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Period study of TW Draconis*

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ABSTRACT

Context. TW Draconis is one of the most well known Algol-type eclipsing binaries. There is significant evidence for miscellaneous interacting physical processes in the system, which manifest themselves as for example period and light curve changes. We study time series data for the star accumulated over the past 150 years to detect changes in minima timings.

Aims. Using 561 available minima timings, we compile an extended O–C diagram analysis. A detailed description of period changes allows us to construct the true phased light curve at any moment.

Methods. By applying weighting nonlinear robust regression the timing residuals were analysed. The weights of individual types of minima were determined iteratively. The ephemeris are expressed in the orthogonal or quasi-orthogonal forms allowing us for example to determine directly uncertainties of photometric phases for any given moments.

Results. The timing residuals (according to the linear ephemeris) display two stages of differing behaviour in 1858–2007. The first part ends around 1942. It is characterised by more or less smooth linear/parabolic course of timing residuals. In 1858–1905, the period was almost constant $P = 2\overset{\mathrm{d}}{.}806513(9)$, but, in 1905–1942, the period increased with the rate of $5.69(5) \times 10^{-8}$ d year^{-1}. We interpret this as a result of continuous mass exchange between components at the mean rate of $6.9 \times 10^{-7}\ M_{\odot}$ year^{-1}. Since 1942, the system has been showing alternating and shortening period changes with the length of a cycle of about two decades, modulated by short-term periodic variations. The latter small variations with the period 6.5 years are caused by the light-time effect due to the presence of a third low-mass body in the system. Major oscillations can be explained as consequences of quadrupole moment variation in the system.

Key words. stars: binaries: eclipsing – stars: individual: TW Dra – stars: fundamental parameters

1. Introduction

TW Draconis (also HD 139319, BD+64 1077, HIP 76196, $\alpha = 15^{\mathrm{h}}33^{\mathrm{m}}51\overset{\mathrm{s}}{.}1$, $\delta = 63°54'26''$, J2000.0) is a well-known, relatively bright and frequently observed Algol-type eclipsing binary. Its variability was discovered by Cannon (see Pickering 1910). The variable star is also an A-component of the visual binary ADS 9706. Light variations of TW Dra ($\Delta B \simeq 2.3$ mag) are caused predominantly by eclipses of a hot main-sequence star A5V by a cooler and fainter sub-giant component K0III in the primary minimum. The relatively long eclipses (11.5 h) repeat with the orbital period close to 2.807 days. The system has been studied both photometrically and spectroscopically several times (e.g. Baglow 1952; Walter 1978; Papoušek et al. 1984; Smith 1949; Popper 1989). In brief, although TW Dra is a known and well-studied eclipsing binary, the complete solution of this unusual system has not been published until now.

The goal of this study is to demonstrate new approaches in a methodology for period changes studies in eclipsing binary systems.

2. Historical summary

In spite of the fact that the eclipsing variability of TW Dra was discovered in 1910, the O–C diagram illustrates changes in orbital period over the past 150 years (see Fig. 1).

Orbital period changes in TW Dra were suggested shortly after the discovery of its variability. However, the first detailed study of period change was completed by Slovokhotova (1954). She described the period change and presented a quadratic term of light ephemeris from all previously published minima timings excluding the two oldest ones:

$$\mathrm{Pri.Min.} = 2\,423\,711.321 + 2\overset{\mathrm{d}}{.}80664\,E + 0\overset{\mathrm{d}}{.}0195\,(E/1000)^2.$$

Schneller (1966) found that a period increase stopped abruptly after 1950. Pohl (1970) described a shortening of orbital period after 1960. Todoran (1972) attempted to explain the period changes using different mechanisms, e.g. using a third body of very high mass. Tremko & Kreiner (1981) proposed the Biermann-Hall (1973) mechanism as an explanation for alternating period change. Walter (1978) uncovered a sudden period change during his photometric study.

Abhyankar & Panchatsaram (1984) explained period changes using the *Light-Time Effect* (hereafter LITE) hypothesis and solutions from a three- up to five-body system were

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* Table 1 is only available in electronic form at the CDS via anonymous ftp to `cdsarc.u-strasbg.fr (130.79.128.5)` or via `http://cdsweb.u-strasbg.fr/cgi-bin/qcat?J/A+A/489/321`

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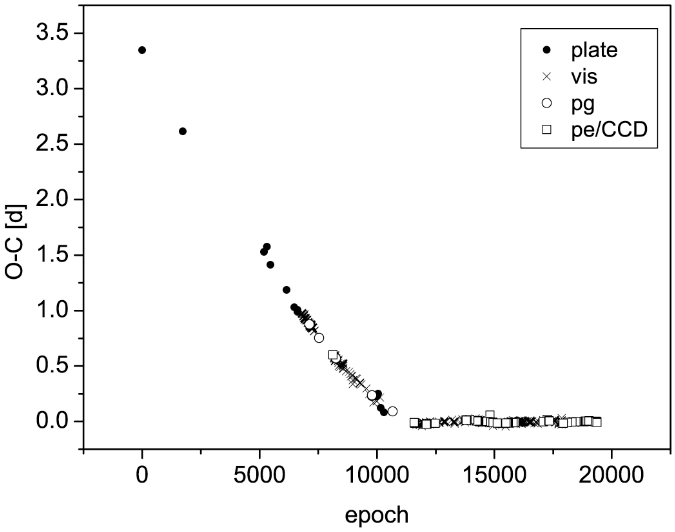

Fig. 1. O–C diagram of TW Dra in 1858–2007. The O–C values were calculated using the following ephemeris: PriMin. = 24 51675.517 + $2\overset{\mathrm{d}}{.}8068559$ (E – 18471).

discussed. Wolf (1990) studied O–C changes in an interval of 4500 epochs and explained their changes by LITE caused by the third body with an orbital period of 8854 days, eccentricity $e = 0.168$ and minimal mass of 0.74 $M_\odot$ (assuming $M_1 = 1.90\ M_\odot$ and $M_2 = 0.82\ M_\odot$). The last period study was published by Qian & Boonrucksar (2002). They used the main trend for the quadratic change according to the following light ephemeris:

$$\text{Pri.Min.} = 2\,444\,136.253 + 2\overset{\mathrm{d}}{.}80676153\,E + 1\overset{\mathrm{d}}{.}59 \times 10^{-8}\,E^2.$$

Subtracting the quadratic term, residuals as a few sudden changes of orbital period were interpreted.

3. Observations

Many sources were investigated for the timings of minima, including the database of Kreiner (2004), Lichtenknecker-Database of BAV [1], O–C gate of VSS of CAS [2], and the personal archive of MZ. The data in archives and databases were checked against those of original sources.

We have used minima timings from the archives and the newly determined minima from the Hipparcos and Tycho datasets and from Baker (1921) (see upper part in Table 2), and results from our observations. In 2001–2007, 44 timings of CCD minima in *BVRI* bands were obtained and 15 mean (in regard to colour) timings were calculated (see Table 2). CCD observations acquired at four observatories were utilised:

- N. Copernicus Observatory and Planetarium in Brno, Czech Republic: CCD camera SBIG ST-7 mounted on 0.4-m reflector (observer Zejda);
- Hlohovec Observatory, Slovakia: 0.6-m reflector and CCD camera SBIG ST-9 (observers Chrastina, Szász);
- private observatory of P. Svoboda, Brno, Czech Republic: 0.034-m refractor with CCD camera SBIG ST-7;
- private observatory of L. Brát, Pec pod Sněžkou, Czech Republic: 0.08-m refractor with CCD camera SBIG ST-8.

Table 2. New times of minimum light of TW Dra.

JDhel –2400000	Epoch	O–C [days]	Min. type	Filter	Observer
20 977.6908	–10937.0	0.7568	p	pg	Bak
48 456.0539	–1147.0	0.0006	p	V_T	Hip
48 464.4761	–1144.0	0.0023	p	Hp	Hip
48 481.3160	–1138.0	0.0010	p	B_T	Hip
53 595.4080	684.0	0.0016	p	*BVRI*	MC, GS
53 609.4421	689.0	0.0014	p	*VRI*	MZ
53 943.4545	808.0	–0.0021	p	*VRI*	MZ
53 957.4872	813.0	–0.0019	p	*B*	PS
53 985.5578	823.0	–0.0022	p	*B*	PS
53 988.3636	824.0	–0.0023	p	*B*	PS
53 988.3642	824.0	–0.0021	p	*VRI*	MZ
53 999.5931	828.0	–0.0016	p	*B*	PS
54 002.3979	829.0	–0.0027	p	*B*	PS
54 002.3986	829.0	–0.0020	p	*VRI*	MZ
54 026.2572	837.5	–0.0016	s	*I*	PS
54 096.4335	862.5	0.0033	s	*I*	PS
54 190.4574	896.0	–0.0025	p	*R*	LB
54 190.4569	896.0	–0.0030	p	*B*	PS
54 197.4690	898.5	–0.0081	s	*I*	PS

Notes: Hip = new timing of minima determined using Hipparcos and Tycho, datasets.
Bak = new timing of minima obtained from photographic observations Baker (1921).
Observers: LB = L. Brát, MC = M. Chrastina, PS = P. Svoboda, GS = G. Szász, MZ = M. Zejda.

The small telescope settings in Brno and Pec observatories are able to acquire frames of sufficient signal-to-noise ratios suitable for photometry using longer exposure times (typically 180–240 s).

We collected 561 minima timings, which are plotted in an O–C diagram in Fig. 1. The complete information on minima timings in heliocentric Julian dates, as summarised in Table 1, is only available in electronic form at the CDS. This contains the following: Col. 1, HJD of minimum; Col. 2, a designation on the accuracy of observations; Col. 3, type of minimum (p – primary, s – secondary); Col. 4, type of observations (plate, visual, photographic, photoelectric, CCD); Col. 5, observer(s); Col. 6, source of information. We should point out, that only 8 of these 561 timings are secondary and 6 of them were obtained in the past few years. The two oldest secondary minima are not sufficiently accurate. The oldest one represents bad superposition of two parts and the other is based on Hipparcos data of only two points on the ascending branch. All secondary minima were omitted from our analysis.

Although the variability was discovered in 1910, older observations, from as early as 1858, are available. There are two significant data gaps corresponding to World War I and II. During the time gap due to World War II, significant changes occurred in the photometric period. All data are therefore separated into two time intervals, divided by the year 1942.

Ephemeris of TW Dra was found by the thorough analysis of O–C diagrams with 15 own, 4 calculated, and all published timings of minimum light obtained during the past 150 years. We found that the time dependence of O–C is complex, which implies that significant changes in the interacting system have occurred. Nevertheless, we first considered the dependence without any a priory presumption about physical reasons for the observed behaviour. For the fit, we use our own weighted robust regression with iteratively adapted weights of different types of observations and observational methods. We used preferably orthogonal

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[1] `http://www.bav-astro.de/LkDB/index.html`
[2] `http://var.astro.cz/ocgate/`

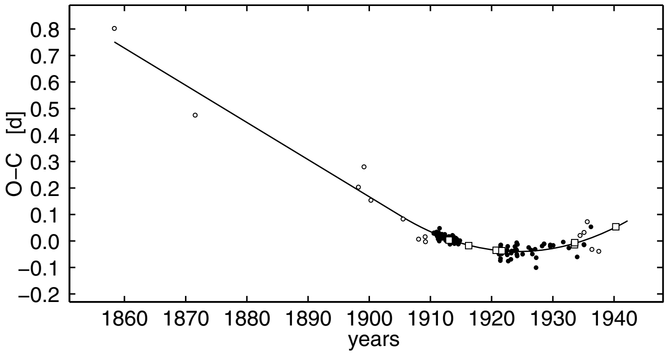

Fig. 2. O–C diagram of TW Dra in 1858–1942. Plate dimming are drawn by small circles, visual observations by dots and sets of photographic or photoelectric measurements by squares.

or quasi-orthogonal models for the mathematical expression of ephemeris (Mikulášek 2007b). This provides straightforward estimates of the uncertainty in phase predictions and the accuracy of individual parameters describing the adopted relations.

4. Time interval before 1942

The time interval before 1942 (JD ≃ 2 417 065–2 430 000) contains 141 analysed timings of light minima: 9 dimming in photographic plates (hereafter plate dimming), 125 visual timings, and 7 photoelectric and photographic minima (see Fig. 2). Nevertheless, the quality of visual observations varied considerably during the time span.

This data-set of visual observations was split into two subgroups by time interval: 1905–1920, which contains 76 visual minima timings and 1920–1942, with 49 observations. Adopting a quadratic model of period change during the latter time interval, evaluation of proper weights for the individual set of observations obtained by different techniques was possible. In the first run, the same weight was assumed for all types of data and the standard deviation s_i for each datasets in our model was calculated. Then, we derived the individual dataset weights $1/s^2$ and calculated a new set of standard deviations. Repeating this way, improved weights for the data in our model were iteratively determined. Relative weights for the following datasets were calculated: plate dimming $w_1 = 1$, visual data 1920–1942 $w_2 = 4$, visual data 1905–1920 $w_3 = 28$, and photoelectric or photographic series $w_4 = 266$(!). We propose that the difference of a factor of 7 between the visual dataset subgroups could be caused by different quality levels of observers and observational methods they used.

The period varied throughout the time interval 1905–1942. Assuming a quadratic model, the proper light ephemeris is given by the following quadratic orthogonal form (for details see Mikulášek 2007a,b)

$$\text{Pri.Min.} = M_0 + \tilde{P}\,E' + \frac{\dot{P}\,\tilde{P}}{2}\left(E'^2 - \frac{\overline{E'^3}}{\overline{E'^2}}\,E' - \overline{E'^2}\right), \tag{1}$$

where the basic minimum of the centre of gravity for the data is $M_0 = 2\,422\,032.9979(7)$, the period $\tilde{P} = 2\overset{\mathrm{d}}{.}8066209(6)$ day, the time derivative of period $\dot{P} = 1.53(5)\times10^{-8} = 5.69\times10^{-8}$ d/year, where numbers in brackets here and hereafter represent the standard errors in the measured parameter values. Furthermore, the epoch $E' = E - \overline{E}$, where $\overline{E} = 7910$, $\overline{E'^3}/\overline{E'^2} = 1360$ and $\overline{E'^2} = 1.39 \times 10^6$. In this time interval the period $P(E')$ varied according to the relation

$$\begin{aligned} P &= \tilde{P}\left[1 + \dot{P}\left(E' - \frac{\overline{E'^3}}{2\,\overline{E'^2}}\right)\right] \\ &= 2\overset{\mathrm{d}}{.}8066209 + 4\overset{\mathrm{d}}{.}294 \times 10^{-8}\ (E' - 680). \end{aligned} \tag{2}$$

The preceding time interval for the years 1858–1905 contains only 9 plate dimmings. Timings residuals in this interval were described in the simplest way – by a linear function sharing one point with the quadratic function described above. In such an approximation only one free parameter is required – the mean orbital period $P_1 = 2.806513(9)$ in 1858–1905, which produces the relevant light ephemeris as follows

$$\begin{aligned} \text{Pri.Min.} &= M_0 + P_1\,(E - E_1) \\ &= 2\,417\,065.3683(29) + 2\overset{\mathrm{d}}{.}806513(9)\,(E - 6140)\,, \end{aligned}$$

where $E = 0$ corresponds to the time of the first dimming in our data in 1858. The period increased by 1.94×10^{-4} day, during the entire epoch 1905–1942. Using the well–known formula (Kwee 1958)

$$\frac{1}{M}\frac{\mathrm{d}M}{\mathrm{d}t} = \frac{q}{3P(q^2-1)}\frac{\mathrm{d}P}{\mathrm{d}t}, \tag{3}$$

where $M = M_1 + M_2$ is a mass of binary system and $q = M_2/M_1$, the rate of mass exchange in the system, according to a prediction of conservative mass transfer and no exchange between rotational and orbital angular momentum, can be estimated. Assuming component masses $M_1 = 1.9\ M_\odot$ and $M_2 = 0.82\ M_\odot$ (Al-Naimiy & Al-Sikab 1984), we found a mass exchange rate of $3.9 \times 10^{-7}\ M_\odot$/year. This short epoch of very high mass transfer was followed by the time interval supposedly called “the relaxation epoch”.

5. Time interval 1942–2007

The second time interval 1942–2007 is shorter than the previous one, but its data coverage is better and the data quality is significantly higher. The O–C curve shows a feature similar to a damped oscillator; however, there is a significant difference. Both the amplitude of timing residuals and their period of oscillations decrease (Fig. 3). The following mathematical model was therefore developed to describe this phenomenon.

The periodicity of timing residual variations can be described simply by a function $\cos(2\pi\vartheta)$, where ϑ is time expressed by the number of elapsed cycles and phase in the actual cycle. The duration Θ of each further cycle is shortened by the same relative part

$$\dot{\Theta} = \frac{\mathrm{d}\Theta}{\mathrm{d}\vartheta}\frac{1}{\Theta} = \text{const.} \tag{4}$$

It is clear that for the chosen reference timing T_0 of O–C curve extremum is $\Theta = \Theta_0$ and $\dot{\Theta} = \dot{\Theta}_0$. Then we find that

$$\Theta = \frac{\mathrm{d}t}{\mathrm{d}\vartheta} = \Theta_0\ \mathrm{e}^{\dot{\Theta}_0\vartheta}. \tag{5}$$

By solving the Eq. (5), the following expression is obtained

$$\vartheta = \frac{1}{\dot{\Theta}_0}\ln\left[1 + \frac{\dot{\Theta}_0}{\Theta_0}\,(t - T_0)\right], \tag{6}$$

where t means flowing time. The decrease in the amplitude of change in the timing residuals with time can be expressed by a

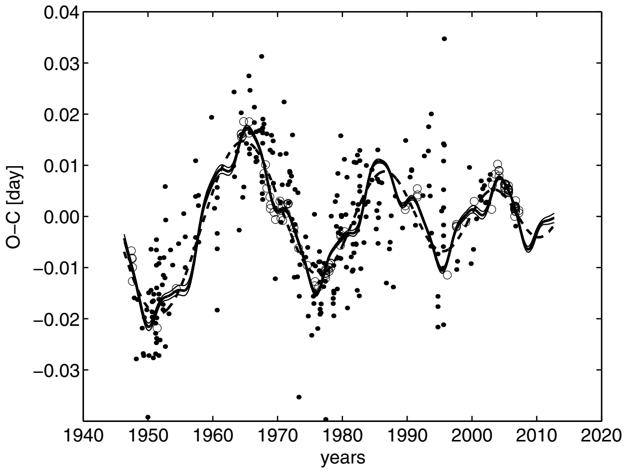

Fig. 3. The O–C diagram of TW Dra in 1942–2007. Visual observations are drawn by points and CCD or photoelectric observations by empty circles. The dashed line shows the fit to major period changes according to Eq. (8) without a quadratic term and the full line shows the fit including changes caused by the third body.

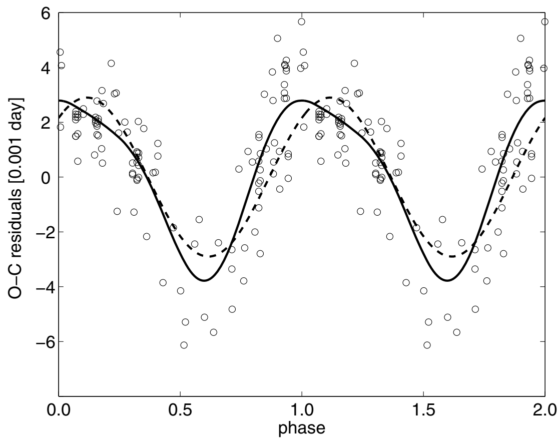

Fig. 4. O–C residuals after subtracting major changes in orbital period for the system TW Dra. Only photoelectric and CCD primary minima are presented here. Phase is calculated according to the ephemeris $T_{02} + P_2 E$ (see Eq. (10)). The continuous line represents the numerical solution according to Eq. (9); broken line shows LITE solution according to Eq. (13).

multiplicative term $B\left(\frac{\Theta}{\Theta_0}\right)^2$, where B is the semi-amplitude of the O–C value change in time T_0. The variations in timing residuals can then be described by the following wavy quadratic function

$$\text{Pri.Min.} = M_0 + \tilde{P}\,E' + \frac{\tilde{P}\dot{P}}{2}\left(E'^2 - \frac{\overline{E'^3}}{\overline{E'^2}}E' - \overline{E'^2}\right) + B\left(\frac{\Theta}{\Theta_0}\right)^2 \cos(2\pi\vartheta), \qquad (7)$$

where E' is the centre of gravity of data in selected interval as mentioned above ($E' = E - \overline{E} = E - 16634$). After some modifications

$$\text{Pri.Min.} = M_0 + \tilde{P}\,E' + \frac{\dot{P}\tilde{P}}{2}\left(E'^2 - \frac{\overline{E'^3}}{\overline{E'^2}}E' - \overline{E'^2}\right) \qquad (8)$$
$$+B\left[1 + \dot{\Theta}_0\frac{E'-E_0}{\Theta_0}\right]^2 \cos\left[\frac{2\pi}{\dot{\Theta}_0}\ln\left(1 + \dot{\Theta}_0\frac{E'-E_0}{\Theta_0}\right)\right],$$

where E_0 is the epoch of a basic extremum, and Θ is expressed in orbital periods.

Applying the robust nonlinear regression, the quadratic term for this epoch can be neglected. However, the O–C residual diagram appears to indicate another periodic term (see Fig. 4). The quadratic term was finally excluded and the new term included, to yield the following equation

$$\begin{aligned}\text{Pri.Min.} &= M_0 + \tilde{P}\,E'\\ &+ B_1\left[1 + \dot{\Theta}_0\frac{E'-E_{01}}{\Theta_0}\right]^2 \cos\left[\frac{2\pi}{\dot{\Theta}_0}\ln\left(1 + \dot{\Theta}_0\frac{E'-E_{01}}{\Theta_0}\right)\right]\\ &+ B_2\cos\left[\frac{2\pi\,(E'-E_{02})}{P_2}\right] \qquad (9)\\ &+ B_2\,b_2\left\{\sin\left[\frac{2\pi\,(E'-E_{02})}{P_2}\right] - \frac{1}{2}\sin\left[\frac{4\pi\,(E'-E_{02})}{P_2}\right]\right\},\end{aligned}$$

where, in the first (original) term describing the major oscillations, index "1" was added, and, in the following new terms describing small oscillations caused probably by the third body, index "2" was used. By applying the robust nonlinear weighted regression ($w_1 = 1$ for visual and $w_2 = 35$ for CCD and photoelectric minima), the following basic light ephemeris were derived:

$M_0 = 2\,446\,519.31969(18)$, $\tilde{P} = 2\overset{\mathrm{d}}{.}80685567(9)$

and the coefficient values

$$\begin{aligned} B_1 &= -0.01135(24)\ \text{days},\\ \dot{\Theta}_0 &= -0.257(7),\\ \Theta_0 &= 2816(16)\tilde{P} = 21.64(12)\ \text{years},\\ E_{01} &= -1247(13),\\ T_{01} &= 2\,443\,019(33) \sim 1976.7,\\ B_2 &= 0.00279(24)\ \text{days}, \qquad (10)\\ b_2 &= 0.51(13),\\ P_2 &= 847(4)\tilde{P} = 2377\ \text{days} = 6.51(3)\ \text{years},\\ E_{02} &= -162(15),\\ T_{02} &= 2\,446\,065(42) \sim 1985.0.\end{aligned}$$

An "effective amplitude" (Mikulášek et al. 2007) can express the true amplitude of variations caused by the possible third body using the equation

$$A_{\text{eff}} = 2B_2\sqrt{1 + \frac{5}{4}b_2^2}, \qquad (11)$$

which yields the value $A_{\text{eff}} = 0.0064(24)$ day.

Regarding the corresponding coefficients, we conclude that the major oscillations in the O–C diagram will disappear in the years 2051 ± 9. We predict that the closest extrema will be in 2010.5 ± 1.5 and 2021.9 ± 2.2, respectively. The hypothetic third body causes a cyclic undulation of the main O–C course of period 6.5 years with constant amplitude. The final fit is shown in Fig. 3.

6. Causes of O–C variations

The exact determination of the period and its variations have provided unique tools in studying changes in stellar structure, geometry, and stellar evolution. This variation is manifested in

Table 3. Changes of quadrupole moments in the past.

Interval years	$\lvert\Delta P\rvert$ [10^{-5} day]	$\lvert\Delta Q\rvert$ [10^{43} kg m^2]
until 1942	19.4	88.6
1952–1965	1.98	9.0
1965–1976	1.75	8.0
1976–1987	1.52	6.9
1987–1996	1.34	6.1
1996–2003	1.18	5.4

the O–C diagram. There are several different mechanisms responsible for variations in O–C values: (i) mass transfer between components; (ii) changes in the quadrupole moment as a consequence of magnetic activity; and a (iii) third body orbiting an eclipsing pair.

6.1. Mass transfer

Mass exchange in semi-detached systems produces significant period changes – the period can both decrease and increase (Kwee 1958). Biermann & Hall (1973) suggested that sudden mass transfer bursts, which store temporarily the orbital angular momentum into the rotation of a hot accretor, can produce cyclic period changes.

A sudden conservative mass transfer in the binary system produces the phase of a rapid period decrease followed by the phase of a slow increase as the tidal forces return angular momentum back to the orbital motion. An unconservative mass transfer leads to the non-cyclic period changes. In the case of TW Dra the Biermann-Hall mechanism was used by Tremko & Bakos (1976); however, recent observations do not support this explanation. In the 90's of the last century, the hypothesis of mass transfer as a cause of cyclic period variations started to be displaced by the models involving magnetic field of binary components.

6.2. Influence of magnetic field

Magnetically induced period variations were widely described in Applegate (1992). The changes in a differential rotation modify the oblateness of the active secondary star and thus the gravitational quadrupole moment varies. New developments in these models can be found in Lanza et al. (1998) and Rüdiger et al. (2002). However, Applegate (1992) already made three basic predictions for his magnetic activity model:

- both the light curve and the time residuals change in cycles of same lenght;
- extrema in brightness coincide with the O–C diagram extrema;
- a star should be the bluest when it is the brightest.

However, these predictions were never confirmed due to the lack of precise photometric data of TW Dra. A period change ΔP corresponds to a change in the quadrupole moment ΔQ as follows

$$\frac{\Delta P}{P} = -9\left(\frac{R_2}{a}\right)^2 \frac{\Delta Q}{M_2 R_2^2}, \tag{12}$$

where M_2 and R_2 are mass and radius of secondary star and a is the orbital separation of components. We calculated $\lvert\Delta Q\rvert$ for six intervals with a good coverage of period change (see Table 3).

These values correspond to typical values of quadrupole moment in Algols and RS CVn systems (Lanza 2006).

6.3. Third body – light time hypothesis

Assuming that the basic run of timing residuals was modulated by the LITE, we can compute the minima timings by the well-known formula (Irwin 1959; Mayer 1990):

$$\begin{aligned}\text{Pri.Min.} &= JD_0 + PE + \frac{a_{12}\sin i}{c}\\ &\quad\times\left[\frac{1-e^2}{1+e\cos\nu}\sin(\nu+\omega) + e\sin\omega\right],\end{aligned} \tag{13}$$

where $a_{12}\sin i$ is the projected semi-major axis, e eccentricity, c speed of light, ω longitude of periastron and ν is the true anomaly of binary orbit around the common center of mass of triple system. To estimate the mass of the third body, we used a mass function

$$\begin{aligned} f\,(M_3) &= \frac{(a_{12}\sin i)^3}{P_2^2} = \frac{1}{P_2^2}\left[\frac{173.15A}{\sqrt{1-e^2\cos^2\omega}}\right]^3\\ &= \frac{(M_3\sin i)^3}{(M_1+M_2+M_3)^2},\end{aligned} \tag{14}$$

where M_1, M_2, M_3 are masses of components in $M_\odot$, P_2 is a period of the third-body orbit in years and A observed semi-amplitude in days.

Subtracting the influence of damped oscillations of timing residuals, described in Sect. 5, we obtained the O–C diagram shown in Fig. 4. The sinusoidal variation of these values is well remarkable and could be caused by LITE. A preliminary analysis of the third-body circular orbit gives the period $P_2 = 2370(300)$ days $= 6.5(8)$ years and semi-amplitude $A = 0.0029(7)$ day (using the accurate photoelectric and CCD timings only). These values were obtained as a least-squares method solution of Eq. (13). The resulting period 6.5 years of the third body is in good agreement with the solution according to the Eq. (9). Assuming component masses $M_1 = 1.9\ M_\odot$ and $M_2 = 0.82\ M_\odot$ (Al-Naimiy & Al-Sikab 1984), we can estimate the minimum third body mass to be $M_{3,\,\mathrm{min}} = 0.3\ M_\odot$ with the assumption of a coplanar orbit.

7. Conclusions

We have found that the first part of O–C diagram, spanning the time interval 1858–1905, is poorly covered by observations. We are therefore only able speculate that the dependence of O–C values was more or less linear, that the period was constant, and that there was no mass-exchange in this time interval. The consecutive time interval 1905–1942, represented by the parabolic course of O–C dependence, can be interpreted as representing a stage of significant mass exchange between components at the rate of $3.9 \times 10^{-7}\ M_\odot$/year. Unfortunately, the most interesting part of the O–C diagram during Second World War is not covered by observations of light minima at all. During this time, we speculate that a significant amount of material was transferred from the secondary component towards the primary one.

Following this violent event, the mass transfer between components ceased and the O–C diagram shows waves that correspond alternatively to a lengthening and shortening of the period. We consider these to represent a changing gravitational quadrupole moment of the system as a consequence of magnetic field variations. The calculated values of $|\Delta Q|$ for time intervals between subsequent incidents of period change are comparable to these predicted values (Lanza 2006) and those derived for a similar system, e.g. XX Cep (Lee et al. 2007). The LITE could explain only small periodic variations in timing residuals after subtracting major cyclic period changes. The period of O–C variations for the third body, measured by robust regression, $P_{02} = 6.51(3)$ years and the corresponding effective amplitude $A_{\rm eff} = 0.0064(24)$ day, are in good agreement with the direct LITE solution $P = 6.5(7)$ years, $A = 0.0058(14)$ day. The predicted third body is (apparently) invisible in the system that has the A5 primary component. Assuming that this third body is a main sequence star of mass 0.3 $M_\odot$, we estimate that the contribution to the total light of TW Dra is lower than 0.0003 mag in V band.

Our description of period changes has allowed us to predict the possible future evolution of period. The next extrema of period will occur in 2010.5 ± 1.5 and 2021.9 ± 2.2, respectively. To confirm this prediction, further observations will be required.

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