A&A 486, 523–528 (2008)
DOI: 10.1051/0004-6361:200809658

Astronomy & Astrophysics

Stability of helium accretion discs in ultracompact binaries

J.-P. Lasota[1,3], G. Dubus[1,2], and K. Kruk[1,3]

[1] Institut d'Astrophysique de Paris, UMR 7095 CNRS, UPMC Univ. Paris 06, 98bis boulevard Arago, 75014 Paris, France
e-mail: lasota@iap.fr
[2] Laboratoire d'Astrophysique de Grenoble, UMR 5571 CNRS, Université J. Fourier, BP 53, 38041 Grenoble, France
[3] Astronomical Observatory, Jagiellonian University, ul. Orla 171, 30-244 Kraków, Poland

Received 26 February 2008 / Accepted 8 May 2008

ABSTRACT

Context. Stellar companions of accreting neutron stars in ultra compact X-ray binaries (UCXBs) are hydrogen-deficient. Their helium or C/O accretion discs are strongly X-ray irradiated. Both the chemical composition and irradiation determine the disc stability with respect to thermal and viscous perturbations. At shorter periods, UCXBs are persistent, whereas longer-period systems are mostly transient.

Aims. To understand this behaviour one has to derive the stability criteria for X-ray irradiated hydrogen-poor accretion discs.

Methods. We use a modified and updated version of the Dubus et al. code describing time-dependent irradiated accretion discs around compact objects.

Results. We obtained the relevant stability criteria and compared the results to observed properties of UCXBs

Conclusions. Although the general trend in the stability behaviour of UCXBs is consistent with the prediction of the disc instability model, in a few cases the inconsistency of theoretical predictions with the system observed properties is weak enough to be attributed to observational and/or theoretical uncertainties. Two systems might require the presence of some amount of hydrogen in the donor star.

Key words. accretion, accretion disks – stars: binaries: close – stars: low-mass, brown dwarfs – X-rays: binaries – stars: white dwarfs – stars: evolution

1. Introduction

It is well known that accretion discs around compact objects are subject to a thermal-viscous instability at temperatures corresponding to partial ionization of hydrogen. The model based on this instability explains the main properties of dwarf nova and low-mass X-ray binary (LMXB) outbursts (see Lasota 2001, for a review). In the case of transient LMXBs (often called soft X-ray transients – SXTs) X-ray irradiation of the accretion disc plays a fundamental role in the stability criteria and outburst physics (van Paradijs 1996; King & Ritter 1998; Dubus et al. 1999, 2001). An analogous instability should be present in ultra-compact binaries ($P_{\rm orb} \lesssim 60$ min) whose accretion discs are known to be hydrogen deficient (see e.g. Nelemans 2005, 2007, for recent reviews). Since some AM CVn binaries (in which matter is accreted onto a white dwarf) show dwarf-nova type outbursts and several Ultra Compact X-ray Binaries (UCXBs – binaries with a neutron-star accreting primary) are transient (see Nelemans et al. 2006; Nelemans & Jonker 2006) it seems that the thermal-viscous instability is operating also in these systems. Stability criteria for hydrogen-deficient discs were first obtained by Smak (1983) and later by Cannizzo (1984), Tsugawa & Osaki (1997), El-Khoury & Wickramasinghe (2000) and Menou et al. (2002). However, since they did not include effects of irradiation, they were not applicable to UCXBs in which irradiation is important (Deloye & Bildsten 2003; Nelemans & Jonker 2006, but they are of course relevant to AM CVn stars).

The formation and evolutionary channels of UCXBs are still subject to discussion (see e.g. Nelemans 2007, and reference therein). In the so-called white-dwarf channel a double white-dwarf or neutron-star – white-dwarf binary evolves due to angular momentum losses through emission of gravitational radiation. Such systems evolve from very short to longer orbital periods with ever decreasing mass-transfer rate (Deloye & Bildsten 2003; Deloye et al. 2007; Nelemans 2007). In the second channel a binary composed of non-degenerate helium star and a white-dwarf or a neutron-star shrinks first to a minimum period then evolves back to long periods with a strongly decreasing mass-transfer rate. Finally, the third scenario involves cataclysmic binaries or LMXBs with evolved companions. Such systems evolve towards shorter periods and when mass-loss from the companion uncovers the helium core their evolution is similar to the previous case (Podsiadlowski et al. 2002). In all three cases the mass-transfer rate decreases with orbital period. On the other hand the critical accretion rate determining the stability of a hot accretion disc increases with the orbital period. This is because in a stationary disc, the temperature decreasing with radius and the critical temperature being practically constant, the stability is determined by the physical parameters of the outer disc's ring. The disc radius in turn increases with orbital period. Therefore one should expect the shorter period UCXBs to be stable and persistent, while longer period systems should be transient. This expectation is roughly confirmed by observations. However, the absence of stability criterion for irradiated hydrogen-deficient accretion discs has not allowed a detailed comparison of the evolution models and the Disc Instability Model (DIM) with observations of UCXBs (Deloye & Bildsten 2003).

In this article we study the stability properties of irradiated hydrogen-poor discs and apply our results to UCXBs. In Sect. 2 we present stability criteria for helium discs and in the case of

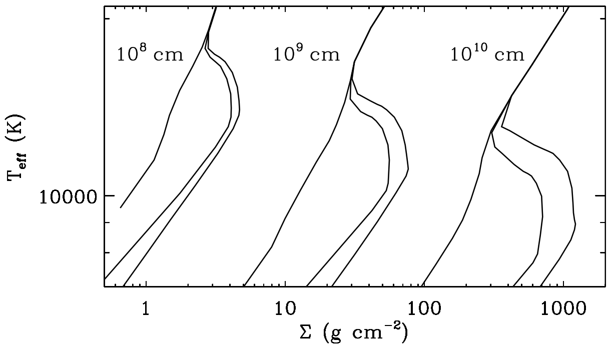

Fig. 1. Example S-curves for a pure helium disc with varying irradiation temperature $T_{\rm irr}$. The plot shows the effective temperature as a function of the surface density for $\alpha = 0.16$. The different sets of S-curves correspond to different radii $R = 10^6$, 10^9 and 10^{10} cm. For each radius, the irradiation temperature $T_{\rm irr}$ is 0 K, 10 000 K and 20 000 K. The instability disappears for high irradiation temperatures.

no irradiation compare our results with those obtained by other authors. Application to observed UCXBs is presented in Sect. 3 and problems are discussed in the last Sect. 4. In the Appendix we included updated results for hydrogen-rich discs.

2. Thermal-viscous stability of helium discs

We used an updated and slightly modified version of the code described in Hameury et al. (1998), Dubus et al. (1999) and Dubus et al. (2001).

2.1. Opacities and EOS

The updated opacities were taken from OPAL (Iglesias & Rogers 1996) and Ferguson et al. (2005). OPAL Rosseland mean opacities run from $\log T = 3.75$ to 8 and the Ferguson et al. opacities run from $\log T = 2.7$ to 4.5. An average of $\log \kappa$ is taken in the temperature regime where the tables overlap, weighted by $\log T$, so that the two tables connect smoothly. In addition to the opacities, the thermodynamical quantities for the gas mixture (pressure, internal energy, etc.) are calculated following Paczyński (1969). Our accretion discs are pure helium ($Y = 1$) except where noted. In the cases where the metal composition was non-zero ($Z \neq 0$), we assume the Grevesse & Noels (1993) metal relative distribution.

2.2. S-curves

The stability condition is found by investigating the local vertical structure of the disc. The vertical structure was calculated following Hameury et al. (1998) and Dubus et al. (1999). The input parameters are R_{10}, $T_{\rm c}$, $T_{\rm irr}$ and α (where $R = R_{10}\, 10^{10}$ cm, $T_{\rm c}$ the midplane temperature, $T_{\rm irr}$ the irradiation temperature and α the viscosity parameter). The returned variables are the surface density Σ and the effective temperature $T_{\rm eff}$ for which local thermal equilibrium is achieved. The solutions form the well-known S-curves in the (Σ, $T_{\rm eff}$, or $T_{\rm c}$, or $\dot{M}$) plane. We computed several thousands of such S-curves for α between 10^{-4} and 1, R between 10^6 and 10^{11} cm, $T_{\rm irr}$ between 0 and 25 000 K, $T_{\rm c}$ between 10^3 and 10^6 K. Example S-curves are shown in Fig. 1. The critical values of Σ and $T_{\rm eff}$ are given by the two inflexion points in the S-curve. Their dependence on the input parameters can then be approximated by a numerical fit, giving the critical Σ, $T_{\rm c}$, $T_{\rm eff}$ as a function of α, R and $T_{\rm irr}$ as in Dubus et al. (2001). The critical mass accretion rate is derived from the above using

$$\dot{M} = \sigma T_{\rm eff}^4 \frac{8\pi R^3}{3GM} \cdot \tag{1}$$

The dependence on M is derived from the dependence on R since the S-curve is unchanged for a constant M/R^3 ratio.

2.3. Non-irradiated helium disc: comparison with other calculations and impact of metallicity

For the non-irradiated case $T_{\rm irr} = 0$ K the critical values for a pure helium disc ($Y = 1$) are

$$\begin{aligned}
\Sigma^+_{\rm crit} &= 589\ \alpha_{0.1}^{-0.78}\ R_{10}^{1.07}\ M_1^{-0.36}\ {\rm g\,cm^{-2}} \\
T^+_{\rm c} &= 76\,000\ \alpha_{0.1}^{-0.21}\ R_{10}^{0.08}\ M_1^{-0.03}\ {\rm K} \\
T^+_{\rm eff} &= 13\,100\ \alpha_{0.1}^{-0.01}\ R_{10}^{-0.08}\ M_1^{0.03}\ {\rm K} \\
\dot{M}^+_{\rm crit} &= 1.05\times 10^{17}\ \alpha_{0.1}^{-0.05}\ R_{10}^{2.69}\ M_1^{-0.90}\ {\rm g\,s^{-1}} \qquad (2) \\
\Sigma^-_{\rm crit} &= 1770\ \alpha_{0.1}^{-0.83}\ R_{10}^{1.20}\ M_1^{-0.40}\ {\rm g\,cm^{-2}} \qquad (3) \\
T^-_{\rm c} &= 16\,000\ \alpha_{0.1}^{-0.14}\ R_{10}^{-0.05}\ M_1^{0.02}\ {\rm K} \\
T^-_{\rm eff} &= 9700\ \alpha_{0.1}^{-0.00}\ R_{10}^{-0.09}\ M_1^{0.03}\ {\rm K} \\
\dot{M}^-_{\rm crit} &= 3.18\times\ 10^{16}\ \alpha_{0.1}^{-0.01}\ R_{10}^{2.65}\ M_1^{-0.88}\ {\rm g\,s^{-1}}
\end{aligned}$$

where M_1 is the mass of the compact object in solar units and $\alpha = 0.1\,\alpha_{0.1}$. The superscript (+) designates a critical value for hot ("upper branch") discs, whereas (−) corresponds to the cold ("lower branch") discs.

We compare our results with those obtained earlier by other authors. Menou et al. (2002; who use a version of the Hameury et al. 1998 code) give $\dot{M}^+_{\rm crit}$ for a pure-helium disc ($Y = 1$)

$$\dot{M}^+_{\rm crit} = 5.9\times 10^{16}\ \alpha_{0.1}^{-0.41}\ R_{10}^{2.62}\ M_1^{-0.87}\ {\rm g\,s^{-1}}. \tag{4}$$

The fit to the critical mass accretion rate is very close to what we find except for the stronger dependence on α in the fit of Menou et al. (2002). This is the result of their assumption of a perfect gas equation of state: as discussed by Menou et al. (2002), the dependence on α disappears when convective transport is included, as we have done.

El-Khoury & Wickramasinghe (2000) model the emission from AM CVn stars and present a few S-curves for some specific values of α, R and M assuming a low hydrogen content. We find good agreement of the critical $T_{\rm eff}$ derived from these S-curves (see their Fig. 19). However, we find our critical Σ are higher by ≈50%.

Smak (1983) who took $Y = 0.98$, $Z = 0.02$ obtains $\Sigma^-_{\rm crit} \approx 790\ {\rm g\,cm^{-2}}$ but in this case a non-standard way of calculating the cold branch was used (Smak 1984, and private communication).

As noticed by Tsugawa & Osaki (1997), the critical values depend on metallicity (see their Fig. 3). This mainly changes the critical Σ, especially $\Sigma^-_{\rm crit}$. The reason is that at $\Sigma^-_{\rm crit}$ the opacity $\tau \sim 1$ and that changing the metallicity has a strong effect on the opacities at low temperature. It is therefore not surprising that our values for $\Sigma^-_{\rm crit}$, found using $Y = 1$, differ from those of Tsugawa & Osaki (1997) whose baseline model uses $Y = 0.97\,Z = 0.03$. Comparing the results of our numerical fits (given above) to their S-curve for $Z = 0.0004$ (see their Fig. 3), we find that our values of $\Sigma^-_{\rm crit}$ are within 15% of their values. We also found very good agreement with their fits to the critical $T_{\rm eff}$ and Σ (Eqs. (4)−(7) of their paper) when using the same composition ($Y = 0.97\,Z = 0.03$).

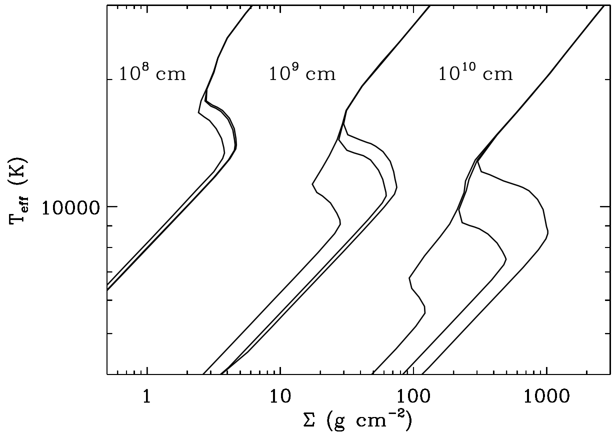

Fig. 2. Example S-curves for a pure helium disc with varying C. The plot shows the effective temperature as a function of the surface density for $\alpha = 0.16$. The different sets of S-curves correspond to different radii $R = 10^8$, 10^9 and 10^{10} cm. For each radius, the irradiation constant C is changed from 10^{-5}, to 10^{-4} to 10^{-3}. Here, the irradiation temperature varies along the S-curve. Irradiation has little influence in the inner regions of the disc but greatly changes the S-curves at the outer radii.

One possible consequence of the metallicity dependence of $\Sigma^-_{\rm crit}$ is that running an unstable disc with α constant can lead to outbursts that have a larger amplitude when Z is low than when Z is high (because $\Sigma^-_{\rm crit}/\Sigma^+_{\rm crit}$ is larger). This is intriguing as large ratios of $\Sigma^-_{\rm crit}/\Sigma^+_{\rm crit}$ are known to be required to obtain realistic lightcurves in the framework of the DIM (Smak 1984). Typically, this is achieved by lowering α in quiescence i.e. assuming less efficient momentum transport in a cold disc. However, although the ratio of $\Sigma^-_{\rm crit}/\Sigma^+_{\rm crit}$ increases with lower metallicities, the values are still far from the amplitudes required for realistic lightcurves (about 10–20 compared to a ratio of about 3 here). Therefore this will not change the conclusion that one needs a lower α in quiescence for the DIM to work (Smak 1984).

2.4. Irradiated helium discs

In determining the stability of irradiated helium discs we followed Dubus et al. (1999): we make a hypothesis on $T_{\rm irr}$ and find $\dot{M}$ for which the disc of given outer radius is stable. We assume that $T_{\rm irr}$ is given by

$$\sigma T^4_{\rm irr} = C\frac{\dot{M}c^2}{4\pi R^2} \tag{5}$$

where $0 \le C \le \infty$ is the irradiation constant defined as in Dubus et al. (1999). The value of C depends upon the radiative efficiency, irradiation geometry, albedo, irradiation spectrum etc.

With Eq. (5) we calculate S-curves with as input parameters R_{10}, $T_{\rm c}$, C and α. Here, the irradiation temperature varies along the S-curve with the mass accretion rate required for thermal equilibrium. The critical points are now given as functions of α, R and C as in Dubus et al. (1999). When one is interested in stability (and not in time-dependent calculations) C is assumed to be constant for a given system. It is therefore more convenient to express the critical points as a function of C rather than of $T_{\rm irr}$ (which itself depends on R and $\dot{M}$). We computed several thousands of S-curves for α between 10^{-4} and 1, R between 10^6 and 10^{11} cm, $T_{\rm c}$ between 10^3 and 10^6 K, M_1 between 0.1 and 100 $M_\odot$ and C between 10^{-10} and 10^{-1}. Examples are given in Fig. 2.

Irradiated pure helium disc

For $C \le 10^{-6}$ one should use the non-irradiated fits given by Eq. (4). For $C \ge 10^{-6}$ the fits to the critical values are given to a good approximation (average relative error $\le$25% for $\Sigma^+_{\rm crit}$, $\le$10% for $T^+_{\rm eff}$ and $\le$2% for $\log \dot{M}^+_{\rm crit}$) by

$$\begin{aligned}
\Sigma^+_{\rm crit} &= 227 C_{-3}^{-0.16}\alpha_{0.1}^{-0.79} R_{10}^{0.96-0.04\log C_{-3}} M_1^{-0.25+0.04\log C_{-3}}\ {\rm g\,cm^{-2}}\\
T^+_{\rm c} &= 4480 C_{-3}^{-0.06}\alpha_{0.1}^{-0.19} R_{10}^{0.02-0.01\log C_{-3}} M_1^{0.02+0.01\log C_{-3}}\ {\rm K}\\
T^+_{\rm eff} &= 8730 C_{-3}^{-0.05}\alpha_{0.1}^{-0.01} R_{10}^{-0.12-0.01\log C_{-3}} M_1^{0.06+0.01\log C_{-3}}\ {\rm K}\\
T^+_{\rm irr} &= 22600 C_{-3}^{0.20}\alpha_{0.1}^{-0.01} R_{10}^{0.13-0.01\log C_{-3}} M_1^{-0.19+0.01\log C_{-3}}\ {\rm K}\\
\dot{M}^+_{\rm crit} &= 2.1\times 10^{16} C_{-3}^{-0.22}\alpha_{0.1}^{-0.03-0.01\log C_{-3}} R_{10}^{2.51-0.05\log C_{-3}}\\
&\quad \cdot M_1^{-0.74+0.05\log C_{-3}}\ {\rm g\,s^{-1}}
\end{aligned} \tag{6}$$

where $C = 10^{-3}C_{-3}$.

We show only the results for the upper-branch (hot discs). Since in quiescent SXTs X-ray irradiation is negligible, the relevant criterion for the lower-branch (cold discs) is the one without irradiation. For example, according to the DIM in a quiescent disc the surface density must satisfy $\Sigma < \Sigma^-_{\rm crit}$ (see e.g. Lasota 2001). Since in such a disc $\dot{M} \sim R^{2.65}$ (Eq. (4)) the ratio of viscous to irradiating fluxes $F_{\rm vis}/F_{\rm irr} \sim R^{1.65}$ and self-irradiation is never important (Dubus et al. 1999).

Irradiated disc with mixed composition

In order to discuss the influence of the hydrogen depletion we also calculated the case with $X = 0.1$ and $Y = 0.9$. We find

$$\begin{aligned}
\Sigma^+_{\rm crit} &= 54.0 C_{-3}^{-0.18}\alpha_{0.1}^{-0.80} R_{10}^{0.92-0.04\log C_{-3}} M_1^{-0.21+0.05\log C_{-3}}\ {\rm g\,cm^{-2}}\\
T^+_{\rm c} &= 18\,700 C_{-3}^{-0.07}\alpha_{0.1}^{-0.21} R_{10}^{-0.05-0.02\log C_{-3}} M_1^{0.06+0.02\log C_{-3}}\ {\rm K}\\
T^+_{\rm eff} &= 4280 C_{-3}^{-0.06}\alpha_{0.1}^{-0.02} R_{10}^{-0.16-0.02\log C_{-3}} M_1^{0.09+0.02\log C_{-3}}\ {\rm K}\\
T^+_{\rm irr} &= 12\,500 C_{-3}^{0.19}\alpha_{0.1}^{-0.02} R_{10}^{0.09-0.02\log C_{-3}} M_1^{-0.16+0.02\log C_{-3}}\ {\rm K}\\
\dot{M}^+_{\rm crit} &= 1.9\times 10^{15} C_{-3}^{-0.25}\alpha_{0.1}^{-0.07+0.01\log C_{-3}} R_{10}^{2.38-0.06\log C_{-3}}\\
&\quad \cdot M_1^{-0.64+0.07\log C_{-3}}\ {\rm g\,s^{-1}}.
\end{aligned} \tag{7}$$

3. Application to ultra-compact binaries

The neutron star's stellar companions in Ultra Compact X-ray Binaries (UCXBs) $P_{\rm orb} \lesssim 60$ min cannot be hydrogen-rich stars since such stars would not fit into the orbit. The 10 confirmed UCXBs (see Table 1) have orbital periods in two ranges: one between ~10 and ~20 min, the second ~40 and ~50 min. The four systems below 20 min. are classified as persistent X-ray sources (except for 4U 1543-624 they are X-ray bursters). Of the six binaries with periods above 40 min. one is classified as persistent: 4U 1916-05 which is an X-ray bursting dipper. The binary 4U 1626-67 (which contains a 7.7 s X-ray pulsar) used to be classified as persistent but its brightness is slowly but clearly decaying (Krauss et al. 2007). The other four systems are accretion-driven millisecond pulsars and are all transient systems. In general therefore UCXBs follow the expected pattern: shorter period are stable, at longer periods discs become unstable. However, as we shall see, comparing actual stability criteria with observations makes things a bit more complicated. In any case one should keep in mind that the division of UCXBs into persistent and transient might be "time dependent". We already mentioned 4U 1626-67 (which should fade out into quiescence in 2–15 years, see Krauss et al. 2007) but also the UCXB candidate 1H 1905+000 that was persistent for at least 11 years has now completely disappeared from the X-ray sky (Jonker et al. 2007).

According to Paczyński (1977) the maximum disc radius can be written as

$$\frac{R_{\rm D}(\max)}{a} = \frac{0.60}{1+q}, \tag{8}$$

(valid for $0.03 < q < 1$), where

$$a = 2.28 \times 10^9 M_1^{1/3} (1+q)^{1/3} P_{\rm min}^{2/3}\ {\rm cm} \tag{9}$$

is the binary separation; $P_{\rm min}$ being the orbital period in minutes. For convenience we will write the disc outer radius as

$$R_{\rm D} = 2.28 \times 10^9 f\, M_1^{1/3}\, P_{\rm min}^{2/3}\ {\rm cm} \tag{10}$$

where

$$f = \frac{0.60}{(1+q)^{2/3}}\,\cdot \tag{11}$$

The value of C is not known a priori. Since it is a measure of the fraction of the X-ray luminosity that heats up the disc it contains information on the irradiation geometry, X-ray albedo, X-ray spectrum, etc. Dubus et al. (1999) found that the observed optical magnitudes and stability properties of persistent low mass X-ray binaries were compatible with a value $C \approx 5 \times 10^{-4}$. It is not clear that the same value describes well the properties of UCXBs. Therefore will take $C = 10^{-3}$ as the results do not vary much for the range $5\ 10^{-4} \leq C \leq 2 \times 10^{-3}$, say. With this value of C, from Eqs. (6), (8) and (9), assuming $M_1 = 1.4$ $\alpha = 0.1$, $f = 0.6$, one obtains the following relation between the critical accretion rate and the orbital period for a pure helium X-ray irradiated disc

$$\dot M^{+}_{\rm crit} = 2.4 \times 10^{-12} f_{0.6}^{2.51} P_{\rm min}^{1.67}\ M_\odot\,{\rm y}^{-1}. \tag{12}$$

In the non-irradiated pure-helium disc the equivalent relation reads

$$\dot M^{+}_{\rm crit} = 8.2 \times 10^{-12} f_{0.6}^{2.69} P_{\rm min}^{1.79}\ M_\odot\,{\rm y}^{-1}. \tag{13}$$

For an irradiated mixed composition ($X = 0.1$, $Y = 0.9$) the critical accretion rate is

$$\dot M^{+}_{\rm crit} = 2.9 \times 10^{-13} f_{0.6}^{2.38} P_{\rm min}^{1.59}\ M_\odot\,{\rm y}^{-1}. \tag{14}$$

and for the solar composition the critical accretion rates for irradiated discs is

$$\dot M^{+}_{\rm crit} = 1.4 \times 10^{-13} f_{0.6}^{2.41} P_{\rm min}^{1.61}\ M_\odot\,{\rm y}^{-1}. \tag{15}$$

These four criteria are plotted in Fig. 3.

4. Discussion

Deloye & Bildsten (2003) studied the structure and evolution of UCXBs and showed that for orbital periods $\gtrsim$30 min the mass-transfer rate cannot be $\gtrsim 10^{-10}\ M_\odot\,{\rm y}^{-1}$ if they evolved adiabatically from systems filling their Roche-lobes at $P_{\rm orb} \approx 10$ min. As seen in Fig. 3 one arrives at a similar conclusion using full stellar models (Deloye, private communication). These models are parameterized by the total binary mass and the initial degeneracy parameter of the secondary given by $\psi = E_{\rm F}/k\,T_{\rm c}$, where $E_{\rm F}$ is the Fermi energy and $T_{\rm c}$ the central temperature (see Deloye et al. 2007). In general the evolutionary tracks are consistent with

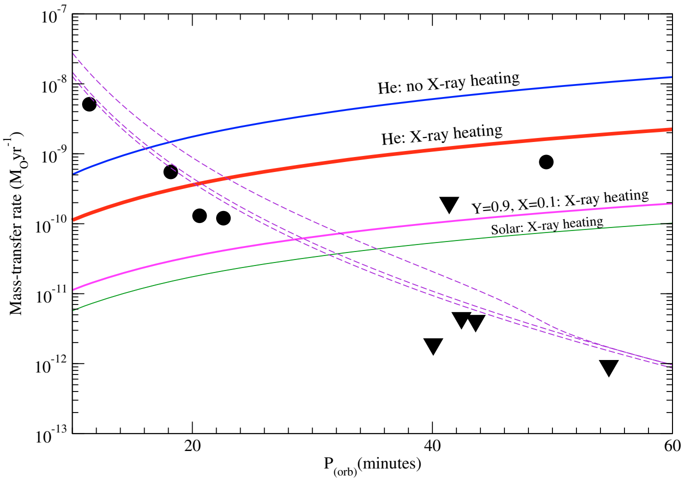

Fig. 3. Stability criteria for accretion discs in UCXBs. (Compare with Fig. 11 in Deloye & Bildsten 2003.) Four criteria are plotted (*from top to bottom*): (i) for non-irradiated pure-helium discs; (ii) for irradiated pure-helium discs; (iv) for an irradiated mixed-composition disc and (v) for an irradiated solar-composition disc. The accretion rates of persistent UCXBs are marked by circles. Transient systems are identified by triangles. Dashed lines correspond to three UCXB evolutionary tracks by Deloye (private communication). From bottom to top they represent evolution of binaries with a 1.4 $M_\odot$ neutron star and a secondary with initial mass M_2 and degeneracy parameter ψ (see text) equal respectively: 0.1 $M_\odot$, 3.5; 0.325 $M_\odot$, 3.0 and 0.325 $M_\odot$, 1.5.

the stability properties of individual systems but there are interesting cases where this can be questioned.

All UCXBs known to be transient (the millisecond pulsars and the young pulsar binary 4U 1626-67) have mass transfer rates well below the stability limit for irradiated helium discs. However, a word of caution is needed as it is not clear that the criterion in form of Eq. (12) can be applied to MSP binary systems as their secondary masses are supposed to be $<0.01 M_\odot$. As mentioned in Yungelson et al. (2006), for lower values of q matter transferred from the Roche lobe circularizes on unstable orbits and it is unclear how disc formation proceeds. The actual outer disc radius may therefore be smaller (but not by much, see below) than we have assumed above, which would act to make the systems stabler than anticipated. On the other hand according to Eq. (12) the maximum outburst luminosity of helium UCXB MSPs can be expressed as (see also Lasota 2008)

$$L_{\max} \simeq 3.5 \times 10^{37} \left(\frac{P_{\rm orb}}{1{\rm h}}\right)^{1.67} {\rm erg\ s}^{-1}, \tag{16}$$

which agrees well with observations. In fact in all UCXBs mass-ratios could be less than 0.03 but the fact that luminosities are consistent with the size of a standard disc model suggests that the stability criterion is applicable even in such extreme binary systems.

4.1. Persistent systems

The mass-transfer rate were calculated by using the luminosities quoted in the references mentioned in Table 1 assuming a 1.4 $M_\odot$ neutron star with a 10 km radius. Among the systems classified as persistent, two (4U 1820-30 and 4U 1543-624) are stable according to the stability criterion for irradiated helium discs but three other binaries (4U 1850-087, M 15 X-2 and 4U 1916-05) should be transient according to this criterion but apparently are

Table 1. UCXBs ($P_{orb} \lesssim 60$ min).

System	P_{orb} (min)	$\dot{M}$ ($M_\odot$ y^{-1})[a]	Type[b]	Comment[c]
4U 1820-30[1]	11.42	5.1×10^{-9}	P	(in GC)
4U 1543-624[2]	18.2	5.5×10^{-10}	P	
4U 1850-087[3]	20.6	1.3×10^{-10}	P	(in GC)
M15 X-2[4]	22.58	1.2×10^{-10}	P	(in GC)
XTE J1807-294[5]	40.07	1.9×10^{-12}	T	(MSP)
4U 1626-67[6]	41.4	2.0×10^{-10}	T	(young pulsar)
XTE J1751-305[7]	42.42	4.5×10^{-12}	T	(MSP)
XTE J0929-314[8]	43.6	4.1×10^{-12}	T	(MSP)
4U 1916-05[9]	49.48	7.6×10^{-10}	P	
SWIFT J1756.9-2508[10]	54.7	9.3×10^{-13}	T	(MSP)

[a] See comments in the text; [b] P – persistent, T – transient; [c] GC – globular cluster, MSP – millisecond pulsar.
[1] Zdziarski et al. (2007); [2] Wang & Chakrabarty (2004); [3] Sidoli et al. (2006); [4] Dieball et al. (2005); [5] Markwardt et al. (2003); Falanga et al. (2005); [6] Krauss et al. (2007); [7] Markwardt et al. (2002); Gierliński & Poutanen (2005); [8] Galloway et al. (2002); Juett et al. (2003); [9] Juett & Chakrabarty (2006); [10] Krimm et al. (2007).

not. One can try to explain this apparent contradiction in three ways.

First, the mass-transfer rates (bolometric luminosities) could be underestimated. Second, the outer disc radius has been overestimated. Third, the companions are not pure-helium stars but (still) contain some amount of hydrogen. Let us note that C/O – donors would make things even worse for the DIM as the corresponding stability curves would be *above* the helium curves (Menou et al. 2002; Deloye & Bildsten 2003) because of their higher ionization potentials.

Considering the first possibility one remarks that of the three outliers the two with $P_{orb} \sim 20$ min are located in globular clusters, so one could assume the distance to these sources is well known. However, the distance to the peculiar globular cluster NGC 6712 first believed to be 6.8 kpc (Cudworth 1988) is now measured to be ~8 kpc by Paltrinieri et al. (2001). Also in the case of M 15 X-2, the mass-transfer rate based on the X-ray luminosity used in Table 1 could be an underestimate and $\dot{M} \gtrsim 4 \times 10^{-10}\, M_\odot\, y^{-1}$ (Dieball et al. 2005).

At first, the second possibility seems to be more promising since the dependence of stability criterion on the outer disc radius is very strong. One could think that it is enough to take a value smaller than the maximum to get rid of the problem. For example at $P_{orb} \sim 20$ min, an outer disc radius of $0.3\,a$ (a factor ~2 smaller than assumed above) is enough for the X-ray heating of a helium disc to stabilize the system. However, already for $q = 0.1$ the circularization radius is equal to $0.3\,a$ so for the likely smaller mass-ratio of systems of interest it is not a viable solution.

It seems finally that the third possibility is the most likely way out of the stability problem for the two systems. A small fraction of hydrogen left ($X \gtrsim 0.05$, see Fig. 3) would make them stable. For 4U 1820-30 Cumming (2003) found that $X \sim 0.1$ is compatible with the X-ray burst properties of this system. This conclusion, however, does not apply directly to the two systems of interest but evolutionary models of Podsiadlowski et al. (2002) would allow them to have $X \gtrsim 0.1$ (rather paradoxically these models predict mass-transfer rates higher than observed, in fact in the helium stability range.)

The evolution of 4U 1916-05 is rather controversial. Nelson et al. (1986) suggested that mass-transfer rate deduced for 4U 1916-05 points to an evolved secondary as donor in this system. This would allow the presence of hydrogen. However, Nelemans et al. (2006) challenge this conclusion finding a He donor and a high N abundance. However, it is not clear how this is consistent with the white-dwarf channel.

Finally, the 20 min systems could have evolved through the so-called "magnetic capture" (van der Sluys et al. 2005). In such a (very unlikely) case they still contain enough hydrogen to fulfill the stability criterion for $X \lesssim 0.1$.

4.2. The unstable UCXBs

Among the five systems in this category, four are accreting millisecond pulsars (MSP). The mass-transfer rate was estimated from the formula

$$\dot{M}_{tran} \lesssim 4\pi \left(\frac{D}{c}\right)^2 F_{outb} \frac{t_{outb}}{t_{rec}}, \tag{17}$$

where D is the distance to the binary, F_{outb} is the mean bolometric flux during outburst; t_{outb} and t_{rec} are respectively the outburst duration and the recurrence time. Except for XTE J1751-305 where the recurrence time is known to be 2–3 years, t_{rec} was assumed to be 10 years and $\dot{M}_{tran}$ is therefore an upper limit. Our estimates are close to those of Watts et al. (2004). In all cases the distance to the sources is poorly known so in addition to the unknown recurrence times this makes mass-transfer estimates somewhat uncertain. From the point of view of the instability criterion this is of no importance as all the MSP are comfortably well below (two orders of magnitudes) the threshold. Figure 3 shows also evolutionary tracks calculated by Deloye (private communication) and one might suspect that MSP donors are rather C/O than He stars (see Deloye & Bildsten 2003).

The case of 4U 1626-67 is different. The neutron star in this UCXB is a young pulsar and it is transient in a different sense: its "outburst" does not last tens of days but tens of years and it is not clear at all that this behaviour has anything to do with the thermal-viscous instability of the DIM. The value of mass-transfer rate given in Table 1 is just a reference value given by Eq. (4) of Krauss et al. (2007). The source has been on since 1977 and seems to be decaying but the estimate that it will off in 2–15 years is based on dubious premises. One can say only two things:

- The mass-transfer rate of $2.0 \times 10^{-10}\, M_\odot\, y^{-1}$ is incompatible with evolutionary tracks of Deloye & Bildsten (2003). (See, however, Yungelson 2008.)

– The mass accreted during ∼30 years was $\sim 5 \times 10^{24}$ g (for $D = 3$ kpc) which is compatible with the maximum mass in a cold quiescent helium disc:

$$M_{\rm D,max} \approx 3.5\times 10^{25}\left(\frac{\alpha_{\rm cold}}{0.01}\right)^{-0.83} f_{0.6}^{3.2} \times\left(\frac{M_{\rm ns}}{1.4{\rm M}_\odot}\right)^{0.67}\left(\frac{P_{\rm orb}}{1{\rm h}}\right)^{2.13} {\rm g}, \tag{18}$$

(see Eq. (3)). However, the recurrence time could be longer than 1000 years, say and the mean accretion rate over the cycle $\lesssim 2.5\times 10^{-12}\, M_\odot\,{\rm y}^{-1}$, compatible with evolutionary tracks of Deloye & Bildsten (2003) and Deloye et al. (2007).

Acknowledgements. Lars Bilsten inspired this work and contributed invaluable advice and comments. We thank Chris Deloye for providing UCXB evolutionary tracks. J.P.L. is grateful to Lev Yungelson for very helpful discussions and remarks. This work was supported by the Centre National d'Études Spatiales (CNES), the CNRS GDR PCHE and by the National Science Foundation under Grant No. PHY05-51164.

Appendix A: Solar-composition discs

For comparison with results for helium discs we present here the fits to critical values for solar composition ($X = 0.7$, $Y = 0.28$ and $Z = 0.02$) using the same procedure as described in the main text.

Non-irradiated solar-composition disc

$$\begin{aligned}
\Sigma^+_{\rm crit} &= 39.9\,\alpha_{0.1}^{-0.80}\,R_{10}^{1.11}\,M_1^{-0.37}\,{\rm g\,cm^{-2}}\\
T^+_{\rm c} &= 30\,000\,\alpha_{0.1}^{-0.18}\,R_{10}^{0.04}\,M_1^{-0.01}\,{\rm K}\\
T^+_{\rm eff} &= 6890\,R_{10}^{-0.09}\,M_1^{0.03}\,{\rm K}\\
\dot M^+_{\rm crit} &= 8.07\times 10^{15}\,\alpha_{0.1}^{-0.01}\,R_{10}^{2.64}\,M_1^{-0.89}\,{\rm g\,s^{-1}} \qquad \text{(A.1)}\\
\Sigma^-_{\rm crit} &= 74.6\,\alpha_{0.1}^{-0.83}\,R_{10}^{1.18}\,M_1^{-0.40}\,{\rm g\,cm^{-2}}\\
T^-_{\rm c} &= 8240\,\alpha_{0.1}^{0.14}\,R_{10}^{-0.10}\,M_1^{0.04}\,{\rm K}\\
T^-_{\rm eff} &= 5210\,R_{10}^{-0.10}\,M_1^{0.04}\,{\rm K}\\
\dot M^-_{\rm crit} &= 2.64\times 10^{15}\,\alpha_{0.1}^{0.01}\,R_{10}^{2.58}\,M_1^{-0.85}\,{\rm g\,s^{-1}}.
\end{aligned}$$

Irradiated solar-composition discs

The fits were obtained with the parameters varied in the range $10^{-5} \le \alpha \le 1$, $10^{-4} \le R_{10} \le 10$, $0.1 \le M_1 \le 100$ and $10^{-6} \le C \le 1$. The average relative uncertainty on $\log \dot M^+_{\rm crit}$ is $\lesssim 2\%$.

$$\begin{aligned}
\Sigma^+_{\rm crit} &= 8.7 C_{-3}^{-0.28}\alpha_{0.1}^{-0.78+0.01\log C_{-3}} R_{10}^{0.92-0.07\log C_{-3}}\\
&\quad\cdot M_1^{-0.19+0.06\log C_{-3}}\,{\rm g\,cm^{-2}}\\
T^+_{\rm c} &= 16\,300 C_{-3}^{-0.10}\alpha_{0.1}^{-0.17} R_{10}^{-0.03-0.03\log C_{-3}} M_1^{0.05+0.02\log C_{-3}}\,{\rm K}\\
T^+_{\rm eff} &= 4040 C_{-3}^{-0.09}\alpha_{0.1}^{0.01} R_{10}^{-0.15-0.02\log C_{-3}} M_1^{0.09+0.02\log C_{-3}}\,{\rm K}\\
T^+_{\rm irr} &= 10\,500 C_{-3}^{0.16}\alpha_{0.1}^{0.01} R_{10}^{0.10-0.02\log C_{-3}} M_1^{-0.16+0.02\log C_{-3}}\,{\rm K}\\
\dot M^+_{\rm crit} &= 9.5\times 10^{14} C_{-3}^{-0.36}\alpha_{0.1}^{0.04+0.01\log C_{-3}} R_{10}^{2.39-0.10\log C_{-3}}\\
&\quad\cdot M_1^{-0.64+0.08\log C_{-3}}\,{\rm g\,s^{-1}}. \qquad \text{(A.2)}
\end{aligned}$$

References

Cannizzo, J. K. 1984, Nature, 311, 443
Cudworth, K. M. 1988, AJ, 96, 105
Cumming, A. 2003, ApJ, 595, 1077
Deloye, C. J., & Bildsten, L. 2003, ApJ, 598, 1217
Deloye, C. J., Taam, R. E., Winisdoerffer, C., & Chabrier, G. 2007, MNRAS, 381, 525
Dieball, A., Knigge, C., Zurek, D. R., et al. 2005, ApJ, 634, L105
Dubus, G., Lasota, J.-P., Hameury, J.-M., & Charles, P. 1999, MNRAS, 303, 139
Dubus, G., Hameury, J.-M., & Lasota, J.-P. 2001, A&A, 373, 251
El-Khoury, W., & Wickramasinghe, D. 2000, A&A, 358, 154
Falanga, M., Bonnet-Bidaud, J. M., Poutanen, J., et al. 2005, A&A, 436, 647
Ferguson, J. W., Alexander, D. R., Allard, F., et al. 2005, ApJ, 623, 585
Galloway, D. K., Chakrabarty, D., Morgan, E. H., & Remillard, R. A. 2002, ApJ, 576, L137
Gierliński, M., & Poutanen, J. 2005, MNRAS, 359, 1261
Grevesse, N., & Noels, A. 1993, Origin and evolution of the elements: proceedings of a symposium in honour of H. Reeves, held in Paris, June 22–25, 1992, ed. N. Prantzos, E. Vangioni-Flam, & M. Casse (England, Cambridge: Cambridge University Press), 14
Hameury, J.-M., Menou, K., Dubus, G., Lasota, J.-P., & Hure, J.-M. 1998, MNRAS, 298, 1048
Hameury, J.-M., Lasota, J.-P., & Dubus, G. 1999, MNRAS, 303, 39
Iglesias, C. A., & Rogers, F. J. 1996, ApJ, 464, 943
Jonker, P. G., Steeghs, D., Chakrabarty, D., & Juett, A. M. 2007, ApJ, 665, L147
Juett, A. M., & Chakrabarty, D. 2006, ApJ, 646, 493
Juett, A. M., Galloway, D. K., & Chakrabarty, D. 2003, ApJ, 587, 754
King, A. R., & Ritter, H. 1998, MNRAS, 293, L42
Krauss, M. I., Schulz, N. S., Chakrabarty, D., Juett, A. M., & Cottam, J. 2007, ApJ, 660, 605
Krimm, H. A., et al. 2007, ArXiv e-prints, 709 [arXiv:0709.1693]
Lasota, J.-P. 2001, New Astron. Rev., 45, 449
Lasota, J.-P. 2008, New Astron. Rev., 51, 752
Markwardt, C. B., Swank, J. H., Strohmayer, T. E., in't Zand, J. J. M., & Marshall, F. E. 2002, ApJ, 575, L21
Markwardt, C. B., Juda, M., & Swank, J. H. 2003, IAU Circ., 8095, 2
Menou, K., Perna, R., & Hernquist, L. 2002, ApJ, 564, L81
Nelemans, G. 2005, The Astrophysics of Cataclysmic Variables and Related Objects , 330, 27
Nelemans, G. 2007, in proceedings Hydrogen-Deficient Stars, ed. K. Werner, & T. Rauch, ASP Conf Ser., in press [arXiv:0711.0602]
Nelemans, G., & Jonker, P. G. 2006, in proceedings of A life with stars a conference in honour of Ed van den Heuvel's 60th birthday, New Ast. Rev. [arXiv:astro-ph/0605722]
Nelemans, G., Jonker, P. G., & Steeghs, D. 2006, MNRAS, 370, 255
Nelson, L. A., Rappaport, S. A., & Joss, P. C. 1986, ApJ, 304, 231
Paczyński, B. 1969, Acta Astron., 19
Paczyński, B. 1977, ApJ, 216, 822
Paltrinieri, B., Ferraro, F. R., Paresce, F., & De Marchi, G. 2001, AJ, 121, 3114
Podsiadlowski, P., Rappaport, S., & Pfahl, E. D. 2002, ApJ, 565, 1107
Sidoli, L., Paizis, A., Bazzano, A., & Mereghetti, S. 2006, A&A, 460, 229
Smak, J. 1983, Acta Astron., 33, 333
Smak, J. 1984, Acta Astron., 34, 161
Tsugawa, M., & Osaki, J. 1997, PASJ, 49, 75
van der Sluys, M. V., Verbunt, F., & Pols, O. R. 2005, A&A, 431, 647
van Paradijs, J. 1996, ApJ, 464, L139
Wang, Z., & Chakrabarty, D. 2004, ApJ, 616, L139
Watts, A. L., Krishnan, B., Schutz, B. F. & Bildsten, L. 2008, MNRAS, submitted [arXiv:0803.4097]
Yungelson, L. R. 2008, Astron. Lett., 34, 9
Yungelson, L. R., Lasota, J.-P., Nelemans, G., et al. 2006, A&A, 454, 559
Zdziarski, A. A., Gierliński, M., Wen, L., & Kostrzewa, Z. 2007, MNRAS, 377, 1017