Detection of real periodicity in the terrestrial impact crater record: quantity and quality requirements

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Received 22 October 2008 / Accepted 30 January 2009

ABSTRACT

Aims. To determine the quantity and quality requirements for the terrestrial impact crater data which would allow reliable detection of real periodicity.

Methods. Artificial impact crater data samples of different size and accuracy are simulated. Erosion is considered, as well as the effect of the unknown ratio between periodic and aperiodic impacts. The probabilities for detecting real and false periodicities are solved with the Rayleigh test from these simulated data.

Results. Reliable detection of real periodicity is currently impossible – unless all impacts on Earth have been periodic.

Key words. Earth – minor planets, asteroids – Oort cloud – methods: statistical – Galaxy: solar neighborhood – comets: general

1. Introduction

Alvarez & Muller (1984) detected a 28.4 Myr periodicity in the ages of terrestrial impact craters. In the same volume of *Nature*, two theories on the possible astronomical process capable of triggering periodic comet showers from the Oort cloud were presented. Davis et al. (1984) and Whitmire & Jackson (1984) suggested that an unseen solar companion “Nemesis” causes periodic perturbances to the Oort cloud, and Rampino & Stothers (1984) and Schwartz & James (1984) argued for perturbances caused by the oscillation of the Solar System through the Galactic plane. For a quarter of a century, evidence that confirms the existence of real periodicity in these catastrophic events has been published, e.g. by Stothers (1998), Moon et al. (2003), Yabushita (2004), Chang & Moon (2005), Stothers (2006) and Wickramasinghe & Napier (2008). Evidence against such periodicity has also been presented, e.g. by Bailey & Stagg (1988), Grieve et al. (1988), Deutsch & Schaerer (1994), Grieve & Pesonen (1996), Jetsu (1997) and Jetsu & Pelt (2000).

We decided to determine the quantity and quality requirements for the terrestrial impact crater data that would allow reliable detection of real periodicity, if present. We also noted the presence of erosion and different values for the ratio between periodic and aperiodic impacts.

2. Real data

2.1. Detection rate

Several geological processes reduce the detectability of terrestrial impact craters as a function of time. This decrease in detectability was estimated from the crater data in *The Earth Impact Database*¹ maintained by the Planetary and Space Science Centre at the University of New Brunswick. These data

contained $n = 174$ impact structures. The errors for the ages were available for $n = 89$ craters. Simultaneous pairs of events were combined as in Jetsu (1997). As in that particular study, the age $t_i \pm \sigma_i$ [Myr] and the diameter D_i [km] of each crater were then used for selecting the two subsamples of data:

C_2 : $t_i \leq 250$, $\sigma_i \leq 20$, $D \geq 5$ ($n = 33$)

C_3 : $5 \leq t_i \leq 300$, $\sigma_i \leq 20$ ($n = 40$).

These data are given in Table 1. It contains the following information for each crater: name (Col. 1), location (Col. 2), age (Col. 3), diameter (Col. 4), belongs (“Yes”) or does not belong (“No”) to the sub-sample C_2 or C_3 (Cols. 5 and 6), and the cycle number $k_i = 1, 2, \dots, 10$ in C_2 and C_3 (Cols. 7 and 8). For each t_i , this cycle number is $k_i = 1 + \text{INT}[K(t_i - t_1)/(t_n - t_1)]$, where $\text{INT}[x]$ removes the decimal part of its argument x and $K = 10$. In other words, the data in C_2 and C_3 were divided into ten subintervals of equal length $(t_n - t_1)/10$ and the value of k_i indicates the particular subinterval where each t_i belongs. The k_i values were used for calculating the fractional estimates A_k for the detectability of the craters as a function of their age. The results are given in Table 2. First, the number of craters within each cycle k was divided by the total number of craters in samples C_2 and C_3 (Table 2: Cols. 2 and 3). Because these estimates fluctuated considerably, the averages for each pair of consecutive cycles were then calculated (Table 2: Cols. 4 and 5). The final values for the fractional detectability estimates A_k were fixed to the mean of both samples (Table 2: Col. 6). Note that these fractional estimates satisfy the relation $\sum_{k=1}^{K=10} A_k = 1$.

2.2. Age uncertainty

For K cycles of equal length over the whole time span of the data $t_1 \pm \sigma_1, t_2 \pm \sigma_2, \dots, t_n \pm \sigma_n$, the period is $P_K = (t_n - t_1)/K$. Our definition for the average error of these data is

$$\sigma_{P'} = \frac{1}{n} \sum_{i=1}^n \frac{\sigma_i}{P_K}, \quad (1)$$

¹ Planetary and Space Science Centre. The Earth Impact Database, <http://www.unb.ca/passc/ImpactDatabase/> (accessed: 11 Aug. 2008).

Table 1. The most recent terrestrial impact crater data obtained in August 11th, 2008 from *The Earth Impact Database*. The combined simultaneous pairs of impacts are marked with “★”. The following information of these craters is given: name (Col. 1), location (Col. 2), age and its error (Col. 3: $[t_i \pm \sigma_i] = \text{My}$), diameter (Col. 4: $[D_i] = \text{km}$) and the crater belongs (“Yes”) or does not belong (“No”) to the sub-sample C_2 or C_3 (Cols. 5 and 6). The value of k_i is the cycle number in C_2 and C_3 (Cols. 7 and 8).

Crater	Location	$t_i \pm \sigma_i$	D_i	C_2	C_3	k_i in C_2	k_i in C_3
Zhamanshin	Kazakhstan	0.9 ± 0.1	14	Yes	No	1	–
El'gygytgyn	Russian	3.5 ± 0.5	18	Yes	No	1	–
Bigach/Karla★	Kazakhstan/Russia	5.0 ± 0.7	12.8	Yes	Yes	1	1
Steinheim	Germany	15 ± 1	3.8	No	Yes	–	1
Ries	Germany	15.1 ± 0.1	24	Yes	Yes	1	1
Chesapeake Bay	USA	35.5 ± 0.3	90	Yes	Yes	2	2
Popigai	Russia	35.7 ± 0.2	100	Yes	Yes	2	2
Mistastin	Canada	36.4 ± 4.0	28	Yes	Yes	2	2
Wanapitei	Canada	37.2 ± 1.2	7.5	Yes	Yes	2	2
Beyenchime-Salaatin/Logancha★	Russia/Russia	40 ± 14	21.5	Yes	Yes	2	2
Logoisk	Belarus	42.3 ± 1.1	15	Yes	Yes	2	2
Shunak	Kazakhstan	45 ± 10	2.8	No	Yes	–	2
Chiyl/Ragozinka★	Kazakhstan/Russia	46.0 ± 2.1	10.5	Yes	Yes	2	2
Gusev/Kamensk★	Russia/Russia	49.00 ± 0.14	25.2	Yes	Yes	2	2
Montagnais	Canada	50.50 ± 0.76	45	Yes	Yes	3	2
Marquez	USA	58 ± 2	12.7	Yes	Yes	3	2
Chicxulub	Mexico	64.98 ± 0.05	170	Yes	Yes	3	3
Boltysh	Ukraine	65.17 ± 0.64	24	Yes	Yes	3	3
Kara	Russia	70.3 ± 2.2	65	Yes	Yes	3	3
Lappajärvi	Finland	73.3 ± 5.3	23	Yes	Yes	3	3
Manson	USA	73.8 ± 0.3	35	Yes	Yes	3	3
Zeleny Gai	Ukraine	80 ± 20	3.5	No	Yes	–	3
Wetumpka	USA	81.0 ± 1.5	6.5	Yes	Yes	4	3
Dellen	Sweden	89.0 ± 2.7	19	Yes	Yes	4	3
Steen River	Canada	91 ± 7	25	Yes	Yes	4	4
Deep Bay	Canada	99 ± 4	13	Yes	Yes	5	4
Carswell	Canada	115 ± 10	39	Yes	Yes	5	4
Rotmistrovka	Ukraine	120 ± 10	2.7	No	Yes	–	5
Mien	Sweden	121.0 ± 2.3	9	Yes	Yes	5	5
Tookoonooka	Australia	128 ± 5	55	Yes	Yes	6	5
Mjølner	Norway	142.0 ± 2.6	40	Yes	Yes	6	5
Gosses Bluff	Australia	142.5 ± 0.8	22	Yes	Yes	6	5
Morokweng	South Africa	145.0 ± 0.8	70	Yes	Yes	6	5
Tabun-Khara-Obo	Mongolia	150 ± 20	1.3	No	Yes	–	6
Zapadnaya	Ukraine	165 ± 5	3.2	No	Yes	–	6
Puchezh-Katunki	Russia	167 ± 3	80	Yes	Yes	7	6
Obolon'	Ukraine	169 ± 7	20	Yes	Yes	7	6
Viewfield	Canada	190 ± 20	2.5	No	Yes	–	7
Rochechouart/Manicouagan★	France/Canada	214.0 ± 0.7	103	Yes	Yes	9	8
Araguainha	Brazil	244.40 ± 3.25	40	Yes	Yes	10	9
Ternovka	Ukraine	280 ± 10	10	No	Yes	–	10
Clearwater East/West★	Canada/Canada	290 ± 14	43.6	No	Yes	–	10

Table 2. The fractional crater detectability estimates calculated from data of Table 1: Col. 1 (k) is the cycle number. Columns 2 (a_1) and 3 (a_2) give the number of craters within cycle k divided by the total number of craters in samples C_2 ($n = 33$) and C_3 ($n = 40$). Columns 4 (a_3) and 5 (a_4) give the averages for pairs of two consecutive cycles in samples C_2 and C_3 . Column 6 ($A_k = (a_3 + a_4)/2$) is the mean of both samples.

k	$C_2 : a_1$	$C_3 : a_2$	$C_2 : a_3$	$C_3 : a_4$	$A_k = (a_3 + a_4)/2$
1	4/33	3/40	6/33	7/40	$A_1 = 0.1784$
2	8/33	11/40	6/33	7/40	$A_2 = 0.1784$
3	7/33	8/40	5/33	5.5/40	$A_3 = 0.1445$
4	3/33	3/40	5/33	5.5/40	$A_4 = 0.1445$
5	3/33	6/40	3.5/33	5/40	$A_5 = 0.1155$
6	4/33	4/40	3.5/33	5/40	$A_6 = 0.1155$
7	2/33	1/40	1/33	1/40	$A_7 = 0.0277$
8	0/33	1/40	1/33	1/40	$A_8 = 0.0277$
9	1/33	1/40	1/33	1.5/40	$A_9 = 0.0339$
10	1/33	2/40	1/33	1.5/40	$A_{10} = 0.0339$

where $\sigma_{P'}$ is measured in the units of P_K . If the subsamples C_2 and C_3 of Table 1 both contained $K = 10$ cycles, the respective errors would be $\sigma_{P'} = 0.11$ and $\sigma_{P'} = 0.17$. In our simulations, it was assumed that the error of the simulated data has a Gaussian distribution

$$f_G(t, \mu, \sigma_P) = \frac{1}{\sigma_P \sqrt{2\pi}} e^{-\frac{1}{2} \left(\frac{t-\mu}{\sigma_P} \right)^2} \quad (2)$$

$$F_G(t, \mu, \sigma_P) = \frac{1}{\sigma_P \sqrt{2\pi}} \int_{-\infty}^t e^{-\frac{1}{2} \left(\frac{t'-\mu}{\sigma_P} \right)^2} dt', \quad (3)$$

where $f_G(t, \mu, \sigma_P)$ is the probability density function, $F_G(t, \mu, \sigma_P)$ is the cumulative distribution function and the mean is $\mu = 0$. The error σ_P for the simulated data is measured in the units of the simulated P . Note that we calculated the error $\sigma_{P'}$ from the *real* data (Eq. (1)), while the error used for the *simulated* data was σ_P (Eqs. (2) and (3)).

2.3. Origin of periodicity

The periodicity detected by [Alvarez & Muller \(1984\)](#) in the terrestrial impact crater record has been suggested to be caused by regular “comet showers” from the Oort cloud towards the inner solar system ([Davis et al. 1984](#); [Whitmire & Jackson 1984](#); [Rampino & Stothers 1984](#); [Schwartz & James 1984](#)). However, direct evidence for such comet showers is lacking. Although [Farley \(1998\)](#) argued for a cometary shower hitting the Earth 35 Myr ago, his findings have been disputed by [Tagle & Claeys \(2004\)](#).

Since 1984, the proposed periodicities for the comet showers have ranged between 20 and 40 Myr. [Hills \(1981\)](#), among others, has shown that a comet survives less than 1 Myr in the inner Solar System. Each aforementioned comet shower could therefore be considered as “an instantaneous event” in the simulation of the periodic part in Sect. 3.1. Finally, we emphasize that no evidence have been presented for *all* comet impacts on Earth having been periodic.

2.4. Origin of aperiodicity

According to [Neukum & Ivanov \(1994\)](#), the impact cratering rate of the Earth-Moon system has been nearly constant for approximately 3 billion years. [Gladman et al. \(1997\)](#) have described how collisions in the main-asteroid-belt can eventually transport asteroids to Earth-crossing orbits. Some fragments of these colliding bodies drift into the gravitational resonance areas of the belt, where their orbits decay into planet-crossing orbits. [Vokrouhlicky & Farinella \(2000\)](#) and [Bottke et al. \(2002a\)](#) have shown how the combination of the Yarkovski effect and the aforementioned collisional dynamics cause a continuous drift of objects from the main-asteroid-belt. Thus the observed Near Earth Object (NEO) population would consist mainly of former main-asteroid-belt objects, but some NEOs might also be ancient comet nuclei. In short, the currently known physical processes indicate that *all* asteroid impacts on Earth have been aperiodic.

3. Simulated data

In this section, we will formulate the simulation of a fully periodic case (Sect. 3.1) and a fully aperiodic case (Sect. 3.2), as well as their combination (Sect. 3.3).

3.1. Periodic component

In the fully periodic case we assumed that periodic comet showers have occurred and simulated $K = 10$ such showers at the epochs $t_k = k - 1/2$, where $k = 1, 2, \dots, K$. It was assumed that the duration of each comet shower is much shorter than the period $P_K = P = 1$ and can be approximated with the Dirac delta function. This gave the probability density and the cumulative distribution functions

$$f_P(t, \sigma_P) = \sum_{k=1}^K A_k f_G(t, t_k, \sigma_P). \quad (4)$$

$$F_P(t, \sigma_P) = \sum_{k=1}^K A_k F_G(t, t_k, \sigma_P). \quad (5)$$

These forms represent the convolution of the Gaussian error (Eqs. (2) and (3)) with a Dirac delta function at each t_k . The multipliers A_k ensured that $\int_{-\infty}^{\infty} f_P(t, \sigma_P) dt = 1$. The subscript “P” in $f_P(t)$ and $F_P(t)$ was adopted to denote the periodic

Fig. 1. The simulated case with $S = 1/3$ and $\sigma_P = 0.1$: **a)** the dashed line denotes the probability density function $f_{AP}(t, S, \sigma_P)$ of Eq. (8). The continuous line shows the aperiodic component $S f_A(t, \sigma_P)$, while the dotted line shows the periodic component $(1-S) f_P(t, \sigma_P)$. **b)** The respective cumulative distribution functions are $F_{AP}(t, S, \sigma_P)$, $S F_A(t, \sigma_P)$ and $(1-S) F_P(t, \sigma_P)$ of Eq. (9).

component in the simulated crater distribution. One example of these two functions $f_P(t, \sigma_P)$ and $F_P(t, \sigma_P)$ is shown in Figs. 1a,b (dotted lines), where $\sigma_P = 0.1$ and both functions have been multiplied with the constant $(1-S) = 2/3$.

3.2. Aperiodic component

In the fully aperiodic case, we first defined the following probability density function

$$f_2(t) = \sum_{k=1}^K A_k f_1(t, a_k, b_k),$$

$$\text{where } a_k = k-1, b_k = k \text{ and } f_1(t, a_k, b_k) = \begin{cases} 0, & t < a_k \\ (b_k - a_k)^{-1}, & a_k \leq t < b_k \\ 0, & t \geq b_k. \end{cases}$$

Again, the A_k values were used to represent the fraction of craters detected within the k th cycle in this aperiodic case

(i.e. uniform distribution within each cycle) and to ensure that $\int_{-\infty}^{\infty} f_A(t, \sigma_P) = 1$. The cumulative distribution function was

$$F_2(t) = \sum_{k=1}^K A_k F_1(t, a_k, b_k),$$

$$\text{where } F_1(t, a_k, b_k) = \begin{cases} 0, & t < a_k \\ (t - a_k)/(b_k - a_k), & a_k \leq t < b_k \\ 1, & t \geq b_k. \end{cases}$$

The convolution of $f_2(t)$ and $F_2(t)$ with the Gaussian error (Eqs. (2) and (3)) gave the following probability density and cumulative distribution functions

$$f_A(t, \sigma_P) = \sum_{k=1}^K A_k [F_G(t, k-1, \sigma_P) - F_G(t, k, \sigma_P)] \quad (6)$$

$$F_A(t, \sigma_P) = \sum_{k=1}^K A_k \{F_G(t, k, \sigma_P) + (t - k + 1) [F_G(t, k-1, \sigma_P) - F_G(t, k, \sigma_P)] + \sigma_P^2 [f_G(t, k-1, \sigma_P) - f_G(t, k, \sigma_P)]\}, \quad (7)$$

where the subscript “A” in $f_A(t)$ and $F_A(t)$ was adopted to denote the aperiodic component. One example of these two functions $f_A(t, \sigma_P)$ and $F_A(t, \sigma_P)$ is shown in Figs. 1a,b (continuous lines), where $\sigma_P = 0.1$ and both functions have been multiplied with the constant $S = 1/3$.

3.3. Combined components

Finally, the combination of the periodic and aperiodic cases was simulated with the functions

$$f_{AP}(t, S, \sigma_P) = S f_A(t, \sigma_P) + (1 - S) f_P(t, \sigma_P) \quad (8)$$

$$F_{AP}(t, S, \sigma_P) = S F_A(t, \sigma_P) + (1 - S) F_P(t, \sigma_P), \quad (9)$$

where the constant $0 \leq S \leq 1$ modified the relative contribution of each component. The subscript “AP” in $f_{AP}(t)$ and $F_{AP}(t)$ was used to denote the combination of periodic and aperiodic cases. These two functions are plotted for the particular case $S = 1/3$ and $\sigma_P = 0.1$ in Figs. 1a,b (dashed lines).

In this paper, we have simulated the cases $S = 0$ (fully periodic), $S = 1/3$ or $2/3$ (partly periodic) and $S = 1$ (fully aperiodic). For all these S values, all combinations of $n = 10, 25, 50, 75, 100$ or 200 and $\sigma_P = 0.05, 0.1, 0.2$ or 0.3 were simulated. The simulation process was simple. First the values of n, σ_P and S were fixed. Then a series of $x_1, x_2, \dots, x_n$ random numbers were generated from a uniform distribution $[0, 1]$. Finally, the cumulative distribution function was used to solve the simulated data values $\bar{t}^* = [t_1, t_2, \dots, t_n]$, which fulfilled the relation $x_i = F_{AP}(t_i, \sigma_P, S)$. Note that these simulated $\bar{t}^*$ data and their error σ_P were both measured in the units of $P = 1$.

4. Rayleigh test

The Rayleigh test is formulated as in [Jetsu & Pelt \(2000\)](#). The phases for the time points $t_1 \leq t_2 \leq \dots \leq t_n$ are

$$\phi_i = \text{FRAC}[(t_i - t_0)P^{-1}] = \text{FRAC}[(t_i - t_0)f],$$

where $\text{FRAC}[x]$ removes the integer part of its argument x , $f = P^{-1}$ is the tested frequency and t_0 is an arbitrary epoch. The “null hypothesis” is

H_0 : “the phases ϕ_i calculated with an arbitrary tested period P have a random distribution between $[0, 1)$ ”.

The Rayleigh test statistic is

$$z(f) = \frac{1}{n} \left[\left(\sum_{i=1}^n \cos \theta_i \right)^2 + \left(\sum_{i=1}^n \sin \theta_i \right)^2 \right], \quad (10)$$

where $\theta_i = 2\pi\phi_i \equiv 2\pi f t_i$ is the phase angle. Note that z does not depend on the chosen t_0 . Under H_0 , the probability density function is $f(z) = e^{-z}$, while the cumulative distribution function is $P(z \leq z_0) = F(z_0) = 1 - e^{-z_0}$ ([Kruger et al. 2002](#)). We performed the Rayleigh test between $f_{\min} = P_{\max}^{-1} = 0.5$ and $f_{\max} = P_{\min}^{-1} = 2.0$, i.e. the periodogram $z(f)$ was calculated within the frequency range $[0.5, 2.0]$. The estimate for the number of independent statistical tests was $m = \text{INT}[(f_{\max} - f_{\min})/f_0]$, where $f_0 = (t_n - t_1)^{-1}$ was the distance between two independent tested frequencies and $\text{INT}[x]$ removed the decimal part of its argument x . The probability that z reached z_0 at least once in all these m independent statistical tests was

$$Q = P(z \geq z_0) = 1 - (1 - e^{-z_0})^m. \quad (11)$$

This parameter Q is hereafter called the *critical level*. The null hypothesis H_0 was rejected if, and only if

$$Q < \gamma, \quad (12)$$

where γ is hereafter called the *preassigned significance level*. This parameter γ expressed the probability of falsely rejecting H_0 when it was in fact true.

The best period $P_{\text{best}} = f_{\text{best}}^{-1}$ for each simulated data sample $\bar{t}^*$ was determined from the highest peak of the periodogram, $z(f_{\text{best}}) = \max[z(f)]$. In all our tests for the simulated data, only the f_{best} values between $f_{\min} + f_0/2$ and $f_{\max} - f_0/2$ were accepted, which prevented the “detection” of possibly false f_{best} values outside the tested range $[f_{\min}, f_{\max}]$.

5. Rayleigh test for the simulated data

The fully aperiodic simulations ($S = 1$) are discussed in Sect. 5.1. The fully periodic ($S = 0$) and the partly periodic ($S = 1/3$ and $2/3$) simulations are discussed in Sect. 5.2.

5.1. Aperiodic cases ($S = 1$)

The statistics of the Rayleigh test were based on the “null hypothesis” H_0 formulated in Sect. 4. There would have been no reason to suspect these statistics if the detectability of the craters had remained constant in our data. Since, however, the detectability of older craters is lower, the statistics based on H_0 are not necessarily valid. To study this question, we performed simulations of fully aperiodic cases ($S = 1$ in Eq. (9)), where the A_k values were used to simulate this detectability effect. The Rayleigh test statistics based on the following four fully aperiodic hypotheses

H_1 : “the cumulative distribution function for the time points t_i is $F_{AP}(t, S = 1, \sigma_P = 0.05)$ ”.

H_2 : “the cumulative distribution function for the time points t_i is $F_{AP}(t, S = 1, \sigma_P = 0.1)$ ”.

H_3 : “the cumulative distribution function for the time points t_i is $F_{AP}(t, S = 1, \sigma_P = 0.2)$ ”.

Table 3. Results of the fully aperiodic simulations for the preassigned significance levels $\gamma = 0.01$ and 0.001 .

n	H_1		H_2		H_3		H_4	
	$F_{AP}(t, S = 1, \sigma_P = 0.05)$		$F_{AP}(t, S = 1, \sigma_P = 0.10)$		$F_{AP}(t, S = 1, \sigma_P = 0.20)$		$F_{AP}(t, S = 1, \sigma_P = 0.30)$	
	$\gamma = 0.01$	$\gamma = 0.001$	$\gamma = 0.01$	$\gamma = 0.001$	$\gamma = 0.01$	$\gamma = 0.001$	$\gamma = 0.01$	$\gamma = 0.001$
	$P(z \geq z_0) < \gamma$	$P(z \geq z_0) < \gamma$	$P(z \geq z_0) < \gamma$	$P(z \geq z_0) < \gamma$	$P(z \geq z_0) < \gamma$	$P(z \geq z_0) < \gamma$	$P(z \geq z_0) < \gamma$	$P(z \geq z_0) < \gamma$
10	$z_0 = 6.88$	$z_0 = 8.12$	$z_0 = 6.84$	$z_0 = 8.12$	$z_0 = 6.84$	$z_0 = 8.12$	$z_0 = 6.82$	$z_0 = 8.02$
25	$z_0 = 7.75$	$z_0 = 9.80$	$z_0 = 7.73$	$z_0 = 9.80$	$z_0 = 7.67$	$z_0 = 9.68$	$z_0 = 7.63$	$z_0 = 9.68$
50	$z_0 = 8.33$	$z_0 = 10.71$	$z_0 = 8.20$	$z_0 = 10.47$	$z_0 = 8.04$	$z_0 = 10.21$	$z_0 = 7.94$	$z_0 = 10.22$
75	$z_0 = 8.59$	$z_0 = 10.98$	$z_0 = 8.48$	$z_0 = 11.01$	$z_0 = 8.18$	$z_0 = 10.41$	$z_0 = 8.04$	$z_0 = 10.38$
100	$z_0 = 8.82$	$z_0 = 11.42$	$z_0 = 8.70$	$z_0 = 11.18$	$z_0 = 8.35$	$z_0 = 10.82$	$z_0 = 8.10$	$z_0 = 10.52$
200	$z_0 = 9.81$	$z_0 = 12.71$	$z_0 = 9.51$	$z_0 = 12.25$	$z_0 = 8.83$	$z_0 = 11.41$	$z_0 = 8.34$	$z_0 = 10.70$

Table 4. The results of the fully periodic ($S = 0$) simulations.

	“Real”	“False”	“Real”	“False”	“Real”	“False”	“Real”	“False”
Case 4A								
t_i simulations with $F_{AP}(t, S = 0, \sigma_P = 0.05)$								
n	H_1 rejected with $\gamma = 0.01$	H_0 rejected with $\gamma = 0.01$	H_1 rejected with $\gamma = 0.001$	H_0 rejected with $\gamma = 0.001$	H_1 rejected with $\gamma = 0.001$	H_0 rejected with $\gamma = 0.001$	H_1 rejected with $\gamma = 0.001$	H_0 rejected with $\gamma = 0.001$
10	1.00	0.00	1.00	0.00	0.993	0.000	0.559	0.000
25	1.00	0	1.00	0	1.000	0	1.000	0
50	1.00	0	1.00	0	1.000	0	1.000	0
75	1.00	0	1.00	0	1.000	0	1.000	0
100	1.00	0	1.00	0	1.000	0	1.000	0
200	1.00	0	1.00	0	1.000	0	1.000	0
Case 4B								
t_i simulations with $F_{AP}(t, S = 0, \sigma_P = 0.1)$								
n	H_2 rejected with $\gamma = 0.01$	H_0 rejected with $\gamma = 0.01$	H_2 rejected with $\gamma = 0.001$	H_0 rejected with $\gamma = 0.001$	H_2 rejected with $\gamma = 0.001$	H_0 rejected with $\gamma = 0.001$	H_2 rejected with $\gamma = 0.001$	H_0 rejected with $\gamma = 0.001$
10	0.70	0.01	0.67	0.01	0.267	0.002	0.030	0.000
25	1.00	0.00	1.00	0.00	1.000	0.000	1.000	0.000
50	1.00	0	1.00	0	1.000	0	1.000	0
75	1.00	0	1.00	0	1.000	0	1.000	0
100	1.00	0	1.00	0	1.000	0	1.000	0
200	1.00	0	1.00	0	1.000	0	1.000	0
Case 4C								
t_i simulations with $F_{AP}(t, S = 0, \sigma_P = 0.2)$								
n	H_3 rejected with $\gamma = 0.01$	H_0 rejected with $\gamma = 0.01$	H_3 rejected with $\gamma = 0.001$	H_0 rejected with $\gamma = 0.001$	H_3 rejected with $\gamma = 0.001$	H_0 rejected with $\gamma = 0.001$	H_3 rejected with $\gamma = 0.001$	H_0 rejected with $\gamma = 0.001$
10	0.03	0.01	0.03	0.01	0.005	0.001	0.000	0.000
25	0.31	0.02	0.44	0.03	0.119	0.003	0.173	0.006
50	0.83	0.01↓	0.92	0.01↓	0.637	0.003	0.767	0.006
75	0.98	0.00↓	0.99	0.00↓	0.935	0.002↓	0.973	0.002↓
100	1.00	0.00↓	1.00	0.00↓	0.992	0.000↓	0.998	0.000↓
200	1.00	0↓	1.00	0↓	1.000	0↓	1.000	0↓
Case 4D								
t_i simulations with $F_{AP}(t, S = 0, \sigma_P = 0.3)$								
n	H_4 rejected with $\gamma = 0.01$	H_0 rejected with $\gamma = 0.01$	H_4 rejected with $\gamma = 0.001$	H_0 rejected with $\gamma = 0.001$	H_4 rejected with $\gamma = 0.001$	H_0 rejected with $\gamma = 0.001$	H_4 rejected with $\gamma = 0.001$	H_0 rejected with $\gamma = 0.001$
10	0.00	0.01	0.00	0.01	0.000	0.001	0.000	0.000
25	0.01	0.01	0.02	0.03	0.001	0.001	0.002	0.003
50	0.02	0.02	0.06	0.05	0.005	0.002	0.013	0.007
75	0.05	0.02	0.11	0.07	0.013	0.003	0.033	0.011
100	0.09	0.03	0.19	0.08	0.027	0.004	0.066	0.015
200	0.35	0.04	0.53	0.09	0.178	0.012	0.314	0.032

H_4 : “the cumulative distribution function for the time points t_i is $F_{AP}(t, S = 1, \sigma_P = 0.3)$ ”.

were simulated for the sample sizes $n = 10, 25, 50, 75, 100$ and 200 . Under each hypothesis H_1, H_2, H_3 or H_4 , we performed the Rayleigh test for $Z = 100\,000$ random samples of $\tilde{t}^* = [t_1, \dots, t_n]$ simulated from the corresponding fully aperiodic $F_{AP}(t, S, \sigma_P)$. The periodogram $z(f)$ was calculated between $f_{\min} = 0.5$ and $f_{\max} = 2.0$ for each of these $\tilde{t}^*$ samples. The highest peak $z_0 = z(f_{\text{best}}) = \max[z(f)]$ of the periodogram for each simulated $\tilde{t}^*$ sample, i.e. the global maximum of the periodogram, was then chosen. Finally, the ranking of all these $Z = 100\,000$ values of z_0 determined the cumulative distribution function $P(z > z_0) = F(z)$, i.e. the probability that z reached a

value equal to, or higher than, z_0 under the hypothesis H_1, H_2, H_3 or H_4 . The results of all these fully aperiodic simulations are compiled into Table 3 for the preassigned significance levels $\gamma = 0.01$ and 0.001 .

5.2. Periodic cases ($S = 0, 1/3$ and $2/3$)

In this section, we determine how often the Rayleigh test detects the “real” ($P = 1$) or the “false” period ($P \neq 1$) in the simulated crater data. The probability for the “reliable” detection of the “real” period is also investigated.

The cases $S = 0$ (fully periodic), $1/3$ (partly periodic) and $2/3$ (partly periodic) were simulated for the sample sizes

Table 5. Simulation results for the partly periodic cases ($S = 1/3$), otherwise as in Table 4.

	“Real”	“False”	“Real”	“False”	“Real”	“False”	“Real”	“False”
Case 5A								
t_i simulations with $F_{AP}(t, S = 1/3, \sigma_P = 0.05)$								
n	H_1 rejected with $\gamma = 0.01$	H_0 rejected with $\gamma = 0.01$	H_1 rejected with $\gamma = 0.001$	H_0 rejected with $\gamma = 0.001$	H_1 rejected with $\gamma = 0.001$	H_0 rejected with $\gamma = 0.001$	H_1 rejected with $\gamma = 0.001$	H_0 rejected with $\gamma = 0.001$
10	0.20	0.01	0.21	0.01	0.104	0.002	0.032	0.000
25	0.79	0.01	0.86	0.01	0.604	0.001	0.663	0.004
50	0.99	0.00	1.00	0.00	0.978	0.000	0.992	0.001↓
75	1.00	0.00	1.00	0.00	1.000	0.000	1.000	0.000↓
100	1.00	0	1.00	0	1.000	0	1.000	0↓
200	1.00	0	1.00	0	1.000	0	1.000	0↓
Case 5B								
t_i simulations with $F_{AP}(t, S = 1/3, \sigma_P = 0.1)$								
n	H_2 rejected with $\gamma = 0.01$	H_0 rejected with $\gamma = 0.01$	H_2 rejected with $\gamma = 0.001$	H_0 rejected with $\gamma = 0.001$	H_2 rejected with $\gamma = 0.001$	H_0 rejected with $\gamma = 0.001$	H_2 rejected with $\gamma = 0.001$	H_0 rejected with $\gamma = 0.001$
10	0.10	0.01	0.10	0.01	0.027	0.002	0.003	0.000
25	0.58	0.01	0.68	0.02	0.341	0.003	0.408	0.005
50	0.97	0.00	0.99	0.00↓	0.893	0.001↓	0.950	0.002↓
75	1.00	0.00	1.00	0.00↓	0.992	0.000↓	0.998	0.000↓
100	1.00	0.00	1.00	0.00↓	1.000	0.000↓	1.000	0.000↓
200	1.00	0	1.00	0↓	1.000	0↓	1.000	0↓
Case 5C								
t_i simulations with $F_{AP}(t, S = 1/3, \sigma_P = 0.2)$								
n	H_3 rejected with $\gamma = 0.01$	H_0 rejected with $\gamma = 0.01$	H_3 rejected with $\gamma = 0.001$	H_0 rejected with $\gamma = 0.001$	H_3 rejected with $\gamma = 0.001$	H_0 rejected with $\gamma = 0.001$	H_3 rejected with $\gamma = 0.001$	H_0 rejected with $\gamma = 0.001$
10	0.01	0.01	0.01	0.01	0.001	0.001	0.000	0.000
25	0.06	0.02	0.09	0.03	0.013	0.002	0.019	0.004
50	0.23	0.02	0.38	0.05	0.094	0.004	0.168	0.011
75	0.48	0.02	0.65	0.05	0.266	0.006	0.411	0.014
100	0.69	0.02	0.83	0.03↓	0.456	0.006	0.645	0.014
200	0.99	0.00↓	1.00	0.00↓	0.949	0.002↓	0.986	0.003↓
Case 5D								
t_i simulations with $F_{AP}(t, S = 1/3, \sigma_P = 0.3)$								
n	H_4 rejected with $\gamma = 0.01$	H_0 rejected with $\gamma = 0.01$	H_4 rejected with $\gamma = 0.001$	H_0 rejected with $\gamma = 0.001$	H_4 rejected with $\gamma = 0.001$	H_0 rejected with $\gamma = 0.001$	H_4 rejected with $\gamma = 0.001$	H_0 rejected with $\gamma = 0.001$
10	0.00	0.01	0.00	0.02	0.000	0.001	0.000	0.000
25	0.00	0.01	0.01	0.03	0.000	0.001	0.001	0.002
50	0.01	0.01	0.02	0.04	0.001	0.001	0.003	0.005
75	0.01	0.01	0.03	0.05	0.002	0.002	0.007	0.007
100	0.02	0.02	0.05	0.06	0.003	0.002	0.012	0.009
200	0.07	0.03	0.16	0.10	0.020	0.005	0.055	0.021

$n = 10, 25, 50, 75, 100$ and 200 . For every fixed combination of S , σ_P , and n , we simulated $Z = 100\,000$ random samples of $\tilde{t}^* = [t_1, t_2, \dots, t_n]$ from the respective cumulative distribution function $F_{AP}(t, S, \sigma_P)$. The Rayleigh test periodogram $z(f)$ of each $\tilde{t}^*$ sample was calculated between $f_{\min} = 0.5$ and $f_{\max} = 2.0$. The detected best period $P_{\text{best}} = f_{\text{best}}^{-1}$ was identified from the one particular periodogram peak located at $f = f_{\text{best}}$ (i.e. the global maximum), which fulfilled $z(f_{\text{best}}) = \max[z(f)]$.

For each particular sample $\tilde{t}^*$ simulated with some combination of S , n and σ_P , the value of $z(f_{\text{best}})$ was then compared to the numerical z_0 limit in the corresponding fully aperiodic $S = 1$ case having the same n and σ_P values in Table 3. If the criterion $z(f_{\text{best}}) \geq z_0$ was satisfied, the corresponding fully aperiodic hypothesis H_1 , H_2 , H_3 or H_4 was rejected.

The results of all simulations for the fully periodic ($S = 0$) case are given in Table 4. The respective results for the partly periodic cases ($S = 1/3$ and $2/3$) are given in Tables 5 and 6. All notations in Tables 4–6 are the same. Column 1 (n) is the number of simulated time points $\tilde{t}^* = [t_1, t_2, \dots, t_n]$ generated with corresponding cumulative distribution function $F_{AP}(t, S, \sigma_P)$. Column 2 (“real”) is the probability for rejecting one of the hypothesis H_1 , H_2 , H_3 or H_4 with $\gamma = 0.01$ for the “real” period $P = 1$. Detections marked with 0.99 or 1.00 (in bold face) are “reliable” as explained later. Column 3 (“false”) is the probability for rejecting one of the hypothesis H_1 , H_2 , H_3 or H_4 with

$\gamma = 0.01$ for the “false” periods $P \neq 1$. An integer value “0” indicates that the corresponding hypothesis was not rejected even once in all simulations. Columns 4 (“real”) and 5 (“false”) give the probability for rejecting H_0 with $\gamma = 0.01$, otherwise as in Cols. 2 and 3. Columns 6 (“real”), 7 (“false”), 8 (“real”) and 9 (“false”) were the results with $\gamma = 0.001$, otherwise as in Cols. 2–5. The symbol “↓” denotes the cases where the probability of detecting “false” periodicities first increased and then decreased as a function of the sample size n , i.e. the cases where the “real” period was emerging, as explained in Sect. 6.

For clarity, the cases of all S and σ_P combinations in these periodic simulations have been named in the corresponding tables. These names provide an easy way to refer to any particular simulated case. For example, the z_0 values of H_1 (Table 3: $S = 1, \sigma_P = 0.05$) were compared to the $z(f_{\text{best}})$ values of Case 4A (Table 4: $S = 0, \sigma_P = 0.05$), Case 5A (Table 5: $S = 1/3, \sigma_P = 0.05$) and Case 6A (Table 6: $S = 2/3, \sigma_P = 0.05$).

The analytical estimate for the critical level Q for all $z(f_{\text{best}})$ values were also calculated from Eq. (11). The “null hypothesis” H_0 was rejected, if and only if $Q < \gamma$ (Eq. (12)).

When H_0 , H_1 , H_2 , H_3 , or H_4 was rejected with $\gamma = 0.01$ or 0.001 for some particular simulated sample $\tilde{t}^*$, we also checked if the inverse $P_{\text{best}} = f_{\text{best}}^{-1}$ was equal to the period $P = 1$

Table 6. Simulation results for the partly periodic cases ($S = 2/3$), otherwise as in Table 4.

	“Real”	“False”	“Real”	“False”	“Real”	“False”	“Real”	“False”
Case 6A								
t_i simulations with $F_{AP}(t, S = 2/3, \sigma_P = 0.05)$								
n	H_1 rejected with $\gamma = 0.01$		H_0 rejected with $\gamma = 0.01$		H_1 rejected with $\gamma = 0.001$		H_0 rejected with $\gamma = 0.001$	
10	0.02	0.01	0.02	0.01	0.004	0.001	0.001	0.000
25	0.08	0.01	0.12	0.04	0.024	0.002	0.034	0.004
50	0.26	0.02	0.42	0.05	0.114	0.003	0.219	0.012
75	0.50	0.02	0.68	0.05	0.289	0.004	0.470	0.015
100	0.69	0.01↓	0.84	0.03↓	0.472	0.005	0.688	0.015
200	0.98	0.00↓	1.00	0.00↓	0.930	0.001 ↓	0.987	0.003 ↓
Case 6B								
t_i simulations with $F_{AP}(t, S = 2/3, \sigma_P = 0.1)$								
n	H_2 rejected with $\gamma = 0.01$		H_0 rejected with $\gamma = 0.01$		H_2 rejected with $\gamma = 0.001$		H_0 rejected with $\gamma = 0.001$	
10	0.01	0.01	0.01	0.01	0.001	0.001	0.000	0.000
25	0.04	0.01	0.07	0.04	0.010	0.002	0.016	0.003
50	0.15	0.02	0.27	0.06	0.053	0.003	0.112	0.011
75	0.31	0.02	0.50	0.06	0.137	0.004	0.279	0.016
100	0.48	0.02	0.69	0.05	0.272	0.005	0.471	0.017
200	0.91	0.01↓	0.97	0.01↓	0.779	0.004↓	0.930	0.009↓
Case 6C								
t_i simulations with $F_{AP}(t, S = 2/3, \sigma_P = 0.2)$								
n	H_3 rejected with $\gamma = 0.01$		H_0 rejected with $\gamma = 0.01$		H_3 rejected with $\gamma = 0.001$		H_0 rejected with $\gamma = 0.001$	
10	0.01	0.01	0.01	0.01	0.000	0.001	0.000	0.000
25	0.01	0.01	0.01	0.03	0.001	0.001	0.002	0.003
50	0.01	0.01	0.04	0.05	0.003	0.002	0.008	0.006
75	0.03	0.02	0.08	0.06	0.007	0.002	0.021	0.010
100	0.05	0.02	0.13	0.07	0.013	0.003	0.041	0.013
200	0.20	0.03	0.39	0.10	0.074	0.006	0.199	0.028
Case 6D								
t_i simulations with $F_{AP}(t, S = 2/3, \sigma_P = 0.3)$								
n	H_4 rejected with $\gamma = 0.01$		H_0 rejected with $\gamma = 0.01$		H_4 rejected with $\gamma = 0.001$		H_0 rejected with $\gamma = 0.001$	
10	0.00	0.01	0.00	0.01	0.000	0.001	0.000	0.000
25	0.00	0.01	0.00	0.03	0.000	0.001	0.000	0.002
50	0.00	0.01	0.01	0.04	0.000	0.001	0.001	0.004
75	0.00	0.01	0.01	0.04	0.000	0.001	0.001	0.006
100	0.00	0.01	0.01	0.05	0.000	0.001	0.002	0.006
200	0.01	0.01	0.02	0.07	0.001	0.002	0.004	0.010

used in our simulations. The criterion for accepting the detected period P_{best} as the “real” one was

$$1 - \frac{f_0}{2} \leq f_{\text{best}} \leq 1 + \frac{f_0}{2}.$$

Hence the criterion for “false” detections was

$$f_{\text{best}} < 1 - \frac{f_0}{2} \text{ or } 1 + \frac{f_0}{2} < f_{\text{best}}.$$

Finally, the number of simulated samples, where the “real” period was detected, was divided by the number of all simulated samples $Z = 100\,000$. The detection of the “real” period was accepted as “reliable”, if the following criterion was satisfied:

The hypothesis H_0 , H_1 , H_2 , H_3 , or H_4 was rejected with the “real” periodicity at the preassigned significance level γ in at least $(1 - \gamma)Z$ of all Z simulated $\bar{t}^$ samples.*

One particular example of all our simulations is given in Fig. 1, where the cumulative distribution function $F_{AP}(t, S, \sigma_P)$ is displayed for $S = 1/3$ and $\sigma_P = 0.1$. This cumulative distribution function was used to simulate one arbitrary random sample $\bar{t}^* = [t_1, t_2, \dots, t_n]$ ($n = 25$). The periodogram $z(f)$ for this random sample $\bar{t}^*$ is shown in Fig. 2. The highest peak of the periodogram $z(f)$ is within the frequency range $[1 - f_0/2, 1 + f_0/2]$, i.e. the best period P_{best} detected from these particular simulated data is “real”. When the critical level Q of this peak is

calculated from the Eq. (11), the hypothesis H_0 has to be rejected with $\gamma = 0.01$ (i.e. the peak exceeds the lower continuous horizontal line), but not with $\gamma = 0.001$ (i.e. the peak does not exceed the higher continuous horizontal line). The corresponding simulated fully aperiodic hypothesis is H_2 , where the cumulative distribution was $F_{AP}(t, S = 1, \sigma_P = 0.1)$. Under H_2 , the probabilities that $z(f_{\text{best}})$ reaches $z_0 = 7.73$ or 9.80 are $\gamma = 0.01$ or 0.001 (Table 3: Cols. 4 and 5, $n = 25$). This highest peak of the periodogram at f_{best} exceeds only the former level (i.e. lower dotted horizontal line), but not the latter (i.e. higher dotted horizontal line). Hence H_2 is rejected with $\gamma = 0.01$, but not with $\gamma = 0.001$. Finally, the results for all $Z = 100\,000$ data samples $\bar{t}^* = [t_1, t_2, \dots, t_n]$ ($n = 25$) simulated with $F_{AP}(t, S = 1/3, \sigma_P = 0.10)$ are given in Table 5 (Case 5B: $n = 25$). The probability for detecting the “real” periodicity with $\gamma = 0.01$ is 0.58 (under H_2) and 0.68 (under H_0), while the respective probabilities for detecting “false” periodicities are 0.01 (under H_2) and 0.02 (under H_0). With $\gamma = 0.001$, the probability for detecting the “real” periodicity is 0.341 (under H_2) and 0.408 (under H_0). Table 5 also reveals that the detection of the “real” periodicity $P = 1$ would be “reliable”, if the sample size reached $n \geq 75$ or $n \geq 50$ (Case 5B: H_2 or H_0 rejected with $\gamma = 0.01$: marked 1.00 or 0.99) and $n \geq 100$ (Case 5B: H_2 or H_0 rejected with $\gamma = 0.001$: both marked 1.000), respectively.

Jetsu (1997) showed that the rounding of the impact crater data induces a “human-signal”, i.e. the detection of spurious

Fig. 2. One arbitrary random sample $\tilde{t}^* = [t_1, t_2, \dots, t_n]$ ($n = 25$) was simulated from the cumulative distribution function $F_{AP}(t, S, \sigma_P)$ of Fig. 1. The thick continuous line is the Rayleigh test periodogram $z(f)$ of these data between $f_{\min} = 0.5$ and $f_{\max} = 2.0$. The global maximum $z(f_{\text{best}})$ (denoted with a diamond) is located between $1 - f_0/2$ and $1 + f_0/2$ (denoted with two vertical dashed lines), i.e. the best period $P_{\text{best}} = f_{\text{best}}^{-1}$ fulfills the criterion of the “real” periodicity. This maximum $z(f_{\text{best}})$ reaches the $\gamma = 0.01$ levels for rejecting H_0 (lower continuous horizontal line) or H_2 (lower dotted horizontal line $\equiv z_0 = 7.73$ in Table 3). However, the $\gamma = 0.001$ levels for rejecting H_0 (higher continuous horizontal line) or H_2 (higher dotted horizontal line $\equiv z_0 = 9.80$ in Table 3) are not reached.

periodicities like 3, 5, 10 or 20 Myr. The effects of this rounding to the statistics and to the significance estimates of the detected periodicities were then studied in Jetsu & Pelt (2000). We decided to round the simulated t_i values into integer multiples of $(3/\pi)\sigma_P \approx 0.95\sigma_P$. The factor $3/\pi$ was applied, because this prevented that some simulated rounded t_i would have been integers, i.e. multiples of the “real” periodicity $P = 1$. The results of these simulations for rounded data were compiled into Tables 7–9. For example, the simulations of Case 4A in Table 4 were otherwise same as those of Case 7A in Table 7, except that in the latter case the t_i were rounded into integer multiples of $(3/\pi)\sigma_P$, where $\sigma_P = 0.05$.

6. Results and discussion

The simulated periodicity was $P = f^{-1} = 1$. The Rayleigh tests were applied between $f_{\min} = 0.5$ and $f_{\max} = 2.0$. This combination was ideal. Let us assume that the tested frequency range had remained the same, but the simulated periodicity would have been $P = 1.8 \equiv f = 0.56$. In this case, the $z(f)$ periodogram would peak at $f = 0.56 = 1.8^{-1}$, $f = 1.11 = 0.9^{-1}$ and $f = 1.66 = 0.6^{-1}$, because these frequencies give similar phase distributions. In the fully periodic case, it might have been

possible to notice the effect that these fractions of the correct periodicity concentrated the t_i on every other ($P = 0.9$) or every third ($P = 0.6$) round. But this effect would have gone unnoticed, if there also was an aperiodic component in the data. With this $P = 1.8$ periodicity, the detection of the fractions of the “real” periodicity would certainly have increased the probability of detecting “false” periodicities. Hence the location of “real” periodicity within the tested range influences the probability for “real” or “false” detections. Furthermore, if the tested range had been longer, there would have been more problems with these fractions, as well as problems connected to the rounding of the data. For example, the rounding of t_i with $\sigma_P = 0.3$ would have introduced a spurious frequency $f \approx 3.33$, which was “conveniently” outside our tested range. For obvious reasons, no “reliable” detections of “false” periods occurred in Tables 4–9. But the above rounding with $\sigma_P = 0.3$ would have caused a “reliable” detection of the “false” spurious frequency $f \approx 3.33$, because the phases ϕ_i calculated with this f would have been equal for all rounded t_i .

Even though we studied only some limited combinations of K , n , σ_P , S , P , $P_{\min}$ and $P_{\max}$, we have in fact formulated a method to determine the probability of “reliable” detection of any periodicity in any type of impact crater data. This method also includes effects of erosion and the presence of an aperiodic component. It reveals, if the fractions of “real” periodicity influence the probability of “reliable” detection, or enhance detection “false” periodicities. Let us assume that the tested data are $t_1 \pm \sigma_1$, $t_2 \pm \sigma_2$, ..., $t_n \pm \sigma_n$, while the period P_K has been detected in a Rayleigh test between $f_{\min} = P_{\max}^{-1}$ and $f_{\max} = P_{\min}^{-1}$. A suitable estimate for the number of cycles is $K = \text{INT}[(t_n - t_1)/P_K]$. The A_k values can now be calculated as in Table 2. The error in the simulations is fixed to $\sigma_P = \sigma_{P'}$ of Eq. (1). The simulations are first performed with $S = 1$, i.e. for the fully aperiodic case. A large number (e.g. $Z = 100\,000$) of samples of n simulated time point samples $\tilde{t}^* = [t_1, t_2, \dots, t_n]$ are generated from $F_{AP}(t, S = 1, \sigma_P = \sigma_{P'})$ of Eq. (9). Since the simulated periodicity is $P = 1$, the Rayleigh test range must be fixed to $f'_{\min} = (P_{\max}/P_K)^{-1} + f_0/2$ and $f'_{\max} = (P_{\min}/P_K)^{-1} - f_0/2$, where $f_0 = P_K/(t_n - t_1)$ is the distance between independent frequencies. The highest peak $z_0 = \max[z(f)]$ of the Rayleigh test is identified for each $\tilde{t}^*$ sample. All these Z values of z_0 are used to determine the limits for $z(f)$ reaching the chosen pre-assigned significance level γ , like in Table 3. Finally, the probability of “reliable” detection for any value of $S < 1$ can be obtained with the same approach as already outlined in Sect. 5.2 (see also Fig. 2). The “real” periodicity in all these simulations is $P = 1$. Because the contribution of the aperiodic component is unknown, it is logical to begin these simulations from $S = 0$. This will reveal the possibility for “reliable” detection, if all data were fully periodic.

If all impacts in Table 1 were periodic, “reliable” periodicity detection would be possible in C_2 with $\gamma = 0.01$ and 0.001 (comparable to $n = 25$ of Case 4B in Table 4), and impossible in C_3 (comparable to $n = 50$ of Case 4C in Table 4). But “reliable” detections would be impossible, if one or two thirds of these impacts were aperiodic (see Tables 5 and 6).

As the first example, we performed the Rayleigh test for the subsample C_2 of data in Table 1, because “reliable” detection could be possible in the fully periodic case. The tested period range was between 15 and 60 Myr, because this combination was comparable to the frequency range $[0.5, 2.0]$ used in our simulations, if the “real” periods were close to 30 Myr. The best period $P_K = 34.2$ reached the critical level $Q = 0.29$ (Eq. (11)), i.e. H_0 was not rejected. Nor was H_2 . The maximum value for the test

Table 7. Simulation results for the fully periodic cases ($S = 0$) for the rounded data, otherwise as in Table 4.

	“Real”	“False”	“Real”	“False”	“Real”	“False”	“Real”	“False”
Case 7A								
t_i simulations with $F_{AP}(t, S = 0, \sigma_P = 0.05)$ rounded to multiples of $(3/\pi)\sigma_P$								
n	H_1 rejected with $\gamma = 0.01$		H_0 rejected with $\gamma = 0.01$		H_1 rejected with $\gamma = 0.001$		H_0 rejected with $\gamma = 0.001$	
10	1.00	0.00	1.00	0.00	0.993	0.000	0.510	0.000
25	1.00	0	1.00	0	1.000	0	1.000	0
50	1.00	0	1.00	0	1.000	0	1.000	0
75	1.00	0	1.00	0	1.000	0	1.000	0
100	1.00	0	1.00	0	1.000	0	1.000	0
200	1.00	0	1.00	0	1.000	0	1.000	0
Case 7B								
t_i simulations with $F_{AP}(t, S = 0, \sigma_P = 0.1)$ rounded to multiples of $(3/\pi)\sigma_P$								
n	H_2 rejected with $\gamma = 0.01$		H_0 rejected with $\gamma = 0.01$		H_2 rejected with $\gamma = 0.001$		H_0 rejected with $\gamma = 0.001$	
10	0.64	0.01	0.62	0.01	0.228	0.002	0.024	0.000
25	1.00	0.00	1.00	0.00	1.000	0.000	1.000	0.000
50	1.00	0	1.00	0	1.000	0	1.000	0
75	1.00	0	1.00	0	1.000	0	1.000	0
100	1.00	0	1.00	0	1.000	0	1.000	0
200	1.00	0	1.00	0	1.000	0	1.000	0
Case 7C								
t_i simulations with $F_{AP}(t, S = 0, \sigma_P = 0.2)$ rounded to multiples of $(3/\pi)\sigma_P$								
n	H_3 rejected with $\gamma = 0.01$		H_0 rejected with $\gamma = 0.01$		H_3 rejected with $\gamma = 0.001$		H_0 rejected with $\gamma = 0.001$	
10	0.03	0.01	0.03	0.01	0.003	0.001	0.000	0.000
25	0.24	0.02	0.33	0.03	0.085	0.003	0.123	0.005
50	0.74	0.01↓	0.86	0.02↓	0.511	0.004	0.656	0.008
75	0.95	0.00↓	0.98	0.01↓	0.865	0.003	0.934	0.004↓
100	0.99	0.00↓	1.00	0.00↓	0.974	0.001↓	0.992	0.001↓
200	1.00	0↓	1.00	0↓	1.000	0↓	1.000	0↓
Case 7D								
t_i simulations with $F_{AP}(t, S = 0, \sigma_P = 0.3)$ rounded to multiples of $(3/\pi)\sigma_P$								
n	H_4 rejected with $\gamma = 0.01$		H_0 rejected with $\gamma = 0.01$		H_4 rejected with $\gamma = 0.001$		H_0 rejected with $\gamma = 0.001$	
10	0.00	0.01	0.00	0.01	0.000	0.003	0.000	0.002
25	0.01	0.01	0.01	0.03	0.001	0.002	0.002	0.004
50	0.01	0.02	0.04	0.05	0.003	0.002	0.007	0.008
75	0.03	0.02	0.07	0.07	0.007	0.004	0.018	0.011
100	0.05	0.02	0.12	0.08	0.014	0.004	0.037	0.016
200	0.21	0.04	0.37	0.11	0.087	0.010	0.180	0.031

statistic was $z_0 = 3.56$, which was much lower than the closest comparable value in Table 3 (H_2 rejection with $\gamma = 0.01$ would have required $z_0 = 7.73$ for $\sigma_P = 0.10$ and $n = 25$).

As the second example, we studied the data in Alvarez & Muller (1984), which consisted of eleven impact craters. Because the ages of Logoisk and Boltsh craters were equal in the data given in their article, the actual sample size was $n = 10$. Their detected period $P_K = 28.4$ Myr and the errors for the crater ages gave $\sigma_P = 0.240 \approx 0.2$ (Eq. (1)). In the fully periodic case, the probability of “reliable” detection with $\gamma = 0.01$ was only 0.03 (Table 4: Case 4C, $n = 10$). In the partly aperiodic cases, the respective probability was 0.01 for both cases (Table 5: Case 5C and Table 6: Case 6C, $n = 10$). Hence the detection of the 28.4 Myr periodicity was far from “reliable”. The above probabilities were in fact even lower, because Alvarez & Muller (1984) tested the range between 10 and 100 Myr, i.e. their range was $10/28.4 = 0.4$ and $100/28.4 = 3.5$ in the units of P_K , which was a much longer range than in our simulations. Jetsu & Pelt (2000) performed a Rayleigh test between 10 and 100 Myr for the same data and detected the 28.54 Myr period, which reached the critical level of $Q = 0.034$.

As the third example, a subsample

$$S_1: t_i \leq 250, \sigma_i \leq 10, D_i \geq 35.$$

was selected from the data in Table 1. Stothers (2006) detected a 35 Myr periodicity from such a subsample. They applied

this criterion, because Shoemaker et al. (1990) had argued that larger craters were more probably caused by comet impacts. The S_1 criterion actually gave a further subsample of the subsample selected with our C_2 criterion. We performed the Rayleigh test to the $n = 13$ craters selected with S_1 from Table 1. The tested range was between 22 and 50 Myr, as in Stothers (2006). The best period $P_K = 34.94$ Myr reached the critical level $Q = 0.22$ (Eq. (11)). Hence there was no significant periodicity in these data, besides what could be described as “noise”. The test statistic maximum was only $z_0 = 3.02$. One could argue that the statistics based on H_0 were not reliable for such a small sample ($n = 13$). But even this argument would fail, because the rejection of H_1 with $\gamma = 0.01$ would have required values close to $z_0 = 6.88$ (see Table 3: $\sigma_P = 0.05, n = 10$). Furthermore, the tested range was short, between $22/34.94 = 0.63$ and $50/34.94 = 1.43$ in units of P_K . Therefore “reliable” detection with $\gamma = 0.01$ would have succeeded in the fully periodic case ($S = 0$), because these data were very accurate having $\sigma_P = 0.064$ with $P_K = 34.94$ (comparable to Case 4A, $n = 10$ in Table 4). In conclusion, had significant periodicity been present, we would certainly have detected it.

A sequence of random variables is called heteroscedastic, if the variances of these variables are different. This was more or less obvious in our study, because the accuracy decreased for older craters. It might have been possible to simulate this effect by using a different σ_P value during each cycle $k = 1, \dots, K$.

Table 8. Simulation results for the partly periodic cases ($S = 1/3$) for the rounded data, otherwise as in Table 4.

	“Real”	“False”	“Real”	“False”	“Real”	“False”	“Real”	“False”
Case 8A								
t_i simulations with $F_{AP}(t, S = 1/3, \sigma_P = 0.05)$ rounded to multiples of $(3/\pi)\sigma_P$								
n	H_1 rejected with $\gamma = 0.01$	H_0 rejected with $\gamma = 0.01$	H_1 rejected with $\gamma = 0.001$	H_0 rejected with $\gamma = 0.001$	H_1 rejected with $\gamma = 0.01$	H_0 rejected with $\gamma = 0.01$	H_1 rejected with $\gamma = 0.001$	H_0 rejected with $\gamma = 0.001$
10	0.20	0.01	0.20	0.01	0.101	0.002	0.029	0.000
25	0.79	0.01	0.85	0.01	0.599	0.002	0.659	0.004
50	0.99	0.00	1.00	0.00	0.977	0.000	0.992	0.000↓
75	1.00	0.00	1.00	0.00	1.000	0.000	1.000	0.000↓
100	1.00	0	1.00	0	1.000	0	1.000	0↓
200	1.00	0	1.00	0	1.000	0	1.000	0↓
Case 8B								
t_i simulations with $F_{AP}(t, S = 1/3, \sigma_P = 0.1)$ rounded to multiples of $(3/\pi)\sigma_P$								
n	H_2 rejected with $\gamma = 0.01$	H_0 rejected with $\gamma = 0.01$	H_2 rejected with $\gamma = 0.001$	H_0 rejected with $\gamma = 0.001$	H_2 rejected with $\gamma = 0.01$	H_0 rejected with $\gamma = 0.01$	H_2 rejected with $\gamma = 0.001$	H_0 rejected with $\gamma = 0.001$
10	0.10	0.01	0.09	0.01	0.023	0.001	0.002	0.000
25	0.56	0.01	0.66	0.02	0.319	0.002	0.385	0.005
50	0.96	0.00	0.98	0.00↓	0.876	0.001↓	0.941	0.002↓
75	1.00	0.00	1.00	0.00↓	0.991	0.000↓	0.998	0.000↓
100	1.00	0.00	1.00	0.00↓	1.000	0.000↓	1.000	0.000↓
200	1.00	0	1.00	0↓	1.000	0↓	1.000	0↓
Case 8C								
t_i simulations with $F_{AP}(t, S = 1/3, \sigma_P = 0.2)$ rounded to multiples of $(3/\pi)\sigma_P$								
n	H_3 rejected with $\gamma = 0.01$	H_0 rejected with $\gamma = 0.01$	H_3 rejected with $\gamma = 0.001$	H_0 rejected with $\gamma = 0.001$	H_3 rejected with $\gamma = 0.01$	H_0 rejected with $\gamma = 0.01$	H_3 rejected with $\gamma = 0.001$	H_0 rejected with $\gamma = 0.001$
10	0.01	0.01	0.01	0.01	0.001	0.001	0.000	0.000
25	0.04	0.01	0.07	0.03	0.010	0.002	0.015	0.003
50	0.18	0.02	0.31	0.05	0.064	0.003	0.123	0.009
75	0.38	0.02	0.56	0.04↓	0.189	0.004	0.318	0.011
100	0.59	0.01↓	0.76	0.03↓	0.342	0.004	0.539	0.011
200	0.97	0.00↓	0.99	0.00↓	0.889	0.002↓	0.966	0.003↓
Case 8D								
t_i simulations with $F_{AP}(t, S = 1/3, \sigma_P = 0.3)$ rounded to multiples of $(3/\pi)\sigma_P$								
n	H_4 rejected with $\gamma = 0.01$	H_0 rejected with $\gamma = 0.01$	H_4 rejected with $\gamma = 0.001$	H_0 rejected with $\gamma = 0.001$	H_4 rejected with $\gamma = 0.01$	H_0 rejected with $\gamma = 0.01$	H_4 rejected with $\gamma = 0.001$	H_0 rejected with $\gamma = 0.001$
10	0.00	0.01	0.00	0.01	0.000	0.003	0.000	0.002
25	0.00	0.01	0.01	0.03	0.000	0.002	0.001	0.004
50	0.00	0.02	0.01	0.04	0.001	0.002	0.002	0.006
75	0.01	0.02	0.02	0.05	0.001	0.002	0.004	0.008
100	0.01	0.02	0.03	0.06	0.002	0.002	0.007	0.010
200	0.04	0.02	0.10	0.08	0.009	0.003	0.029	0.016

The distributions of Eqs. (2) and (3) with these particular σ_P values could then have been convolved separately to the aperiodic and periodic components within each cycle. But in this case there would have been no unique simulated σ_P value that could have been compared with the unique $\sigma_{P'}$ value calculated from the data (Eq. (1)). Deutsch & Schaerer (1994) noted that the error estimates in the crater data at the time of the initial discovery of the alleged periodicity might have been too small. This view is supported by the comparison of Tables 1 and 10. The more accurate crater ages derived using modern Ar-Ar dating techniques, described e.g. by Kelley (2003a) and Kelley (2003b), clearly indicate that some errors in the earlier ages exceeded the $\pm 3\sigma_i$ estimate (e.g. Shunak: from 12 ± 5 to 45 ± 10 Myr, Boltysh: from 88 ± 3 to 65.17 ± 0.64 Myr, or Zapdanaya: from 115 ± 7.1 to 165 ± 5 Myr). Although the accuracy of the dating has improved over the years, these large and heteroscedastic errors still prevent “reliable” detection of periodicity.

Our simulations clearly revealed that the statistics based on H_0 were less reliable than those based on H_1, H_2, H_3 or H_4 . This can be seen from the following points:

Firstly, Case 6D in Table 6 was dominated by the aperiodic component ($S = 2/3$) and the error was large ($\sigma_P = 0.3$), i.e. this case was very close to being fully aperiodic. Under H_4 , the sums for the probabilities for detecting the “false” or “real” periodicity were close to $\gamma = 0.01$ or 0.001. Under H_0 , these sums clearly exceeded γ .

Secondly, the rejection of H_0 was always more probable than that of H_1, H_2, H_3 or H_4 , except for $n = 10$. This effect occurred for both $\gamma = 0.01$ and 0.001 in all Tables 4–9. For example, Brazier (1994) have noted that with small samples ($n < 50$) the assumption of asymptotic probability density distribution $f(z) = e^{-z}$ of the Rayleigh power z does not hold. This explained the above exception with $n = 10$.

Thirdly, when erosion was introduced into the simulations, the statistics differed from those based on H_0 . The effect of erosion was simulated with the A_k estimates. We chose to give one example which should clarify how erosion can alter the statistics. If A_1 were much larger than $A_2, A_3, \dots, A_{10}$, then values of $0 < t_i < 1$ would be more probable than in the case $A_1 = A_2 = \dots = A_{10} = 0.1$, where the data are evenly distributed over $0 < t_i < 10$. For P values tested close to 2, the phases of all these $0 < t_i < 1$ values would be within 0.5 in phase. This effect would yield larger simulated $z(f)$ values than the statistics based on H_0 do predict. In conclusion, erosion altered statistics and this had to be included into the simulations.

In most of the simulations, the “real” period was detected more often than the “false” periods. Only for $S = 2/3$ and $\sigma_P = 0.3$, the detection “false” periods was more likely. As expected, the probability of detecting the “real” period always increased when n increased. There were clear cases, where the “real” period began to dominate as the sample size increased, and this lowered the probability of detecting “false” periods. In

Table 9. Simulation results for the partly periodic cases ($S = 2/3$) for the rounded data, otherwise as in Table 4.

	“Real”	“False”	“Real”	“False”	“Real”	“False”	“Real”	“False”
Case 9A								
t_i simulations with $F_{AP}(t, S = 2/3, \sigma_P = 0.05)$ rounded to multiples of $(3/\pi)\sigma_P$								
n	H_1 rejected with $\gamma = 0.01$		H_0 rejected with $\gamma = 0.01$		H_1 rejected with $\gamma = 0.001$		H_0 rejected with $\gamma = 0.001$	
10	0.02	0.01	0.02	0.02	0.004	0.001	0.001	0.000
25	0.08	0.01	0.12	0.03	0.024	0.002	0.035	0.004
50	0.25	0.01	0.42	0.05	0.110	0.002	0.214	0.010
75	0.48	0.01	0.68	0.04↓	0.281	0.003	0.462	0.012
100	0.68	0.01	0.84	0.03↓	0.463	0.002↓	0.682	0.009
200	0.98	0.00↓	1.00	0.00↓	0.922	0.001↓	0.986	0.001↓
Case 9B								
t_i simulations with $F_{AP}(t, S = 2/3, \sigma_P = 0.1)$ rounded to multiples of $(3/\pi)\sigma_P$								
n	H_2 rejected with $\gamma = 0.01$		H_0 rejected with $\gamma = 0.01$		H_2 rejected with $\gamma = 0.001$		H_0 rejected with $\gamma = 0.001$	
10	0.01	0.01	0.01	0.01	0.001	0.001	0.000	0.000
25	0.04	0.01	0.07	0.03	0.009	0.002	0.015	0.003
50	0.14	0.01	0.26	0.05	0.048	0.002	0.103	0.008
75	0.29	0.01	0.48	0.05	0.122	0.002	0.260	0.010
100	0.45	0.01	0.67	0.04↓	0.246	0.003	0.442	0.011
200	0.89	0.00↓	0.97	0.01↓	0.743	0.001↓	0.918	0.005↓
Case 9C								
t_i simulations with $F_{AP}(t, S = 2/3, \sigma_P = 0.2)$ rounded to multiples of $(3/\pi)\sigma_P$								
n	H_3 rejected with $\gamma = 0.01$		H_0 rejected with $\gamma = 0.01$		H_3 rejected with $\gamma = 0.001$		H_0 rejected with $\gamma = 0.001$	
10	0.00	0.01	0.00	0.01	0.000	0.001	0.000	0.000
25	0.00	0.01	0.01	0.03	0.001	0.001	0.002	0.002
50	0.01	0.01	0.03	0.04	0.002	0.001	0.006	0.005
75	0.02	0.01	0.06	0.05	0.005	0.001	0.015	0.006
100	0.04	0.01	0.10	0.05	0.008	0.001	0.029	0.007
200	0.14	0.01	0.32	0.06	0.046	0.002	0.142	0.011
Case 9D								
t_i simulations with $F_{AP}(t, S = 2/3, \sigma_P = 0.3)$ rounded to multiples of $(3/\pi)\sigma_P$								
n	H_4 rejected with $\gamma = 0.01$		H_0 rejected with $\gamma = 0.01$		H_4 rejected with $\gamma = 0.001$		H_0 rejected with $\gamma = 0.001$	
10	0.00	0.01	0.00	0.01	0.000	0.003	0.000	0.002
25	0.00	0.01	0.00	0.03	0.000	0.002	0.000	0.004
50	0.00	0.01	0.00	0.04	0.000	0.002	0.001	0.006
75	0.00	0.01	0.01	0.04	0.000	0.002	0.001	0.007
100	0.00	0.01	0.01	0.05	0.000	0.002	0.001	0.008
200	0.00	0.01	0.01	0.05	0.000	0.002	0.002	0.009

most cases, the probability of detecting “false” periods either increased or decreased monotonically when n increased. In some cases, however, the probability of detecting “false” periods first increased, but then began to decrease. Such cases were marked with “↓” in Tables 4–9. In many such cases, this trend did lead to a “reliable” detection with larger sample sizes (e.g. Case 4C of Table 4). But there were also “↓” cases, where “reliable” detection was not achieved even with $n = 200$ (e.g. H_1 was not rejected with $\gamma = 0.01$ in Case 6A of Table 6). Thus the notation “↓” in Tables 4–9 indicates that the sample size has increased so much that the “real” period has begun to dominate over the “false” periods, but sometimes this effect was not yet strong enough to induce a “reliable” detection even with $n = 200$.

Comparison of pairs of tables (i.e. Tables 4 and 7, Tables 5 and 8, Tables 6 and 9) revealed the effects of rounding. Rounding reduced the probability of detecting the “real” periodicity. As expected, this effect was strongest when the data were rounded with $\sigma_P = 0.3$. On the other hand, rounding with $\sigma_P = 0.05$ or 0.10 caused only a minor reduction in the probability of detecting “real” periodicity. There were only three cases, where rounding prevented “reliable” detection (i.e. $n = 75$ in Case 7C of Table 7, $n = 50$ in Case 8B of Table 8 and $n = 200$ in Case 8C of Table 8). Rounding had no strong influence to the probability of “false” detections. But as already noted, the tested frequency range “conveniently” excluded frequencies like $f = 3.33$ connected to rounding with $\sigma_P = 0.3$. Furthermore, the Rayleigh

test was sensitive only to unimodal phase distributions. The detection of the multimodal phase distributions induced by rounding would have been possible with other nonparametric methods, such as applied in Jetsu (1997) or Jetsu & Pelt (2000).

In order to estimate the improvement in the quality and quantity of the impact crater data in recent years, we compared the data in our Table 1 to the data in Jetsu (1997), which are presented in our Table 10. The sample size n of criterion C_2 has decreased from 34 to 33, while that of criterion C_3 has increased from 35 to 40. The $P_K = (t_n - t_1)/K$ values of the old and new subsamples were calculated with $K = 10$. The accuracy σ_P of the data had improved: from 0.17 to 0.11 in C_2 and from 0.20 to 0.17 in C_3 .

The main aim of this study was to determine the quantity (n) and quality (σ_P) of terrestrial impact crater data that would allow certain detection of periodicity – if present. Such “reliable” detections were listed in Tables 4–9 (numbers marked in bold). There were four Cases (A–D) in each of these six tables, where the results were determined for two preassigned significance levels $\gamma = 0.01$ and 0.001 and for six different n values, i.e. results were determined for $4 \times 6 \times 2 \times 6 = 288$ different alternatives. These “reliable” detections did not depend strongly on the chosen hypothesis, i.e. testing of H_0 versus H_1, H_2, H_3 or H_4 . In only 5 out of all 288 alternatives, there were minor differences. These were the Cases 4C ($\gamma = 0.01, n = 75$), 5B ($\gamma = 0.01, n = 50$), 6A ($\gamma = 0.01, n = 200$), 8C ($\gamma = 0.01, n = 200$) and

Table 10. The data subsamples C_2 ($n = 34$) and C_3 ($n = 35$) selected from the terrestrial impact crater data in Jetsu (1997). The selection criteria as well as all notations, are the same as in Table 1.

Crater	Location	$t_i \pm \sigma_i$	D_i	C_2	C_3	k_i in C_2	k_i in C_3
Zhamanshin	Kazakhstan	0.9 ± 0.1	13.5	Yes	No	1	–
Bosumtwi	Ghana	1.03 ± 0.02	10.5	Yes	No	1	–
El'gygytgyn	Russia	3.5 ± 0.5	18	Yes	No	1	–
Bigach	Kazakhstan	6 ± 3	7	Yes	Yes	1	1
Shunak	Kazakhstan	12 ± 5	3.1	No	Yes	–	1
Ries/Steinheim★	Germany/Germany	15.0 ± 0.7	24.3	Yes	Yes	1	1
Haughton	Canada	23 ± 1	24	Yes	Yes	1	1
Logancha	Russia	25 ± 20	20	Yes	Yes	1	1
Popigai	Russia	35 ± 5	100	Yes	Yes	2	2
Chesapeake Bay	USA	35.5 ± 0.6	85	Yes	Yes	2	2
Wanapitei	Canada	37 ± 2	7.5	Yes	Yes	2	2
Mistastin	Canada	38 ± 4	28	Yes	Yes	2	2
Logoisk	Belarus	40 ± 5	17	Yes	Yes	2	2
Chiyli	Kazakhstan	46 ± 7	5.5	Yes	Yes	2	2
Kamensk/Gusev★	Russia/Russia	49.00 ± 0.14	25.2	Yes	Yes	2	2
Montagnais	Canada	50.50 ± 0.76	45	Yes	Yes	3	2
Ragozinka	Russia	55 ± 5	9	Yes	Yes	3	2
Marquez	USA	58 ± 2	13	Yes	Yes	3	2
Chicxulub	Mexico	64.98 ± 0.05	170	Yes	Yes	3	3
Kara/Ust-Kara★	Russia/Russia	73.0 ± 2.1	69.6	Yes	Yes	3	3
Manson	USA	73.8 ± 0.3	35	Yes	Yes	3	3
Lappajärvi	Finland	77.3 ± 0.4	23	Yes	Yes	4	3
Boltysh	Ukraine	88 ± 3	24	Yes	Yes	4	3
Dellen	Sweden	89.0 ± 2.7	19	Yes	Yes	4	3
Steen River	Canada	95 ± 7	25	Yes	Yes	4	4
Carswell/Zapadnaya★	Canada/Ukraine	115.0 ± 7.1	39.2	Yes	Yes	5	4
Zeleny Gai	Ukraine	120 ± 20	2.5	No	Yes	–	5
Mien	Sweden	121.0 ± 2.3	9	Yes	Yes	5	5
Tookoonooka	Australia	128 ± 5	55	Yes	Yes	6	5
Romistrovka	Ukraine	140 ± 20	2.7	No	Yes	–	5
Gosses Bluff	Australia	142.5 ± 0.5	22	Yes	Yes	6	5
Mjølnir	Norway	143 ± 20	40	Yes	Yes	6	5
Puchenzh-Katunki	Russia	175 ± 3	80	Yes	Yes	8	6
Rochechouart	France	186 ± 8	23	Yes	Yes	8	7
Wells Creek/Red Wing★	USA/USA	200 ± 18	15	Yes	Yes	9	7
Manicouagan	Canada	214 ± 1	100	Yes	Yes	9	8
Araguainha Dome	Brazil	247.0 ± 5.5	40	Yes	Yes	10	9
Clearwater West/East★	Canada/Canada	290 ± 14	44.4	No	Yes	–	10

Table 11. Compilation of the results for simulations using the hypotheses H_1 , H_2 , H_3 , or H_4 .

	$\sigma_P = 0.05$	$\sigma_P = 0.1$	$\sigma_P = 0.2$	$\sigma_P = 0.3$	$\sigma_P = 0.05$	$\sigma_P = 0.1$	$\sigma_P = 0.2$	$\sigma_P = 0.3$
	H_1	H_2	H_3	H_4	H_1	H_2	H_3	H_4
	Rejected with $\gamma = 0.01$				Rejected with $\gamma = 0.001$			
Table 4: $S = 0$	$n \geq 10$	$n \geq 25$	$n \geq 100$	†	$n \geq 25$	$n \geq 25$	$n \geq 200$	†
Table 5: $S = 1/3$	$n \geq 50$	$n \geq 75$	$n \geq 200$	†	$n \geq 75$	$n \geq 100$	↓	†
Table 6: $S = 2/3$	↓	↓	†	†	↓	↓	†	†
Table 7: $S = 0$ and t_i rounded	$n \geq 10$	$n \geq 25$	$n \geq 100$	†	$n \geq 25$	$n \geq 25$	$n \geq 200$	†
Table 8: $S = 1/3$ and t_i rounded	$n \geq 50$	$n \geq 75$	↓	†	$n \geq 75$	$n \geq 100$	↓	†
Table 9: $S = 2/3$ and t_i rounded	↓	↓	†	†	↓	↓	†	†

9A ($\gamma = 0.01$, $n = 200$). But even in these cases, the “real” period was emerging (i.e. marked with ↓) regardless of the chosen hypothesis, except for the Case 5B ($\gamma = 0.01$, $n = 50$). As already discussed, the statistics based on H_0 were found to be less reliable. Therefore we shall hereafter concentrate only on the results based on H_1 , H_2 , H_3 , or H_4 . These results for “reliable” detections have been compiled into Table 11, which shows the lowest number n of simulated time points t_i , that was needed for “reliable” detection of the “real” periodicity with the preassigned significance levels of $\gamma = 0.01$ or 0.001 . The symbol “↓”

is used to denote the cases where the “real” period was emerging with $n = 200$, but the detection was not yet “reliable”. The cases where “reliable” detection would require $n > 200$ have been denoted with “†”.

7. Conclusions

If all impacts were periodic ($S = 0$), “reliable” detection of periodicity would be possible for numerous combinations, where $\sigma_P \leq 0.2$ and $n \leq 200$ (Table 11). In fact, very accurate data

having $\sigma_P = 0.05$ would yield “reliable” detections even for small samples like $n = 10$ ($\gamma = 0.01$) or $n = 25$ ($\gamma = 0.001$). If this accuracy deteriorates to $\sigma_P \geq 0.3$, “reliable” detections become impossible even if all impacts were periodic and $n = 200$.

If one third of the impacts were aperiodic ($S = 1/3$), “reliable” detections would require from three to five times larger samples ($n \geq 50$) than in the fully periodic case ($S = 0$). Furthermore, the accuracy would have to be at least $\sigma_P \leq 0.1$.

The results for $S = 2/3$ were, mildly put, discouraging. There was no chance of “reliable” detection even with $\sigma_P = 0.05$ and $n = 200$. One may find indications of periodicity only in very large ($n \geq 100$) and very accurate ($\sigma_P \leq 0.1$) samples (Table 11: $S = 2/3$ cases marked with ↓). In other words, if two thirds of all impacts are caused by asteroids, then real periodicity due to comet impacts could not be reliably detected (even if present).

Currently, the ratio between periodic and aperiodic impacts is unknown. For example, Bailey & Stagg (1988) have argued that 98% of terrestrial impact craters have been produced by Earth-crossing asteroids. Bottke et al. (2002b) have estimated that approximately 6% of NEOs are comets originating in the Jupiter-family comet region, and the rest are asteroids from different regions of the main asteroid belt. All asteroid impacts on Earth have most probably been aperiodic and they seem to dominate the possibly periodic comet impacts. Moreover, there are no direct evidence of any terrestrial comet showers and there are no reasons for assuming that *all* comet impacts would have been periodic. In the presence of an aperiodic part, “reliable” detection is possible only with very accurate and large samples. Comparison of Tables 1 and 10 in Sect. 6 revealed only minor quantity and quality improvements during the past decade, i.e. between the years 1997 and 2008. Hence a “reliable” detection is highly unlikely in the near future.

Our main conclusion is: “reliable” detection of “real” periodicity from the currently available terrestrial impact crater data is impossible – unless all impacts have been due to periodic comet showers.

Acknowledgements. The authors wish to thank Professors Philippe Claeys and Karri Muinonen for valuable comments on the manuscript.

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