Type Ia supernovae and the formation of single low-mass white dwarfs

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ABSTRACT

Context. There is still considerable debate over the progenitors of type Ia supernovae (SNe Ia). Likewise, it is not agreed how single white dwarfs with masses ≤0.5 M⊙ can be formed in the field, even though they are known to exist.

Aims. We consider whether single low-mass white dwarfs (LMWDs) could have been formed in binary systems where their companions have exploded as an SN Ia. In this model, the observed single LMWDs are the remnants of giant-branch donor stars whose envelopes have been stripped off by the supernova explosion.

Methods. We investigate the likely remnants of SNe Ia, including the effects of the explosion on the envelope of the donor star. We also use evolutionary arguments to examine alternative formation channels for single LMWDs. In addition, we calculate the expected kinematics of the potential remnants of SNe Ia.

Results. SN Ia in systems with giant-branch donor stars can naturally explain the production of single LMWDs. It seems difficult for any other formation mechanism to account for the observations, especially for those single LMWDs with masses ≤0.4 M⊙. Independent of those results, we find that the kinematics of one potentially useful population containing single LMWDs is consistent with our model. Studying remnant white-dwarf kinematics seems to be a promising way to investigate SN Ia progenitors.

Conclusions. The existence of single LMWDs appears to constitute evidence for the production of SNe Ia in binary systems with a red-giant donor star. Other single white dwarfs with higher space velocities support a second, probably dominant, population of SN Ia progenitors which contained main-sequence or subgiant donor stars at the time of explosion. The runaway stars LP 400–22 and US 708 suggest the possibility of a third formation channel for some SNe Ia in systems where the donor stars are hot subdwarfs.

Key words. stars: binaries: close – stars: supernovae: general – stars: white dwarfs – stars: kinematics

1. Introduction

Type Ia supernovae (SNe Ia) are of major astrophysical importance. They have acquired particular cosmological significance since they have been used to measure the expansion history of the Universe (Riess et al. 1998; Perlmutter et al. 1999; Riess et al. 2004). Understanding their nature is also of importance for understanding the metallicity evolution and star-formation history of galaxies (e.g. Canal et al. 1996; Matteucci & Recchi 2001). Despite their importance, there is still no agreement on the nature of their progenitors.

There is broad agreement that the destruction of a white dwarf (WD) in a thermonuclear explosion constitutes the supernova event itself, but there are two main classes of competing models for the events which lead to the explosion. In the single-degenerate scenario, the doomed WD accretes matter from a non-degenerate companion (Whelan & Iben 1973; Nomoto & Kondo 1991; but see also Yoon et al. 2007). At present we do not know whether all WD–WD mergers do leave remnants – in which case the single degenerate scenario could not be responsible for SNe Ia – and it seems unlikely that this will become clear in the near future (but see, e.g. Levan et al. 2006). Hansen (2003) first noticed that observed high-velocity WDs (Oppenheimer et al. 2001) could have been produced through SNe Ia; such WDs would be the descendants of non-degenerate mass donors in the pre-supernova binaries. Hansen’s idea seems to be consistent with more detailed work on the ages of the WDs in the Oppenheimer et al. sample by Bergeron et al. (2005) and deserves further attention, but by itself it is not a clinching argument for the single-degenerate channel. Nor has the evidence that the SN Ia rate is different for different stellar populations (Mannucci et al. 2005) led to firm conclusions. The strongest direct evidence that non-degenerate donor stars can lead to normal type Ia supernovae has been

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provided by Patat et al. (2007), who observed circumstellar material around SN 2006X which seems extremely hard to reconcile with a double-degenerate progenitor.

Here we suggest that the observed, apparently single, low-mass white dwarfs (LMWDs) provide evidence that at least some SN Ia explosions have occurred with non-degenerate donor stars. We define LMWDs as WDs which are too low in mass to have been produced by single-star evolution as we currently understand it. A population of single LMWDs has been implied by, e.g., the work of Maxted et al. (2000a). We also investigate the apparently single ultra-cool white dwarfs (UCWDs) as potentially containing a useful subset of the LMWD population and indicate how further observations of the kinematics of this and other populations could lead to constraints on the progenitors of SNe Ia.

In Sects. 2 and 3 we argue that the existence of single LMWDs is most naturally explained by the single-degenerate model for SNe Ia. In Sect. 4 we introduce UCWDs, discuss to what extent the observed single UCWDs might be a useful sample of LMWDs, and consider in what way the observed single UCWD population is consistent with an SN Ia origin.

### 2. Formation channels for single LMWDs

Current evidence suggests that 1 $M_\odot$ zero-age main-sequence (ZAMS) stars, left to evolve in isolation, produce white dwarfs $\geq 0.55$ $M_\odot$ (e.g. Han et al. 1994; Weidemann 2000). In order to produce low-mass helium WDs (with masses $\leq 0.5$ $M_\odot$), it is necessary to remove the envelope of a star before it is able to ignite helium. Within the age of the Universe, it is almost certainly impossible for a single star to produce WDs with masses close to 0.3 $M_\odot$, or even 0.17 $M_\odot$, as recently inferred for the runaway WD LP 400–22 (Kawka et al. 2006), or 0.23 $M_\odot$, the estimated mass of the apparently single UCWD LHS 3250 (Bergeron & Leggett 2002).

#### 2.1. Production of single LMWDs by single stars?

It would be simplistic to conclude from the existence of currently single LMWDs that single stars can produce LMWDs. However, arguments have been made in favour of a single-star channel for the production of some LMWDs, at least in significantly metal-rich populations (Kalirai et al. 2007; Kilic et al. 2007c; see also the production of some LMWDs, at least in significantly metal-rich populations (Kalirai et al. 2007; Kilic et al. 2007c). We also investigate the apparently single ultra-cool white dwarfs (UCWDs) as potentially containing a useful subset of the LMWD population and indicate how further observations of the kinematics of this and other populations could lead to constraints on the progenitors of SNe Ia.

In Sects. 2 and 3 we argue that the existence of single LMWDs is most naturally explained by the single-degenerate model for SNe Ia. In Sect. 4 we introduce UCWDs, discuss to what extent the observed single UCWDs might be a useful sample of LMWDs, and consider in what way the observed single UCWD population is consistent with an SN Ia origin.

### 2.2. Production of single LMWDs by binary stars?

In binary systems we can invoke mass transfer (and sometimes ablation by a pulsar companion) in order to explain the observed binary LMWDs (e.g. van Kerkwijk et al. 2000; Liebert et al. 2004). Apparently single LMWDs must also be formed within an interacting binary system – either we have not detected their companion or the binary has been disrupted. One attractive formation channel stands out: the formation of the LMWD in a binary where the binary companion exploded in an SN Ia. Before examining this channel in more detail, we discuss possible alternative explanations for single LMWDs.

#### 2.2.1. Alternatives to SN Ia: Core-collapse supernovae?

The natural alternative to an SN Ia in explaining the disruption of a binary is a core-collapse supernova (chosen such that the system becomes unbound). However, forming an LMWD in such a system is challenging. The binary must remain intact long enough for the WD progenitor to lose its envelope such that it will later form an LMWD; this implies that the initially more massive star has transferred its envelope to the secondary and then becomes a WD. In order to form an LMWD, the primary mass must be $\leq 4.5$ $M_\odot$, which our own stellar calculations find to have a core mass at the end of the main sequence of $\approx 0.52$ $M_\odot$. In order to produce a core-collapse supernova (requiring $M_{\text{ZAMS}} \approx 8$ $M_\odot$), the initially less massive star would thus have to accrete the large majority of the envelope of the primary soon after the primary has left the main sequence, and the initial mass ratio would have to be close to 1. This highly optimistic scenario does not produce a distinctly low-mass WD; to produce a 0.3 $M_\odot$ WD in this way requires a ZAMS mass for the primary of $\approx 3$ $M_\odot$, precluding a core-collapse supernova in the binary system.

Our arguments above are generous; Davies et al. (2002) found a much more restrictive result. In a different context they investigated the evolution of systems where a WD is formed before a core-collapse supernova occurs in the system. They concluded that “... the mass of the white dwarfs generated in this way, $M_{\text{WD}} \geq 1$ $M_\odot$.”

#### 2.2.2. Alternatives to SN Ia: Circumbinary discs?

If cataclysmic-variable (CV) evolution is driven by circumbinary discs, the donor star may eventually be entirely consumed (the “White Widow” scenario; see Spruit & Taam 2003, following Spruit & Taam 2001; Taam & Spruit 2001). For this mechanism

1. See also Marsh et al. (1995); their study was originally motivated by their interest in finding double degenerate binaries as potential SN Ia progenitors.
2. Note that mass segregation tends to move low-mass objects outwards in such clusters, so LMWDs might be formed in the core and observed far from the centre.
3. Bedin et al. argue that there may not, after all, be an unusual single LMWD population in NGC 6791 and Van Loon et al. see no evidence for enhanced stellar mass loss in infrared observations of the cluster.
4. We use Eggleton’s stellar evolution code (Eggleton 1971; Pols et al. 1995) with a metallicity of 0.02 along with the convective overshooting calibration of Pols et al. (1998).
to explain single LMWDs, the WD in the progenitor CV must also have been an LMWD; the WD may not gain much mass, as the matter it accretes can be ejected via nova explosions, but it is unlikely to become significantly less massive.

It is not clear whether this mechanism operates in CVs: attempts to detect circumbinary discs have inferred disc masses several orders of magnitude below the required values (Dubus et al. 2004; Muno & Mauerhan 2006). Hence in the absence of further supporting evidence we consider this potential formation channel unlikely at present.

2.2.3. Alternatives to SN Ia: accretion-induced collapse?

Some systems will contain an accreting WD which succeeds in reaching the Chandrasekhar mass but fails to produce a supernova as the WD is predominantly composed of oxygen, neon & magnesium (ONeMg) rather than carbon and oxygen (CO). This can occur either because the WD began accreting as an ONeMg WD or because the accretion rate onto the WD did not allow the WD to remain a CO WD (e.g. Nomoto & Iben 1985; Nomoto & Kondo 1991; Martin et al. 2006). Such WDs will produce a neutron star (NS) via AIC. Currently it does not appear likely that AIC produces sufficiently large kicks to disrupt such close binaries (see, e.g. Podsiadlowski et al. 2004).

2.2.4. Alternatives to SN Ia: white-dwarf mergers?

Single LMWDs may be the product of the merger of two low-mass He WDs, with formation rates comparable to or greater than the SN Ia rate (see, e.g. Han et al. 2002, and references therein). However, Han et al. predict masses in excess of 0.4 $M_\odot$.

3. Single-degenerate SN Ia populations

3.1. Expected formation channels

We do not present an exhaustive description of the full evolutionary histories for single-degenerate SN Ia progenitors (see, e.g. Whelan & Iben 1973; Nomoto 1982; van den Heuvel et al. 1992; Rappaport et al. 1994; Hachisu et al. 1996, 1999; Li & van den Heuvel 1997; Langer et al. 2000; Hachisu & Kato 2001; Han & Podsiadlowski 2004). There is no clear consensus on which donor stars are likely to produce a type Ia supernova. The favoured options involve either donors on the main sequence (MS) or the subgiant branch (known as the supersoft channel), or red-giant (RG) donors.

While the supersoft channel (e.g. Han & Podsiadlowski 2004) is arguably the favoured channel for the majority of SNe Ia, Hachisu et al. (1996, 1996) and Hachisu & Kato (2001) suggest situations in which a low-mass giant star may take a WD to the Chandrasekhar mass $M_{\text{Ch}}$ at long orbital periods\(^5\). Sokoloski et al. (2006) used the 2006 outburst of RS Ophiuchi to confirm the conclusions of Hachisu & Kato by inferring that RS Oph contains a very massive WD ($M_{\text{WD}} \approx 1.4 M_\odot$). It is worth noting that we cannot be sure that RS Oph contains a CO WD rather than an ONeMg one and so we cannot be sure that it will explode rather than collapse. Observational support for a giant donor in a system which produced an SN Ia has been provided via the observations by Patat et al. (2007) of SN 2006X.

\(^5\) The point at which an explosive nuclear runaway occurs in a non-rotating CO WD is slightly below the Chandrasekhar mass: Nomoto et al. (1984) calculated a mass of $\sim 1.378 M_\odot$.

King et al. (2003) have also suggested that an accreting WD may not reach $M_{\text{Ch}}$ via the supersoft channel alone, but that a later phase of WD growth could occur in long-period dwarf novae. They argue that, even though the average mass-transfer rate does not reach the steady-burning band (Paczynski & Zytkow 1978; Nomoto & Kondo 1991), the accretion rate may be high enough for the WD to grow during dwarf nova outbursts driven by the thermal-viscous disc instability (Cannizzo et al. 1982).

Providing the correct mass-accretion rate for the CO WD to grow to $M_{\text{Ch}}$ is a significant uncertainty in all these models.

3.2. Remnant mass

In order to understand the formation of LMWDs in systems which produce SN Ia explosions, we must consider the mass and evolutionary stage of the donor star at the point of the explosion and also the extent to which the donor loses mass because of the explosion. There is a clear division between pre-giant and giant donor stars, with giant donors apparently able to leave LMWD remnants.

Marietta et al. (2000) performed numerical simulations of the effect of an SN Ia explosion on the companion star. They found that 0.15 to 0.17 $M_\odot$ is stripped away from a 1 $M_\odot$ main-sequence or subgiant companion by the high-velocity ejecta. Han & Podsiadlowski (2004) found in their population synthesis simulations of the supersoft channel that, at the time of the explosion, the companion has a mass between $\geq 0.5 M_\odot$ and $2.2 M_\odot$, with a typical mass of 1 $M_\odot$ (for more details see also Han 2008). Applying the results of Marietta et al. as a percentage – 15% of the donor mass – leads to a lowest estimated remnant mass of $\geq 0.42 M_\odot$. If the WD explodes as it reaches $M_{\text{Ch}}$, then this remnant mass is a lower limit for the MS channel, assuming negligible subsequent mass loss in a wind. Hachisu & Kato (2001) found a lower limit on the mass of the donor from the supersoft channel (at the time of the SN) of $>1.3 M_\odot$ (assuming an initial white dwarf mass of 1 $M_\odot$). Despite these differences, both studies suggest that it is difficult to produce LMWDs via main-sequence or subgiant donors.

Marietta et al. also found that a red-giant donor will lose almost its entire envelope (96%–98%) due to the impact of the SN Ia explosion and leave only the core of the star, providing a possible pathway for the formation of a subset of single, low-mass He WDs\(^6\). For the RG channel, Hachisu & Kato (2001) found a lower limit on the total donor mass of $\geq 0.4 M_\odot$. If the RG channel produces SNe Ia, then ram-pressure stripping of the donor’s envelope would be expected to lead to the formation of LMWDs. The remnant WD mass is dependent on the core mass of the donor at explosion and is therefore strongly correlated with the orbital period (see Sect. 3.4).

One formation channel that is rarely discussed in the literature is one where the donor star is a hot subdwarf star (see, e.g. Geier et al. 2007). We do not expect significant stripping of the donor by the supernova ejecta in this case, as the donor star will be tightly bound, but the mass of the donor star could easily be low enough for a single LMWD to be formed by the natural evolution of the donor star.

3.3. Sub-Chandrasekhar mass explosions

A variation on the above models for SNe Ia involves the explosion of sub-$M_{\text{Ch}}$ CO WDs covered with a thick helium layer (Woosley & Weaver 1994). In that model, the detonation of the
helium layer is responsible for triggering the supernova. Fink et al. (2007) found that sub-\( M_{\text{Ch}} \) explosions were unlikely to be able to explain either normal or subluminous SNe Ia, but there could be implications for our LMWD formation channel if a significant fraction of SNe Ia were found to be produced by sub-\( M_{\text{Ch}} \) detonations. Qualitatively there would be little change to our model, as the RG donors would still be stripped of their envelopes and produce LMWDs. The quantitative remnant mass distribution may be different. For example, these sub-\( M_{\text{Ch}} \) explosions might plausibly happen when the donor stars are lower on the giant branch than for the standard model. In that case, the typical remnant WDs may be less massive, the orbital periods at explosion lower and the runaway remnant velocities higher than for detonations at \( M_{\text{Ch}} \). The remnant velocity is considered here-after as a diagnostic of the orbital period at explosion, assuming Chandrasekhar-mass explosions.

3.4. Binary orbits and runaway velocities

If single LMWDs have been released from binary systems in which the other component has exploded as a type Ia supernova, the space velocity of the remnant should be a useful diagnostic of the orbital period at explosion. As our arguments above suggest that the LMWDs are most likely to originate in systems with red-giant donor stars, the relationship between core mass and orbital period in such systems can act as a further constraint.

In what follows we assume that the donor stars are filling their Roche lobes, as it seems to us that the mass transfer in systems which produce an SN Ia is most likely to be due to Roche-lobe overflow. However, it is not known whether this is the case as, for example, it is unclear whether the donor star in RS Ophiuchi is rotating synchronously with the binary orbit (see, e.g. Murset & Schmid 1999; Zamanov et al. 2007). If the donor stars do not fill their Roche lobes, then the following method may slightly overestimate the runaway velocities for a given remnant mass.

If we define \( q \) as \( M_2/M_1 \) (where \( M_1 \) is the mass of the SN Ia progenitor and \( M_2 \) is the mass of the companion producing the LMWD), write the total mass of the system (in solar units) as \( M_{\text{tot,0}} \) and the pre-SN orbital period (in days) as \( P_{\text{days}} \), we can write the pre-SN orbital velocity as:

\[
V_{\text{orb}} \approx \frac{213}{1 + q} \left( \frac{M_{\text{tot,0}}}{P_{\text{days}}} \right)^{1/3} \text{ km s}^{-1}.
\]

This shows that, for \( M_2 \ll M_1 \), the orbital velocity is relatively insensitive to the donor mass.

Furthermore, assuming that the remnant WD mass equates to the core mass of the donor at the time of the SN explosion, then the well-defined relationship between core mass and radius for giant-branch donors leads to an expression for the orbital period of the system at the supernova stage. Rappaport et al. (1995) found the period-mass relation:

\[
P_{\text{orb}} \approx 0.374 \left( \frac{R_0 M_{\text{wd}}^{1.5}}{1 + 4 M_{\text{wd}}^{4}} + 0.5 \right)^{3/2} M_{\text{wd}}^{-1/2} \text{ days}, \tag{2}
\]

where \( M_{\text{wd}} \) is the mass of the future WD (currently the core of the giant star) in units of solar masses. Their preferred value for the fitting parameter \( R_0 \) was 4950 \( R_\odot \), which we also adopt. If

7 If such sub-\( M_{\text{Ch}} \) explosions occur but are not seen as SN Ia, it is not clear to us whether such an event would remove the RG donor’s envelope in order to produce a LMWD.

where \( V_{\text{orb}} \) is the total orbital velocity. Using the same component mass, we record for comparison that \( P_{\text{orb}} = 1 \) d corresponds to \( V_{\text{orb}} \approx 194 \text{ km s}^{-1} \) and \( P_{\text{orb}} \approx 1 \) h to \( V_{\text{orb}} \approx 560 \text{ km s}^{-1} \). The orbital velocities at explosion in the simulations of Han & Podsiadlowski (2004) range from 80–230 km s\(^{-1}\) for MS and subgiant donors.

RS Ophiuchi has an orbital period of \( \approx 457 \) d. Inverting Eq. (2) above, this corresponds to a core mass for the donor of slightly over 0.4 \( M_\odot \), still within the mass range for a LMWD should the envelope be removed.

The arguments in Sect. 3.2 suggest that LMWDs are produced by giant donors, as long as the orbital period is not so long that the core has already grown to 0.5 \( M_\odot \) by the time their tenuous envelopes are stripped by the supernovae ejecta. So, in contrast to the high-velocity WDs observed by Oppenheimer et al. (2001) and interpreted by Hansen (2003) as remnants of SNe Ia with main-sequence donors, it would be consistent to find that the single LMWD population was not significantly kinematically heated.

In Sect. 3.2 we identified two potential SN Ia formation channels able to produce LMWDs. We note that whilst one of those set of donor stars (red giants) would leave LMWD remnants with low runaway velocities, the other (hot subdwarfs) would result in high-velocity LMWDs.

4. Single UCWDs as LMWDs and SN Ia remnants?

We have argued that single LMWDs can be produced from single-degenerate SNe Ia with red-giant donors. Single LMWDs are inferred to exist and the most natural explanation, especially for the lower-mass LMWDs, seems to be that some single-degenerate SNe Ia occur with red giant donors. However, there is no obvious collected sample of LMWDs to examine as potential SN Ia remnants. Independently of our arguments above, it may well be that the known set of apparently single UCWDs constitutes or contains a useful sample of single LMWDs. When a suitable sample of single LMWDs becomes available our work should be extended.

4.1. The UCWD sample

In selecting sub-samples of the WD population, UCWDs (see, e.g. Harris et al. 1999 & 2001; Gates et al. 2004; Wolf 2005) are clear outliers in a colour-colour diagram (see Fig. 1). Their optical colours distinctly separate them from the normal WD population, and they are an interesting curiosity in appearing to become bluer as they cool down, possibly due to the effects of collisionally induced absorption (CIA) by hydrogen molecules in the atmosphere (Bergeron et al. 1994; see also Kowalski & Saumon 2006). When CIA affects only the infrared part of the spectrum, WDs are classed as cool; if CIA also affects the optical colours, then the WD is admitted into the select group of UCWDs (see Fig. 1; also Wolf 2005). The transition temperature between cool and ultra-cool is \( \lesssim 4000 \) K.
The known UCWDs constitute a clean observational sample; they are easy to identify, and with such low luminosities (notably in the ultraviolet and near infrared) and line-free spectra, it would be hard to hide a light-emitting close companion that is anything but another UCWD (see Sect. 4.3). Table 1 contains the UCWD sample we use. The estimates of the tangential velocities for these objects depend on their assumed distances, and Table 1 shows the range of velocities obtained for assumptions taken from the literature. For that sample we shall adopt the tangential velocities obtained by taking the absolute magnitude of LHS 3250 (which is the only UCWD with a known parallax) to be representative of the whole sample.

Since we first began this work, a new sample of twenty-four UCWD candidates has been presented by Vidrih et al. (2007). We are not convinced that they are cold enough to conform to our strict criteria as UCWDs, which may mean that this sample is more likely to be contaminated by non-LMWD objects (see Sect. 4.2). However, in our later figures we shall show for comparison this new, independent, sample alongside the smaller set from Table 1.

4.2. Are the observed UCWDs mostly LMWDs?

The one UCWD with a known parallax (LHS 3250) has an absolute magnitude of $M_V = 15.72$. This is brighter than expected for anything other than an LMWD; hence Bergeron & Leggett (2002) conclude that the mass of LHS 3250 is $0.23 M_\odot$. Unfortunately we do not have such good mass estimates for all single UCWDs.

The argument that the observed UCWDs are mostly LMWDs is partly built upon theoretical WD cooling tracks. Whereas a $0.6 M_\odot$ WD takes more than 9 Gyr to cool to 4000 K (after the formation of the WD), the $0.3 M_\odot$ WD of equivalent composition takes less than 4 Gyr (Bergeron et al. 2001; Bergeron et al. 2005). As the cooling of WDs is a function of composition it is likely that not all UCWDs are LMWDs (see, e.g. Hansen 1999, who requires WD masses $\lesssim 0.25 M_\odot$ in order for those objects to cool to 4000 K within 7 Gyr$^{10}$). Extremely low-mass WDs ($\lesssim 0.17 M_\odot$) seem to cool more slowly than more massive WDs, due to the retention of a relatively thick hydrogen envelope (e.g. Panei et al. 2007). It is not clear whether the LMWD remnants that have been formed by having their envelopes forcibly removed by a supernova shockwave will retain a thick hydrogen envelope.

Hence we expect that the observed, apparently single, UCWDs are dominated by single LMWDs if they exist, partly as they are significantly more luminous than massive WDs and hence more likely to be discovered. Although it is unfortunate that we cannot prove what fraction of single UCWDs are LMWDs, in Sect. 4.4 we show that the observed numbers of single UCWDs could all be single LMWDs produced via an SN Ia explosion.

4.3. Are the apparently single UCWDs mostly single?

Seven of the eight UCWDs in Fig. 1 have no known companion. The exception – SDSS J0947 – has a common proper motion companion (Gates et al. 2004). The 20 arcsec angular separation of J0947 from its potential companion implies a projected separation of over $2 \times 10^3 R_\odot$ for a distance of 47 pc; if that companion really forms a binary with J0947 (rather than being chance projection), it could not have influenced the evolution of the progenitor of the J0947 UCWD.

We do, however, need to consider whether these apparently single UCWDs really are single. We fully expect that UCWDs should exist in binary systems, but in the following we argue that these are unlikely to contaminate our sample; in many cases, a companion would even completely hide a UCWD.

4.3.1. Non-degenerate companions

An M-dwarf with an absolute $V$ magnitude of $\sim 16$ – similar to the UCWD LHS 3250 – would have a mass of $\sim 0.1 M_\odot$ (Delfosse et al. 2000). Such a star would be bright in the infrared, where the emission of UCWDs is strongly suppressed$^{11}$. Furthermore, M-dwarfs are rich in spectral lines, so could be easily detected.

4.3.2. White-dwarf companions

A binary containing a non-ultra-cool WD should be identifiable: an advantage of UCWDs is that their low luminosity and featureless spectrum makes it hard for them to possess an undetectable hotter WD companion. Their characteristic spectral energy distributions mean that a non-UCWD companion would be brighter than the UCWD in either near infrared or ultraviolet light. However, a spectroscopic UCWD-UCWD binary would be difficult to distinguish from a single UCWD. We have no reason

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8 We have not included an extremely recent sample of seven very cool white dwarfs (Harris et al. 2008).
9 Harris et al. (2001) also suggest that LHS 3250 is probably a low-mass helium core WD based on its absolute magnitude; however, the earlier paper by Harris et al. (1999) noted that a pair of UCWDs in a binary might help explain the higher luminosity without recourse to a low-mass WD.
10 We note in advance that this mass of $0.25 M_\odot$ is in good agreement with our results in Sect. 4.6 combined with the core-mass orbital-period relation; it is thus self-consistent.
11 For example, a $\sim 0.1 M_\odot$ star is about six magnitudes brighter in the $J$ band than the $V$ band (Delfosse et al. 2000), whereas LHS 3250 and SDSS 1337 are dimmer in the $J$ than $V$ bands (Harris et al. 2001). This trend is greater at longer wavelengths.
for thinking that such binary UCWDs do not exist, and Harris et al. (2008) have discovered one system which might eventually be expected to become such a binary. Although we do not expect that they are common enough to dominate the population, the possibility that the apparently single UCWDs have extremely cool or faint WD companions should be studied further.

4.3.3. Neutron-star companions

From the arguments in Sect. 2.1 it is clear that, in order to make a LMWD in a system where the NS was produced in a core-collapse supernova, the LMWD progenitor must lose mass after the formation of the NS. Hence, either mass transfer onto the collapse supernova, the LMWD progenitor must lose mass after this would suggest that many more pulsars exist than we currently expect, significantly worsening any mismatch between the inferred birthrates of LMXBs and MSPs (e.g. Kulkarni & Narayan 1988; Lorimer 1995; Pfahl et al. 2003)\textsuperscript{12}.

If a NS was formed in the system through AIC and was not subsequently spun up, it could reasonably be expected not to emit pulsar radiation. A black-hole companion would, of course, not be expected to emit pulsar radiation, but a local space density for such black-hole binaries of \(10^{-5}\) pc\(^{-3}\) (see Sect. 4.4) is highly unexpected (see, e.g. Romani 1998).

4.4. UCWD population numbers

Gates et al. (2004) estimated a space density for UCWDs of \(3 \times 10^{-5}\) pc\(^{-3}\) from a sample of 6 objects found in the Sloan digital sky survey\textsuperscript{13}. This rough figure does compare with an estimate of the SN Ia rate integrated over time and space. The local stellar density of 0.1 M\(_{\odot}\) pc\(^{-3}\) (Binney & Merrifield 1998), combined with a mass for the thin disc of \(~4 \times 10^{10}\) M\(_{\odot}\) and the assumption that the mass fraction of UCWDs is constant throughout the disc, produces an estimate for the number of UCWDs of \(~10^{7}\).

We approximate the current SN Ia rate in the Galactic disc as \(~7 \times 10^{-3}\) /yr (using the same disc mass as above and the SN Ia rate per unit mass of Mannucci et al. 2005). Multiplying this rate by a Galactic age of \(~10^{10}\) yr leads to an estimate of a total of \(7 \times 10^{7}\) remnants\textsuperscript{14}.

\textsuperscript{12} If we repeat this estimate for the new Viridi et al. sample of 24 UCWDs, which extends to a distance of 180 pc but only 250 square degrees of the sky, then we have \(~600/200\) (24/2)/250, i.e. single UCWDs outnumber those with MSPs by a factor of 70000 (over 27000 if UCWDs are twice as bright as Viridi et al. assume and their survey depth extends to 250 pc).

\textsuperscript{13} For the Viridi et al. sample, we estimate a space density of \(~16 \times 10^{-3}\) pc\(^{-3}\) for a survey depth of 180 pc, and \(~6 \times 10^{-3}\) pc\(^{-3}\) for a survey depth of 250 pc.

\textsuperscript{14} The supernova rate may well have been different in the past. Hansen (2003) made estimates for the number of SN Ia remnants in the Milky Way based on the amount of iron in the Galaxy and found a range between \(~4 \times 10^{7}\) and \(~2.2 \times 10^{7}\) objects.

<table>
<thead>
<tr>
<th>Name</th>
<th>Estimated properties for (M_V = 15.7^{m})</th>
<th>Estimated properties for (M_V = 16.5 \pm 1.0^{h})</th>
</tr>
</thead>
<tbody>
<tr>
<td>LHS 3250(^{(1)})</td>
<td>Distance (pc) 30</td>
<td>Tangential velocity (km s(^{-1})) 81</td>
</tr>
<tr>
<td>LHS 1402(^{(2)})</td>
<td>24(^{c})</td>
<td>56</td>
</tr>
<tr>
<td>SDSS J0947(^{(3)})</td>
<td>47</td>
<td>18</td>
</tr>
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<td>SDSS J1001(^{(4)})</td>
<td>64</td>
<td>107</td>
</tr>
<tr>
<td>SDSS J1220(^{(5)})</td>
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</tr>
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<td>SDSS J1337(^{(4)})</td>
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<td>SDSS J1403(^{(5)})</td>
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<td>60</td>
</tr>
<tr>
<td>COMBO-17 J1143(^{(5)})</td>
<td>169</td>
<td>42</td>
</tr>
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</table>

For all objects except LHS 3250, the distance is inferred from an assumed absolute magnitude. \(^{*}\) i.e. taking LHS 3250, with a known trigonometric parallax, as representative of all UCWDs; \(^{b}\) i.e. using a conservative assumption adopted by Salim et al. (2004), Gates et al. (2004) and Wolf (2005); \(^{c}\) for LHS 1402, we compare a \(B\) magnitude of 18.32 (Oppenheimer et al. 2001) with LHS 3250 (18.85; Harris et al. 1999); \(^{d}\) correcting for different photometry, we compare a \(B\) magnitude of 18.2 with the \(g\) magnitude (20.04) of SDSS J1001.

References. \(^{(1)}\) Harris et al. (1999); \(^{(2)}\) Oppenheimer et al. (2001); \(^{(3)}\) Gates et al. (2004); \(^{(4)}\) Harris et al. (2001); \(^{(5)}\) Wolf (2005).

Table 1. Distances and tangential velocities of the UCWD sample (as in Fig. 1).
Even if we halve this number of remnants to allow for the WD cooling time (see Sect. 4.2), the number of single UCWDs is easily consistent with them being produced through SN Ia. Indeed, these numbers suggest that only a subset of SNe Ia produces single UCWDs, which is as expected if only a subset of the SN Ia formation channels can produce LMWDs (see Sects. 3.1 and 3.2).

4.5. Population kinematics: method

The lack of lines in UCWD spectra means that we do not know the radial velocities for our sample. However, we can examine the population kinematics using only the information from the tangential velocities. We now investigate whether their observed space velocities are consistent with single LMWDs released from binaries with a range of orbital periods.

For a range of initial parameters, we integrated the motion of 10^5 assumed SN Ia remnants for up to 10 Gyr through the Galactic potential (using a similar procedure to Brandt & Podsiadlowski 1995), orientating the orbital velocity vector of the donor at random when the binary is disrupted. For each integrated population, we used a single value of orbital period (and hence orbital velocity) at the time of the explosion. For calculating the orbital velocity at a given orbital period, donor stars are assumed to be 0.5 M☉ at the time of the explosion, and the WDs are assumed to explode at a mass of 1.4 M☉. Equation (1) shows that our results should be relatively insensitive to those assumptions, but in the future we intend to perform this procedure using the output of our binary population synthesis calculations.

Each remnant is initially located at random within an axisymmetric Galaxy modelled by two exponential scale-heights (vertical and radial). The axisymmetry is also exploited for computational efficiency: at each integration time-step the view from Earth is calculated at all points on the solar circle. A further assumption is that the remnants can be observed to a distance of 300 kpc from the solar circle. A reasonable conclusion is that the remnants should be observed to a distance of 100 pc – broadly appropriate for UCWDs. Within such a small volume, the space velocities should only be a very weak function of distance.

The Galactic potential was taken from Paczynski (1990), using the parameters in Brandt & Podsiadlowski (1995), with the addition of scattering from giant molecular clouds (GMCs) randomly distributed within the Galactic disc. The total mass in these GMCs is assumed to be 1.2 × 10^9 M☉, with a mass spectrum exponent such that dN/dlog m ∝ m^{-1.7} (see, e.g. Digel et al. 1996; Binney & Merrifield 1998).

We do not include any kick imparted by the supernova ejecta in our simulations, since the simulation of Marietta et al. (2000) show that the kicks due to the supernova interaction (86 km s^{-1} and 49 km s^{-1} for their main-sequence and subgiant donors, respectively, with no kick given to the core of the giant donor) are generally small compared the orbital-velocity kick from the break-up of the binary.

Since the initial disc scale-height of the progenitor population is uncertain, we present our results for a range of values. The vertical scale-height of massive stars is 75 pc (van der Kruit 1987), and ~200 pc is an approximation to a more generic thin disc population (e.g. Ojha et al. 1996; who find a scaleheight of 260 ± 50 pc for the Galactic thin disc and 760 ± 50 for the thick disc; see also Kroupa et al. 1993). Given previous speculations that UCWDs are so cool because they are very old objects, we also modelled initial scale-heights of 500 pc, 1 kpc and 4 kpc.\footnote{Note that the collective tangential velocities of the sample in Table 1 are clearly inappropriate for objects from a halo population.}

4.6. Population kinematics: Results

Figures 2 and 3 present the results of our integrations for orbital periods at the time of the supernova of 1, 10 and 100 days, as well as a population which received no kick. The population which received no kick seems to be difficult to reconcile with the observed tangential velocities, except for the most extreme range of luminosities consistent with the literature combined with an initial vertical scale height of 4 kpc (or greater). The population released from a one day orbital period also appears inconsistent with the data, whichever initial scale-height is assumed. These conclusions are supported by applying the Kolmogorov-Smirnov test to compare our simulations with the tangential velocity distribution produced by assuming LHS 3250 is a typical UCWD (Table 1). Each plot contains a Kolmogorov-Smirnov acceptance probability (Pₓ) for each curve.\footnote{Here 1 – Pₓ gives the probability that the data and model x are drawn from different distributions, so we can reject a model with P = 0.01 with 99% confidence.}

Given the considerable uncertainties, we consider that the extremely favourable Kolmogorov-Smirnov test for the population released from 100 day orbits with a 4 kpc scale height should not be over-interpreted, especially as a real SN Ia population would be expected to have a range of orbital periods. Both the uncertainty in the distances to UCWDs and in the formation kinematics restricts our ability to draw quantitative conclusions. However, both the observational samples presented in Figs. 2 and 3, with their different assumptions, seem to suggest that these apparently single LMWDs have experienced some kick.

The broadly favoured period range at explosion is between 10 and 100 days, with the longer orbital periods preferred for a larger initial scale-height. The sample presented by Vidrih et al. leads to a lower set of tangential velocities, and their sample strongly prefers 100 over 10 d. This may be because they assume fainter absolute magnitudes than we adopt, and hence systematically produce lower velocities than us. Alternatively, their larger but less cool sample may be more likely to be contaminated by objects which are not single LMWDs.

This period range of 10–100 d approximately encompasses the core masses appropriate for the production of LMWDs, exactly as might be expected if the stellar evolution was truncated by the loss of a giant star’s envelope. This is consistent with an explanation of these UCWDs as being descended from giant donors. We note, however, that we cannot exclude a thick disc origin for these objects with an arbitrarily small kick. Although we cannot use this apparent kick as a definitive signature of an origin in type Ia supernovae, a thick disc origin would not falsify our SN Ia hypothesis, but it would suggest longer orbital periods at the point of explosion.

5. Discussion

The main conclusion of this study is that single LMWDs constitute indirect evidence that SNe Ia are formed through the single-degenerate channel, specifically from systems with red-giant donors. A field population of truly single LMWDs, especially with masses less than ∼0.4 M☉, would seem to require an SN Ia origin, or a significant revision of our understanding...
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Fig. 2. Comparison of the tangential velocities resulting from our Galactic integrations (smooth black curves), with the tangential velocities of the observed UCWDs (grey step functions). The solid light grey step function uses the distance estimates from the assumption that LHS 3250 is representative of the UCWD population, and the broken grey step functions encompass the wider range of distances in Table 1. The dark grey step function uses the new sample of UCWD candidates from Vidrih et al. (2007), adopting their assumptions for the UCWD distances. The solid black curves in each panel are for orbital periods at the time of the explosion of 100, 10 and 1 days (left to right). The dashed black curve represents a population which receives no “kick” due to the break-up of a binary system. The Kolmogorov-Smirnov acceptance probabilities for the individual models are given in each panel, compared to both the objects in Table 1 and the Vidrih et al. sample. Here $P_\infty$ refers to the model with no added “kick” velocity from binary break-up and $P_{100}$, $P_{10}$ and $P_1$ to the curves representing the 100, 10 and 1 day orbital-period populations, respectively (see text). The simulations assumed initial disc scale-heights of 75 pc, 500 pc and 4 kpc (left to right). If these objects were once the cores of red-giant donor stars in SN Ia producing systems, the simplest expectation would be for orbital periods of $\sim100$ d (see text).

Fig. 3. As Fig. 2, for simulations that do not include any scattering from giant molecular clouds. The plots assume initial disc scale-heights of 75 pc, 200 pc and 1000 pc (left to right).

of either CV evolution or AIC\textsuperscript{18}. That is based on evolutionary arguments (Sects. 2 and 3) and does not rely on any kinematic information.

We have simultaneously demonstrated that there is a natural formation channel for single LMWDs which does not require any unexpected modification of single-star evolution.

Those conclusions are independent of the nature of apparently single UCWDs. However the currently observed UCWD sample may be dominated by LMWDs, and we find that the kinematics of the known single UCWDs seem to be consistent with an origin in SNe Ia with red-giant donor stars. For reasonably broad assumptions, they even suggest that the orbital periods at the time of explosion were $\leq 100$ d. The population of UCWDs require further study in order to strengthen those specific conclusions.

5.1. SN Ia progenitor evolution

The implication of the existence of single LMWDs is that at least some single-degenerate SNe Ia occur with the donor star on the giant branch at the time of the explosion.

Both the high-velocity WD and single LMWD populations seem to contain members originating in SN Ia explosions. Dwarf star donors at the time of explosion should acquire higher space velocities than giant donors (due to their shorter orbital periods), but the less extended donors should be less stripped by the

\textsuperscript{18} It may be that explosions of sub-Chandrasekhar-mass WDs occur which are not seen as SN Ia but are capable of stripping the envelopes of RG donor stars. That would provide a further possible channel for the formation of single LMWDs.
ram pressure of the explosion (Marietta et al. 2000) and hence go on to produce higher-mass WDs than a giant which loses its envelope prior to helium ignition. Hence the Oppenheimer et al. (2001) sample of high-velocity WDs considered by Hansen (2003) do not need to be low-mass for an SN Ia origin to be reasonable, nor do single LMWDs need to have high space velocities to invoke an SN Ia explanation for their production. LP 400–22 (Kawka et al. 2006) is notable for being an extreme object using either selection criterion.

A simplistic comparison of the remnant space densities quoted for the Oppenheimer et al. (2001) high-velocity sample ($1.8 \times 10^{-3}$ pc$^{-3}$) and the Gates et al. (2004) UCWDs ($\sim 3 \times 10^{-5}$ pc$^{-3}$) suggests that the supersoft channel is almost an order of magnitude more important than the red-giant channel, though presumably some single LMWDs destined to be single UCWDs have not yet cooled sufficiently to become UCWDs, and some UCWDs may not be LMWDs. The relative importance of these formation channels will be an interesting quantity to constrain with future data and compare with binary population synthesis models. Perhaps more decisively, the estimates of Hansen (2003) and Sect. 4.4 imply that the total number of observed remnants is consistent with the expected number of past SNe Ia in our Galaxy. If this is confirmed by future work, it would leave room for only a minority of SNe Ia to result from double-degenerate systems.

One curiosity of our study of UCWDs is that the range of orbital periods we infer for the SN Ia progenitors at explosion is only broadly consistent with that predicted by Hachisu & Kato (2001) for systems with red-giant donors. Their models produce no SNe Ia with final orbital periods 2.5 d $\leq P_{\text{orb}}$ $\leq$ 60 d; most of the final parameter space for red-giant donors has 100 d $\leq P_{\text{orb}}$ $\leq$ 1000 d. Our favoured assumptions seem to indicate orbital periods on the shorter side of 100 d, i.e. the kinematic signal seems to be a little stronger than expected. A possible resolution could be that our use of UCWDs is biased and preferentially selects systems with shorter orbital periods because they produce lower-mass WDs, which cool more rapidly. Or perhaps the progenitor population is preferentially from the thick disc, and hence the remnants falsely appear kinematically hotter than in our models. It may also be that some non-LMWD single UCWDs from the thick disc or Galactic halo could be contaminating these results. Alternatively, this may be an indication that the models need some modification; for example the systems containing red-giant donors might produce SNe Ia earlier than expected, or perhaps a subset of systems from the supersoft channel does not explode until their donor stars have evolved more than the current models predict.

5.2. The nature of UCWDs

We encourage further work on the nature of UCWDs. It is important to confirm that the apparently single UCWDs really are single. High signal-to-noise searches for any spectral features that allow radial-velocity measurements would be worthwhile. GAIA astrometry is a long-term hope for examining the single status of UCWDs, as well as providing accurate distances to all these objects.

It is also important to understand the mass distribution of the UCWD population. The inferred mass of the best-studied UCWD, LHS 3250, (0.23 $M_\odot$ (Bergeron & Leggett 2002)) makes it a clear LMWD and hence a good candidate SN Ia remnant, but no other apparently single UCWD has a well-constrained mass estimate that we are aware of.

5.3. Outlook: a definitive signature?

A sample of non-UCWD single LMWDs has distinct advantage over the UCWDs in that the presence of spectral lines allows for radial velocity measurements (see, e.g. Maxted et al. 2000a). The significant difficulty for these objects is in selecting a large sample: whilst UCWDs stand out from survey photometry, hotter LMWD do not. Eisenstein et al. (2006) used the Sloan digital sky survey to produce a catalogue of over 9000 white dwarfs, and identify 13 WDs with masses $<0.3 M_\odot$, of which 7 have masses $\leq 0.2 M_\odot$. These objects should be investigated for signs of a companion NS, or for radial velocity variations.

With a substantial sample of single LMWDs, it would make sense to use directional information about the space velocities of the objects rather than just the magnitudes of the transverse velocities. We expect that the use of such information will help distinguish between a large initial scale-height, long orbital period population and a thinner, shorter-period population. Our conclusions would be considerably stronger if we were sure about the initial kinematics of these stars; it is important to try to determine whether the remnants come from the thick disc.

Once the single LMWD sample becomes large enough, it could potentially be split into sub-samples with different WD masses. If our model is correct, then less massive single LMWDs should be kinematically hotter than more massive single LMWDs (see Sect. 3.4).

Since we began this work, van Leeuwen et al. (2007) and Kilic et al. (2007a) have searched for companions to LMWDs in the radio and optical wavebands, respectively. The search by van Leeuwen et al. of LMWDs for radio pulsations found none "down to flux densities of 0.6–0.8 mJy kpc$^{-2}$", and concluded that "a given low-mass helium-core white dwarf has a probability of $<0.18 \pm 0.05$ of being in a binary with a radio pulsar". For four WDs with masses $<0.2 M_\odot$, Kilic et al. found: "None of these white dwarfs show excess emission from a binary companion, and radial velocity searches will be necessary to constrain the nature of the unseen companions". Our paper suggests that the assumption that there are unseen companions is not necessary.

5.4. Runaway hot subdwarfs

Perhaps the most interesting possibility for the evolutionary state of the donor in a single-degenerate SN Ia is that of a hot subdwarf (sdO or sdB) star. This would naturally allow short or-tal periods ($\sim 1$ h) and also naturally produce extremely low-mass WDs, as recently observed in the runaway WD LP 400–22 (Kawka et al. 2006), which is inferred to have a mass of 0.17 $M_\odot$ and a tangential velocity of $414 \pm 43$ km s$^{-1}$. This would also provide a natural explanation for stars like the runaway hot subdwarf US 708 (discovered by Hirsch et al. 2005). We feel that an SN Ia origin for this object is more satisfying than a scenario combining dynamical ejection from the supermassive black hole in the Galactic centre with the simultaneous merger of two helium WDs (as speculated by Hirsch et al.). The orbital velocity in this case may well be augmented by a kick due to an impulse from the supernova shock (e.g. Marietta et al. 2000).

The evolution of the WD–sdB binary system KPD 1930+2752 (see, e.g. Maxted et al. 2000b) has been investigated by Ergma et al. (2001). They conclude that this system is likely to eventually result in a merger of two WDs (see also

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19 A companion to one of these Kilic et al. LMWDs has been found by such a radial velocity search (Kilic et al. 2007b).
6. Summary and conclusions

We have considered the formation of apparently single LMWDs in general, concluding that the most natural scenario for the formation of single LMWDs is that they are the remnants of donor stars in single-degenerate SNe Ia. Indeed, lone LMWDs should be expected if some single-degenerate SNe Ia do occur with giant donor stars, as inferred from the observations of Patat et al. (2007), notably if the donors lose a significant fraction of their envelopes, as predicted for giant donors (Marietta et al. 2000; Iben & Tutukov 1984). The observations of Maxted et al. (2000a), van Leeuwen et al. (2007) and Kilic et al. (2007a) are all in support of the existence of a population of genuinely single LMWDs.

It seems difficult for the majority of apparently single UCWDs to possess companions, and we have adopted them as a useful sample of single LMWDs. We have integrated a population of SNe Ia donor remnants through a simple Galactic potential and compared the results of those calculations to the known space velocities of apparently single UCWDs. Our results are consistent with the single low-mass UCWDs having once been red-giant donor stars at the time of an SN Ia explosion, as predicted for single LMWDs.

A unified picture emerges in which the high-velocity WDs are remnants of main-sequence donors in SNe Ia (as suggested first by Hansen, 2003), and a kinematically cooler population of single LMWDs were once giant donors in long-period SN Ia progenitors: their longer orbital periods led to a lower runaway velocity whereas their tenuous envelopes were stripped more easily by the supernova ejecta to produce LMWDs.

Furthermore, it seems plausible that runaway LMWDs such as LP 40–22 and runaway hot subdwarf stars such as US 708 originate from donor stars in short-period (∼1 h) SN Ia systems. We will explore this idea more in detail in a future paper.

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References


Geier et al. 2007). However we see no reason why similar systems could not produce an SN Ia via a single-degenerate channel, hence producing such objects as US 708 and then LP 400–22.