

# Constraints on cosmic-ray positron excess and average pulsar parameters

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## ABSTRACT

Recent, accurate  $e^+/(e^++e^-)$  ratio measurements in cosmic rays allow us to distinguish among different estimates of secondary positron production in the interstellar medium (ISM), provided the effect of solar modulation and solar polarity are properly taken into account. Data above a few GeV indicate that a possible extra component of positrons could be required in addition to the secondaries. This positron excess is compatible with the hypothesis of pair production at the polar cap of mature pulsars. Assuming only pulsar contributions without any exotic contributions such as dark-matter annihilation, the average parameters of Galactic pulsars contributing to positron and electron interstellar fluxes were obtained. These parameter values are found near the peak of the distributions of the observed characteristics of radio pulsars. The studied gamma-ray pulsar sample is too small to make any conclusion. The expected  $e^+/(e^++e^-)$  ratio from the PAMELA experiment currently in orbit is reported in this paper. The GLAST mission will allow us to double-check our findings about the role of pair production at the pulsar polar cap and outer gap.

**Key words.** cosmic rays – Sun: activity – Sun: magnetic fields – Sun: solar-terrestrial relations – stars: pulsars: general

## 1. Introduction

Early observations of positrons and antiprotons (Muller & Tang 1987; Golden et al. 1979; Bogomolov et al. 1979) seem to indicate a major excess of antimatter with respect to the estimated secondary component produced by primary cosmic-ray collisions in the interstellar medium. These observations generated many different speculations about the extra sources of positrons (see for example, Grimani 2004 and references therein). Due to the improvement of particle detectors and data analysis techniques,  $e^+$  and  $\bar{p}$  measurements carried out during the past fifteen years present lower values compared to the past (Grimani et al. 2002; Hof et al. 1996; Asaoka et al. 2002). At present, there is general agreement about  $e^+$  and  $\bar{p}$  essentially having secondary origin. However, various estimates of  $e^+$  and  $e^-$  secondary fluxes in the ISM differ by more than a factor of two as a function of the energy (see Protheroe 1982; Moskalenko & Strong 1998; Stephens 2001a,b). Moreover, the solar activity and drift of opposite-charge particles in the heliosphere modulate cosmic-ray observations near Earth (Potgieter & Langner 2004), and only an accurate evaluation of the drift process during opposite polarity periods allows us to distinguish among different models of secondary electron and positron calculations. Finally, only a reliable secondary-component evaluation leads to identifying possible extra positron sources. As an example, it was proposed in Couto et al. (1999), among other possibilities, that the feature observed above 6 GeV in the  $e^+/(e^++e^-)$  HEAT data was due to supersymmetric particle annihilation. This possibility could have been ruled out in the case of the secondary calculation by Stephens was assumed (instead of the models by Protheroe or Moskalenko & Strong), as can be noticed in Fig. 1.

We have used experimental results on protons, helium, electrons, and positrons to estimate the overall effect of the solar modulation. We find that, once secondary  $e^+$  and  $e^-$  are properly

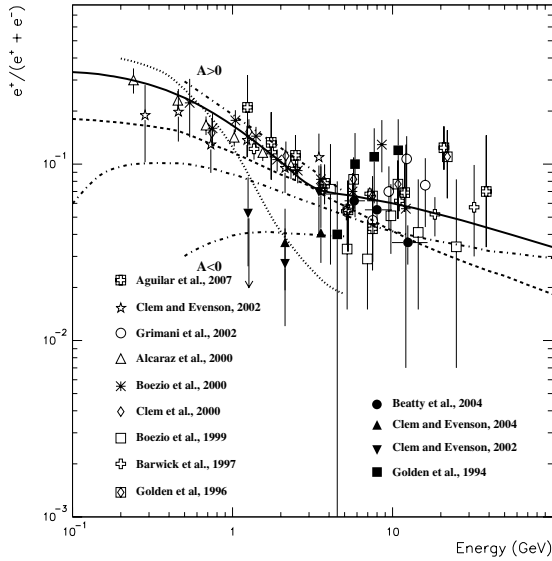
taken into account, data indicate that an additional component of positrons must be claimed above a few GeV. This excess is found to be compatible with a model of pair production at the pulsar polar cap (Harding & Ramaty 1987, hereafter H&R87). Recent, low-error pulsar birthrate (PB) measurements (Faucher-Giguère & Kaspi 2005) allow us to determine the average parameters of pulsars contributing to interstellar positron and electron fluxes. We improve on our previous work, where we set an upper limit to the PB using positron measurements (Grimani 2004).

## 2. Comparison of $e^+$ measurements to secondary positron and electron production models

In Grimani (2004), data revealed that only experiments carried out during the past fifteen years present reliable electron and positron identification with proton rejection factors against positrons of  $10^5$  (we recall that  $e^+/p \approx 10^{-3}$  in cosmic rays). In Fig. 1 we report the most recent  $e^+/(e^++e^-)$  ratio measurements. We compare the data and secondary  $e^+$  and  $e^-$  production models separately below and above 4 GeV where the solar modulation plays a minor role in affecting the  $e^+/(e^++e^-)$  ratio measurements.

### 2.1. Positron fraction below 4 GeV

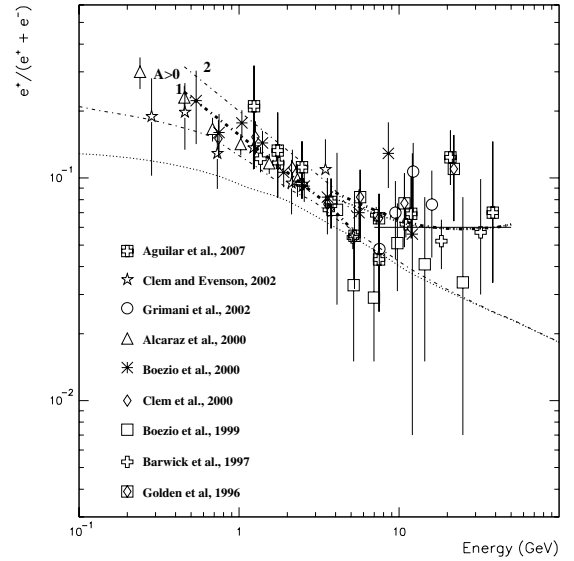
Long-term cosmic-ray variations can be correlated with the 11-year solar cycle and the 22-year global solar magnetic field (GSMF) polarity reversal. The most widely used model for solar modulation is the symmetric model in the *force field approximation* by Gleeson & Axford (1968). In it, the cosmic-ray flux intensity  $J(r, E, t)$  at the radial distance  $r$  from the Sun at time  $t$  for a total energy  $E$  of the cosmic-ray particles is related to the time-independent interstellar intensity  $J(\infty, E)$ .



**Fig. 1.** Positron fraction measurements in cosmic rays carried out during the past fifteen years. Continuous and dashed lines represent the calculated secondary  $e^+/(e^++e^-)$  ratio according to Stephens (2001a,b) and Moskalenko & Strong (1998), respectively, near Earth when an average solar modulation (no drift;  $\phi = 550$  MV/c) is considered. The middle dot-dashed curve down to 100 MeV represents the secondary positron fraction estimated by Protheroe (1982) on the basis of a *Leaky Box Model* at ISM. The top and bottom dot-dashed curves have been determined by Clem et al. (2000) assuming the Protheroe model with the inclusion of solar modulation and solar polarity effects. The dotted curve represents the positron fraction estimated by Potgieter & Langner (2004) at solar minimum during a positive polarity period.

This simple model does not take into account that the GSMF polarity influences the drift of positive and negative particles in the heliosphere. During positive heliomagnetic field polarity ( $A > 0$ ), positive charge particles most likely reach the Earth from the polar regions of the heliosphere, while negative charge particles ( $e^-$ ,  $\bar{p}$ ) come mainly from the ecliptic regions along the heliospheric current sheet (HCS). The opposite situation holds during negative magnetic field polarity epochs ( $A < 0$ ). Particle fluxes propagating along the HCS are more modulated with respect to those coming from the poles. Modelizations of electron and positron fluxes for different conditions of solar modulation and solar polarity have been carried out by Potgieter & Langner (2004). However, these authors use ISM electron and positron energy spectra about 30% higher than other authors below 1 GeV, while the  $e^+$  spectrum becomes lower above 1 GeV. The result is that, at solar minimum, the  $e^+/(e^++e^-)$  prediction during a positive polarity epoch lie well below observations gathered both in positive and negative polarity periods (see Fig. 1). Therefore, when possible, we prefer to infer the role of the solar modulation and, in particular, of the drift process, from experimental data gathered by the same experiment in different conditions of solar modulation and solar polarity.

Impressive agreement is found between the Gleeson and Axford model and the BESS proton-flux observations gathered in positive polarity (Shikaze et al. 2006). Conversely, this model does not allow reproduction of the data of the negative polarity flights from the BESS (Shikaze et al. 2006) and MASS (Grimani et al. 2007) experiments. Drift effects have been observed to determine a maximum reduction of the Galactic cosmic-ray proton and helium fluxes during a negative polarity epoch of 40% at 100 MeV, 30% at 200 MeV, 25% near 1 GeV and of a few % up to 4 GeV at solar minimum (Boella et al. 2001). At solar

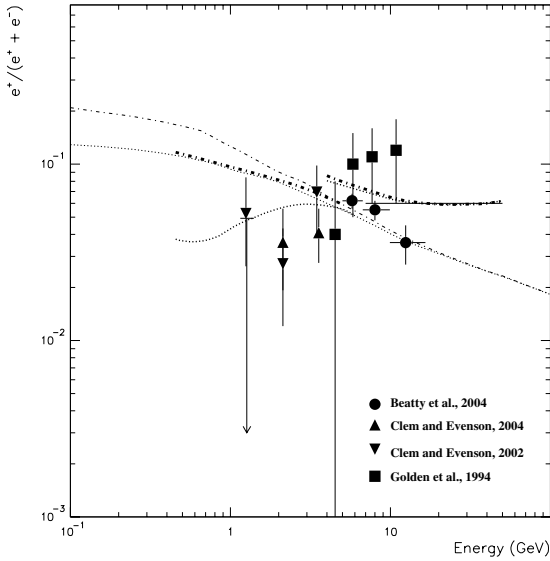


**Fig. 2.** Estimates of the positron fraction near Earth at solar minimum ( $\phi = 350$  MV/c; thin dot-dashed line) and maximum ( $\phi = 1200$  MV/c; thin dotted line) using the model by Gleeson & Axford (1968) on the basis of the Moskalenko & Strong (1998) calculations. The thick dot-dashed curve 1 represents the expected solar minimum positron fraction observations during positive polarity periods. The curve 2 is the estimated upper limit for the  $e^+/(e^++e^-)$  ratio during positive polarity epochs. Thick dot-dashed and dotted lines above 4 GeV indicate the expected  $e^+/(e^++e^-)$  ratio at solar minimum and maximum, respectively, when  $e^+$  and  $e^-$  of secondary and pulsar polar cap origin are considered.

maximum, the drift process is found to be ineffective (Boella et al. 2001); however, large fluctuations in the opposite charge particle ratio have been observed before and after the polarity change (see for example Moraal et al. 1991). In particular, the minimum values for the  $e^+/(e^++e^-)$  ratio are supposed to be detected after a polarity change from + to - at solar maximum.

Figure 1 shows the positron fraction estimated near Earth on the basis of the secondary  $e^+e^-$  calculations according to Stephens (2001a; 2001b) and Moskalenko & Strong (1998). We modulated the electron and positron fluxes using the Gleeson and Axford model. The calculations by Protheroe (1982) were carried out at ISM assuming a *Leaky Box Model*. The effect of solar modulation, including drift, was estimated by Clem et al. (2000). These authors assumed an identical effect on  $e^+$  and  $e^-$  fluxes for the solar modulation (defined by the modulation parameter,  $\phi$ , only) and evaluated the drift effect separately. However,  $e^+$  and  $e^-$  would suffer the same modulation in the case of an identical energy spectrum shape, which is not the case (see for example Grimani et al. 2002). Moreover, the top curve in Fig. 1 overestimates recent, accurate data gathered during a positive polarity epoch, while the errors of negative polarity period data are too large to make any conclusion.

The comparison of the other two models to data indicates that the Stephens calculations overestimate most of the data below 1 GeV. The inclusion of the drift effect during a positive polarity period would enhance this trend. When using the Moskalenko and Strong calculations, we modulated the  $e^+$  and  $e^-$  fluxes estimated by these authors at ISM on the basis of a reasonable diffusion model, in conditions of solar minimum ( $\phi = 0.35$  GV/c) and solar maximum ( $\phi = 1.2$  GV/c) (Figs. 2 and 3). To evaluate the drift effect, we considered a positive particle flux reduction during a negative polarity period at a solar minimum analogous to that reported by Boella et al. for protons



**Fig. 3.** Same as Fig. 2 for negative polarity periods. The bottom, thick-dotted curve has been inferred for  $e^+/(e^++e^-)$  from the Moskalenko et al. (2001) model developed for the  $\bar{p}/p$  ratio at polarity change from + to -.

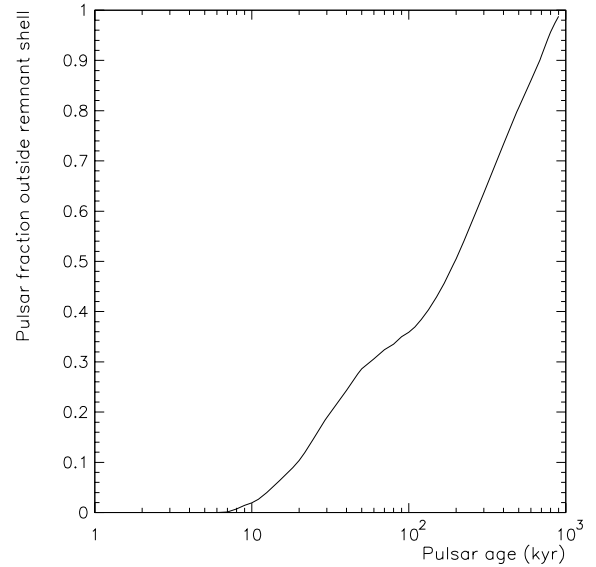
(Fig. 3). Since both modulation and drift are essentially rigidity-dependent processes, we have properly evaluated the electron and positron rigidity corresponding to the proton kinetic energy given by Boella et al. In addition we assumed an analogous drift effect on positive and negative particles during negative and positive polarity epochs, respectively. The Boella et al. results agree with those published by Moraal et al. (1991) for He/e ratio measurements in the electron energy range 600–1000 MeV and the observations by Clem et al. (2000) of the  $e^+/(e^++e^-)$  ratio at 1.3 GeV.

Our results are reported in Fig. 2 (curve 1), in conditions of positive polarity at solar minimum. Curve 1 goes through the data as expected when good agreement is found between the model and observations within measurement statistical errors. We also set an upper limit to the expected  $e^+/(e^++e^-)$  measurements below a few GeV (curve 2 in Fig. 2) before the polarity change from + to - (see Fig. 5 in Moraal et al. 1991). The  $e^+/(e^++e^-)$  ratio trend expected soon after a polarity change from + to - at solar maximum is shown in Fig. 3. This estimate was carried out from the Moskalenko et al. (2001) modelization of the  $\bar{p}/p$  ratio developed for different polarity epochs and various solar modulation levels. This model agrees very closely with the BESS  $\bar{p}/p$  (Asaoka et al. 2002) and the LEE/AESOP (Clem & Evenson 2004)  $e^+/(e^++e^-)$  observations, both carried out in 2000. Our results allow us to predict low-energy observations, for an experiment such as PAMELA (Boezio et al. 2005) at solar minimum during a negative polarity epoch (see Sect. 4).

## 2.2. Positron fraction above 4 GeV

We have found that recent  $e^+/(e^++e^-)$  ratio observations above 7 GeV are best-fitted by the constant value 0.06 (Figs. 2 and 3). The comparison among low-energy  $e^+/(e^++e^-)$  data and theoretical calculations indicates the secondary component by Moskalenko and Strong to be preferred to the others.

When the secondary component is subtracted from the best fit, the extra positron component appears to be consistent with an  $e^+$  production at the polar cap of young radio pulsars (typically Crab and Vela, H&R87) when a PB of one pulsar born every



**Fig. 4.** Estimated fraction of pulsars outside host remnants as a function of age (Arzoumanian et al. 2001).

33 years and a pulsar lifetime for pair production of  $10^4$  years are considered (see Fig. 2 in Grimani 2005). The pulsar polar cap electron and positron fluxes originally estimated in H&R87 at the interstellar medium have been normalized to  $1/PB = 33$  years and propagated in the heliosphere. Our results are reported in Figs. 2 and 3 at both solar minimum and maximum above 4 GeV after adding the  $e^+-e^-$  secondary component estimated by Moskalenko and Strong.

The  $1/PB = 33$  year assumption is in very good agreement with recent radio pulsar observations (Faucher-Giguère & Kaspi 2005); however, against the H&R87 model, it can be argued that  $e^+$  and  $e^-$  suffer severe energy losses in the remnants surrounding young pulsars. Polar cap models predict that all radio pulsars are capable of emitting gamma rays to some extent. Moreover, an increasing number of pulsars escape their host remnant shells as a function of age. Consequently, it was suggested that electrons and positrons produced at the polar cap of mature pulsars are favored for reaching the interstellar medium (Chi et al. 1996; Zhang & Cheng 2001). In the next paragraph, we determine the average parameters of pulsars contributing to  $e^+$  and  $e^-$  interstellar fluxes assuming this last scenario.

## 3. Pulsar parameters from positron observations

Arzoumanian et al. (2001) have estimated the fraction of pulsars escaped from host remnants versus age (Fig. 4). We used the results of their work to evaluate whether pair production at the polar cap of pulsars escaped from host remnants leads to a result similar to what is reported in the H&R87 original paper for young pulsars. In the H&R87 model, only 0.0625% of the Galactic sample was considered if a pulsar average age of  $16 \times 10^6$  years was reasonably assumed. In the same work, an  $e^+$  luminosity per young pulsar proportional to  $B_{12} P^{-1.7}$  above a few GeV (where  $B_{12}$  is the pulsar surface magnetic field in terms of  $10^{12}$  Gauss and  $P$  is the period in seconds) was considered using the parameters of Crab and Vela. To take the whole sample of Galactic pulsars into account, the average luminosity per pulsar should be reduced by about two orders of magnitude compared to what is assumed in H&R87. The parameters of the

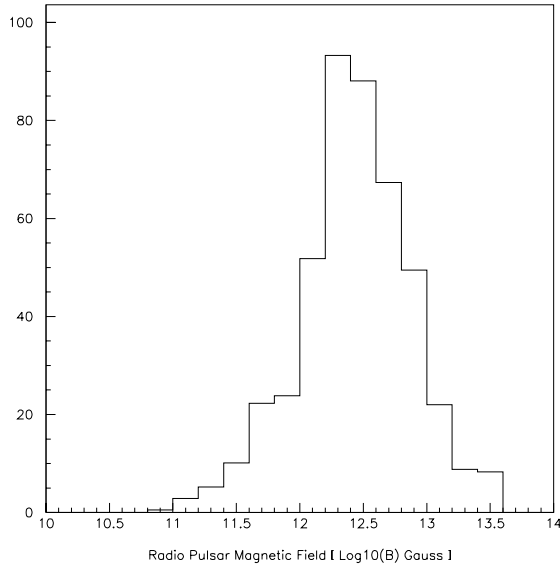


Fig. 5. Distribution of radio pulsar observed magnetic field.

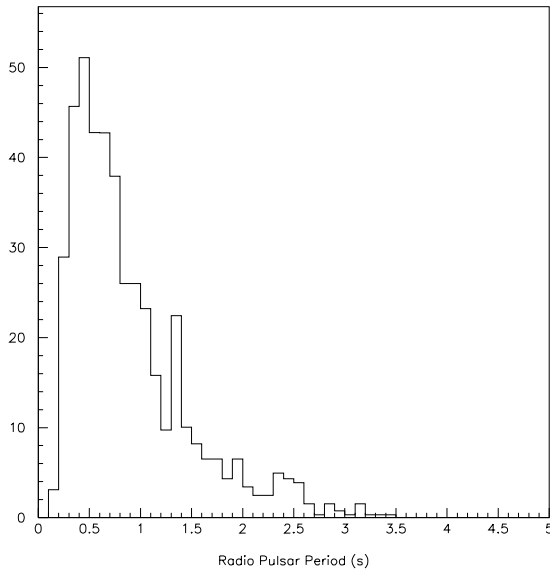


Fig. 6. Distribution of radio pulsar observed period.

Crab ( $B_{12} = 3.8$ ;  $P = 0.033$  s) and Vela ( $B_{12} = 3.4$ ;  $P = 0.089$  s) pulsars should be changed in the calculations accordingly.

The observed characteristics of Galactic radio pulsars (Gonthier et al. 2002) are shown in Figs. 5 and 6. It can be noticed that the magnetic fields of the Crab ( $\log B = 12.58$ ) and Vela ( $\log B = 12.53$ ) pulsars lie near the peak of the distribution. Conversely, the observed pulsar period distribution peaks at about 0.5 s, so the period values assumed in H&R87 appear at the far left edge of the distribution in Fig. 6. If the average magnetic field of radio pulsars contributing to  $e^+$  and  $e^-$  fluxes in the interstellar medium is similar to the one used in H&R87, the average pulsar period is constrained to 300 ms. This period value is characteristic of pulsars having about 2% of the Crab luminosity. However, approximately half of these pulsars are predicted as lying outside host remnants. Therefore, they are expected to contribute about 1% of Crab-like pulsar luminosity.

Average pulsar periods closer to 200 ms would be required to best-fit the positron fraction data in the case of a slightly smaller average magnetic field is assumed (for example  $B_{12} = 1.6$  for

Table 1. Observed gamma-ray pulsar characteristics.

Pulsar	Age (years)	Magnetic Field ( $10^{12}$ G)	Period (ms)
Crab	1300	3.8	33
B1509 – 58	1500	15.4	150
Vela	11 000	3.4	89
B1706 – 44	17 000	1.165	102
B1951+32	110 000	1.1	40
Geminga	340 000	1.6	237
B1055 – 52	530 000	0.97	197

Geminga with a period of 237 ms). These period values are closer to the peak of the observed radio pulsar period distribution. Moreover, a period of 200–300 ms might indicate that the major contribution to interstellar electron and positron fluxes is given by pulsars maintaining a good efficiency for pair production (the youngest among those escaped from host remnants).

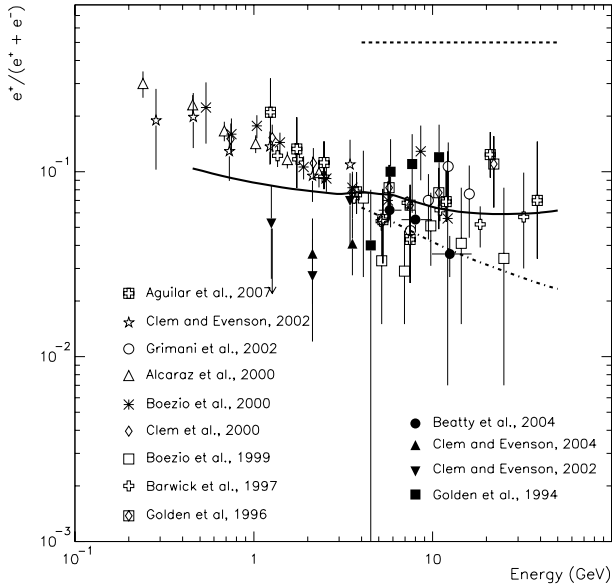
The characteristics of the observed gamma-ray pulsars are summarized in Table 1. The comparison is encouraging but not conclusive because of the uncertainties due to the small available sample.

#### 4. Experimental clues on positron pulsar polar cap origin

Low-error  $e^+$  measurements and pulsed gamma-ray observations from a large sample of pulsars are needed in order to support or to reject the pulsar polar cap  $e^+$  origin. An equal flux of electrons and positrons is supposed to be produced at the pulsar polar cap. The overall positron fraction trend depends on the role played by secondary and polar cap electron and positron fluxes as a function of the energy (see Grimani 2004, 2005). Below 20 GeV, the secondary fluxes dominate the positron fraction, while an overwhelming pulsar component compared to the secondary one gives a flat trend to the  $e^+/(e^+ + e^-)$  ratio above this energy towards the asymptotic value of 0.5 (Fig. 7).

On the basis of this scenario, we can predict the PAMELA experiment observation trend for the period 2006–2007. This time is of particular interest since it corresponds to a negative-polarity, solar-minimum epoch. For obvious reasons no recent, highly reliable data are available under these conditions. The expected  $e^+/(e^+ + e^-)$  PAMELA measurement trend appear in Fig. 7. We modulated the electron and positron fluxes according to an average solar parameter of 550 MV/c, reasonably assumed for the period 2006–2007 ([http://odysseus.uchicago.edu/Neutron-Monitor/neutron\\_mon.html](http://odysseus.uchicago.edu/Neutron-Monitor/neutron_mon.html)), and taken the polarity effect into account on the basis of the results reported in Fig. 3.

A very interesting comparison will be made with the CAPRICE94 (Boezio et al. 2000) observations gathered under the same solar modulation conditions but during a positive polarity period. We point out that, in case our speculations are found correct, the positron flux should present a feature at a few GeV when the pulsar positron component becomes a relevant fraction of the secondary one. The PAMELA data will be available shortly. Consequently, it will be possible to discriminate our speculations from others where a possible dark matter origin for positrons above a few GeV was suggested (see, in particular, Coutu et al. 1999; Lionetto et al. 2005; Picozza et al. 2006). Finally, we expect positron observations to be consistent with those of pulsed gamma-ray measurements from the GLAST experiment that will clarify the role of electromagnetic



**Fig. 7.** PAMELA experiment expected positron fraction observations (thick solid line) during the first period of data taking (2006–2007) at solar minimum during a negative polarity period ( $\phi = 550$  MV/c). The positron fraction resulting from the secondary ( $e_s^+/(e_s^+ + e_s^-)$ ); dot-dashed line) and pulsar ( $e_p^+/(e_p^+ + e_p^-)$ ); dashed line) components are indicated. The PAMELA expected measurements include both components  $[(e_s^+ + e_p^+)/(e_s^+ + e_p^+ + e_s^- + e_p^-)]$ .

energy losses at the polar cap or at the outer gap in the pulsar magnetosphere.

## 5. Conclusions

We have compared the  $e^+/(e^+ + e^-)$  ratio data published during the past fifteen years to various sets of secondary calculations. Low-energy data are affected by different modulations on  $e^+$  and  $e^-$  during opposite solar-polarity epochs. Above a few GeV we determined the positron flux in excess with respect to the best secondary component estimate. We found this excess in agreement with a model of  $e^+$  and  $e^-$  production at the polar cap of mature pulsars. We determined the average magnetic field and period for pulsars contributing to the  $e^+$  and  $e^-$  interstellar fluxes. These parameters were found to be in good agreement with peak radio pulsar observations. Future, low-error positron and pulsed gamma-ray observations from a large sample of pulsars will allow us to confirm or to reject the scenario presented in this paper.

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