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LETTER TO THE EDITOR

Convective envelopes in rotating OB stars

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ABSTRACT

Aims. We study the effects of rotation on the outer convective zones of massive stars.

Methods. We examine the effects of rotation on the thermal gradient and on the Solberg-Hoiland term by analytical developments and by numerical models.

Results. Writing the criterion for convection in rotating envelopes, we show that the effects of rotation on the thermal gradient are much larger and of opposite sign to the effect of the Solberg-Hoiland criterion. On the whole, rotation favors convection in stellar envelopes at the equator and to a smaller extent at the poles. In a rotating 20 $M_\odot$ star at 94% of the critical angular velocity, there are two convective envelopes, with the bigger one having a thickness of 13.2% of the equatorial radius. In the non-rotating model, the corresponding convective zone has a thickness of only 4.6% of the radius. The occurrence of outer convection in massive stars has many consequences.

Key words. stars: evolution – convection – stars: rotation

1. Introduction

It is generally considered that the Cowling model applies to massive OB stars: i.e., a convective core surrounded by a large radiative envelope. However, long since stellar models have shown that massive stars have an outer convective envelope encompassing several percent of the stellar radius (Maeder 1980). Also, Langer (1997) has shown that an Eddington factor $\Gamma = \kappa L/(4\pi c G M)$ tending toward 1.0 implies convection. Our aim is to show that fast rotation amplifies the size of the convective envelope in OB stars as well as to develop anisotropic convective envelopes.

Various limits can be considered about the effects of rotation and high luminosity on the stellar stability (Langer 1997; Maeder & Meynet 2000): the Γ-limit, which is the Eddington limit for $\Gamma \to 1$; the Ω-limit, which is reached by stars at rotational break-up with a small or negligible effect of the Eddington factor Γ; the ΩΓ-limit, which applies to stars where both luminosity and rotation play significant roles. We show that not only the stars at the Γ-limit, but also the stars at the ΩΓ-limit and at the Ω-limit, have amplified external convective zones.

The occurrence of outer convective envelopes in OB stars and their anisotropic structure lead to many astrophysical consequences:

- convection generates acoustic modes that may allow asteroseismic observations of OB stars;
- convective motions may play a role in driving mass loss by stellar winds;
- for stars close to the critical rotation, convective motions lower the effective break-up velocities;
- an outer convective envelope may make a dynamo and contribute to some chromospheric activity generating an X-ray emission from OB-stars;
- a convective envelope transports chemical elements and angular momentum;
- the occurrence of outer convection may modify the von Zeipel theorem (von Zeipel 1924).

A closer investigation is justified. We start by an analytical approach (Sect. 2), and finish by two-dimensional models of 20 $M_\odot$ rotating envelopes (Sect. 3).

2. Convection in rotating stars

In a rotating star of mass M, luminosity L, and angular velocity Ω (supposed to be shellular, i.e. constant on shells), the total gravity is the sum of the gravitational, centrifugal, and radiative accelerations:

$$\boldsymbol{g}_{\rm tot} = \boldsymbol{g}_{\rm eff} + \boldsymbol{g}_{\rm rad} = \boldsymbol{g}_{\rm grav} + \boldsymbol{g}_{\rm rot} + \boldsymbol{g}_{\rm rad}. \quad (1)$$

The vector $\boldsymbol{g}_{\rm eff}$ has both radial and tangential components, the radial component at colatitude ϑ is

$$g_{{\rm eff},r} = -\frac{GM_r}{r^2}\left(1 - \frac{\Omega^2 r^3}{GM_r}\sin^2\vartheta\right), \quad (2)$$

where r is the radius at colatitude ϑ. The radiative acceleration is directed outward

$$\boldsymbol{g}_{\rm rad} = \frac{1}{\rho}\boldsymbol{\nabla} P_{\rm rad} = \frac{\kappa(\vartheta)\boldsymbol{F}}{c}, \quad (3)$$

where $\boldsymbol{F}$ is the flux. On an isobaric surface, $\boldsymbol{F}$ is given by the von Zeipel theorem (von Zeipel 1924),

$$\boldsymbol{F} = -\frac{L(P)}{4\pi G M_\star}\boldsymbol{g}_{\rm eff}, \quad (4)$$

$$\text{with} \quad M_\star(r) = M_r\left(1 - \frac{\Omega^2}{2\pi G\rho_{\rm m}}\right), \quad (5)$$

and $L(P)$ is the luminosity on the isobar, $\rho_{\rm m}$ the internal average density. In baroclinic stars, there are other terms (Maeder 1999), however they are small and neglected here. The flux is proportional to the effective gravity $\boldsymbol{g}_{\rm eff}$. The effective mass $M_\star$ is the mass reduced by the centrifugal force. Let us note that one has $\Omega^2/(2\pi G\rho_{\rm m}) \approx (4/9)(v/v_{\rm crit})^2 \approx (16/81)\omega^2$, where $\omega = \Omega/\Omega_{\rm crit}$, v is the rotation velocity at the level considered and $v_{\rm crit} = (2/3)[GM/R_{\rm crit}(\vartheta = 0)]^{1/2}$.

2.1. Effect of rotation on the thermal gradient

Formally the Solberg-Hoiland criterion is to be considered in a rotating star, as in Sect. 2.2. However, the radiative gradient $\nabla_{\rm rad}$ is also modified by rotation, an effect generally not accounted for in Schwarzschild's criterion. The local flux and the equation of hydrostatic equilibrium are

$$\boldsymbol{F} = -\chi \boldsymbol{\nabla} T \quad \text{and} \quad \boldsymbol{\nabla} P = \varrho\, \boldsymbol{g}_{\rm eff}, \tag{6}$$

with $\chi = 4acT^3/(3\kappa\varrho)$. Radiation pressure is included in P, the total pressure. The local radiative gradient becomes in a rotating star,

$$\nabla_{\rm rad} = \frac{{\rm d}T}{{\rm d}n}\frac{{\rm d}n}{{\rm d}P}\frac{P}{T} = \frac{3}{16\,\pi\,a\,c\,G}\,\frac{\kappa\,L(P)\,P}{M_\star(r)\,T^4}, \tag{7}$$

where the derivatives are computed along a direction $\boldsymbol{n}$ perpendicular to the isobars. Except $L(P)$, the terms are local and thus have to be taken at a given (r, ϑ). We ignore the horizontal thermal gradient and take $L(P)$ as constant in the envelope. With (5) and the expression of the Eddington factor Γ, we get

$$\nabla_{\rm rad} = \frac{\Gamma}{4\,(1-\beta)\left(1-\frac{\Omega^2}{2\pi G\rho_{\rm m}}\right)}, \tag{8}$$

where $\beta = P_{\rm g}/P$ is the ratio of gas pressure to the total pressure, thus $P/(aT^4) = 1/[3(1-\beta)]$. The adiabatic gradient $\nabla_{\rm ad}$ is

$$\nabla_{\rm ad} = \frac{8-6\beta}{32-24\beta-3\beta^2}. \tag{9}$$

As T varies with ϑ, β also varies with colatitude, and we write $\beta(\vartheta)$. That $\beta(\vartheta)$ is higher at the equator favors equatorial convection. The criterion for convective instability $\nabla_{\rm rad} > \nabla_{\rm ad}$ becomes

$$\frac{\Gamma(\vartheta)}{\left(1-\frac{\Omega^2}{2\pi G\rho_{\rm m}}\right)} > 4\,[1-\beta(\vartheta)]\,\nabla_{\rm ad}, \tag{10}$$

where the ϑ-dependence of Γ comes only through $\kappa(\vartheta)$ (Maeder & Meynet 2000). Equation (10) has various interesting consequences:

- in the absence of rotation, expression (8) is equivalent to Langer's result (Langer 1997). The right-hand side of Eq. (10) is always smaller than 1.0, thus if $\Gamma \to 1$, the criterion is satisfied. Convection is present in layers close to the Eddington limit;
- in a rotating star, inequality (10) is more easily satisfied. Thus, rotation favors convection in stellar envelopes;
- the occurrence of convection depends on both $\kappa(\vartheta)$ and $\beta(\vartheta)$. Equatorial ejection is always favored, even for electron scattering opacity caused by the higher β;
- when the centrifugal force can be derived from a potential (conservative case), the temperature and density are constant on isobars and so that Γ and β are also constant on isobars. In that case, rotation also favors convection as can be seen from Eq. (9).

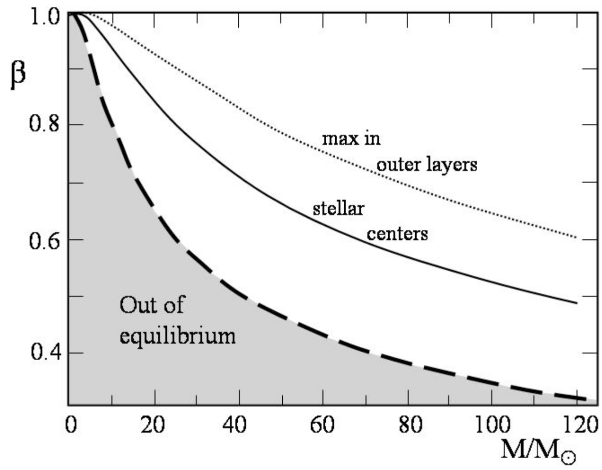

Fig. 1. The ratio $\beta = P_{\rm gas}/P$ as a function of the stellar masses on the zero-age main-sequence with $Z = 0.02$. The continuous line shows the value at the stellar centers, the dotted line the maximum value of β inside these models. The long-dashed line indicates the minimum value of β permitted by equilibrium conditions (Chandrasekhar 1984; Mitalas 1997).

One can wonder whether rotating stars that are neither at the critical nor at the Eddington limit may develop a convective envelope. The ratio β decreases with mass (Fig. 1), while the Eddington factor Γ increases, e.g. $\Gamma = 2.5 \times 10^{-5}$, 0.0047, 0.021, 0.098, 0.239, 0.343, 0.544 for 1, 5, 9, 20, 40, 60, and 120 $M_\odot$ stars on the ZAMS. The parameter β is at a minimum in the stellar centers, reaches a maximum in the envelope, and is zero at the stellar surface. One has the following relation between the maximum β-values in the outer layers and Γ,

$$\beta = 1.0 - s\,\Gamma \qquad \text{or} \qquad \Gamma = \frac{1}{s}(1-\beta), \tag{11}$$

with $s = 0.72 \pm 0.01$ between 20 $M_\odot$ and 120 $M_\odot$. Using Eq. (11), one eliminates Γ from criterion (10) and gets

$$\frac{1}{s\left(1-\frac{\Omega^2}{2\pi G\rho_{\rm m}}\right)} > 4\,\frac{8-6\beta}{32-24\beta-3\beta^2}. \tag{12}$$

This relation indicates above which value of ω there is convection for a given value of β at the maximum (for lower β in the envelope, the inequality is evidently satisfied more easily). For example, for $\beta \to 1.0$, the inequality becomes $\frac{\Omega^2}{2\pi G\rho_{\rm m}} > 0.132$ or $(v/v_{\rm crit}) > 0.54$. This is an approximation owing to the simplified relation adopted and because the various parameters also vary with depth. However, it shows that rotating massive stars not even at the critical limit may have enhanced convection.

2.2. The Solberg-Hoiland criterion

A fluid element displaced in a rotating star is also subject to the restoring effect of angular momentum conservation. This leads to the Solberg-Hoiland criterion for stability (Kippenhahn & Weigert 1990), which is (for constant mean molecular weight μ)

$$\nabla_{\rm ad} - \nabla_{\rm rad} + \nabla_\Omega \sin\vartheta > 0 \tag{13}$$

$$\text{with} \quad \nabla_\Omega = \frac{H_P}{g_{\rm grav}\delta}\,\frac{1}{\varpi^3}\,\frac{{\rm d}(\Omega^2\varpi^4)}{{\rm d}\varpi}, \tag{14}$$

where $\varpi = r\sin\vartheta$ is the distance to the rotation axis and $\delta = -(\partial\ln\varrho/\partial\ln T)_P$. The quantity ∇_Ω depends on the distribution of the specific angular momentum $j = \varpi^2\Omega$, which results from transport processes. As j decreases outward, ∇_Ω generally has a stabilizing effect. Let us consider the two extreme cases for $\Omega(r)$:

1. *Constant specific angular momentum*: a distribution $\Omega \sim r^{-2}$ may result from the Rayleigh-Taylor instability. This distribution is also sometimes considered in convective regions, with the argument that the plumes rapidly redistribute the angular momentum. If so, $\nabla_\Omega = 0$, and one is brought back to Schwarzschild's criterion.
2. *Constant angular velocity*: this assumption is also used in convective regions, with the argument that turbulent viscosity favors solid rotation. If so, ∇_Ω simplifies to

$$\nabla_\Omega = 4\frac{\Omega^2}{g_{\rm grav}}\frac{H_P}{\delta} = \frac{4\,\Omega^2}{g_{\rm grav}\,\varrho}\,\frac{P}{g_{\rm eff}\delta}. \qquad (15)$$

We can simplify this expression further. In the outer layers, as long as $\kappa \approx$ const. and $g_{\rm eff} \approx$ const., at an optical depth τ one has $P \approx (g_{\rm eff}/\kappa)\,\tau$. This gives

$$\nabla_\Omega \approx 4\,\frac{\Omega^2}{g_{\rm grav}\,\varrho\,\kappa\delta}\,\tau \approx 4\left(\frac{\Omega^2 R^3}{G\,M}\right)\left[\frac{\tau}{\varrho\,\kappa\,R\delta}\right]. \qquad (16)$$

The term in the first parenthesis is ω^2, while that in square brackets is just the ratio $(R - r)/R$ (assuming $\delta = 1$), which is small in the envelope. The Solberg-Hoiland criterion becomes in this approximation,

$$\frac{\Gamma}{\left(1 - \frac{\Omega^2}{2\pi G\rho_{\rm m}}\right)} > 4\,(1-\beta)\left(\nabla_{\rm ad} + \omega^2\left[\frac{R-r}{R}\right]\sin\vartheta\right), \qquad (17)$$

where as above the various quantities are local ones. At low rotation, the Solberg-Hoiland term ∇_Ω is negligible with respect to the other terms. At high rotation for constant Ω, it is not negligible, but in general smaller than the other terms because the convective zone lies very close to the surface and the term $(R - r/R)$ is small.

Since the actual rotation laws are likely between the two extreme cases $\Omega(r) =$ const. and $\Omega \sim r^{-2}$, we conclude that the main effect of rotation on convection in stellar envelopes is not the inhibiting effect due to the Solberg-Hoiland criterion, but the effect of rotation on the thermal gradient (Eq. (10)), which enhances convection.

3. Numerical models

We do some 2D models of the outer regions of a 20 $M_\odot$ fast-rotating star with $X = 0.70$ and $Z = 0.020$ (Fig. 2). At each latitude we integrate the equations of the structure for the corresponding effective gravity and $T_{\rm eff}$ of the Roche model of the given rotation, also taking the effect of the reduced mass into account. In the envelope, we suppose that Ω is a constant as a function of depth (the problem is conservative). We first consider only the effect of rotation on the thermal gradient and then the complete Solberg-Hoiland criterion to see the differences.

3.1. Effects of rotation on the thermal gradient

Without rotation, a 20 $M_\odot$ model at the end of the MS evolution has two outer convective zones. The first one is very close to the surface and is due to an increase of the opacity caused by partial He ionization. It extends from $r/R = 0.992$ to 0.999, i.e. only 0.7% of the radius, and contains a very small fraction of the total stellar mass (2.5×10^{-9}). The second one is deeper, between

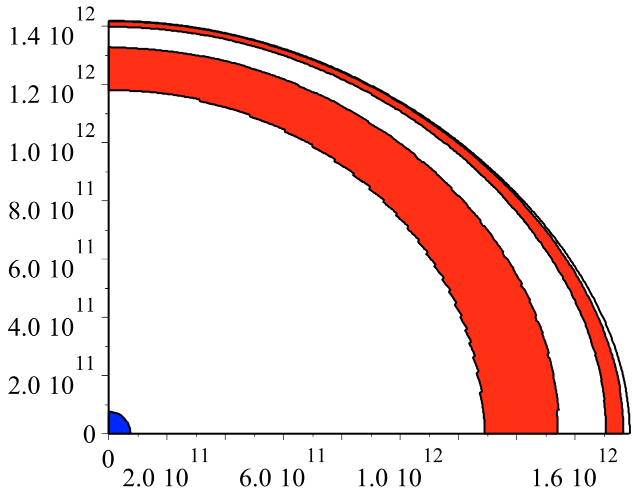

Fig. 2. 2D model of the external convective zones and of the convective core (dark areas) in a model of 20 $M_\odot$ with $X = 0.70$ and $Z = 0.020$ at the end of MS evolution with fast rotation ($\omega = \Omega/\Omega_{\rm crit} = 0.94$). The axes are in units of cm.

$r/R = 0.915$ to 0.962 (4.7% of the total radius), and contains a fraction 7.4×10^{-7} of the total mass. Both convective zones are associated with opacity enhancements.

For fast rotation, with a ratio $\Omega/\Omega_{\rm crit} = 0.94$, where $\Omega_{\rm crit}$ is the critical angular velocity, the convective layers are shown in Fig. 2. They are more extended than without rotation. The thin upper convective zone extends from $r/R = 0.987$ to 0.999 at the pole, i.e., over 1.2% of the stellar radius, and it contains 1.3×10^{-8} of the stellar mass. The deeper convective zone covers the region between $r/R = 0.836$ to 0.936 (i.e. 10.0% of the polar radius), and its mass fraction is 2.8×10^{-6}. At the equator, we have the following sizes for the two convective layers: between $r/R = 0.958$ and $r/R = 0.988$ for the first one (3.0% of the equatorial radius) and between $r/R = 0.727$ and $r/R = 0.862$ for the deeper one (13.5% of the radius). The included masses are the same as at the pole.

In agreement with Sect. 2.1, convection is more extended in the rotating model than in the non-rotating one. Figure 2 shows that contrarily to the classical Cowling model, massive rotating stars have a large-size convective envelope. Interesting is that, if we look at the structure of the envelope of the star as a function of the pressure, we see that this structure is independent of the colatitude. This is expected since we have supposed Ω constant in the envelope. In that case, as recalled above, Γ and β are constant on isobars and the extensions of the convective zones, expressed in term of *differences of pressure* between the bottom and the top of the convective zone are the same at the pole and at the equator. The *spatial* extensions are, however, greater in the equatorial region than in polar ones. This comes from the variations in the spatial gradients of the pressure and temperature with the colatitude imposed by the hydrostatic equilibrium (gradients of pressure have to balance $\rho g_{\rm eff}$). Another result of the constancy of pressure and temperature (and thus density) on an isobar is that the radiative gradient is also constant on this isobar, except for the change in the effective mass as given by M_*, which is lower than M in rotating stars. Thus, for rotating stars the radiative gradient is larger and convection is favored not only at the equator, but at each colatitude, compared with the non-rotating model.

The mass loss rate in the considered model is $6.2 \times 10^{-7}\ M_\odot\ {\rm yr}^{-1}$. This means that within one year about 4 times all

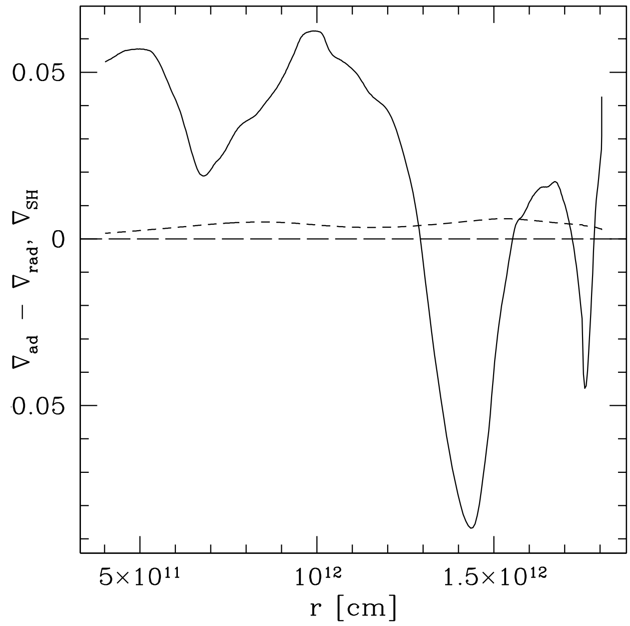

Fig. 3. $\nabla_{\rm ad} - \nabla_{\rm rad}$ (solid line) and $\nabla_\Omega \sin\vartheta$ (short-dashed line) at the equator in a 20 $M_\odot$ model with $\Omega/\Omega_{\rm crit} = 0.94$ at the end of the MS. The long-dashed line indicates the zero level.

the matter in the thin convective zone is lost in the stellar winds! Thus, the matter carried by the winds is continuously passing through the superficial convective zones in a dynamical process.

3.2. Solberg-Hoiland criterion

We also computed the envelope structure with the Solberg-Hoiland criterion for convection. The values of $(\nabla_{\rm ad} - \nabla_{\rm rad})$ and $\nabla_\Omega \sin\vartheta$ are shown in Fig. 3 for a 20 $M_\odot$ model at the equator, i.e. where the effect of the Solberg-Hoiland criterion is the strongest. The solid line shows the difference between the adiabatic and the radiative gradient (identical to the values obtained in the model computed with the Schwarzschild criterion). The short-dashed line shows the Solberg-Hoiland term. The limits of the convective zones when the Soilberg-Hoiland criterion is used are inside the regions where $(\nabla_{\rm ad} - \nabla_{\rm rad})$ is negative. Indeed, the limits are where $(\nabla_{\rm ad} - \nabla_{\rm rad}) + \nabla_\Omega \sin\vartheta = 0$. Convective zones are thus smaller than those obtained with the Schwarzschild criterion. We find the following values for the extension of the convective zones at the equator when the Solberg-Hoiland criterion is used: between $r/R = 0.960$ and $r/R = 0.988$ for the superficial one (2.8% of the equatorial radius) and between $r/R = 0.727$ and $r/R = 0.859$ for the deeper one (13.2% of the total radius), i.e. values slightly smaller than but very similar to those obtained in the model computed with the Schwarzschild criterion. This numerical example confirms that the Solberg-Hoiland term $\nabla_\Omega \sin\vartheta$ has a very limited influence in stellar envelopes, as discussed in Sect. 2.3.

4. Conclusions

In stellar envelope of rotating stars, the effects of rotation on the thermal gradient arestronger and with the opposite sign with respect to the Solberg-Hoiland criterion, so that rotation favors convection instead of inhibiting it. The increase of the convective zone occurs mainly at the equator and also a bit at the poles. In a fast-rotating 20 $M_\odot$ Pop I star, there are two equatorial zones covering a total of 16% of the stellar radius at the equator.

There are several consequences of thees results to be examined in future. The outer convective motions may lower the escape velocity as well as the critical rotation velocity. The matter accelerated in the winds continuously goes through the convective zone in a dynamical process, suggesting that convection plays a role in accelerating the stellar winds and in producting the clumps in the winds. The convective pistons generate acoustic waves of periods of several hours to a few days. The density is very low, and it is thus likely that convection injects oscillations into the wind rather than into the interior.

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