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Gravitational excitation of high frequency QPOs *(Research Note)*

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ABSTRACT

We discuss the possibility that high-frequency QPOs in neutron-star binary systems may result from forced resonant oscillations of matter in the innermost parts of the accretion disc, excited by gravitational perturbations coming from asymmetries of the neutron star or from the companion star. We find that neutron-star asymmetries could, in principle, be effective for inducing both radial and vertical oscillations of relevant amplitude while the binary companion might possibly produce significant radial oscillations but not vertical ones. Misaligned neutron-star quadrupole moments of a size advocated elsewhere for explaining limiting neutron star periods could be large enough also for the present purpose.

Key words. accretion, accretion disks – binaries: general – X-rays: binaries – stars: neutron

1. Introduction

Quasi-periodic oscillations of X-ray brightness (QPOs) have been observed in a number of accreting binary systems containing compact objects, both with neutron stars (see van der Klis 2000; Barret et al. 2005, for a review) and with black holes (Remillard & McClintock 2006). They can have low frequencies (Hz) or high frequencies (kHz). The observed kHz frequencies are comparable with the Keplerian and epicyclic frequencies in the inner parts of the accretion disc (Török 2005) and the question arises of whether they might be associated with forced resonant oscillations of the inner disc material. In order to initiate these, some perturbation mechanism would be required. Here, we focus on neutron-star systems and investigate the possibility that gravitational perturbations caused either by the binary companion or by asymmetries of the neutron star might provide this mechanism. (The case of the binary companion could be relevant also for black hole systems.) We note that perturbations coming from the neutron star can only be relevant for the innermost parts of the disc (because of the rapid fall-off of the force with distance) and hence are mainly associated with the picture for kHz QPO frequencies rather than with that for the lower frequencies. It is necessary that the mechanism should be a resonant one since otherwise the response produced would certainly be much too small to be of interest.

The two types of perturbation (from the neutron star and from the binary companion) clearly induce different behaviours: the frequency of the varying force arising from the influence on the disc of the binary companion is essentially equal to the disc rotation-frequency, whereas the main frequency of the force caused by asymmetries of the neutron star is equal to the difference between the rotation frequencies of the disc and the neutron star (Pétri 2006). Resonance occurs at those points where one of the intrinsic oscillation frequencies of the disc matter coincides with the forcing frequency. Following the discussion by Landau & Lifshitz (1976) for test particle motion (which would need to be modified for fluid elements), the growth in amplitude of the oscillations of the particle within the linear regime of forced resonance is given by

$$a(t) = \frac{f_{\rm p}}{2m_0\omega}\,t, \tag{1}$$

where $f_{\rm p}$ is the amplitude of the variations of the force, ω is the frequency and m_0 is the mass of the particle. This linear regime ends when the oscillation amplitude $a(t)$ becomes large enough so that non-linear phenomena and/or dissipative processes become relevant. Note that $a(t)$ grows linearly with time in this regime and so can become quite large even when the variations in the perturbing force are small.

2. Gravitational perturbing force

We consider here the situation illustrated in Fig. 1 with a basically isotropic neutron star of mass $M_{\rm A}$ and radius $R_{\rm A}$ spinning about its rotation axis with angular velocity $\Omega_{\rm A}$ and with the symmetry plane of the accretion disc being orthogonal to the rotation axis. We use spherical polar coordinates (R, θ, φ) with the origin at the centre of the neutron star. Possible neutron-star asymmetry is approximated by a point-like source with mass m located on the surface of the star at angle $\theta = \theta_{\rm A}$. The binary companion

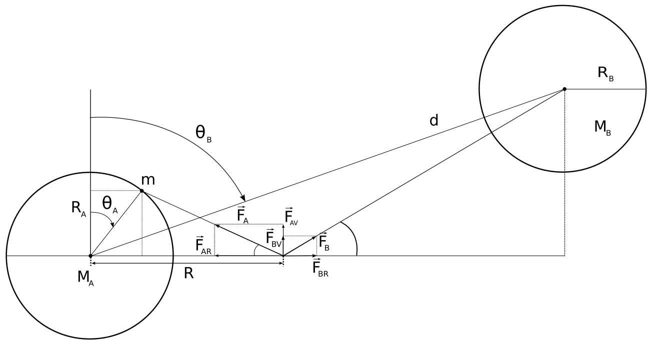

Fig. 1. Schematic picture illustrating the generation of a gravitational perturbing force in an equatorial accretion disc by a single "mountain" on the surface of the neutron star (M_A, R_A) or by the binary companion (M_B, R_B).

(with mass M_B) is taken to be moving on an orbit at a constant distance d from the neutron star at $\theta = \theta_B$. For thin discs, the angular velocity profile $\Omega_d(R)$ is generally well-approximated as being Keplerian (Novikov & Thorne 1973) but for thick (or slim) discs, there is a deviation away from this because of the action of pressure forces (Jaroszyński et al. 1980). We here determine the radial and vertical components of the gravitational force produced by the perturbing sources in a purely Newtonian way. This is an approximation, but we do not expect that a relativistic analysis would greatly change the qualitative features of our results. For simplicity the force will be determined in the equatorial plane, i.e., in the symmetry plane of the disc; this is completely correct for thin, Keplerian discs, and gives good estimates for slim discs. We determine the time evolution of the perturbing force components for a fixed point on the disc with coordinates $(R, \theta = \pi/2, \varphi = \Omega_d t)$, making the restriction $R_A < R < 10\,R_A$ since kHz QPOs are being considered. For the binary companion, we assume $d \gg R, R_B$.

2.1. Neutron-star asymmetry

Here, and in the following, we consider the force acting on a co-moving unit mass element of the accretion disc at a given radius R. Using the quantities $x \equiv R_A/R$ and $\omega_A \equiv |\Omega_A - \Omega_d|$, the vertical component of the perturbing gravitational force is given by

$$F_{AV}(t) = \left(Gm/R_A^2\right) x^3 \cos\theta_A \times \left(1 - 2x\sin\theta_A \cos\omega_A t + x^2\right)^{-3/2}, \quad (2)$$

while the radial component is given by

$$F_{AR}(t) = \left(Gm/R_A^2\right) x^2 (1 - x\sin\theta_A \cos\omega_A t) \times \left(1 - 2x\sin\theta_A \cos\omega_A t + x^2\right)^{-3/2}. \quad (3)$$

The vertical force oscillates around its mean value with amplitude

$$A_V = \left(Gm/R_A^2\right) x^3 \cos\theta_A \times \left[\left(1 - 2x\sin\theta_A + x^2\right)^{-3/2} - \left(1 + x^2\right)^{-3/2}\right] \quad (4)$$

and the radial force oscillates with amplitude

$$A_R = \left(Gm/R_A^2\right) x^2 \left[(1 - x\sin\theta_A)\left(1 - 2x\sin\theta_A + x^2\right)^{-3/2} - \left(1 + x^2\right)^{-3/2}\right]. \quad (5)$$

These oscillations have an anharmonic character which means that, when Fourier analysed, they show both the basic frequency ω_A and also some additional frequencies related to it.

2.2. Binary companion

The binary companion is taken to be orbiting the neutron star at a constant distance d, with angular velocity Ω_B. The vertical and radial components of the perturbing force acting on the accreting material are then given by

$$F_{BV}(t) = \left(GM_B/d^2\right)\cos\theta_B \times \left[1 - 2(R/d)\sin\theta_B \cos\omega_B t + (R/d)^2\right]^{-3/2} \quad (6)$$

$$F_{BR}(t) = (GM_B/d^2)\left[\sin\theta_B \cos\omega_B t - (R/d)\right] \times \left[1 - 2(R/d)\sin\theta_B \cos\omega_B t + (R/d)^2\right]^{-3/2}, \quad (7)$$

where the relative angular velocity $\omega_B = |\Omega_B - \Omega_d| \approx \Omega_d$. In general, these relations represent anharmonically oscillating forces but we are taking $R/d \ll 1$ and they then reduce to an approximate form which is harmonic with frequency $\omega_B \approx \Omega_d$. The vertical force oscillates around its mean value with amplitude

$$B_V \approx 3(GM_B/d^2)(R/d)\cos\theta_B \sin\theta_B \quad (8)$$

and the radial force oscillates with amplitude

$$B_R \approx (GM_B/d^2)\sin\theta_B. \quad (9)$$

3. Magnitudes of the neutron-star asymmetries

In this section we give estimates for the magnitudes of neutron-star asymmetries arising in different ways. We first consider classical crystalline mountains and magnetically-confined accretion columns; both of these are found to be inadequate for the present purposes, however. We then turn to some different observationally-motivated possibilities which seem to be more promising.

3.1. Isolated crystalline mountains

Assuming that the basic nature of a mountain on the surface of a neutron star is the same as for mountains on planets, the pressure at the base of the mountain needs to be less than the maximum shear stress of the surface material. This pressure is given by $P_{mnt} = \rho_{mnt} g_{ns} h_{mnt}$, where ρ_{mnt} is the average density of the material in the mountain, g_{ns} is the surface gravity of the neutron star and h_{mnt} is the height of the mountain. The base of the mountain would be located at the outermost solid surface layer of the neutron star. The relevant density to take for this layer is rather uncertain; we will take it as being $\sim 10^6\,\mathrm{g\,cm^{-3}}$ and put ρ_{mnt} equal to that. The surface gravity is given by $g_{ns} = GM_A/R_A^2$. For a neutron star of mass $1.4\,M_\odot$ and radius 10 km, g_{ns} is $1.87 \times 10^{14}\,\mathrm{cm\,s^{-2}}$.

Following Strohmayer et al. (1991), the shear modulus of the neutron star surface material is taken to be

$$\mu = \frac{0.1194}{1 + 1.781 \times (100/\Gamma)^2} \frac{n(Ze)^2}{a}, \quad (10)$$

where n is the number density of ions, a is the inter-ionic distance, Z is the atomic number of the dominant ionic species, and Γ is the Coulomb coupling parameter ($\Gamma > 10^3$ for all practical purposes). The maximum shear strain in the surface of the neutron star has been calculated to be $\Theta \sim 10^{-5}$–10^{-3} (Smoluchowski & Welch 1970), although there are suggestions that it might also be as high as 10^{-2} (Ushomirsky et al. 2000). Taking typical values $Z = 26$, $n \simeq 10^{28}\,\mathrm{cm}^{-3}$ and $a \simeq 7\times10^{-10}$ cm, the corresponding maximum shear stress is then $S = \mu\Theta \simeq 10^{18}$–$10^{21}\,\mathrm{dyn\,cm}^{-2}$. The maximum height of the mountain is obtained by setting $P_{\rm mnt} = S$, which gives $h^{\rm max}_{\rm mnt}$ in the range 0.01–10 cm. Taking the highest of these values, we then get the maximum possible mass of the mountain as being

$$m_{\rm mnt} \sim \rho_{\rm mnt}(h^{\rm max}_{\rm mnt})^3 \sim 10^9\,\mathrm{g} \sim 10^{-24}\text{–}10^{-25}\,M_{\rm A} \tag{11}$$

which is too small to be relevant here.

3.2. Accretion columns

For a neutron star with a strong magnetic field, accreting matter close to it can be diverted away from the equatorial plane and form accretion columns above the magnetic poles (Woosley & Wallace 1982; Hameury et al. 1983). If the amount of matter in the columns is sufficiently large, this can provide another source of neutron-star asymmetry which could again be modelled in terms of "mountains" (probably two symmetric ones in this case). This has been invoked in connection with gravitational-wave emission (Melatos & Payne 2005) and we check here whether it could also be relevant in the present context.

The column height can be determined from the condition that the flow will start to spread out sideways when the pressure of the matter in the column $P_{\rm ac}$ becomes large enough to bend the magnetic field lines outwards, typically when it is about a hundred times greater than the confining magnetic pressure, i.e. when

$$P_{\rm ac} \sim 4\times10^{24}\,\mathrm{dyn\,cm}^{-2}\left(B_{\rm s}/10^{12}\,\mathrm{G}\right)^2, \tag{12}$$

where $B_{\rm s}$ is the strength of the surface dipole field (Brown & Bildsten 1998). For having hydrostatic equilibrium, this pressure should be the same as that elsewhere at the same level in the crust. In the density range $10^6\,\mathrm{g\,cm}^{-3} \le \rho \le 10^{10}\,\mathrm{g\,cm}^{-3}$ the pressure can be expressed using the fitting formula $\log P = 13.65 + 1.45\log\rho$ (Baym et al. 1971) and the relation between the field strength and the density at the bottom of the column is then given by $(\rho_{\rm bot}/10^6\,\mathrm{g\,cm}^{-3}) \sim 36(B_{\rm s}/10^{12}\,\mathrm{G})^{1.38}$. Using this, the scale height of the column, $h_{\rm ac}$ is then

$$h_{\rm ac} \sim P_{\rm ac}/(\rho_{\rm bot}\,g_{\rm ns}) \sim 10^3\mathrm{cm}\left(B_{\rm s}/10^{12}\,\mathrm{G}\right)^{0.62}. \tag{13}$$

Following Shapiro & Teukolsky (1983), the cross-sectional area of the column is estimated as

$$A_{\rm xs} \sim 10^{10}\,\mathrm{cm}^2\left(\frac{B_{\rm s}}{10^{12}\,\mathrm{G}}\right)^{-4/7}\left(\frac{M_{\rm A}}{1.4\,M_\odot}\right)^{1/7} \times\left(\frac{R_{\rm A}}{10^6\,\mathrm{cm}}\right)^{9/7}\left(\frac{\dot M}{10^{-9}\,M_\odot/\mathrm{yr}}\right)^{2/7}, \tag{14}$$

where $\dot M$ is the accretion rate, which we normalise to a typical value for LMXBs. The mass of the column is then given by

$$\begin{aligned} m_{\rm ac} &\sim \rho_{\rm bot}A_{\rm xs}h_{\rm ac}\\ &\sim 10^{-13}\,M_{\rm A}\left(\frac{B_{\rm s}}{10^{12}\,\mathrm{G}}\right)^{10/7}\left(\frac{M_{\rm A}}{1.4\,M_\odot}\right)^{-6/7}\\ &\times\left(\frac{R_{\rm A}}{10^6\,\mathrm{cm}}\right)^{9/7}\left(\frac{\dot M}{10^{-9}\,M_\odot/\mathrm{yr}}\right)^{2/7} \end{aligned} \tag{15}$$

which is again very small, even for a field as high as 10^{12} G. The continued existence of a roughly Keplerian accretion disc down to small radii, as required for our QPO picture, needs the magnetic field to be suitably low and so magnetically-confined accretion columns do not seem to be relevant for the present purposes.

3.3. Quadrupole moments inferred from limiting neutron star spin rates

Bildsten (1998) noted that the spin frequencies for many accreting weakly-magnetised neutron stars were thought to lie in a rather narrow range around 300 Hz, which is a much lower frequency than that corresponding to centrifugal break-up. Since these objects are thought to have been accreting for long enough so as to gain sufficient angular momentum to reach the break-up limit, it seemed that some mechanism was halting the spin-up. Despite the fact that the spread of spin frequencies is now thought to be less peaked (see Manchester et al. 2005), the issue of why spin-up should be halted before the break-up limit is reached remains a relevant one. Bildsten suggested that this might be caused by the accretion torque becoming balanced by a gravitational-wave torque resulting from an asymmetry of the neutron star. Taking the asymmetry to be represented by an $l = m = 2$ perturbation, one finds that the magnitude of the misaligned quadrupole moment required for attaining this equilibrium at a frequency $\nu_{\rm s}$ is given by

$$Q_{\rm eq} = 3.5\times10^{37}\,\mathrm{g\,cm}^2\left(\frac{M_{\rm A}}{1.4\,M_\odot}\right)^{1/4}\left(\frac{R_{\rm A}}{10^6\,\mathrm{cm}}\right)^{1/4} \times\left(\frac{\dot M}{10^{-9}\,M_\odot/\mathrm{yr}}\right)^{1/2}\left(\frac{\nu_{\rm s}}{300\,\mathrm{Hz}}\right)^{-5/2}. \tag{16}$$

In terms of our simplified model of representing the asymmetry by means of point masses on the surface of an otherwise spherical neutron star, this corresponds to

$$m_{\rm quad} \sim 10^{-8}\,M_{\rm A}\left(\frac{M_{\rm A}}{1.4\,M_\odot}\right)^{-3/4}\left(\frac{R_{\rm A}}{10^6\,\mathrm{cm}}\right)^{-7/4} \times\left(\frac{\dot M}{10^{-9}\,M_\odot/\mathrm{yr}}\right)^{1/2}\left(\frac{\nu_{\rm s}}{300\,\mathrm{Hz}}\right)^{-5/2}. \tag{17}$$

Various mechanisms have been suggested in the literature for producing such a value (or even higher), involving global deformations of the neutron star rather than an isolated mountain on the surface (Bildsten 1998; Ushomirsky et al. 2000; Haskell et al. 2006, etc.). We will not enter into details of this here but we note that values of the deformation large enough to be capable of explaining the limitation of neutron star spin-up, as suggested by Bildsten, may also be large enough for the present purposes: in the next section, we take the value for the point mass given by Eq. (17) with the canonical parameter values, and check whether this same value $\sim10^{-8}\,M_{\rm A}$ might *also* be large enough to be relevant for inducing the QPO behaviour.

4. Discussion and conclusions

Here, we discuss whether the effect of the neutron-star asymmetries or of the binary companion could be large enough to account for excitation of the QPO phenomenon. In our simplified picture (cf. Eq. (1)), the excitation time for the amplitude a of resonant oscillations to grow to a particular value is given by

$$t_{\rm ex} = \left(\frac{a}{R}\right)\left(\frac{\alpha}{\pi}\right)\left(\frac{f_{\rm p}}{f_0}\right)^{-1} \tau_{\rm K}, \qquad (18)$$

where $f_{\rm p}$ is the amplitude of the perturbing force acting on unit mass, $f_0 = GM_{\rm A}/R^2$ is the main gravitational force from the central object, $\tau_{\rm K} = 2\pi/\Omega_{\rm K}$ is the period of circular Keplerian motion at the location being considered and the epicyclic frequency of the perturbation being excited is $\omega = \alpha\Omega_{\rm K}$. The dimensionless radial and vertical "epicyclic functions" satisfy $\alpha \leq 1$ everywhere. Note that since $\tau_{\rm K} \sim 10^{-3}$ s, the amplitude amplification in 1 s is $\sim 10^3$. The ratio a/R needs to grow to $> 10^{-3}$ in order to potentially explain the QPO behaviour and it would need to do that within $\sim 10^3$ s to account for the QPO phenomena seen in atoll sources.

In the case of a binary companion, for the radial perturbing force one has

$$\frac{f_{\rm p}}{f_0} \sim \left(\frac{M_{\rm B}}{M_{\rm A}}\right)\left(\frac{R}{d}\right)^2 \sim 10^{-9}\left(\frac{M_{\rm B}}{0.1\,M_{\rm A}}\right)\left(\frac{R}{10^6\,{\rm cm}}\frac{10^{10}\,{\rm cm}}{d}\right)^2, \qquad (19)$$

where we have normalised to typical parameter values. This could produce $a/R \sim 10^{-3}$ at times $t_{\rm ex} \lesssim 10^3$ s, but there is a problem for it producing resonances in the innermost parts of the disc because of having $\omega_{\rm B} \approx \Omega_{\rm K}$. The corresponding vertical force is smaller by at least four orders of magnitude and is clearly irrelevant. (Note that the vertical force is a tidal force whereas the radial one is a direct gravitational attraction; also, $\theta_{\rm B}$ is probably rather close to $\pi/2$.)

For the neutron star asymmetries, we focus on the case of the misaligned quadrupole moments where $f_{\rm p}/f_0$ can be $\sim 10^{-8}$ in the inner parts of the disc for both radial and vertical oscillations (taking $m_{\rm quad} = 10^{-8}\,M_{\rm A}$). For this, a/R could reach 10^{-3} in $\lesssim 10^2$ s, which encourages further investigation of this scenario. We conclude that at least one of the types of gravitational perturbation considered in this paper might provide a plausible mechanism for inducing kHz QPO behaviour although many details remain to be worked out (in particular concerning the response of the fluid medium and the production of the luminosity variations). The influence of the binary companion could possibly be effective in providing the perturbations but the more likely possibility is that they might be produced by the neutron-star asymmetries. It is striking that the same magnitude for the misaligned quadrupole moment as advocated elsewhere for explaining limiting neutron star periods, seems also to give a plausible mechanism for inducing QPO behaviour.

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