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The frequency separations of stellar p-modes

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ABSTRACT

Aims. The purpose of this work is to investigate the characteristics of a new frequency separation of stellar p-modes.

Methods. Frequency separations are deduced from the asymptotic formula of stellar p-modes. Then, using the theoretical adiabatic frequencies of stellar model, we compute the frequency separations.

Results. A new separation $\sigma_{l-1l+1}(n)$, which is similar to the scaled small separation $d_{ll+2}(n)/(2l+3)$, is obtained from the asymptotic formula of stellar p-modes. The separations $\sigma_{l-1l+1}(n)$ and $d_{ll+2}(n)/(2l+3)$ have the same order. And like the small separation, $\sigma_{l-1l+1}(n)$ is mainly sensitive to the conditions in the stellar core. However, with the decrease in the central hydrogen abundance of stars, the separations σ_{02} and σ_{13} deviate more and more from the scaled small separation. This characteristic could be used to extract the information on the central hydrogen abundance of stars.

Key words. star: oscillations – stars: interiors

1. Introduction

Helioseismology has proved to be a powerful tool for probing the structure of the Sun and has given us information on the interior of the Sun. The investigation of asteroseismology is stimulated by the success of the helioseismology and the verification of the solar-like oscillations in several stars, including α Cen A (Bouchy & Carrier 2001; Bedding et al. 2004), α Cen B (Carrier & Bourban 2003), η Boo (Kjeldsen et al. 1995), Procyon (Martić et al. 1999), and β Hyi (Bedding et al. 2001), etc.

The goal of asteroseismology is to extract knowledge of the stellar internal structure that can be used to test and develop our understanding of stellar evolution from oscillation frequencies. The observation of solar-like oscillations is very difficult because of their small amplitude. Only a very limited number of modes ($\ell = 0, 1, 2, 3$) are likely to be observed in solar-like oscillations due to geometrical cancellation effects. How to extract the maximum information on the stellar internal structure from the limited modes is an important problem.

The frequency separations including small separation and large separation have been successfully applied to extract information on stellar interior from oscillation frequencies in asteroseismology and have been investigated by many authors (Christensen-Dalsgaard 1984, 1988, 1993; Ulrich 1986, 1988; Gough 1987, 1990, 2003; Gough & Novotny 1990; Roxburgh & Vorontsov 1994a,b, 2000, 2003; Audard & Provost 1994; Roxburgh 2005; Oti Floranes et al. 2005). The usual frequency separations are the large separation defined by

$$\Delta_l(n) \equiv \nu_{n,l} - \nu_{n-1,l} \tag{1}$$

and the small separation defined by

$$d_{ll+2}(n) \equiv \nu_{n,l} - \nu_{n-1,l+2}. \tag{2}$$

The second difference is given by (Gough 1990; Monteiro & Thompson 1998; Vauclair & Théado 2004)

$$\delta_l(n) \equiv \nu_{n+1,l} + \nu_{n-1,l} - 2\nu_{n,l}, \tag{3}$$

and the difference is defined by Roxburgh (1993, 2003) as

$$d_{01}(n) \equiv (-\nu_{n,1} + 2\nu_{n,0} - \nu_{n-1,1})/2. \tag{4}$$

Moreover, the ratio of small separation to large separation, e.g.,

$$r_l(n) = \frac{d_{ll+2}(n)}{\Delta_l(n)}, \tag{5}$$

was first pointed out to be independent of the outer layer of star and can be used to measure the stellar age by Ulrich (1986). Roxburgh & Vorontsov (2003), Roxburgh (2005), and Oti Floranes et al. (2005) studied $d_{02}(n)/\Delta_1(n)$ and $d_{13}(n)/\Delta_0(n)$ in more detail and demonstrated that the ratio is essentially independent of the structure of the outer layer and is only determined by the interior structure.

The frequency separations mentioned above have been used to diagnose the element diffusion in solar type stars (Vauclair & Théado 2004; Théado et al. 2005; Mazumdar 2005; Castro & Vauclair 2006) and the structure of stellar convective core (Roxburgh & Vorontsov 2001; Mazumdar et al. 2006).

Are there any other frequency separations? If there are other separations, what are their characteristics? In this paper, we focus mainly on investigating another frequency separation and some of its characteristics. In Sect. 2 we give the formulas for frequency separations. In Sect. 3 we present numerical calculation and results. Then, we discuss our results and conclude in Sect. 4.

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2. Asymptotic formula and frequency separations

The asymptotic formula for the frequency $\nu_{n,l}$ of a stellar p-mode of order n and degree l was given by Tassoul (1980)

$$\nu_{n,l} \sim \left(n + \frac{l}{2} + \varepsilon\right)\nu_0 - [Al(l+1) - B]\nu_0^2 \nu_{n,l}^{-1}, \tag{6}$$

for $n/(l+\frac{1}{2}) \to \infty$, where

$$\nu_0 = \left(2\int_0^R \frac{\mathrm{d}r}{c}\right)^{-1} \tag{7}$$

and

$$A = \frac{1}{4\pi^2\nu_0}\left[\frac{c(R)}{R} - \int_0^R \frac{1}{r}\frac{\mathrm{d}c}{\mathrm{d}r}\mathrm{d}r\right], \tag{8}$$

in which c is the adiabatic sound speed at radius r, and R is some fiducial radius of the star; ε and B are the quantities that are independent of the mode of oscillation but depend predominantly on the structure of the outer parts of the star; ν_0 is related to the sound travel time across the stellar diameter; A is a measure of the sound-speed gradient. In Eq. (8), the integral is large compared with the surface term $c(R)/R$ (Gough & Novotny 1990); therefore, A is most sensitive to conditions in the stellar core and is invariant under homologous transformation. Consequently, the nonhomologous changes brought by the nuclear transmutation can be indicated by the variation in A (Gough & Novotny 1990, 2003; Christensen-Dalsgaard 1993). Formula (6) is a second-order asymptotic description of low-degree p-modes under the Cowling approximation, in which the effects of gravitational perturbations are neglected. This formula is inaccurate except for very high frequencies (Roxburgh & Vorontsov 1994a,b; Audard & Provost 1994).

Using the definitions (1), (2), and the asymptotic formula (6), Gough & Novotny (1990) obtained the large separation

$$\begin{aligned}\Delta_l(n) &= \nu_{n,l} - \nu_{n-1,l} \\ &= \nu_0\left(\frac{\nu_{n-1,l} - [Al(l+1)-B]\nu_0^2\nu_{n,l}^{-1}}{\nu_{n-1,l}}\right)^{-1} \\ &\simeq \nu_0,\end{aligned} \tag{9}$$

and the small separation

$$\begin{aligned}d_{ll+2}(n) &= \nu_{n,l} - \nu_{n-1,l+2} \\ &\simeq \frac{[Al(l+1)-B]\nu_0^2(\nu_{n,l} - \nu_{n-1,l+2})}{\nu_{n,l}\nu_{n-1,l+2}} + \frac{2A(2l+3)\nu_0^2}{\nu_{n-1,l+2}} \\ &\simeq \frac{2A(2l+3)\nu_0^2}{\nu_{n-1,l+2}} \\ &\simeq \frac{2A(2l+3)\nu_0}{n+l/2+\varepsilon}.\end{aligned} \tag{10}$$

The ν_0 depends on the mean density of the star, hence on the mass and radius of the star. Thus $\Delta_l(n)$ puts a constraint on the radius of the star. The small separation $d_{ll+2}(n)$ is proportional to quantity A, therefore it is sensitive to the structure of the stellar core and the chemical compositions in the core. Thus it is related to the evolutionary stage (Gough 1987; Ulrich 1986).

We define another difference in the frequencies,

$$\sigma_{l-1l+1}(n) \equiv -\nu_{n,l-1} + 2\nu_{n,l} - \nu_{n,l+1}. \tag{11}$$

Using asymptotic formula (6), we can get

$$\begin{aligned}\sigma_{l-1l+1}(n) &= 2\nu_{n,l} - \nu_{n,l+1} - \nu_{n,l-1} \\ &\simeq -\frac{\nu_0}{2} + \frac{[Al(l+1)-B]\nu_0^2(\nu_{n,l}-\nu_{n,l+1})}{\nu_{n,l}\nu_{n,l+1}} + \frac{2A(l+1)\nu_0^2}{\nu_{n,l+1}} \\ &\quad + \frac{\nu_0}{2} + \frac{[Al(l+1)-B]\nu_0^2(\nu_{n,l}-\nu_{n,l-1})}{\nu_{n,l}\nu_{n,l-1}} - \frac{2Al\nu_0^2}{\nu_{n,l-1}} \\ &= \frac{[Al(l+1)-B]\nu_0^2(\nu_{n,l}-\nu_{n,l+1})}{\nu_{n,l}\nu_{n,l+1}} \\ &\quad - \frac{[Al(l+1)-B]\nu_0^2(\nu_{n,l-1}-\nu_{n,l})}{\nu_{n,l}\nu_{n,l-1}} \\ &\quad + \frac{2A(l+1)\nu_0^2}{\nu_{n,l+1}} - \frac{2Al\nu_0^2}{\nu_{n,l-1}}.\end{aligned} \tag{12}$$

Equation (12) can be rewritten as

$$\begin{aligned}&(\nu_{n,l} - \nu_{n,l+1})\left(1 - \frac{[Al(l+1)-B]\nu_0^2}{\nu_{n,l}\nu_{n,l+1}}\right) \\ &+(\nu_{n,l} - \nu_{n,l-1})\left(1 - \frac{[Al(l+1)-B]\nu_0^2}{\nu_{n,l}\nu_{n,l-1}}\right) \\ &= \frac{2A(l+1)\nu_0^2}{\nu_{n,l+1}} - \frac{2Al\nu_0^2}{\nu_{n,l-1}}.\end{aligned} \tag{13}$$

Because $[Al(l+1)-B]\nu_0^2/(\nu_{n,l}\nu_{n,l\pm1}) \ll 1$, then

$$\begin{aligned}\sigma_{l-1l+1}(n) &\simeq \frac{2A(l+1)\nu_0^2}{\nu_{n,l+1}} - \frac{2Al\nu_0^2}{\nu_{n,l-1}} \\ &= \frac{2A\nu_0^2}{\nu_{n,l+1}}\left(1 + \frac{l(\nu_{n,l-1}-\nu_{n,l+1})}{\nu_{n,l-1}}\right).\end{aligned} \tag{14}$$

For the low-degree p-modes, $|l(\nu_{n,l-1} - \nu_{n,l+1})/\nu_{n,l-1}| < 1$, we can therefore get

$$\sigma_{l-1l+1}(n) \approx \frac{2A\nu_0^2}{\nu_{n,l+1}}. \tag{15}$$

On one hand, comparing Eq. (15) with Eq. (10), the difference $\sigma_{l-1l+1}(n)$ and the scaled small separation $d_{ll+2}(n)/(2l+3)$ should have similar characteristics, which are proportional to quantity A and sensitive to the conditions in the stellar core. The nonhomologous changes brought on by nuclear transmutation should thus be indicated by the variations in the scaled small separation and the difference $\sigma_{l-1l+1}(n)$. On the other hand, Roxburgh & Vorontsov (1994a,b) and Audard & Provost (1994) have shown that the asymptotic formula (6) is inaccurate except for very high frequencies. In Fig. 1, we compare the separations $d_{ll+2}(n)/(2l+3)$ and $\sigma_{l-1l+1}(n)$ obtained from the asymptotic formulas (10) and (15) with the separations obtained from the numerically computed frequencies. The discrepancy between the asymptotic and the numerical values is large, as has been shown by Roxburgh & Vorontsov (1994a) and Audard & Provost (1994). Equations (10) and (15) obtained from the asymptotic formula (6) are inaccurate. There may thus be misleading in the conclusion obtained from comparing Eq. (15) with Eq. (10). Moreover, the term $l(\nu_{n,l-1} - \nu_{n,l+1})/\nu_{n,l-1}$ neglected in Eq. (15) depends on degree l and order n. Consequently, the difference $\sigma_{l-1l+1}(n)$ should be somewhat different from the scaled small separation $d_{ll+2}(n)/(2l+3)$ and more dependent on degree l than $d_{ll+2}(n)/(2l+3)$.

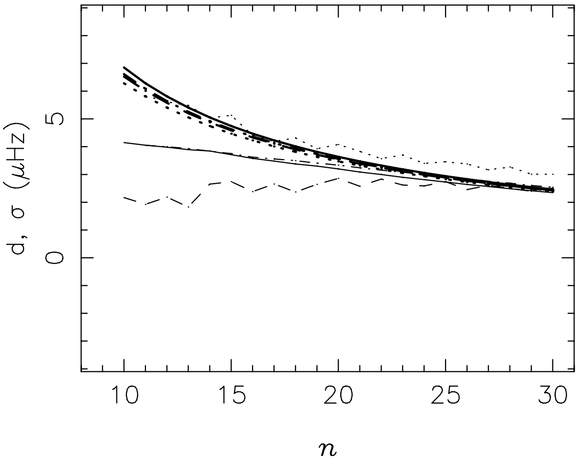

Fig. 1. Frequency separations for model M1.0 at the age of 4.5 Gyr. The solid line refers to $d_{02}(n)/3$. The dash-dot line shows $d_{13}(n)/5$. The dashed line indicates $\sigma_{02}(n)$. The dotted line corresponds to $\sigma_{13}(n)$. The data of bold lines are computed using the asymptotic formulas (10) and (15), but the data of other lines are computed using jig6 code (Guenther et al. 1992).

The more accurate eigenfrequency equation was given by Roxburgh & Vorontsov (2000, 2003)

$$2\pi T \nu_{n,l} = (n + l/2)\pi + \alpha(2\pi\nu_{n,l}) - \varphi_l(2\pi\nu_{n,l}), \tag{16}$$

where $T = \int_0^R \frac{dr}{c}$ is acoustic radius, $\alpha(2\pi\nu)$ the surface phase shift, and $\varphi_l(2\pi\nu)$ the internal phase shift that only depends on the interior structure of the star. Using Eq. (16), Roxburgh & Vorontsov (2003) get

$$\Delta_l(n) = \frac{1}{2T}, \tag{17}$$

and

$$d_{ll+2}(n) = \frac{\varphi_{l+2} - \varphi_l}{2\pi T}. \tag{18}$$

Using Eq. (16), we can get

$$\sigma_{l-1l+1}(n) = \frac{\varphi_{l+1} + \varphi_{l-1} - 2\varphi_l}{2\pi T}. \tag{19}$$

Comparing Eq. (18) with Eq. (19), one can find that the difference $\sigma_{l-1l+1}(n)$ should be different from the small separation $d_{ll+2}(n)$ or the scaled small separation $d_{ll+2}(n)/(2l+3)$ because they rely on the different internal phase shifts φ_l, which strongly depends on the degree l (Roxburgh & Vorontsov 2000). However, if one assumes $\varphi_l \sim l(l+1)D_\varphi$, where D_φ is a quantity determined only by the refractive properties of the stellar core (Roxburgh & Vorontsov 2000), one can get

$$d_{ll+2}(n) \sim \frac{(2l+3)D_\varphi}{\pi T}, \tag{20}$$

and

$$\sigma_{l-1l+1}(n) \sim \frac{D_\varphi}{\pi T}. \tag{21}$$

From Eqs. (20) and (21), we can find that the separations both $\sigma_{l-1l+1}(n)$ and $d_{ll+2}(n)/(2l+3)$ mainly depend on D_φ, and they should have some common characteristics. However, this conclusion can be obtained only under the approximation of $\varphi_l \sim l(l+1)D_\varphi$, which is inaccurate (Roxburgh & Vonrontsov 2000).

Table 1. Model parameters.

Parameter	M1.0	M1.1	M1.2
Mass	1.0 $M_\odot$	1.1 $M_\odot$	1.2 $M_\odot$
α	1.720	1.720	1.720
X_0	0.706	0.706	0.706
Z_0	0.020	0.020	0.020

Note. The α is the mixing-length parameter; X_0 and Z_0 are the initial hydrogen and metal abundance, respectively.

3. Numerical calculation and results

We use the Yale Rotation Evolution Code (YREC7) to construct the stellar models in its nonrotating configuration. All models are evolved from the pre-main sequence to somewhere near the end of the main sequence. The newest OPAL EOS-2005[1] (Rogers & Nayfonov 2002), OPAL opacity (Iglesias & Rogers 1996), and the Alexander & Ferguson (1994) opacity for low temperature are used. Element diffusion is incorporated for helium and metals (Thoul et al. 1994). The parameters of the models calculated are listed in Table 1. The mixing-length parameter α and hydrogen abundance are scaled to obtain the solar radius and luminosity, respectively, at the age of 4.5 Gyr for model M1.0. Adiabatic oscillation frequencies of all models are computed using Guenther & Demarque pulsation code jig6 (Guenther et al. 1992).

In Fig. 2, we present the scaled small separations $d_{02}/3$ and $d_{13}/5$ and the differences σ_{02} and σ_{13} computed from the numerically computed frequencies of model M1.0 as a function of frequency $\nu_{n,l}$ at the different evolutionary stage labeled by the central hydrogen mass fraction X_c. The errorbars indicate 1σ errors obtained assuming errors of 1 part in 10^4 in frequencies. In Table 2, we list the assumed errors of $\nu_{n,0}$ and the errors of $d_{02}(n)/3$ and $\sigma_{02}(n)$ of model M1.0 at the age of 4.5 Gyr. The errors in $\sigma_{02}(n)$ are larger than the errors in $d_{02}(n)/3$. At the early evolutionary stage, showed in Figs. 2A–C, the differences σ_{02} and σ_{13} cannot be distinguished from the scaled small separations $d_{02}/3$ and $d_{13}/5$. From $X_c \sim 0.5$ to $X_c \sim 0.003$, the difference $\sigma_{l-1l+1}(n)$ deviates more and more from the scaled small separation $d_{ll+2}(n)/(2l+3)$; the σ_{02} is less than the $d_{02}/3$ and $d_{13}/5$, but the σ_{13} becomes larger than the $d_{02}/3$ and $d_{13}/5$. The less central the hydrogen, the more deviation. We also plot the quantity $d_{01}(n)$ in Fig. 2. The values of $d_{01}(n)$ are between the values of $\sigma_{13}(n)$ and of $d_{02}(n)/3$. With the decrease in X_c, the changes of the separation σ_{13} in Fig. 2 are small. But the separations $d_{ll+2}/(2l+3)$ and σ_{02} in Fig. 2 decrease with decrease in X_c. The difference $\sigma_{02}(n)$ is somewhat more sensitive to the X_c than the differences $d_{02}(n)/3$ and $d_{13}(n)/5$. In Fig. 2, the scaled small separation is smoother than the differences $\sigma_{02}(n)$, $\sigma_{13}(n)$, and $d_{01}(n)$. The scatter of the differences $\sigma_{02}(n)$ and $\sigma_{13}(n)$ may be related to $\sigma_{l-1l+1}(n)$ depending on the conditions not only in the stellar core but also in the envelope just as $d_{01}(n)$ (Oti Floranes et al. 2005).

In Figs. 3A and B, we compare the scaled small separation $d_{ll+2}(n)/(2l+3)$ with the difference $\sigma_{l-1l+1}(n)$ computed using the frequencies of degree as high as 8 at the evolutionary stage of X_c = 0.342 of the model M1.0. The difference $\sigma_{l-1l+1}(n)$ has the order of the scaled small separation $d_{ll+2}(n)/(2l+3)$. Moreover, on the one hand, for a given l and $l \geq 4$, $\sigma_{l-1l+1}(n)$ is almost invariant; on the other hand, $\sigma_{l-1l+1}(n)$ is more dependent on the degree l than $d_{ll+2}(n)/(2l+3)$. From Eqs. (18) and (19), one can

[1] www-pat.llnl.gov/Research/OPAL/

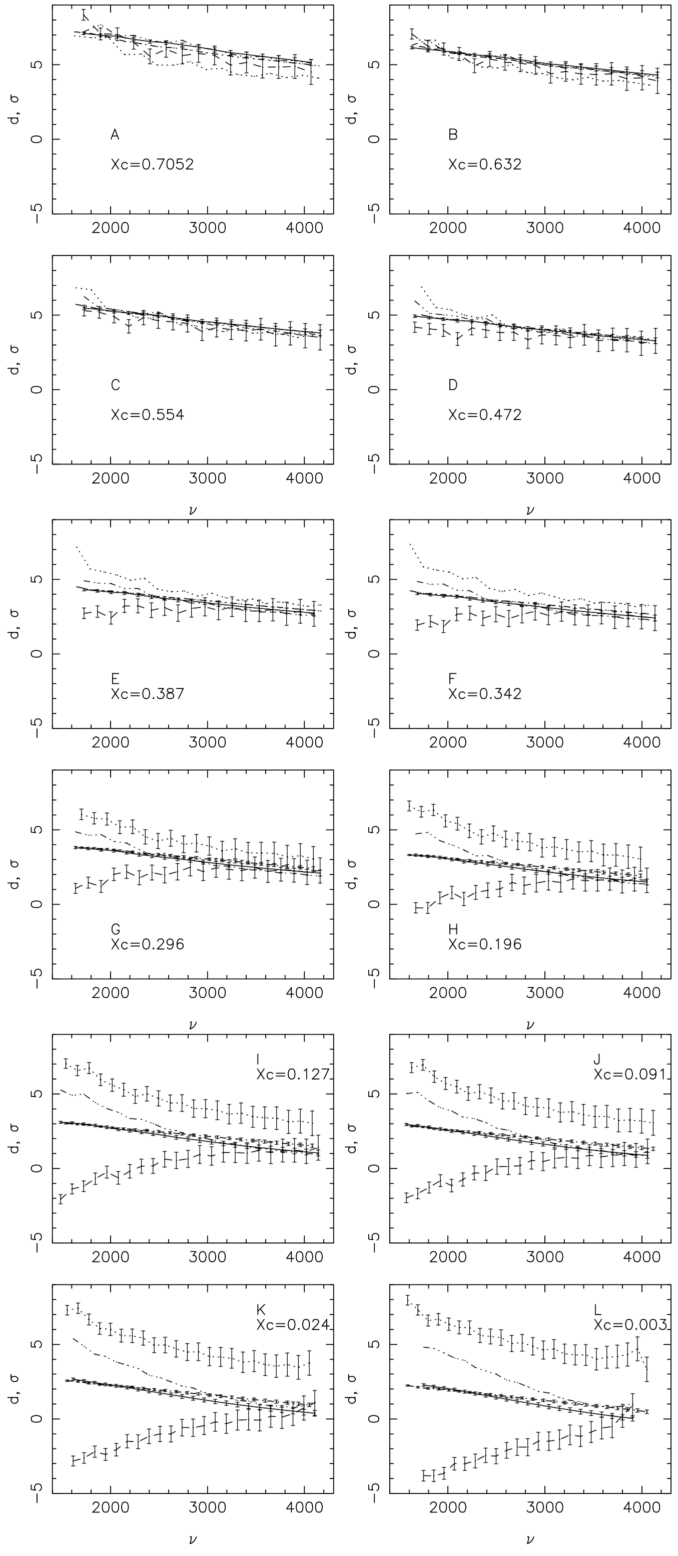

Fig. 2. Frequency separations $d_{02}(n)/3$: solid line, $d_{13}(n)/5$: dash-dotted line, $\sigma_{02}(n)$: dashed line, $\sigma_{13}(n)$: dotted line, and $d_{01}(n)$: triple-dot-dashed line as a function of frequencies $\nu_{n,l}$ for model M1.0 at the different evolutionary stages. The errorbars represent 1σ errors obtained assuming errors of 1 part in 10^4 in frequencies. For distinguishability, we plot only the errorbars of the $d_{02}/3$ and σ_{02} between A and F.

find that the difference between $d_{ll+2}(n)/(2l+3)$ and $\sigma_{l-1l+1}(n)$ is obvious. $d_{ll+2}(n)/(2l+3)$ and $\sigma_{l-1l+1}(n)$ depend on the different

Table 2. Some of errors of model M1.0 at the age of 4.5 Gyr.

n	$\nu_{n,0}$	$d_{02}(n)/3$	$\sigma_{02}(n)$
11	1683.346 ± 0.168	4.053 ± 0.079	1.929 ± 0.349
12	1818.643 ± 0.182	3.979 ± 0.085	2.183 ± 0.376
13	1954.024 ± 0.195	3.895 ± 0.092	1.828 ± 0.403
14	2089.951 ± 0.209	3.846 ± 0.098	2.655 ± 0.431
15	2225.364 ± 0.223	3.712 ± 0.105	2.737 ± 0.458
16	2360.143 ± 0.236	3.577 ± 0.111	2.384 ± 0.485
17	2494.034 ± 0.249	3.482 ± 0.117	2.657 ± 0.511
18	2628.444 ± 0.263	3.377 ± 0.124	2.368 ± 0.538
19	2763.917 ± 0.276	3.300 ± 0.130	2.648 ± 0.566
20	2899.730 ± 0.290	3.200 ± 0.136	2.855 ± 0.593
21	3036.027 ± 0.304	3.088 ± 0.143	2.575 ± 0.620
22	3172.335 ± 0.317	2.999 ± 0.149	2.835 ± 0.647
23	3308.821 ± 0.331	2.896 ± 0.156	2.617 ± 0.675
24	3445.909 ± 0.345	2.812 ± 0.162	2.590 ± 0.702
25	3583.206 ± 0.358	2.730 ± 0.169	2.741 ± 0.730
26	3720.942 ± 0.372	2.640 ± 0.175	2.455 ± 0.757
27	3858.897 ± 0.386	2.568 ± 0.182	2.606 ± 0.785
28	3996.970 ± 0.400	2.487 ± 0.188	2.495 ± 0.813

Note. The errors of 1 part in 10^4 in frequencies are assumed. The errors of $d_{02}/3$ and σ_{02} are obtained using the error propagation formula and the assumed errors in frequencies.

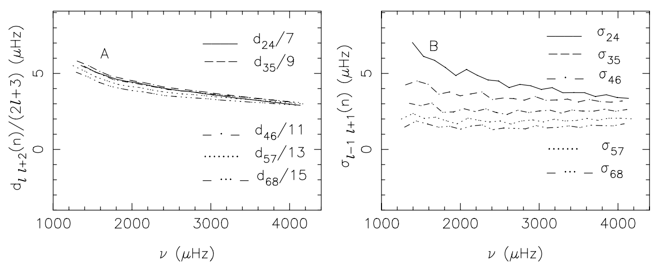

Fig. 3. (A) shows the small separations $d_{24}/7$, $d_{35}/9$, $d_{46}/11$, $d_{57}/13$, $d_{68}/15$ of model M1.0 at the stage of $X_c = 0.342$. (B) shows the differences σ_{24}, σ_{35}, σ_{46}, σ_{57}, σ_{68} of model M1.0 at the stage of $X_c = 0.342$.

phase shifts φ_l, which strongly depend on the degree l (Roxburgh & Vorontsov 1994a, 2000, 2003).

Furthermore, in Fig. 4, we represent the separations $\sigma_{l-1l+1}(n)$ and $d_{ll+2}(n)/(2l+3)$ of model M1.2 as a function of the order n. As results of the model M1.0, the separations $d_{ll+2}(n)/(2l+3)$ and $\sigma_{l-1l+1}(n)$ cannot be distinguished at the early evolutionary stage; with the decrease in X_c, the difference $\sigma_{l-1l+1}(n)$ deviates more and more from the scaled small separation too. For the models with $X_c < 0.423$, the difference $\sigma_{l-1l+1}(n)$ for $n < 25$ clearly deviates from the scaled small separation. The deviation between $\sigma_{l-1l+1}(n)$ and $d_{ll+2}(n)/(2l+3)$ is related to order n.

In Figs. 5A1–A3, we depict the quantities $\langle d_{02}(n)/3\rangle$, $\langle d_{13}(n)/5\rangle$, $\langle\sigma_{02}(n)\rangle$, and $\langle\sigma_{13}(n)\rangle$ as a function of X_c, where $\langle\rangle$ implies averaging over a fixed range in frequencies or in orders. We choose the range of $1600 \le \nu \le 4100$ μHz for the model M1.0, $1200 \le \nu \le 3300$ μHz for the model M1.1, and $11 \le n \le 29$ for the model M1.2. The choice of the range for averaging, as pointed out by Mazumdar et al. (2006), is governed by the availability of the data in the observation. It should be noted that the fixed domain in n does not correspond to a fixed domain in frequency for models with different X_c. These ranges for averaging will be used in the rest of this paper. With the decrease in X_c, the average separations $\langle d_{02}/3\rangle$, $\langle d_{13}/5\rangle$, and $\langle\sigma_{02}\rangle$ decrease;

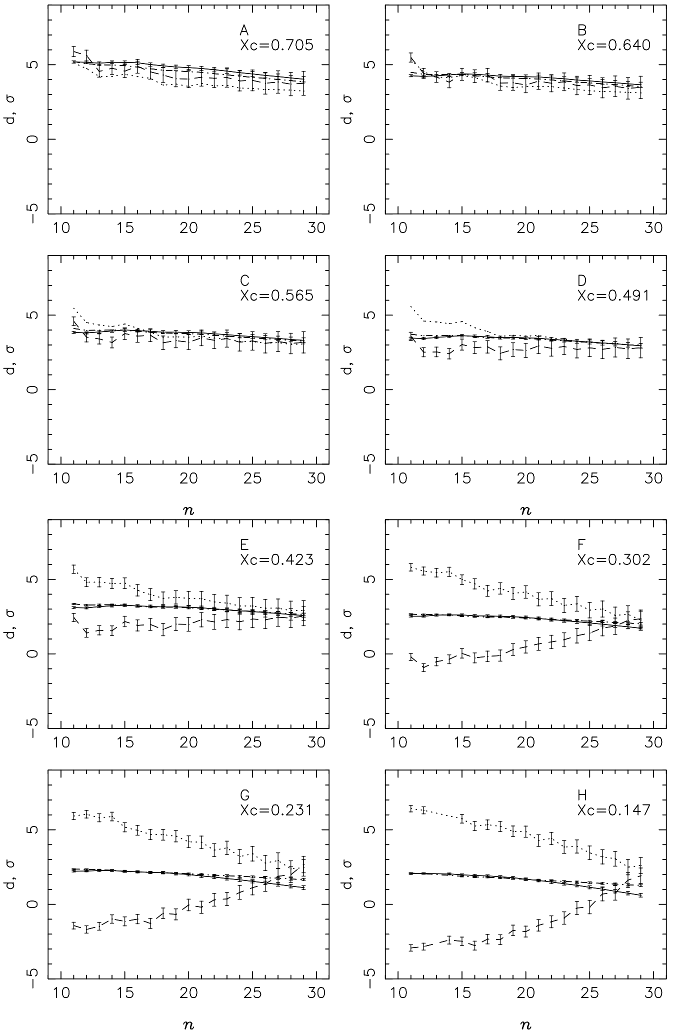

Fig. 4. Frequency separations $d_{02}(n)/3$: solid line, $d_{13}(n)/5$: dash-dotted line, $\sigma_{02}(n)$: dashed line, and $\sigma_{13}(n)$: dotted line as a function of order n for model M1.2 at the different evolutionary stages. The errorbars represent 1σ errors obtained assuming errors of 1 part in 10^4 in frequencies.

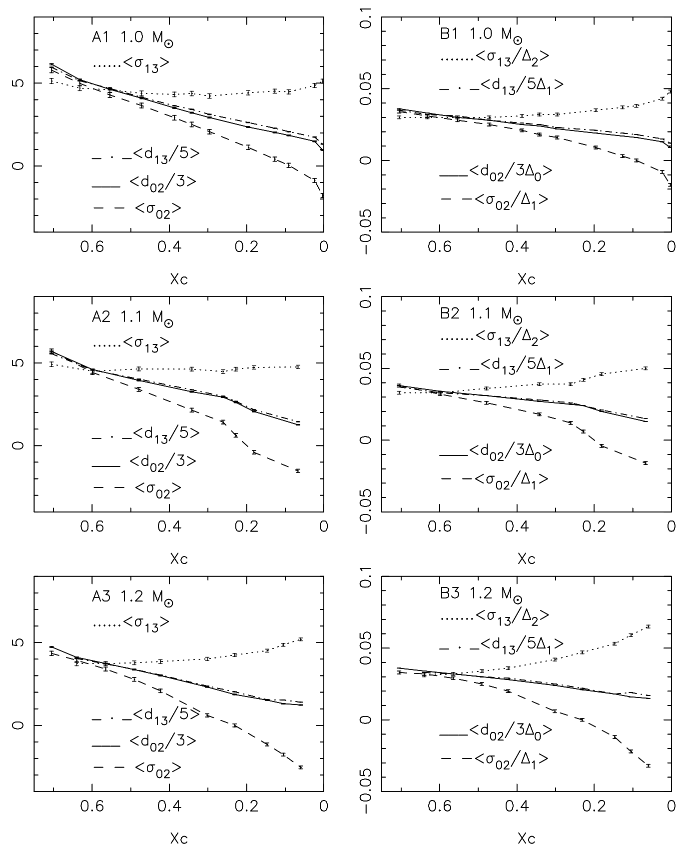

Fig. 5. The average separations as a function of X_c. The errorbars represent 1σ errors obtained assuming errors of 1 part in 10^4 in frequencies.

however, the average separation $\langle\sigma_{13}\rangle$ increases slightly. Moreover, the average separation $\langle\sigma_{02}\rangle$ is more sensitive to X_c than the average separations $\langle d_{02}/3\rangle$ and $\langle d_{13}/5\rangle$. Thus the $\langle\sigma_{02}\rangle$ and $\langle\sigma_{13}\rangle$ deviate more and more from the $\langle d_{02}/3\rangle$ and $\langle d_{13}/5\rangle$.

It can be found from Eqs. (10) and (15) that the difference $\sigma_{l-1l+1}(n)$ and the scaled small separation $d_{ll+2}(n)/(2l+3)$ are dependent on the characteristic frequency ν_0, which is affected by the outer layer of stars. The changes in ν_0 must affect $\sigma_{l-1l+1}(n)$ and $d_{ll+2}(n)/(2l+3)$. Noting that quantity A is related to ν_0^{-1} in Eq. (8) and the large separation is almost equal to ν_0, we can use the large separation to eliminate the effect of ν_0 on the difference $\sigma_{l-1l+1}(n)$ and the scaled small separation. Thus, in Figs. 5B1–B3, we show the average ratio of scaled small separation to large separation, $\langle d_{ll+2}(n)/[(2l+3)\Delta_l(n)]\rangle$, and the average ratio of the difference $\sigma_{l-1l+1}(n)$ to large separation, $\langle\sigma_{l-1l+1}(n)/\Delta_l(n)\rangle$, as a function of the central hydrogen X_c. The value of $\langle\sigma_{l-1l+1}(n)/\Delta_l(n)\rangle$ deviates from $\langle d_{ll+2}(n)/[(2l+3)\Delta_l(n)]\rangle$ with the decrease in the central hydrogen abundance, as the behaviors between $\langle\sigma_{l-1l+1}(n)\rangle$ and $\langle d_{ll+2}(n)/(2l+3)\rangle$. However, the $\langle\sigma_{13}/\Delta_2\rangle$ obviously increases with the decrease in X_c compared with the $\langle\sigma_{13}\rangle$.

4. Discussion and conclusions

Only a very limited number of modes ($l = 0, 1, 2, 3$) are likely to be observed in solar-like oscillations. Thus, our calculation mainly concentrated on these modes.

Equations (10) and (15) hint that both the difference $\sigma_{l-1l+1}(n)$ and the scaled small separation $d_{ll+1}(n)/(2l+3)$ are sensitive to the central conditions of stars, and they should have the same magnitude and characteristics. However, the expressions of the scaled small separation and of the $\sigma_{l-1l+1}(n)$ are obtained from an approximative expression of frequency $\nu_{n,l}$, which is inaccurate (Roxburgh & Vorontsov 1994a; Audard & Provost 1994). Although Eqs. (10) and (15) can present some characteristics of the scaled small separation and the difference $\sigma_{l-1l+1}(n)$, there must be some characteristics covered by Eqs. (10) and (15). From more accurate expressions (18) and (19), one can find that both $\sigma_{l-1l+1}(n)$ and $d_{ll+1}(n)$ are mainly determined by the internal phase shifts φ_l, which only depends on the interior structure of the star (Roxburgh & Vorontsov 1994a, 2000, 2003). The separations $\sigma_{l-1l+1}(n)$ and $d_{ll+1}(n)/(2l+3)$ depend on the different phase shifts, and $\sigma_{l-1l+1}(n)$ must therefore be different from $d_{ll+1}(n)/(2l+3)$. Only under the approximation $\varphi_l \sim l(l+1)D_\varphi$, $\sigma_{l-1l+1}(n)$ and $d_{ll+1}(n)/(2l+3)$ depend on the same quantities.

The separation $\sigma_{l-1l+1}(n)$ depends on the term $l(\nu_{n,l-1} - \nu_{n,l+1})/\nu_{n,l-1}$, neglected in Eq. (15), which relies on order n and degree l. Therefore, the separations σ_{02} and σ_{13} are more dependent on order n than separation $d_{ll+2}(n)/(2l+3)$ and σ_{02} is different from σ_{13}, which is shown in Figs. 2 and 4.

The separation $\sigma_{l-1l+1}(n)$ is more uncertain than the scaled small separation $d_{ll+2}(n)/(2l+3)$. In Table 2, we show the errors of $d_{02}/3$ and σ_{02} obtained from assuming errors in frequencies. The errors of $\sigma_{02}(n)$ are 4 times larger than the errors of $d_{02}(n)/3$. At the early evolutionary stage, the difference $\sigma_{l-1l+1}(n)$ cannot be distinguished from the scaled small separations. With the decrease in central hydrogen, the difference $\sigma_{02}(n)$ becomes smaller, but the $\sigma_{13}(n)$ is larger than the scaled small separation. At the late evolutionary stage, the difference $\sigma_{l-1l+1}(n)$ can thus be distinguished from the scaled small separation except for the separations of the high-order frequencies. The separations $d_{02}/3$, $d_{13}/5$, and σ_{02} decrease with the decrease in X_c. These separations are good indicators of X_c.

The separations σ_{02} and σ_{13} are easily distinguished from the separations $d_{02}/3$ and $d_{13}/5$ of model M1.2 in Fig. 4 compared with those of model M1.0 in Fig. 2. There is a convective core in model M1.2. Furthermore, model M1.1 also has a convective core for $X_c \lesssim 0.261$. In Fig. 5A2, the average separation $\langle\sigma_{02}\rangle$ has an obvious change at $X_c \approx 0.261$. Therefore the separation σ_{02} may be sensitive to the convective core.

The difference $\sigma_{l-1l+1}(n)$ is similar to the scaled small separation $d_{ll+2}/(2l+3)$. They have the same asymptotic formula for the low-degree p-modes and are mainly determined by the conditions of stellar core. However, $\sigma_{l-1l+1}(n)$ is somewhat different from $d_{ll+2}(n)/(2l+3)$, particularly the $\sigma_{02}(n)$ and $\sigma_{13}(n)$. With the decrease in the central hydrogen, $\sigma_{02}(n)$ and $\sigma_{13}(n)$ deviate more and more from the scaled small separation. This characteristic provides us with a possibility for probing the central hydrogen abundance of stars.

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