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Roche lobe effects on the atmospheric loss from "Hot Jupiters"

N. V. Erkaev[1], Yu. N. Kulikov[2], H. Lammer[3], F. Selsis[4], D. Langmayr[3], G. F. Jaritz[5], and H. K. Biernat[3,5]

[1] Institute for Computational Modelling, Russian Academy of Sciences, and Siberian Federal University, Krasnoyarsk 36, Russian Federation
e-mail: erkaev@icm.krasn.ru
[2] Polar Geophysical Institute, Russian Academy of Sciences, Khalturina 15, 183010 Murmansk, Russian Federation
e-mail: kulikov@pgi.ru
[3] Space Research Institute, Austrian Academy of Sciences, Schmiedlstrasse 6, 8042 Graz, Austria
e-mail: [helfried.biernat; helmut.lammer; daniel.langmayr]@oeaw.ac.at
[4] Université de Lyon, 69003 Lyon , France; École Normale Supérieure de Lyon, 46 allée d'Italie, 69007 Lyon, France; CNRS, UMR 5574, Centre de Recherche Astrophysique de Lyon; Université Lyon 1, 69622 Villeurbanne, France
e-mail: franck.selsis@ens-lyon.fr
[5] Institute for Physics, University of Graz, Universitätsplatz 5, 8010 Graz, Austria
e-mail: gerald.jaritz@stud.uni-graz.at

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ABSTRACT

Context. A study of the mass loss enhancement for very close "Hot Jupiters" due to the gravitational field of the host star is presented.
Aims. The influence of the proximity to a planet of the Roche lobe boundary on the critical temperature for blow-off conditions for estimating the increase of the mass loss rate through hydrodynamic blow-off for close-in exoplanets is investigated.
Methods. We consider the gravitational potential for a star and a planet along the line that joins their mass centers and the energy balance equation for an evaporating planetary atmosphere including the effect of the stellar tidal force on atmospheric escape.
Results. By studying the effect of the Roche lobe on the atmospheric loss from short-periodic gas giants we derived reasonably accurate approximate formulas to estimate atmospheric loss enhancement due to the action of tidal forces on a "Hot Jupiter" and to calculate the critical temperature for the onset of "geometrical blow-off", which are valid for any physical values of the Roche lobe radial distance. Using these formulas, we found that the stellar tidal forces can enhance the hydrodynamic evaporation rate from TreS-1 and OGLE-TR-56b by about 2 fold, while for HD 209458b we found an enhancement of about 50%. For similar exoplanets which are closer to their host star than OGLE-TR-56b, the mass loss enhancement can be even larger. Moreover, we showed that the effect of the Roche lobe allows "Hot Jupiters" to reach blow-off conditions at temperatures which are less than expected due to the stellar X-ray and EUV heating.

Key words. hydrodynamics – atmospheric effects – stars: activity – planets and satellites: general

1. Introduction

Because hydrogen-rich upper atmospheres of short-periodic gas giants in orbits with semi-major axes less than 0.1 AU can be heated up to temperatures of more than 10 000 K by X-rays and EUV (XUV) radiation of the central star (Lammer et al. 2003; Yelle 2004), they may experience hydrodynamic blow-off. According to Öpik (1963), atmospheric blow-off is defined as a regime of hydrodynamic escape which replaces gas-kinetic escape, when the thermal energy kT per atom or molecule exceeds its gravitational potential energy. Since the observation of an extended hydrogen corona around the planet HD 209458b by Vidal-Madjar et al. (2003), different blow-off scenarios for evaporating hydrogen-rich atmospheres are discussed in the literature (Sasselov 2003; Lammer et al. 2003; Lecavelier des Etangs et al. 2004; Yelle 2004; Vidal-Madjar et al. 2004; Baraffe et al. 2004; Tian et al. 2005; Yelle 2006; Lecavelier des Etangs 2007; Penz et al. 2007).

Lecavelier des Etangs et al. (2004) and Jaritz et al. (2005) discussed the Roche lobe effects on hydrodynamic mass loss from the atmospheres of "Hot Jupiters" and argued that for some close-in gas giants, due to expected high exospheric temperatures, the exobase level $r_{\rm exo}$ can reach the Roche lobe boundary $r_{\rm Rl}$ before classical hydrodynamic blow-off conditions may develop and, thus, affect their atmospheric loss rates. Lecavelier des Etangs et al. (2004) found that at high temperatures the exobase can be actually superimposed on the Roche lobe boundary and, since the atmosphere beyond the Roche lobe can escape unhampered, the authors argued that the stellar tidal forces can increase the evaporation rate from an exoplanet by one or two orders of magnitude.

In a recent study Lecavelier des Etangs (2007) proposed a method for quick estimation of the thermal escape rate and a life time of an extrasolar planet against its atmosphere evaporation and applied it to almost two hundred already discovered planets. The method is based on an energy diagram where the potential energy per unit mass of the atmosphere is plotted as a function of the stellar EUV-energy flux absorbed by the atmosphere. This method is actually similar to the energy-limited approximation, first suggested by Watson et al. (1981). However, Lecavelier des Etangs (2007) modified the planetary gravitational potential energy field disturbed by the parent star tidal

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forces for estimating evaporation rates from close-in hot exoplanets.

In his estimations Lecavelier des Etangs (2007) assumed that atmospheric evaporation from extrasolar planets always occurs in the blow-off regime where the bulk of the input stellar EUV energy is taken away from the planet by the hydrodynamic outflow of its atmosphere, regardless of the actual thermal balance and atmospheric structure of a specific planet. However, this assumption may oversimplify the atmospheric loss picture, particularly for exoplanets which are more massive and distant from their parent stars and for which blow-off may not occur when the mean thermal velocity of the atmospheric particles at the exobase is less than the escape velocity from the planet. In such a case only Jeans kinetic escape is possible which is typically orders of magnitude slower than hydrodynamic blow-off.

The aim of this study is to propose approximate formulas to estimate the blow-off loss rate enhancement from a close-in exoplanet, which is expected to occur due to proximity to the planet of the Roche lobe boundary this is in contrast with with the "reference" case when the Roche lobe boundary is infinitely distant from the planet and the effect of its parent star tidal forces can be neglected. We also aim to calculate the critical temperature for the onset of the "geometrical blow-off" from close-in "Hot Jupiters". Applying the obtained formulas we study atmospheric stability conditions against blow-off in the presence of the stellar tidal forces and show that close-in gas giants with hot and dense upper atmospheres may experience blow-off conditions more readily than the exoplanets which are under a similar XUV energy exposure, but for which their parent star tidal forces can be neglected.

In Sect. 2 we derive a nonlinear equation to estimate a decrease of the planetary gravitational energy potential due to the action of the parent star tidal forces on the planetary atmosphere. In Sect. 3 we apply this equation this equation to estimate the blow-off loss rate increase from seven selected "Hot Jupiters". Here we show that this enhancement should occur regardless of the approach used for estimating the loss rate, either the simplified energy-limited approximation, or more realistic hydrodynamic modelling. However, the ultimate result will depend on the approach used, its accuracy, and limitations. We further find that the applicability of our derived loss rate enhancement factor is more general than the linear approximation suggested by Lecavelier des Etangs (2007). In Sect. 4 we consider the atmospheric stability conditions against blow-off. We derive an equation for estimating the critical temperature for the onset of blow-off from close-in "Hot Jupiters", which is valid for all Roche lobe boundary radii which are larger than a planetary radius. In conclusion we briefly formulate important evolutionary implications of our findings.

2. Roche lobes of close-in exoplanets

We consider two spherical masses, $M_{\rm pl}$ (exoplanet) and $M_{\rm star}$ (star), separated by an orbital distance d, which rotate around their common center of mass. In the rotating coordinate system, the energy per unit mass of a test particle in the ecliptic plane is given by Paczyński (1971)

$$\Phi = -\frac{GM_{\rm pl}}{r_{\rm a}} - \frac{GM_{\rm star}}{r_{\rm b}} - \frac{G\left(M_{\rm pl}+M_{\rm star}\right)s^2}{2d^3} + \text{const.}, \qquad (1)$$

where s is the distance from a given point to the center of mass, $r_{\rm a}$ and $r_{\rm b}$ are distances to the centers of the exoplanet and the star, respectively, and G is Newton's gravitational constant.

The first term in Eq. (1) represents the potential of the exoplanet and the second the potential of the star. The third term is the result of the orbital motion of the whole system. The Roche lobe of a planet is defined as the last equipotential around a planet beyond which the equipotentials are open to infinity or to encompass the star (e.g., Lecavalier des Etangs et al. 2004).

By introducing dimensionless quantities $\delta = M_{\rm pl}/M_{\rm star}$, $\lambda = d/r_{\rm pl}$, $\eta = r_{\rm a}/r_{\rm pl}$, we analyze the variation of the potential along the axis which connects the exoplanet with its host star

$$\Phi(\eta) = \Phi_0\left[-\frac{1}{\eta} - \frac{1}{\delta(\lambda-\eta)} - \frac{1+\delta}{\delta}\left(\lambda\frac{1}{(1+\delta)} - \eta\right)^2 \frac{1}{2\lambda^3}\right], \qquad (2)$$

where

$$\Phi_0 = GM_{\rm pl}/r_{\rm pl}. \qquad (3)$$

For this potential, there are two locations, the Lagrangian point L1 and the Lagrangian point L2 which are saddle points. Both points are close to each other for a small ratio of mass (Gu et al. 2003)

$$r_{\rm L1,L2} = \left(\frac{\delta}{3}\right)^{1/3}\left[1 \mp \frac{1}{3}\left(\frac{\delta}{3}\right)^{1/3}\right] d, \qquad (4)$$

therefore, we consider as the Roche lobe boundary $r_{\rm Rl}$, approximately

$$r_{\rm Rl} \approx \left(\frac{\delta}{3}\right)^{1/3} d. \qquad (5)$$

If we define a dimensionless Roche lobe boundary distance as

$$\xi = r_{\rm Rl}/r_{\rm pl} \approx \left(\frac{\delta}{3}\right)^{1/3} \lambda, \qquad (6)$$

the potential difference between the Roche lobe boundary and the planetary surface can be written as

$$\Delta\Phi = \Phi_0\frac{(\xi-1)}{\xi}\left[1 - \frac{1}{\delta}\frac{\xi}{\lambda^2}\frac{(\lambda(1+\xi)-\xi)}{(\lambda-1)(\lambda-\xi)} - \frac{(1+\delta)\xi(1+\xi)}{2\delta\lambda^3}\right]. \qquad (7)$$

Using some algebraic manipulations and the relationship between the dimensionless quantities λ, δ, and ξ given by Eq. (6), we can transform this expression to the following

$$\Delta\Phi = \Phi_0\frac{(\xi-1)}{\xi}\left[1 - \frac{1+\xi}{2\xi^2} - \frac{1}{\lambda}A(\xi,\lambda) - \frac{(1+\xi)\xi}{2\lambda^3}\right], \qquad (8)$$

where

$$A(\xi,\lambda) = \frac{(1+\xi)(1+\xi-\xi/\lambda)-\xi}{3\xi^2(1-1/\lambda)(1-\xi/\lambda)}. \qquad (9)$$

By assuming $d \gg r_{\rm Rl} > r_{\rm pl}$ and $M_{\rm star} \gg M_{\rm pl}$ which means $\lambda \gg \xi > 1$ and $\delta \ll 1$, we obtain an asymptotic expansion of the potential difference with respect to a small parameter $1/\lambda$

$$\Delta\Phi = \Phi_0\frac{(\xi-1)}{\xi}\left[1 - \frac{1}{2}\frac{(1+\xi)}{2\xi^2} + O\left(\frac{1}{\lambda}\right)\right]. \qquad (10)$$

By neglecting the small term $O(1/\lambda)$, we finally come to a simple approximate expression

$$\Delta\Phi = \Phi_0\frac{(\xi-1)^2(2\xi+1)}{2\xi^3}. \qquad (11)$$

This potential difference depends on the distance from the center of the planet to the Roche lobe boundary, ξ, which is normalized to the planetary radius. Equation (11) can be further expanded with respect to $1/\xi$

$$\Delta\Phi = \Phi_0 \left(1 - \frac{3}{2\xi} + \frac{1}{2\xi^3}\right). \quad (12)$$

If one neglects the term $O(1/\xi^3)$, Eq. (12) yields the formula of Lecavelier des Etangs (2007) who assumed both large ξ and λ (Eq. (B.15)).

3. Implications for atmospheric blow-off

For estimating the mass loss rate from a "hot" exoplanet

$$\Gamma = 4\pi r^2 \rho v, \quad (13)$$

we consider a steady-state hydrodynamic escape from a spherically symmetric atmosphere. Here Γ is the total mass loss rate (g/s), r radius from the center of the planet, ρ the mass density, v the vertical velocity of the atmosphere. Assuming that a "Hot Jupiter" has a very extended atmosphere, such that the exobase distance $r_{\rm exo} \gtrsim r_{\rm Rl}$, we may integrate the 1D energy conservation equation from the base of the thermosphere, $r_0 \simeq r_{\rm pl}$, up to the Roche lobe boundary, $r_{\rm Rl}$. As a result, we have an equation for estimating the loss rate from a "hot" exoplanet

$$\Gamma = \frac{4\pi Q}{\Delta\Phi + \frac{v_{\rm Rl}^2}{2} + c_p\,(T_{\rm Rl} - T_0)}, \quad (14)$$

where $4\pi Q$ is the net radiative power (erg/s) supplied to the whole atmosphere of the planet due to the stellar XUV radiation absorption and loss of a part of it due to atmospheric IR radiative cooling, $v_{\rm Rl}$ is the gas flow velocity at the Roche lobe boundary, c_p is the specific heat at constant pressure per unit mass of gas, $T_{\rm Rl}$ is the temperature of gas at the Roche lobe $r_{\rm Rl}$ and T_0 is the temperature at the base of the thermosphere which is close to $r_{\rm pl}$. The value of T_0 is about equal to the effective radiative temperature $T_{\rm eff}$ of the exoplanet.

The net radiative power per unit solid angle is

$$Q = \int_{r_{\rm pl}}^{r_{\rm Rl}} q r^2 {\rm d}r, \quad (15)$$

where $q = q_{\rm XUV} - q_{\rm IR}$ is the net volume heating rate of the atmosphere. Here $q_{\rm XUV}$ is the heating rate due to the absorption of the stellar XUV radiation and q_{IR} is the cooling rate due to IR emitting molecules, such as H_3^+ (Yelle 2004).

The gravitational potential difference in Eq. (14) between the base of the thermosphere, r_0 and the Roche lobe boundary, $r_{\rm Rl}$ is (Eq. (12))

$$\Delta\Phi = \Phi_0 K(\xi), \quad (16)$$

where $\xi = r_{\rm Rl}/r_{\rm pl}$ and

$$K(\xi) = 1 - \frac{3}{2\xi} + \frac{1}{2\xi^3} < 1 \quad (17)$$

is the potential energy reduction factor due to the stellar tidal forces.

In the energy-limited loss approximation the kinetic and thermal energy terms in Eq. (14) are assumed to be negligible. So, the integrated energy balance Eq. (14) reduces in this case to the following simple equation for estimating the mass loss rate from a "hot" exoplanet

$$\Gamma = \frac{4\pi Q}{\Phi_0 K(r_{\rm Rl}/r_{\rm pl})}, \quad (18)$$

which has a form similar to that derived by Watson et al. (1981).

Introducing the planetocentric distance $r_{\rm XUV}$

$$r_{\rm XUV} = \left(\frac{\int_{r_{\rm pl}}^{r_{\rm Rl}} q r^2 {\rm d}r}{I_{\rm XUV}}\right)^{\frac{1}{2}}, \quad (19)$$

which corresponds to the distance where the bulk of the incoming stellar XUV radiation flux $I_{\rm XUV}$ is absorbed by the atmosphere, and ignoring the IR radiative loss from the atmosphere, we obtain for the total radiative power input due to absorption

$$4\pi Q = 4\pi \int_{r_{\rm pl}}^{r_{\rm Rl}} q r^2 {\rm d}r = 4\pi r_{\rm XUV}^2 I_{\rm XUV}. \quad (20)$$

We note that the radiative energy flux $I_{\rm XUV}$ in this equation should be equal to the actual stellar XUV flux $I_\star$ at the orbital distance of the planet, averaged over the whole surface of the planet as a result of the spherical symmetry assumption adopted in deriving both Eqs. (14) and (18). Taking into account that the averaging factor for $I_\star$ is equal to $1/4$, that is the ratio of the cross-section area of the planet, $\pi r_{\rm pl}^2$ to its whole surface area, $4\pi r_{\rm pl}^2$, we arrive to the following energy-limited equation for the loss rate from a "Hot Jupiter"

$$\Gamma = \frac{\pi r_{\rm XUV}^2 I_\star}{\Phi_0 K(r_{\rm Rl}/r_{\rm pl})}. \quad (21)$$

Here $\pi r_{\rm pl}^2 I_\star$ is the total power input of the stellar XUV radiation absorbed by the atmosphere and $\Phi_0 K(r_{\rm Rl}/r_{\rm pl})$ is the gravitational potential difference between $r_{\rm pl}$ and $r_{\rm Rl}$ for a planet affected by stellar tidal forces.

In the typical blow-off scenarios considered, for example, by Watson et al. (1981), Lammer et al. (2003) and Jaritz et al. (2005) it is assumed that the stellar XUV radiation is absorbed mainly at the so-called expansion radius r_1 which is located for most "Hot Jupiters" at a planetocentric distance close to $r_{\rm Rl}$ or above. However, hydrodynamic model simulations by Yelle (2004) which include photochemistry of a hydrogen-rich atmosphere, indicate that the optical depth for the stellar XUV radiation absorption $\tau_{\rm XUV}$ by evaporating hydrogen is much less than 1 at $r_{\rm Rl}$ or at distances corresponding to Watson's r_1, so that the main part of the XUV radiation is absorbed at lower altitudes of about $(1.1\text{–}1.3)R_{\rm pl}$ which are much closer to $r_{\rm pl}$. We note that Lammer et al. (2003) applied Watson's assumption about large expansion radii for "Hot Jupiters" and, as a result, overestimated the energy-limited loss rate by an order of magnitude.

Table 1 shows the Roche lobe induced mass loss enhancement factor $1/K$ as a function of $r_{\rm Rl}$ expressed in units of planetary radii $r_{\rm pl}$ for seven short-periodic "Hot Jupiters". One can see from both Eqs. (14) and (18) that the loss rate may be substantially enhanced due to the effect of the tidal forces ($\sim 1/K$) when a planet orbits its star at a close distance, as compared with the case when the orbital distance is large and the K factor is close to unity. It can be seen that TreS-1 and OGLE-TR-56b, both at an orbital distance of about 0.023 AU, experience the strongest loss enhancement due to the Roche lobe effect, resulting in a factor of about 2, while the Roche lobe induced amplification of mass

Table 1. Mass loss enhancement factors $1/K$ and stellar and planetary parameters for 7 "Hot Jupiters" for which the planetary mass and radius is known. The planetary and stellar parameters for HD209458b (Vidal-Madjar et al. 2003; Knutson et al. 2007; Ballester et al. 2007), OGLE-TR-56 b (Burrows et al. 2004; Baraffe et al. 2004), OGLE-TR-132 b (Moutou et al. 2004), OGLE-TR-113 b (Bouchy et al. 2004), OGLE-TR-111 b (Pont et al. 2004), OGLE-TR-132 b (Bouchy et al. 2004; Moutou et al. 2004), OGLE-TR-10 b (Santos et al. 2006) and TreS-1 (Alonso et al. 2004) were used for the calculation of $r_{\rm Rl}$ and $1/K$. Also numerical errors are shown in the last two columns which result from neglecting higher order terms in the approximate formulas.

Exoplanet	star type	$M_\star/M_\odot$	$M_{\rm pl}/M_{\rm Jup}$	$r_{\rm pl}/r_{\rm Jup}$	d [AU]	$r_{\rm Rl}/r_{\rm pl}$	$1/K$	ϵ	$1/\lambda$
HD209458b	G0V	1.05	0.69	1.32	0.045	4.3	1.53	0.020	0.014
OGLE-TR-56b	G	1.04	1.45	1.23	0.023	3.0	1.92	0.025	0.025
OGLE-TR-132b	F	1.34	1.01	1.15	0.031	3.5	1.70	0.021	0.018
OGLE-TR-113b	K	0.77	1.35	1.08	0.023	3.7	1.65	0.027	0.022
TreS-1	K0V	0.87	0.75	1.08	0.023	2.9	1.97	0.022	0.022
OGLE-TR-111b	K0	0.82	0.53	1.00	0.047	5.9	1.34	0.020	0.010
OGLE-TR-10 b	G0	1.22	0.57	1.24	0.042	3.8	1.63	0.018	0.014

loss from OGLE-TR-111b, which orbits its star at about twice the distances of 0.047 AU, is only ~34%.

Table 1 also shows in the last two columns the numerical errors resulting from the neglect of higher order terms in Eq. (4) for the Roche lobe boundary distance, $r_{\rm Rl}$, that is $\epsilon = |\delta r_{\rm Rl}|/r_{\rm Rl}$, and the errors due to neglecting the small $O(1/\lambda)$ term in Eq. (10) for the modified gravitational energy difference $\Delta\Phi$. As can be seen, both types of errors for the seven exoplanets presented in Table 1 are about 1−3%, which shows that Eqs. (11) and (12) derived in this study for estimating the loss rate enhancement due to the Roche lobe effect are basically quite accurate over the whole range of the Roche lobe boundary radii of interest ($r_{\rm Rl} \geq r_{\rm pl}$) for the "Hot Jupiters".

Recently Lecavelier des Etangs (2007) also studied the effect of the stellar tidal forces on the atmospheric loss from short-periodic exoplanets. However, Lecavelier des Etangs (2007) employed a technique for estimating the Roche lobe effect on the atmospheric loss which is different from that used in our study. In his calculations of the disturbed planetary potential field he retained only linear terms (see Appendix B in Lecavelier des Etangs 2007), while we have made a basically nonlinear derivation, except for neglecting the small terms of the order of $O(1/\lambda)$, where $\lambda = d/r_{\rm pl} \gg 1$. As a result, we have arrived to nonlinear Eqs. (11) and (12) for the potential energy difference $\Delta\Phi$ of a unit mass of gas between the planetary "surface'" or visual radius in the case of gas giants and the Roche lobe boundary which are reasonably accurate (see Table 1) and valid for all $r_{\rm Rl} \geq r_{\rm pl}$, while the Eq. (B.15) of Lecavelier des Etangs (2007), is valid for cases when $\xi^2 \gg 1$.

A comparison of the atmospheric escape rate enhancement factors $1/K$ resulting from these two different approaches is presented in Fig. 1. The factor $1/K$ which is calculated from the nonlinear Eq. (17) as a function of a dimensionless Roche lobe boundary distance ξ, is shown by the solid curve and that resulting from a linear approximation of Lecavelier des Etangs (2007), is shown by the dashed curve. Figure 1 shows that for the Roche lobe normalized distances ξ which are larger than 3.5, the two curves merge and both approaches yield basically the same results. Thus, the linear derivation is sufficient for the "Hot Jupiters" presented in Table 1 for which $\xi \geqslant 2.9$. However, for very "Hot Jupiters" which are not yet discovered, but for which ξ may be less than ~2.5, the non-linear derivation should be applied.

As we show in the following section, the difference between the linear approximation and the non-linear approach has important consequences if one wishes to apply the calculated potential energy reduction factors K to estimate a decrease of the minimum temperature needed for the onset of hydrodynamic

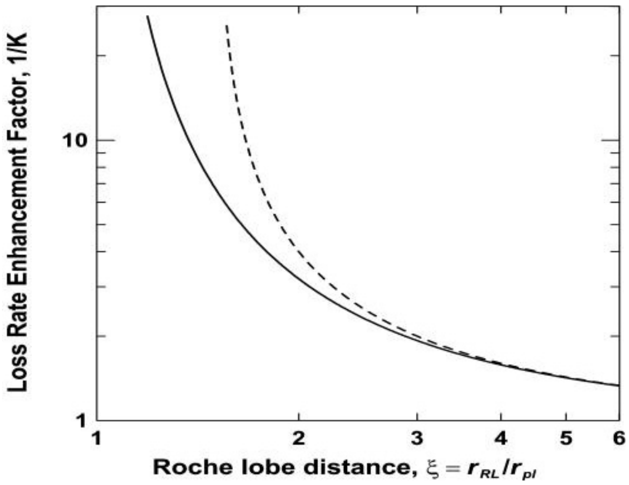

Fig. 1. Atmospheric mass loss enhancement factor $1/K$ as a function of the Roche lobe boundary distance, ξ normalized to $r_{\rm pl}$. Solid line corresponds to the nonlinear Eq. (17) of this study, while the dashed line shows the loss enhancement factor resulting from the linear approximation of Lecavelier des Etangs (2007).

blow-off, the so-called critical temperature, due to the Roche lobe effect as compared with the critical temperature for the Newtonian gravitational potential.

4. Atmospheric stability against blow-off

Here we consider one more important effect which is expected to occur on "Hot Jupiters" due to the stellar tidal forces and which is also related to the gravitational potential energy barrier reduction for escaping particles. It is a decrease of the critical atmospheric temperature, $T_{\rm c}$ at the exobase and above this value the intense hydrodynamic atmospheric escape, i.e. blow-off, starts. Using the conventional definition for the critical temperature $T_{\rm c}$, at which the mean thermal energy of the atmospheric gas at the exobase level $r_{\rm exo}$ is equal to the gravitational potential energy barrier $\Delta\Phi$ at $r_{\rm exo}$ (e.g., Hunten 1973), and assuming that $r_{\rm exo} \leqslant r_{\rm Rl}$, we can write

$$\frac{3}{2}kT_{\rm c} = m\Delta\Phi(r_{\rm exo}). \quad (22)$$

If we apply for the calculation of the potential energy difference $\Delta\Phi(r_{\rm exo})$ between the exobase level and the Roche lobe boundary the nonlinear derivation of Sect. 2 and substitute the

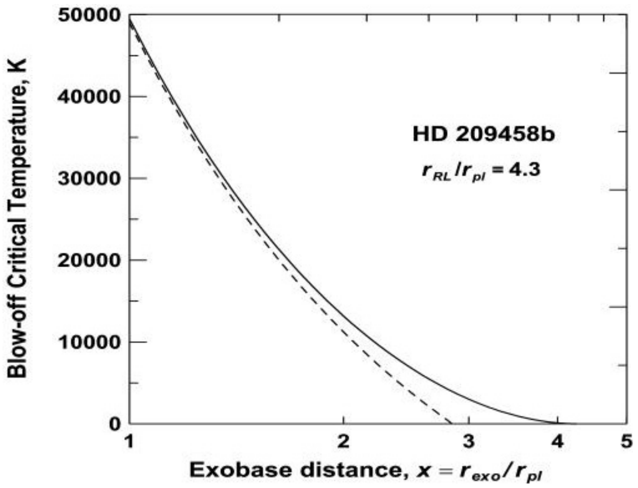

Fig. 2. Critical temperature T_c for the "Hot Jupiter" HD 209458b as a function of the normalized exobase distance, $x = r_{exo}/r_{pl}$. Solid line corresponds to Eq. (17) of this study, dashed line – to Eq. (B.15) of Lecavelier des Etangs (2007). Note that T_c calculated by applying the linear approximation for $x \geq 2.85$ becomes negative.

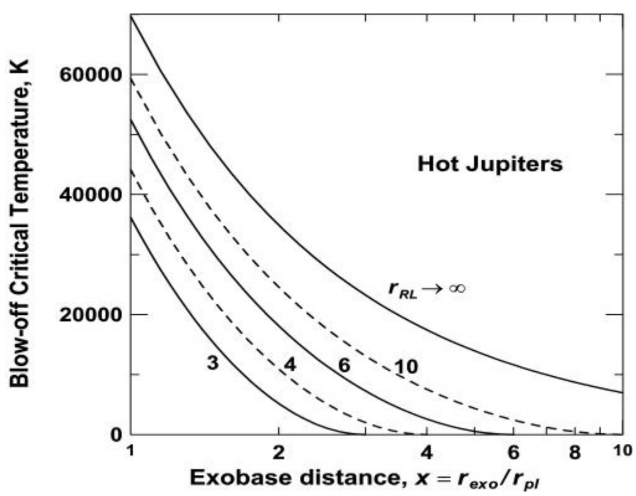

Fig. 3. Critical temperature T_c for the "Hot Jupiters" as a function of the normalized exobase distance, $x = r_{exo}/r_{pl}$. Numbers by the curves indicate the Roche lobe boundary distance, r_{Rl} in the range from 2 to 10 planetary radii. The value of r_{Rl}, according to Eq. (5), is proportional to the planetary orbital distance d, which is taken in the range of 0.021–0.105 AU for this figure. Solid and dashed lines alternate for the clarity of vision only. The figure illustrates that for a "Hot Jupiter" orbiting its star at a closer orbital distance the critical temperature is much reduced within a smaller Roche lobe.

result in Eq. (22), we obtain the following equation for estimating the critical temperature for the onset of blow-off from a "Hot Jupiter"

$$T_c(x) = T^0_{\rm Jup}\left(\frac{M_{\rm pl}}{M_{\rm Jup}}\frac{r_{\rm Jup}}{r_{\rm pl}}\right)\frac{K(x_{\rm Rl}/x)}{x}. \quad (23)$$

Here $x = r_{exo}/r_{pl}$ is a normalized radial distance to the exobase; $x_{Rl} = r_{Rl}/r_{pl}$ is a normalized radial distance to the Roche lobe boundary of a planet; K is the gravitational potential energy reduction factor given by Eq. (17); M_{Jup} and r_{Jup} are Jupiter's mass and radius; $T^0_{Jup} \simeq 1.45 \times 10^5$ K is a critical temperature for the onset of blow-off at the radius of Jupiter (i.e. when $r_{exo} = r_{pl}$).

The value of x in Eq. (23) can vary, depending on the XUV radiation flux of a host star, the planet orbital distance d, and thermospheric density, in the range of $1 \leq x \leq x_{Rl}$. The left boundary of this range ($x = 1$, or $r_{exo} = r_{pl}$) corresponds to low XUV heating and/or very low atmospheric density (a case of a cold and tenuous atmosphere), while the right boundary ($x = x_{Rl}$) corresponds to a hot and dense atmosphere when the exobase reaches the Roche lobe boundary ($r_{exo} = r_{Rl}$). As shown by observations and theoretical studies, "Hot Jupiters" typically have very hot and dense evaporating atmospheres which means that on such planets the exobase superimposes on the Roche lobe boundary (Lecavelier des Etangs et al. 2004). So, the case when $r_{exo} \simeq r_{Rl}$, is the most interesting for a detailed analysis of the conditions for triggering and stopping atmospheric blow-off from "Hot Jupiters".

Although the linear approximation of Lecavelier des Etangs (2007) has not been developed for the domain where $r_{exo} \simeq r_{Rl}$, it is nevertheless instructive to see what are its limitations in this case as compared with our non-linear approach. Figure 2 shows the critical temperature T_c as a function of the normalized radial distance $x = r_{exo}/r_{pl}$ to the exobase of HD 209458b calculated from Eq. (23) for the potential energy reduction factor K given by Eq. (17) (solid curve) and also for the linear approximation (dashed curve). One can see that in the case of a cold and/or tenuous upper atmosphere when $x = r_{exo}/r_{pl} \lesssim 1.3$, i.e. when the exobase is not far from the visual radius of the planet, both modified gravitational potential barriers yield close values of the critical temperature T_c. However, in the contrasting case of a hot and dense upper atmosphere when the exobase is close to the Roche lobe boundary, that is when $x \simeq x_{Rl}$, and which is actually the case for HD 209458b, it is obvious that the linear approximation is unsuitable for estimating the realistic critical temperature for the onset of blow-off. Indeed, for the exobase distances $x > \frac{2}{3}x_{Rl} \simeq 2.85$ the critical temperature even becomes negative. On the other hand, the critical temperature calculated for the modified potential barrier derived in this study approaches zero when the exobase expands to the Roche lobe boundary from below ($x \to x_{Rl}$) remaining ever positive and reasonably accurate in the whole range of the acceptable exobase distances, that is for $1 \leq x \leq x_{Rl}$.

Using Eqs. (17) and (23), we calculated the critical temperature as a function of the normalized exobase distance x for a "Hot Jupiter" like HD209458b with a mass $M_{pl} = 0.69M_{Jup}$ and a visual radius $r_{pl} = 1.32r_{Jup}$ orbiting a solar like G-type star within a range of selected orbital distances from 0.021 to 0.105 AU which correspond to the normalized Roche lobe boundary distances x_{Rl} from 2 to 10 for this planet. The results of our calculations are presented in Fig. 3 which also shows the critical temperature for the same "Hot Jupiter", but for the case when its Roche lobe boundary is moved to the infinite distance from the planet ($r_{Rl} \to \infty$). The latter case corresponds to the classic Newtonian gravitational potential for which all stellar tidal force effects on atmospheric escape are neglected.

Figure 3 shows that the effect of tidal forces on atmospheric loss from the close-in "Hot Jupiters" if compared with the classic blow-off conditions for real Jupiter, can be quite dramatic. For example, for all the exoplanets listed in Table 1 the critical temperature for the onset of blow-off is substantially reduced if one assumes that the exobase for each of these planets is below the Roche lobe boundary and is in the range between 2 and 3 planetary radii. However, as recent detailed simulations of the atmospheric structure and escape from HD 209458b by Yelle (2004), Tian et al. (2005), and Penz et al. (2007) indicate,

r_{exo} can be even larger than $r_{Rl} \simeq 4r_{pl}$, which means that the outer gas envelope of that planet (for $r \gtrsim r_{Rl}$) should be detached from its gravity field and can freely escape to space in a comet-like regime (Schneider et al. 1998).

This result is very important, because it indicates that the effect of the Roche lobe can enhance the possibility that "Hot Jupiters" may reach hydrodynamic blow-off conditions more easily and stay much longer in this regime even if their atmospheres have a high amount of molecules like H_3^+, which act as IR-coolers in the thermosphere. Both effects, the enhanced mass loss and the lower critical temperature for the "Hot Jupiters" at very close orbital distances to their host stars, to be reached more easily that allows a hydrodynamic blow-off regime to be reached more easily, may considerably increase the cumulative atmospheric loss from these planets.

If, on the other hand, we consider a Jupiter-mass exoplanet which orbits its star at a distance larger than about 0.15 AU, its upper atmosphere, depending on the parent star spectral class and age, may be not sufficiently hot for blow-off to occur. In this case the upper atmosphere will remain stable against blow-off and only much less intense Jeans evaporation can take place. Such massive and distant exoplanets belong to the upper right region of the energy diagram of Lecavelier des Etangs (2007) (see his Fig. 2). In this region the loss rates obtained by the simple method of Lecavelier des Etangs (2007), which assumes blow-off to be operating for every exoplanet, can be considerably overestimated.

5. Conclusion

Our study shows that hydrodynamically driven atmospheric mass loss from "Hot Jupiters" at very close orbital distances that are less than 0.05 AU, may be strongly enhanced due to the Roche lobe effect as compared to non-affected exoplanets. We find that atmospheric loss can be enhanced several times if the Roche lobe is located closer to a planet at a distance of a few planetary radii. Furthermore, our study indicates that the Roche lobe effect may also result in hydrodynamic blow-off conditions on "Hot Jupiters" even if their exospheric temperatures are lower than those required for the blow-off to occur in the case of the classic Newtonian gravitational potential of a planet. Both effects may have a strong impact on the atmospheric evolution of short periodic hydrogen-rich gas giants.

Although for close-in exoplanets rough estimates of the atmospheric escape rates obtained by using a method of Lecavelier des Etangs (2007) seem to be reasonable, for more distant and massive exoplanets ($d \gtrsim 0.15$ AU) these simple estimates may be inaccurate. For massive and more distant exoplanets exposed to less intense stellar XUV fluxes the exobase temperatures can be less than the critical temperature for the onset of blow-off. This will result in stable upper atmospheres which experience much slower Jeans thermal escape. The results of our study have to be included in the statistical mass-radius analysis of hot exoplanets expected to be detected during the CoRoT mission in the near future.

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References

Alonso, R., Brown, T. M., Torres, G., et al. 2004, ApJ, 613, 153
Ballester, G. E., Sing, D. K., & Herbert, F. 2007, Nature, 445, 511
Burrows, A., Hubeny, I., Hubbard, W., Sudarsky, D., & Fortney, J. 2004, ApJ, 610, L53
Bouchy, F., Pont, F., Santos, N., et al. 2004, A&A, 421, L13
Baraffe, I., Selsis, F., Chabrier, G., et al. 2004, A&A, 419, L13
Gu, P.-G., Lin, D. N. C., & Bodenheimer, P. H. 2003, ApJ, 588, 509
Hunten, D. M. 1973, J. Atmos. Sci., 30, 1481
Jaritz, G., Endler, S., Langmayr, D., et al. 2005, A&A, 439, 771
Knutson, H., Charbonneau, D., Noyes, R. W., Brown, T. M., & Gilliland, R. L. 2007, ApJ, 655, 564
Lammer, H., Selsis, F., Ribas, I., et al. 2003, ApJ, L121, 598
Lecavelier des Etangs, A. 2007, A&A, 461, 1185
Lecavelier des Etangs, A., Vidal-Madjar, A., McConnell, J. C., & Hébrard, G. 2004, A&A, 418, L1
Moutou, C., Pont, F., Bouchy, F., & Mayor, M. 2004, A&A, 424, L31
Öpik, E. J. 1963, Geophys. J. Roy. Astron. Soc., 7, 490
Paczyński, B. 1971, Ann. Rev. Astr. Ap., 9, 183
Penz, T., Erkaev, N. V., Kulikov, Yu. N., et al. 2007, Planet. Space Sci., submitted
Pont, F., Bouchy, F., Queloz, D., et al. 2004, A&A, 426, L15
Santos, N. C., Pont, F., Bouchy, F., et al. 2006, Colloquium Proceedings Tenth Anniversary of 51 Peg-b, http://www.obs-hp.fr/www/pubs/Coll51Peg/proceedings.html
Sasselov, D. 2003, ApJ, 596, 1327
Schneider, J., Rauer, H., Lasota, J. P., et al. 1998, in Brown Dwarfs and Extrasolar Planets, ed. Rebolo et al., ASP Conf. Ser., 134, 241
Tian, F., Toon, O. B., Pavlov, A. A., & De Sterck, H. 2005, ApJ, 621, 1049
Vidal-Madjar, A., Désert, J.-M., Lecavelier des Etangs, A., et al. 2003, Nature, 422, 143
Vidal-Madjar, A., Lecavelier des Etangs, A., Désert, J.-M., et al. 2004, ApJ, 604, L69
Watson, A. J., Donahue, T. M., & Walker, J. C. G. 1981, Icarus, 48, 150
Yelle, R. V. 2004, Icarus, 170, 167
Yelle, R. V. 2006, Icarus, 183, 508