Dependence of sodium laser guide star photon return on the geomagnetic field

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Received 24 November 2008 / Accepted 10 February 2009

ABSTRACT

Aims. The efficiency of optical pumping that increases the backscatter emission of mesospheric sodium atoms in continuous wave (cw) laser guide stars (LGSs) can be significantly reduced and, in the worst case, eliminated by the action of the geomagnetic field. Our goal is to present an estimation of this effect for several telescope sites.

Methods. Sodium atoms precess around magnetic field lines that cycle the magnetic quantum number, reducing the effectiveness of optical pumping. Our method is based on calculating the sodium magnetic sublevel populations in the presence of the geomagnetic field and on experimental measurements of radiance return from sodium LGS conducted at the Starfire optical range (SOR).

Results. We propose a relatively simple semi-empirical formula for estimating the effect of the geomagnetic field on enhancing the LGSs photon return due to optical pumping with a circularly polarized cw single-frequency laser beam. Starting from the good agreement between our calculations and the experimental measurements for the geomagnetic field effect, and in order to more realistically estimate the sodium LGSs photon return, we introduce the effect of the distance to the mesospheric sodium layer and the atmospheric attenuation. The combined effect of these three factors is calculated for several telescope sites.

Conclusions. In calculating the return flux of LGSs, only the best return conditions are often assumed, relying on strong optical pumping with circularly polarized lasers. However, one can only obtain this optimal return along one specific laser orientation on the sky, where the geomagnetic field lines are parallel to the laser beam. For most of the telescopes, the optimum can be obtained at telescope orientations beyond the observation limit. For the telescopes located close to the geomagnetic pole, the benefit of the optical pumping is much more important than for telescopes located close to the geomagnetic equator.

Key words. instrumentation: adaptive optics – atmospheric effects – atomic processes

1. Introduction

Optical pumping of atomic sodium with circularly polarized light can significantly increase the effective absorption cross section compared to non-polarized excitation of the $F = 2$ hyperfine ground state Milonni et al. (1998). The increase in the return light for circularly over linearly polarized light has been reported by Ge et al. (1998) to be 30%, Rabien et al. (2000) find 30–50%. Drummond et al. (2007) measured a return flux increase of factor 2.25 in the starfire optical range (SOR) when switching from linear to circular polarization. The exact return flux gain reached with optical pumping in practice depends strongly on the continuous wave (cw) laser spectral format, the direction of the magnetic field with respect to the direction of the laser beam propagation Denman et al. (2006a), and atomic effects.

In this paper, we extend our previous analysis of the effect of the geomagnetic field on the enhancement of the sodium LGS return caused by the optical pumping, Moussaoui et al. (2008). In this work, we study the reduction of the LGS return flux due to the geomagnetic field, including the effects of airmass and atmospheric absorption. The geomagnetic field intensities and orientations are obtained from the British Geological Survey.

In Sect. 2, we present the laser excitation of the sodium atoms and the role of the optical pumping by circularly polarized light. In Sect. 3, we calculate the redistribution of the atomic population of the magnetic sublevels relevant to optical pumping due to the geomagnetic field. Section 4 presents the relatively

simple formulation that we propose for estimating the net effect of the geomagnetic field. Section 5 finally shows the results of our calculating the combined effects caused by the geomagnetic field, the distance to the mesospheric sodium layer and, the atmospheric attenuation on the relative enhancement of the sodium LGSs photon return flux due to optical pumping by circularly polarized laser beams for several telescopes.

2. Sodium atom excitation

Atomic sodium has a total of 11 electrons with a single valence electron outside closed shells. The complete term symbol of the ground state is $1s^2 2s^2 2p^6 3s^2 S_{1/2}$, and of the first excited state, $1s^2 2s^2 2p^6 3s^2 P_{1/2,3/2}$. The interaction of the magnetic moment of the electron with the magnetic field associated with the orbital motion of the electron leads to the energy level splitting within the first excited state, $3^2P_{1/2}$, $3^2P_{3/2}$. The two states are separated in energy by about 520 GHz. The transitions between the upper state $3^2P_{3/2}$ and the ground state $3^2S_{1/2}$ cause the sodium D₂ emission or absorption at 589.2 nm, while the transitions between the lower state $3^2P_{1/2}$ and the ground state cause the sodium D₁ emission or absorption at 589.6 nm.

The total electronic angular momentum of the ground state and first excited state are $J = 1/2$ and $J = 1/2, 3/2$, respectively. Naturally occurring sodium is composed virtually 100% of one isotope, which has a nuclear spin of $I = 3/2$.

Fig. 1. Schematic diagram of the sodium *D* transitions (from Hillman et al. 2008).

The interaction of an electron with the nuclear magnetic moment leads to the hyperfine structures associated with the ground and excited states of the sodium atom. The total angular momentum quantum number F is the sum of nuclear spin I and the electron spin J yielding the total momentum

$$F = I + J. \quad (1)$$

The resulting total angular momentum quantum numbers are $F = 1, 2$ for the sodium ground state $3^2S_{1/2}$, $F = 1, 2$ for the $3^2P_{1/2}$ excited state, and $F = 0, 1, 2, 3$ for the $3^2P_{3/2}$ excited state. The energy difference between the hyperfine state $F = 2$ and $F = 1$ in the ground state is 1.772 GHz. The energy separation for the hyperfine splitting in the $J = 1/2$ state of the first excited state is 188.6 MHz, while the energy separations of the $J = 3/2$ state are 15.8, 34.4, and 58.3 MHz for the four hyperfine states with $F = 0, 1, 2, 3$, respectively Steck (2008).

The sodium D_2 transitions are chosen for the generation of sodium laser guide stars because they have a factor of two greater total line strength than the D_1 transitions. Furthermore, the D_2 transitions have much better optical pumping characteristics, which results in more efficient excitation of the sodium atoms in the mesospheric sodium layer.

Each of the F levels splits up into $2F + 1$ magnetic quantum levels. For linearly polarized light, only transitions with $\Delta m = 0$ are permitted, whereas for circularly polarized light, Δm must be either $+1$ or -1 . Atoms at a given m level in the upper state can fall back according to the rule $\Delta m = -1, 0, +1$. In the presence of a $\sigma+$ polarized laser beam, for instance, population migrate toward the $F = 2 \leftrightarrow F = 3$ states, and this transition has the largest relative oscillator strength. This well-known optical pumping process can be used to produce an ensemble of two-state atoms, each of which is in either the $3^2S_{1/2}$ ($F = 2, m = 2$) state or the $3^2P_{3/2}$ ($F = 3, m = 3$) state (Milonni et al. 1999; Bradley 1992; Quivers 1986).

If a single-frequency laser, tuned to the exact frequency of the $(2, 2) \rightarrow (3, 3)$ transition, is used to excite the sodium atoms, unfortunately, it is still possible to optically pump the atoms from the $F = 2$ to the $F = 1$ ground state. While atoms traveling approximately orthogonal to the laser beam can only cycle between the $F = 2$ ground state and the $F = 3$ upper state

Fig. 2. Magnetic substates of the hyperfine levels of the sodium D_2 line. Relative oscillator strengths are indicated (after Ungar 1989).

until they change direction by collision, atoms moving in other directions can excite various atomic transitions to the $F = 2$ or $F = 1$ upper state. Once excited to these levels, they can fall back either to the $F = 2$ or $F = 1$ ground state, and after only a few cycles will end up trapped in the $F = 1$ ground state (see Fig. 2).

A magnetic field causes the sodium atoms to precess, leading to a cyclic probability redistribution of finding the atom in a certain m -state. It is important to have a quantitative assessment of the extent to which the geomagnetic field can reduce the degree of optical pumping.

3. Magnetic sublevel populations in the presence of a geomagnetic field

The interaction Hamiltonian for an atom with angular momentum F in a weak magnetic field B is Milonni et al. (1999)

$$H_{\text{mag}} = g_F \mu_B B \cdot F \quad (2)$$

$$g_F = g_J \left[\frac{F(F+1) + J(J+1) - I(I+1)}{2F(F+1)} \right], \quad (3)$$

$$g_{J=3/2}(3^2P_{3/2}) = 4/3,$$

$$g_{J=1/2}(3^2S_{1/2}) = 2$$

where $\mu_B = 9.274 \times 10^{-24} J/T$ is the Bohr magneton, and g_F is the hyperfine Landé factor.

The weak static field B can only cause transitions between states with different m and the same F . The probability $P_{(F,m,m')}(\theta, t) = |\psi_{(F,m,m')}(\theta, t)|^2$ that an atom, initially in state (F, m) , assumes the state (F, m') after time t without any intermediate perturbation depends on the strength of the magnetic field B and the angle θ between the magnetic field vector and the laser beam direction (Eq. (2))

$$\psi_{(F,m,m')}(\theta, t) = \sum_{k=-F}^{k=F} d_{k,m}^{(F)}(\theta) \exp\left(-2\pi i k \left(\frac{t}{\tau} + \frac{1}{2}\right)\right) d_{k,m'}^{(F)}(-\theta) \quad (4)$$

where $\tau = \frac{2\pi\hbar}{(g_F \mu_B B)}$ is the Larmor precession period (Fig. 3), $d_{k,m}^{(F)}(\theta)$ are rotation matrix elements, Gottfried (1977), and $i = \sqrt{-1}$. Each atomic state (F, m) can be written as a linear combination of the $2F + 1$ different m -states at time t . Equation (4) is a

Fig. 3. Larmor precession of the mesospheric sodium atoms.

Fig. 4. Occupation probabilities of the magnetic sublevels ($F = 2, m' = 2, 1, 0, -1, -2$) as a function of the normalized time $\tilde{t}$ for $\theta = 90^\circ$. At $\tilde{t} = 0$, the atoms are prepared in the ground state magnetic sublevel ($F = 2, m = 2$).

generalization of Morris' (1994) Eq. (23), which is a numerical solution for the simplest nontrivial case ($F = 1, m = 1$), using Milonni's et al. (1999) Eq. (8). We need to know much more about the occupation probabilities of the magnetic sublevels concerned by the optical pumping. In the following we present the evolution of the occupation probabilities of the magnetic sublevels of ground state $3^2S_{1/2}(F = 2)$ and of the magnetic sublevels of the excited state $3^2P_{3/2}(F = 3)$, as functions of t and θ . It is straightforward to use the normalized time $\tilde{t} = t/\tau$. The Larmor precession period equals $\tau = 0.714/(g_F B)$ in μs where B is in Gauss. For a geomagnetic field of 0.23 G, the case of the Very Large Telescope site (Cerro Paranal, northern Chile), we find $\tau_{F=2} = 6.21 \mu s$ and $\tau_{F=3} = 4.66 \mu s$.

Figure 4 presents the probability evolution of the magnetic sublevel $3^2S_{1/2}(F = 2, m = 2)$ population in the presence of a static field of 0.23 G into the other magnetic sublevels $3^2S_{1/2}(F = 2, m' = 1, 0, -1, -2)$ for the case of $\theta = 90^\circ$.

In Fig. 5, we display the probability evolution for the excited state ($F = 3, m = 3$) into the seven m -states $-3, -2, \dots, +3$.

Figures 4 and 5 indicate that the geomagnetic field redistributes the magnetic sublevel populations in a time scale of $\sim 1 \mu s$. In the following we focus on the evolution of the populations of the magnetic sublevels ($F = 2, m = 2$) and ($F = 3, m = 3$). Figure 6 presents the occupation probabilities of

Fig. 5. Occupation probabilities of the magnetic sublevels ($F = 3, m' = 3, 2, 1, 0, -1, -2, -3$) as a function of the normalized time $\tilde{t}$ for $\theta = 90^\circ$. At $\tilde{t} = 0$, the atoms are prepared in the excited state magnetic sublevel ($F = 3, m = 3$).

Fig. 6. Occupation probability of the magnetic sublevels ($F = 2, m = 2$) and ($F = 3, m = 3$) as a function of normalized time $\tilde{t}$ for various values of θ .

respectively ($F = 2, m = 2$) and ($F = 3, m = 3$) as functions of time for various values of θ .

To illustrate the redistribution of the atomic population under the effect of the geomagnetic field, we have calculated the occupation probabilities of the magnetic sublevels ($F = 2, m = 2$) and ($F = 3, m = 3$) as functions of both t and θ . We remind the reader that emission and absorption are ignored in these calculations. The occupation probabilities are presented in Fig. 7.

Fig. 7. Occupation probabilities of the magnetic sublevels ($F = 2, m = 2$) and ($F = 3, m = 3$) as functions of both $\tilde{t}$ and θ .

4. Effect of the geomagnetic field on the LGS photon return

As shown in Sect. 2, the optical pumping process produces an ensemble of two-state atoms, each of which is in either the $3^2S_{1/2}$ ($F = 2, m = 2$) state or the $3^2P_{3/2}$ ($F = 3, m = 3$) state. The photon return flux of sodium laser guide stars pumped with single-frequency cw circularly polarized laser beam is proportional to the atomic population of the upper magnetic sublevel. The atomic population of the lower magnetic sublevel $3^2S_{1/2}$ ($F = 2, m = 2$) constitutes the reservoir from which the laser beam populates the upper magnetic sublevel $3^2P_{3/2}$ ($F = 3, m = 3$). We can see from Fig. 7 that the occupation probabilities of both the lower magnetic sublevel and the upper magnetic sublevel depend on the angle between the laser beam propagation direction and the geomagnetic field lines. Because of the very short lifetime of the sodium excited state $3^2P_{3/2}$ of $\tau = 16.25$ ns compared to the time in which the geomagnetic field starts to redistribute the population of the magnetic sublevels ($\sim 1 \mu s$), the action of the geomagnetic field is much more important on the sodium ground state $3^2S_{1/2}$ than on the excited state, for which the spontaneous emission is faster. Nevertheless the geomagnetic field affects the population of the upper magnetic sublevel by affecting its reservoir (the population of the lower magnetic sublevel).

It is intuitive that the return flux of a sodium atom that is pumped with circularly polarized light and still derives most of its fluorescence from the $3^2P_{3/2}$ ($F = 3, m = 3$) $\rightarrow$ $3^2S_{1/2}$ ($F = 2, m = 2$) transition depends on the occupation probability $P_{(F=2,m=2)}$ of the ground state. The effect of the geomagnetic field on the enhancement of the sodium laser guide star photon return due to optical pumping and temporarily neglecting all other factors could be represented by a geomagnetic field factor f_{mag} . From the above considerations about the geomagnetic field effect on the atomic populations, and starting from

Fig. 8. Geomagnetic field declinations from the World Magnetic Model (WMM2005). Credit: British Geological Survey (Natural Environment Research Council).

Fig. 9. Geomagnetic field inclinations from the World Magnetic Model (WMM2005). Credit: British Geological Survey (Natural Environment Research Council).

our previous work Moussaoui et al. (2008) based on the model proposed by Drummond et al. (2007), we propose the following semi-empirical formula for f_{mag} , Eq. (5). This relatively simple formulation is based on calculation of the atomic sodium population of the lower magnetic sublevel and on the observations of the geomagnetic field effect on the sodium LGS return at SOR, (Denman et al. 2006a, 2004)

$$f_{\text{mag}} = aP_{(F=2,m=2)}(\theta, t) + b \quad (5)$$

where a and b are fitting parameters chosen according to the experimental measurements and $P_{(F=2,m=2)}$ is the occupation probability of the magnetic sublevel ($F = 2, m = 2$). This probability oscillates at the Larmor frequency. If the atom is in the ($F = 2, m = 2$) state at a given time, the probability of finding it again in ($F = 2, m = 2$) state after time t has elapsed, taking into account only the Larmor precession has been calculated using Eq. (4).

$$P_{(F=2,m=m'=2)}(\theta, \tilde{t}) = \frac{1}{16^2} \left[\left\{ (1 + \cos \theta)^4 + (1 - \cos \theta)^4 \right\} \cos(4\pi\tilde{t}) + \frac{1}{4}(\sin \theta)^2 \right]^2 + \frac{1}{16^2} \left[\left\{ (1 - \cos \theta)^4 - (1 + \cos \theta)^4 \right\} \sin(4\pi\tilde{t}) + \frac{1}{4}(\sin \theta)^2 \left\{ (1 - \cos \theta)^2 - (1 + \cos \theta)^2 \right\} \sin(2\pi\tilde{t}) \right]^2 \quad (6)$$

To derive the coefficients of Eq. (5) we calculate the effect of the geomagnetic field on the enhancement of the sodium laser

Fig. 10. a) Polar plot of radiance return from a 30 W, circularly polarized sodium LGS, SOR (Albuquerque, New Mexico, USA) Denman et al. (2006a). The legend indicates the radiance in photon/s/cm². b) Relative radiance enhancement of the sodium LGS due to optical pumping with circularly polarized laser beam in presence of the geomagnetic field. Calculations using Eq. (5) for the SOR.

guide star photon return due to optical pumping with circularly polarized laser beam for the SOR telescope (Albuquerque, New Mexico, USA) (Denman et al. 2006a, 2004).

5. Results

5.1. Effect of the geomagnetic field

The intensity, declination, and inclination of the geomagnetic field depend on the geographic location. The magnetic declination is the angle between true north and the compass needle, where positive values indicate that compass points east of true north, while the inclination is the vertical angle of magnetic field lines against the ground, positive numbers indicate that the compass needle points towards the ground, Figs. 8, 9.

Figure 10a shows experimental measurements of the geomagnetic field's impact on the enhancement of the sodium laser guide star photon return due to optical pumping with a circularly polarized laser beam for the SOR Denman et al. (2006a). The polar diagram represents the sky over the telescope site. The maximum return is observed at the location in the sky where the earth's magnetic field lines are pointing directly at the SOR, hence when the laser beam propagation direction in the mesosphere is parallel with $\mathbf{B}$. The magnetic field has no effect when the LGS is produced with a linearly polarized beam. According to Denman et al. (2006b), these experimental results show that the resultant peak returns are obtained at approximately 198° azimuth and 71° elevation. Starting from this observation, we have used 18° as declination and 71° as inclination instead of the 10° and 62° given by the British Geological Survey website to calculate the relative radiance return for SOR using Eq. (4). Figure 11 shows the results of our calculations for the coefficients $a = 1.25$, $b = 1$ and at normalized time $\tilde{t} = 0.38$.

Fig. 11. Polar plot of the relative radiance enhancement of the sodium LGS due to optical pumping with circularly polarized laser beams in the presence of the geomagnetic field for the VLT telescope, Cerro Paranal (Chile).

Fig. 12. Polar plot of the relative radiance enhancement of the sodium LGS due to optical pumping with circularly polarized laser beams in the presence of the geomagnetic field for the Dome C, Antarctica, telescope.

Figure 10b represents the relative sodium return enhancement of optical pumping with a circularly polarized laser beam in the presence of the geomagnetic field. The red circle indicates the observation limit of the telescope (60° zenith angle). The maximum relative enhancement of the sodium LGSs due to optical pumping with a circularly polarized laser beam is assumed to be 2.25 as measured by Denman et al. (2006b), when switching from linear to circular polarization. No effect of the geomagnetic field on the sodium LGSs produced by linearly polarized laser beams.

We calculate the effect of the geomagnetic field on the enhancement of the circularly polarized sodium laser guide star return for several telescopes. The results of our calculations are presented in the following figures.

We present in Fig. 12, our calculations for the Dome C telescope in Antarctica, for which the angle between the geomagnetic field lines and zenith is less than 10°, and we expect less disruption of the relative radiance enhancement of the sodium LGS due to optical pumping with circularly polarized lasers under the effect of the geomagnetic field compared to the VLT or the SOR telescopes.

5.2. Effect of the geomagnetic field, the airmass and the distance to the Na layer

To model realistic distributions of the sodium LGSs return, we include the effects of the airmass and the atmospheric

Fig. 13. Combined effects of geomagnetic field, the airmass, and the atmospheric attenuation on the relative radiance enhancement of the sodium LGS due to optical pumping with circularly polarized laser beams.

attenuation in our calculations. The specific return flux on the ground per watt of launched laser power is given by

$$F_R(\zeta) = \frac{s_{ce} C_{Na} T_{atmo}^{2 \sec(\zeta)}}{H_{Na}^2 \sec(\zeta)} \quad (7)$$

where s_{ce} is the coupling efficiency of light at 589 nm to the sodium atoms (unit photons $\times$ m²/s/W/atom), C_{Na} is the vertical column density of sodium, $\sec(\zeta)$ the secans of the zenith angle ζ (the airmass), T_{atmo} the single-pass transmission of the

atmosphere at 589 nm at zenith, and H_{Na} the vertical distance from the telescope to the sodium centroid. Note that s_{ce} and thus F_{R} strongly and nonlinearly depend on laser power, spectrum, and pulse format if there is any format.

We now combine Eqs. (5) and (7). To simplify our calculations we consider for all the telescope sites that the coupling efficiency of light at 589 nm to sodium atoms, and the vertical column density of the sodium atoms in the mesosphere C_{Na} are both constant, and we also assume that the vertical distance from the telescope to the sodium centroid H_{Na} is the same for all the telescopes. The single-pass transmission of the atmosphere at 589 nm at zenith T_{atmo} is assumed equal to 0.84, which is slightly worse than the measured value for a photometric night in Paranal (0.89) Patat (2004). The combined effects of the geomagnetic field, the airmass, and the atmospheric attenuation on the enhancement of the sodium laser guide star photon return caused by the optical pumping with circularly polarized laser beams can be represented by the expression

$$f_{\text{comb}} = f_{\text{mag}} F_{\text{R}}(\zeta) \quad (8)$$

where f_{mag} is the geomagnetic field factor defined in Eq. (5), and $F_{\text{R}}(\zeta)$ the specific return flux on the ground per watt of launched laser power. To have a comparative estimate of the combined effects of the geomagnetic field, the airmass and the distance to the mesospheric sodium layer on the enhancement of the LGSs pumped with a single frequency cw laser, we have normalized the factor f_{comb} to unity for the optimum conditions that could be obtained for a launched laser beam simultaneously pointing to zenith and parallel to the geomagnetic field line. We present the results of our calculations in Fig. 13 for several telescope sites around the globe.

6. Conclusion

To estimate the effect of the geomagnetic field on the enhancement of the LGSs photon return, we have proposed a semi-empirical formula for the geomagnetic field factor based on the calculation of the sodium magnetic sublevel population whose coefficients are calibrated using the experimental measurements conducted at the Starfire Optical Range. Starting from the good agreement between our results and the experiments, we extended our calculations to several other astronomical telescopes. Our calculations show that, for most of the telescopes, the maximum relative enhancement of the sodium laser guide stars pumped by circularly polarized laser beam can be obtained at telescope orientations close to the observation limit (60°). For Paranal, the optimum is obtained for an altitude below the observation limit of the VLT telescope.

To gain a more realistic estimate of the sodium LGSs photon return, we introduced the effect of the distance to

mesospheric sodium layer and the effect of the atmospheric attenuation for several telescopes. The results show that, when observing at zenith, the telescope located closer to the geomagnetic pole (Dome C for example) benefits much more from optical pumping than telescopes located close to the geomagnetic equator (Paranal, Cerro Pachon, La Palma, etc.), at least when using a single-frequency cw circularly polarized laser beam.

The intensity of the geomagnetic field varies significantly around the globe. At the VLT, in Paranal, northern Chile, it is quite weak $B = 0.23$ G, and hence optical pumping may be disrupted less severely than for instance at the SOR in Albuquerque, New Mexico (USA) ($B = 0.51$ G), or in Hawaii ($B = 0.35$ G). Experimental results will ultimately be needed to assess the effect of the magnetic field strength. Numerical studies are underway to quantify the return flux further Kibblewhite (2008a). In this context, we acknowledge ongoing helpful interaction with Kibblewhite (2008b).

Acknowledgements. The authors are grateful to anonymous referees for their substantive comments that improved the content and presentation of the paper.

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