids taken from the McMaster Cepheid Photometry and Radial Velocity Archive (Welch 1998), the ratio of $q_{\rm F}/q_{\rm 1OT} = 0.72 \pm 0.17$ can be obtained. Although the ratio itself corresponds to the expectation, its precision is not satisfactory.

Cepheids oscillating simultaneously in two radial modes provide another test. The ratio of the q values determined for the most well observed beat Cepheids, TU Cas and EW Sct, using the photometric data of Berdnikov (2008) and radial velocity data of Gorynya et al. (1992, 1996) is $q_{\rm F}/q_{\rm 1OT} = 0.747 \pm 0.025$ (based on amplitudes listed in Table 6). When performing Fourier decomposition, linear combinations of the frequencies of the two excited modes ($f_0 + f_1$, $2f_0 + f_1$, $3f_0 + f_1$, $f_1 - f_0$, i.e., the most relevant coupling-terms present in the pulsation of the beat Cepheids) have also been taken into account.

Because companions reduce the observable photometric amplitudes, and an unrecognized orbital motion superimposed on pulsational changes results in an increased $A_{V_{\rm RAD}}$, a larger-than-normal value of $A_{V_{\rm RAD}}/A_B$ may indicate a companion. This effect of companions lessens the diagnostic role of q in assigning the mode of pulsation.

A linear relationship would be expected between the radial velocity and photometric amplitudes from the similar pattern of the respective *P-A* plots (as seen in Fig. 1). In spite of the relatively low measurement errors, values plotted in Fig. 4f have significant scatter: extrema may be 20–30 per cent larger or smaller than the mean value of q at any given period. These deviations from the mean greatly exceed the uncertainty in the determination of amplitudes. The reliability of the q values determined from the observations can be estimated with the help of more than 20 Cepheids for which two values of q could be calculated from independent data series. The average difference between the two independently obtained values is about 4%. Therefore, the relative error in the q derived from well covered photometric and radial velocity phase curves does not exceed ±1.7.

Table 5. Average values of the q amplitude ratio.

Sample	q	σ	N
Fundamental-mode Cepheids (solitary)	32.79	4.01	96
$\log P < 1.0$	33.54	3.95	62
$\log P > 1.0$	31.42	3.79	34
Fundamental-mode Cepheids (binary)	33.66	5.68	115
First overtone Cepheids (solitary)	35.23	4.99	26
First overtone Cepheids (binary)	36.58	4.59	24

Table 6. Amplitudes of double-mode Cepheids TU Cas and EW Sct.

Type of	TU Cassiopeiae			EW Scuti		
Ampl.	$A_{\rm F}$	$A_{\rm 1OT}$	$A_{\rm 1OT}/A_{\rm F}$	$A_{\rm F}$	$A_{\rm 1OT}$	$A_{\rm 1OT}/A_{\rm F}$
A_U	0.92	0.34	0.37	0.54	0.36	0.67
A_B	0.89	0.31	0.35	0.50	0.36	0.72
A_V	0.61	0.21	0.34	0.35	0.25	0.71
A_R	0.43	0.15	0.35	0.23	0.16	0.70
A_I	0.27	0.10	0.37	0.21	0.15	0.71
$A_{V_{\rm RAD}}$	29.1	13.3	0.46	13.5	13.3	0.99

Cepheids pulsating in the first overtone occupy a region overlapping that of fundamental pulsators at short periods, although one would expect their occurrence in distinct regions shifted vertically in panels *e* and *f* of Fig. 4. Omitting known binaries, an average ratio, $(q)_{\rm F}/(q)_{\rm 1OT} = 0.93 \pm 0.21$ is obtained, which deviates from the corresponding value obtained using double-mode and Magellanic Cepheids.

A possible cause of the extraordinary ratio of the q values is that some Cepheids classified as first overtone pulsators oscillate, in fact, in the fundamental mode.

Although binaries tend to have larger q values on average than solitary Cepheids, a larger-than-average value of q does not necessarily imply duplicity of the given variable, keeping in mind the width of the interval of normal q values. This conclusion is confirmed numerically by the data listed in Table 5 indicating marginally larger q values for Cepheids in binaries than for their solitary counterparts.

For a given pulsation mode, an extremely large value of this ratio is a hint that the Cepheid may have a companion. According to this duplicity indicator, fundamental pulsators UZ Cas, VW Cas, BP Cas, CT Cas, and V495 Cyg, and the first overtone Cepheid CR Cep certainly belong to binary systems. Further evidence of the duplicity of UZ Cas and V495 Cyg is their relatively low value of the A_B/A_V amplitude ratio indicating a blue companion (see Sect. 4.1). The binarity of VW Cas, CR Cep, and V495 Cyg is also suspected from their m and/or k parameters (see Sect. 4.3.1 and Sect. 4.3.2). In view of these independent hints, UZ Cas, VW Cas, V495 Cyg, and CR Cep are considered as members of binaries throughout this paper.

The q = 47.8 value of LR TrA indicates that this Cepheid either has a companion or pulsates in the second overtone. Single-mode second-overtone Cepheids are known in the Small

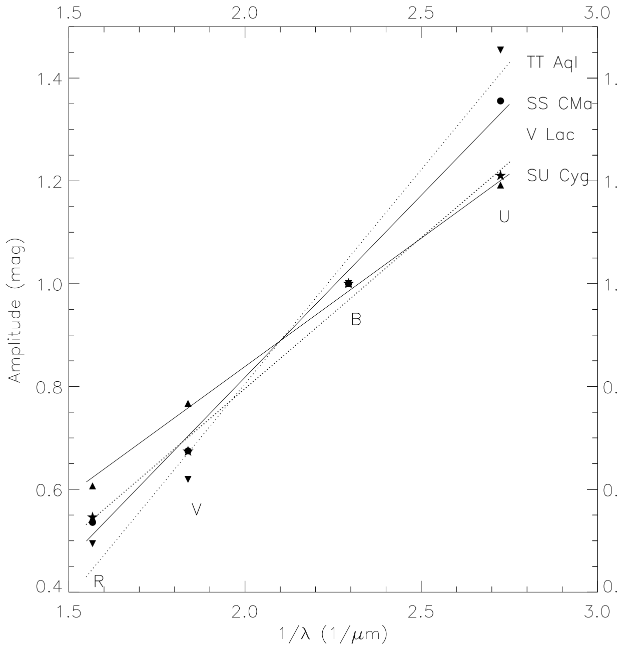

Fig. 5. Dependence of normalized photometric amplitudes on the reciprocal wavelength. The linear fit to the points representing individual Cepheids provides the definition of the m parameter. Two pairs of Cepheids show the usefulness of m: SS CMa (normalized amplitudes are marked by bullets) and SU Cyg (triangles) have blue companions (solid lines), while TT Aql (triangle down) and V Lac (star symbol) are solitary Cepheids (dotted lines) with pulsation period similar to that of SS CMa and SU Cyg, respectively.

Magellanic Cloud (Udalski et al. 1999b) but there is no straightforward method to identify them in our Galaxy.

4.3. Wavelength dependence of amplitudes

It is reasonable to define a single parameter containing information about the wavelength dependence of the individual amplitudes and their ratios. This parameter is more sensitive to the presence of either blue or red companions than ratios of selected pairs of amplitudes.

4.3.1. The *m* parameter

Fernie (1979) conceived the idea of plotting normalized photometric amplitudes versus the reciprocal of the effective wavelength of the given photometric band, which led him to the discovery of a hot companion to SU Cyg.

Amplitudes normalized to A_B as a function of the reciprocal wavelength follow a linear pattern for a given Cepheid if photometric amplitudes observed in U, B, V, and R bands are involved and the wavelength is expressed in μm (Szabados 2000) as

$$A_\lambda / A_B = m \times (1/\lambda) + \text{const.}, \tag{11}$$

where the slope of this line, m, infers that a companion is present, in that a much larger slope than the average value indicates a red companion, while the presence of a blue companion corresponds to a shallower slope. We note that a different slope parameter was used in an earlier paper (Szabados 1993), which was defined to be the slope of the line fitted to the values of the U, B, V, and R amplitudes plotted against the decimal logarithm of the wavelength of the relevant passband.

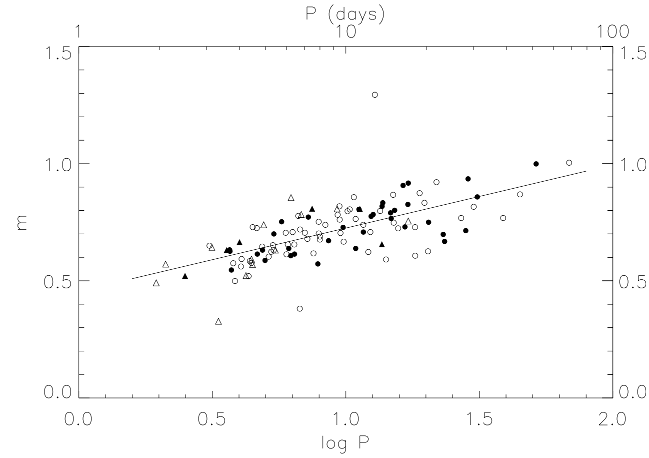

Fig. 6. Plot of the m parameter vs. $\log P$. The meaning of the symbols is the same as in Figs. 1 and 4.

The definition of the m parameter is visualized in Fig. 5 showing normalized amplitudes of four Cepheids: two short-period and two long-period ones. Each pair consists of a solitary Cepheid (V Lac and TT Aql, respectively), while the other Cepheid belongs to a binary system (SU Cyg and SS CMa, respectively) and their pulsation period is pairwise similar. This figure clearly shows that the presence of a blue companion reduces the slope of the fitted line. The m parameter is also period dependent, which is obvious from both Figs. 5 and 6.

The period dependence is related to the temperature dependence of the instability region for various luminosities. The influence of temperature on the value of m is supported by the uniform behaviour of Cepheids pulsating in different (either fundamental or first overtone) modes in the m versus $\log P$ graph (Fig. 6). The linear least squares fit to the 127 data points in this graph (SU Cru has been excluded) provides the relation

$$m = (0.282 \pm 0.027) \times \log P + 0.440 \pm 0.027. \tag{12}$$

Practically the same equation is obtained by taking into account only solitary Cepheids. This infers that, in addition to blue companions identified efficiently by UV-spectroscopy (Evans 1992), red stars also form binaries together with Cepheid variables. Red secondaries are, however, not as luminous as high temperature companions because of the well-known distribution of stars in the H-R diagram. This means that most of these red stars may be line-of-sight optical companions unrelated to the given Cepheid.

The points showing largest deviations from the ridge-line fit in Fig. 6 correspond mostly to known binaries (e.g., SU Cyg, V1334 Cyg, LS Pup). There are, however, outlying points representing Cepheids classified as solitary stars in Table 1. Based on their position in Fig. 6, VY Cyg, VZ Pup, RY Vel, and SW Vel may have a blue companion, while for SZ Aql, X Cyg, and KQ Sco, a red companion is inferred.

4.3.2. The *k* parameter

The advantage of the m parameter is its simple definition. Omission of the I amplitude, an important piece of information, is, however, a drawback. When including the I band amplitude in addition to the four other (U, B, V, and R) amplitudes, the dependence of photometric amplitude on the wave number, $1/\lambda$, is no longer linear. The graph of photometric amplitudes, A_λ, as

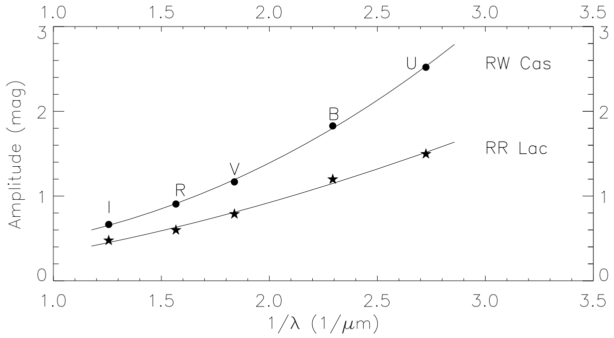

Fig. 7. Dependence of photometric amplitudes on the observational wavelength that defines the k parameter (see text).

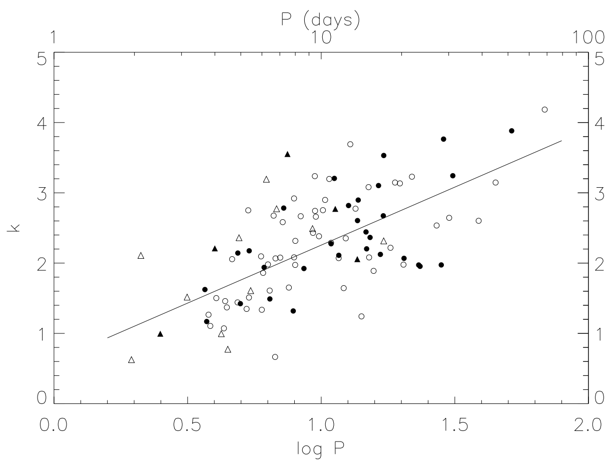

Fig. 8. Plot of k vs. log P. The meaning of the symbols is the same as in Figs. 1 and 4.

a function of the effective wavelength, λ, can be approximated well by a function of the type

$$A_\lambda = m_k \times (1/\lambda)^k + \text{const.} \tag{13}$$

Here the exponent, k, is a useful numerical parameter characterizing the decrease in photometric amplitudes with increasing wavelength. Examples visualizing the effectiveness of fitting this function to the observed amplitudes are shown in Fig. 7, where two solitary Cepheids (the short period RR Lac and the long period RW Cas) serve as templates. It is obvious that the m and k parameters dependent on each other.

The dependence of the k parameter on the decimal logarithm of the pulsation period is shown in Fig. 8. The linear least squares fit to the 114 data points in this graph corresponds to the relation:

$$k = (1.650 \pm 0.173) \times \log P + 0.606 \pm 0.172 \tag{14}$$

we note that k values of the beat Cepheids TU Cas and EW Sct (listed in Table 7) are much lower than the corresponding values determined for single-mode Cepheids pulsating with similar period. As can be seen in Fig. 9, this anomaly is caused mainly by the relatively low U amplitude. This might indicate a blue companion to both TU Cas and EW Sct (without any additional evidence of a companion) or a strange partition in the pulsational energy between the main modes and coupling terms. This latter option has to be studied by hydrodynamic modelling of the pulsating atmosphere.

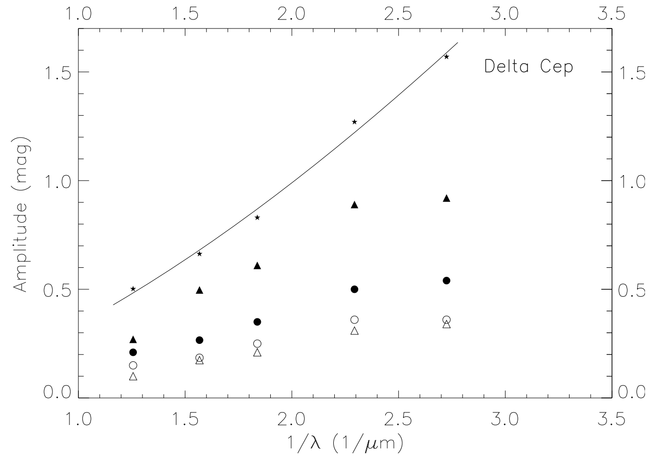

Fig. 9. Strangely low U amplitude of representative double-mode Cepheids. Triangles denote amplitudes of TU Cas, while circles refer to amplitudes of EW Sct. Filled and open symbols represent values for the fundamental mode and first overtone, respectively. Amplitudes of δ Cephei are plotted for comparison purposes.

Table 7. k values for each mode of TU Cas and EW Sct.

Mode	TU Cassiopeiae	EW Scuti
F	$k = 0.038 \pm 0.100$	$k = 0.014 \pm 0.084$
1OT	$k = 0.020 \pm 0.091$	$k = 0.013 \pm 0.112$

The k parameter is also a good indicator of duplicity if the temperature of the companion differs from that of the Cepheid. Most points showing largest deviations from the ridge-line fit in Fig. 8 correspond to known binaries (e.g., SU Cyg, LS Pup). Other outlying points from the fitted line implicitly infer the presence of a companion. The Cepheids classified as solitary stars are VY Cyg, VZ Pup, RY Vel, SW Vel (blue companion is inferred), SZ Aql, V1344 Aql, KQ Sco, and DR Vel (red companion is inferred). Both the m and k parameters of CR Cep and V495 Cyg are indicative of a blue companion, which is consistent with their classification as binaries (see Sect. 4.2).

5. Conclusion

The dependence of amplitudes on period and interdependences of various amplitudes have been investigated based on a homogeneous data set of observed pulsational amplitudes of 369 Galactic Cepheids. Period-amplitude plots were compiled for each of the U, B, V, R, and I photometric amplitudes and for the amplitude of the pulsational radial velocity variations. An upper envelope was determined for each period-amplitude plot. These envelopes enable us to study regularities in the behaviour of the pulsation amplitudes using the subsample of solitary Cepheids (over 200 stars, 60% of the entire sample) since companions falsify observable amplitudes.

The wide range of observable amplitudes at any pulsation period seen in Fig. 1 is only partly caused by companions. As pointed out by Sandage et al. (2004) and Turner et al. (2006a), photometric amplitudes depend on the position of the Cepheid in the instability strip, and this effect causes a spread in amplitudes at a given pulsation period. Nevertheless, companions also contribute to the amplitude range. Cepheids with luminous blue companions (e.g., KN Cen, AX Cir, SY Nor, SV Per) have much smaller photometric amplitudes than the value defined by

Table 8. New indication of binarity from the amplitude behaviour.

Cepheid	q	A_B/A_V	m	k
SZ Aql			+r	+r
V1344 Aql				+
CY Car		+	na	na
FM Car	na	+	na	na
UZ Cas	+	+	na	na
VW Cas	+		+	+
BP Cas	+	+	na	na
CT Cas	+		na	na
AY Cen		+	na	na
CR Cep	+		+	+
X Cyg			+r	
VY Cyg			+	+
GI Cyg	na	+	na	na
V459 Cyg		+	na	na
V495 Cyg	+	+	+	+
V924 Cyg		+	na	na
UY Per		+	na	na
VZ Pup			+	+
V773 Sgr		+		na
KQ Sco			+r	+r
RY Vel		+	+	+
SW Vel			+	+
DR Vel				+r

A + sign means a value indicating a companion. Letter "r" denotes a red companion, otherwise blue companions are inferred (except in the column q). "na" stands for non-applicable because of missing amplitude(s).

the upper envelope at the given pulsation period. Relatively low amplitudes, however, do not necessarily infer binarity.

Cepheids pulsating in the fundamental mode tend to oscillate with an amplitude that does not differ too much from the possible maximum value. However, no such preference is apparent for Cepheids oscillating in the first overtone.

Based on the observed amplitudes, three numerical parameters have been introduced. The period dependence of these parameters, referred to as q, m, and k (see Sects. 4.2, 4.3.1, and 4.3.2, respectively) has been investigated.

Although pulsation theory predicts that q is a good indicator of the mode of pulsation, q values determined from the observed amplitudes are not reliable indicators of the pulsation mode. Exceptionally large q values (for a given mode) are, however, indicative of duplicity of the given Cepheid. The much smaller than predicted separation between the average q values of fundamental mode and first overtone Cepheids may be caused, at least in part, by assigning the incorrect pulsation mode to a number of Cepheids. Fundamental mode pulsators can be present among s-Cepheids (e.g., SZ Cas, AZ Cen, BB Cen, V1726 Cyg, Y Oph) that are usually claimed to be pulsating in the first overtone. This view is shared by Bersier & Burki (1996). Turner et al. (2006b) also doubt that V1726 Cyg pulsates in the first overtone: its carefully determined luminosity is consistent with that of a fundamental mode pulsator. The existence of fundamental mode Cepheids pulsating with small amplitude is also predicted by theoretical computations (Szabó et al. 2007).

The m and k parameters characterizing the wavelength dependence of the observed amplitudes can be used to identify Cepheids with either blue or red companions. Both m and k parameters linearly depend on $\log P$, therefore their true values must be related to the typical values valid for the given pulsation period in duplicity studies.

Quite a few Cepheids classified as solitary stars may have a companion. These Cepheids and the indication of their duplicity by various amplitude parameters are listed in Table 8. The duplicity of VW Cas, CR Cep, and V495 Cyg cannot be doubted based on ample indications. Therefore, a binarity flag 1 has been assigned to these three Cepheids in Table 1. We note, however, that physical relationship between a Cepheid and its companion (i.e., binarity) can be taken as certain if the orbital motion can be identified from radial velocity or astrometric data, or from a strictly periodic light-time effect in the O–C diagram. New spectroscopic studies are recommended for all Cepheids mentioned in Table 8. Other candidate binaries involving a bright Cepheid component (e.g., V950 Sco, LR TrA) not appearing in Table 8 also deserve spectroscopic observations.

A dichotomy was found separating short- and long-period Cepheids (Figs. 2 and 3). This dichotomy was already found for the A_B/A_V amplitude ratio by van Genderen (1974). We have identified similar dichotomies in other amplitude ratios, including $A_{V_{\rm RAD}}/A_V$. This dichotomy was also found in the colour-magnitude diagrams of Cepheids in our Galaxy and the LMC (Sandage et al. 2004), and in the P-L and period-colour relationships for Cepheids in the LMC (Ngeow et al. 2005). A break at $\log P \approx 1$ also appears in the A_V versus $\langle V - I \rangle_0$ diagram of Cepheids in our Galaxy and both Magellanic Clouds (Kanbur & Ngeow 2004, 2006).

We determined the limiting period separating short- and long-period Cepheids to be $10\overset{\rm d}{.}47$ (i.e., $\log P = 1.02$). This period limit is definitely longer than the fundamental period corresponding to the resonance centre for Galactic Cepheids as is obvious from Fig. 1, so the dichotomy and resonance in radial stellar oscillations are unrelated phenomena.

The currently used value of exactly 10 days for the break in the P-L relationship can be accepted only in terms of an anthropomorphic viewpoint. The realistic value of $10\overset{\rm d}{.}47$ will certainly provide more reliable P-L relations.

Finally, we note that when deriving photometric amplitudes, we observed that phase curves of some Galactic s-Cepheids plotted using the correct pulsation period exhibit wider scatter than expected from measurement errors. We suspect that a non-radial mode is weakly excited in these stars, similarly to some Cepheids in the LMC showing such phenomenon (Moskalik & Kołaczkowski 2009).

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