

LNRF-velocity hump-induced oscillations of a Keplerian disc orbiting near-extreme Kerr black hole: a possible explanation of high-frequency QPOs in GRS 1915+105

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ABSTRACT

Context. At least four high-frequency quasiperiodic oscillations (QPOs) at frequencies 41 Hz, 67 Hz, 113 Hz, and 167 Hz were reported in a binary system GRS 1915+105 hosting near-extreme Kerr black hole with a dimensionless spin $a > 0.98$.

Aims. We attempt to explain all four observed frequencies by an extension of the standard resonant model of epicyclic oscillations.

Methods. We use the idea of oscillations induced by the hump of the orbital velocity profile (related to locally non-rotating frames-LNRF) in discs orbiting near-extreme Kerr black holes, which are characterized by a “humpy frequency” ν_h , that could excite the radial and vertical epicyclic oscillations with frequencies ν_r , ν_v . Due to non-linear resonant phenomena, the combinational frequencies are allowed as well.

Results. Assuming mass $M = 14.8 M_\odot$ and spin $a = 0.9998$ for the GRS 1915+105 Kerr black hole, the model predicts frequencies $\nu_h = 41$ Hz, $\nu_r = 67$ Hz, $\nu_h + \nu_r = 108$ Hz, and $\nu_v - \nu_r = 170$ Hz corresponding quite well to the observed ones.

Conclusions. For black-hole parameters being in good agreement with those given observationally, the forced resonant phenomena in non-linear oscillations, excited by the “hump-induced” oscillations in a Keplerian disc, can explain high-frequency QPOs in near-extreme Kerr black-hole binary system GRS 1915+105 within the range of observational errors.

Key words. accretion, accretion disks – black hole physics

1. Introduction

Detailed analysis of the variable X-ray black-hole binary system (microquasar) GRS 1915+105 reveals high-frequency QPOs appearing at four frequencies, namely $\nu_1 = (41 \pm 1)$ Hz, $\nu_2 = (67 \pm 1)$ Hz (Morgan et al. 1997; Strohmayer 2001), and $\nu_3 = (113 \pm 5)$ Hz, $\nu_4 = (167 \pm 5)$ Hz (Remillard 2004). In this range of its errors, both pairs are close to the frequency ratio 3:2 suggesting the possible existence of resonant phenomena in the system. Observations of oscillations with these frequencies have different qualities, but in all four cases the data are quite convincing; see (McClintock & Remillard 2004; Remillard & McClintock 2006).

Several models have been developed to explain the kHz QPO frequencies, and it is usually preferred that these oscillations are related to the orbital motion near the inner edge of an accretion disc. In particular, two ideas based on the strong-gravity properties have been proposed. While Stella & Vietri (1998, 1999) introduced the “Relativistic Precession Model” considering that the kHz QPOs directly manifest the modes of a slightly perturbed (and therefore epicyclic) relativistic motion of blobs in the inner parts of the accretion disc, Kluźniak & Abramowicz (2001) propose models based on non-linear oscillations of an accretion disc that assume resonant interaction between orbital and/or epicyclic modes. In a different context, the possibility of resonant coupling between the epicyclic modes of motion in the Kerr spacetime was also mentioned in the early work of Aliev & Galtsov (1981).

In the case of near-extreme Kerr black holes, it was suggested that the epicyclic oscillations in the disc could be excited by resonances with the so-called “hump-induced” oscillations, see papers of Aschenbach (2004, 2006) and Stuchlík et al. (2004, 2007). This idea was proposed so as to extend standard orbital (resonant) models meant to explain high-frequency QPOs observed in black-hole sources.

Recently, careful and detailed analysis of the spectral continuum from GRS 1915+105 has put a strong limit on the black-hole spin,¹ namely $0.98 < a < 1$ (McClintock et al. 2006), indicating the presence of near-extreme Kerr black hole whose mass has been restricted observationally to $M = (14.0 \pm 4.4) M_\odot$, see McClintock & Remillard (2004) and Remillard & McClintock (2006). Therefore, the microquasar GRS 1915+105 seems to be an appropriate candidate to test the extended resonant model with hump-induced oscillations².

The idea of hump-induced oscillations and their possible resonant coupling with the epicyclic ones is briefly discussed in Sect. 2. The related resonant model, assuming the excitation of epicyclic oscillations by the hump-induced oscillations through

¹ Units $c = G = M = 1$ (M is the total mass of the Kerr black hole) and the Boyer-Lindquist (B-L) coordinates (t, r, θ, φ) are used hereafter.

² However, Middleton et al. (2006) refer to a substantially lower, intermediate value of black-hole spin, $a \sim 0.7$, to which the model of hump-induced oscillations cannot be applied.

non-linear resonant phenomena, is applied to GRS 1915+105 in Sect. 3, concluding remarks are presented in Sect. 4.

2. Hump-induced and epicyclic oscillations in Keplerian discs and possible resonant coupling

In order to describe the local processes in an accretion disc, it is necessary to choose a local observer (characterized by its reference frame). In general relativity there is no preferred observer. On the other hand, if we want to study processes related to the orbital motion of matter in the disc, it is reasonable to choose the observers with zero angular momentum, so-called ZAMOs, as their reference frames do not rotate with respect to the spacetime, and thus ZAMOs should reveal local kinematic properties of the disc in the clearest way. (In rotating-stationary, axisymmetric-spacetimes, they are dragged along with the spacetime.) In the Kerr spacetime, ZAMOs are represented by locally non-rotating frames (LNRF); see Bardeen et al. (1972). Notice that in the Schwarzschild spacetime, LNRF correspond to the static observer frames.

Aschenbach (2004) finds that for near-extreme Kerr black holes with the spin $a > 0.9953$, the test-particle orbital velocity $\mathcal{V}^{(\varphi)}$ related to LNRF reveals a hump in the equatorial plane ($\theta = \pi/2$). This non-monotonicity is located in a small region inside the ergosphere of the black-hole spacetime close to, but above, the marginally stable orbit³. Therefore, it can be relevant for thin accretion discs around near-extreme Kerr black holes, as the inner edge of the disc can extend down to the innermost stable circular orbit (ISCO).

Moreover, Stuchlík et al. (2005) shows that for $a > 0.99979$ the similarly humpy behavior of the orbital velocity in LNRF also takes place for the non-geodesic motion of test perfect fluid in marginally stable barotropic tori characterized by the uniform distribution of the specific angular momentum, $\ell(r, \theta) \equiv -U_\varphi/U_t = \text{const.}$, where the motion of fluid elements is given by the 4-velocity field $U^\mu = (U^t(r, \theta), 0, 0, U^\varphi(r, \theta))$. Outside the equatorial plane, the non-monotonic behavior of $\mathcal{V}^{(\varphi)}$ in marginally stable tori is represented by the topology change of the cylindrical equivelocity surfaces in the region of the hump, because the toroidal equivelocity surfaces centered around the circle corresponding to the local minimum of $\mathcal{V}^{(\varphi)}$ in the equatorial plane exist for $a > 0.99979$ (Stuchlík et al. 2005). This suggests a generation of possible instabilities in radial and vertical directions; see Stuchlík et al. (2004). In the following, we restrict our attention to the case of Keplerian discs.

Heuristic connection between the positive part of the velocity gradient, $\partial\mathcal{V}^{(\varphi)}/\partial r$, and the excitation of epicyclic oscillations in Keplerian discs was suggested by Aschenbach (2004, 2006), who defined the characteristic frequency of oscillations, induced by the humpy profile of $\mathcal{V}^{(\varphi)}$, by the maximum positive slope of the orbital velocity in terms of the coordinate radius, $\nu_{\text{crit}} \equiv (\partial\mathcal{V}^{(\varphi)}/\partial r)_{\text{max}}$. This coordinate-dependent definition was corrected in Stuchlík et al. (2007), where the proper radial distance $d\tilde{r} = \sqrt{g_{rr}} dr$ rather than the coordinate distance dr was used to define the characteristic (critical) frequency $\nu_{\text{crit}}^{\tilde{r}} \equiv (\partial\mathcal{V}^{(\varphi)}/\partial\tilde{r})_{\text{max}}$. Such a locally defined critical frequency was further related to a stationary observer at infinity, obtaining

³ We stress that the Aschenbach effect is frame-dependent, as it is related to LNRF, but recall the arguments for relevance of the LNRF point of view at the beginning of the section.

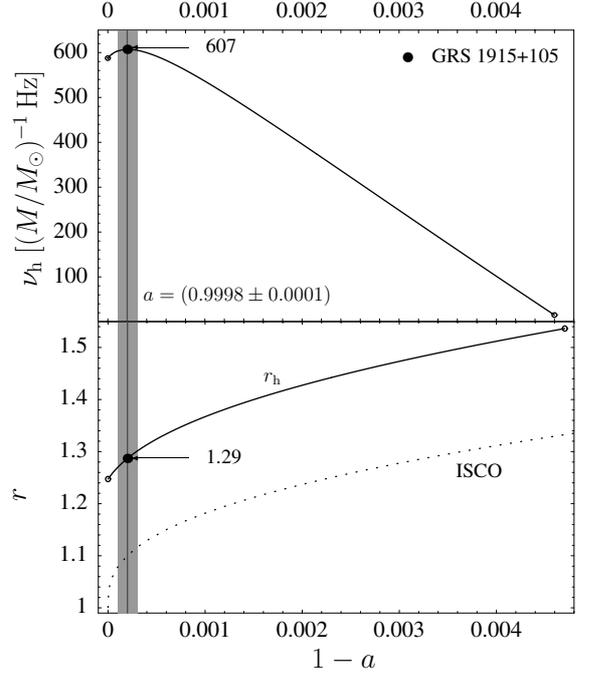


Fig. 1. Spin-dependence of the humpy frequency ν_h and the humpy radius r_h . For completeness, the B-L radius of the innermost stable circular geodesic (ISCO) is plotted.

the so-called “humpy frequency”

$$\begin{aligned} \nu_h &= \sqrt{-(g_{tt} + 2\omega g_{t\varphi} + \omega^2 g_{\varphi\varphi})_{r=r_h} \nu_{\text{crit}}^{\tilde{r}}} \\ &= \sqrt{\frac{1}{r_h} \left[(r_h - 2) - \frac{4a^2}{r_h(r_h^2 + a^2) + 2a^2} \right]} \\ &\quad \times \left[\frac{r_h^5 + a^4(3r_h + 2) - 2a^3 r_h^{1/2}(3r_h + 1)}{2\Delta_h^{3/2} \sqrt{r_h}(r_h^{3/2} + a)^2} \right. \\ &\quad \left. - \frac{2a^2 r_h^2(2r_h - 5) - 2a r_h^{5/2}(5r_h - 9)}{2\Delta_h^{3/2} \sqrt{r_h}(r_h^{3/2} + a)^2} \right] \frac{\sqrt{\Delta_h}}{r_h}, \end{aligned} \quad (1)$$

where $g_{\mu\nu}$ are the metric coefficients of the Kerr geometry and $\omega = -g_{t\varphi}/g_{\varphi\varphi}$ is the angular velocity of the LNRF; see, e.g., Bardeen et al. (1972); $\Delta_h = r_h^2 - 2r_h + a^2$. The analytic formula is given for the equatorial plane ($\theta = \pi/2$). The B-L radius r_h where the positive gradient of the velocity profile in terms of the proper radial distance reaches its maximum, so-called “humpy radius”, is given by the condition

$$\frac{\partial}{\partial r} \left(\frac{\partial\mathcal{V}^{(\varphi)}}{\partial\tilde{r}} \right) = 0 \quad (2)$$

leading to the equation

$$\begin{aligned} &3a^7(r+2) + a^6\sqrt{r}(21r^2 + 18r - 4) - a^5r(33r^2 + 10r + 20) \\ &+ a^4r\sqrt{r}(45r^3 - 62r^2 - 68r + 16) - a^3r^3(83r^2 - 122r - 60) \\ &+ a^2r^4\sqrt{r}(27r^2 - 130r + 136) - 9ar^5(7r^2 - 26r + 24) \\ &+ r^7\sqrt{r}(3r - 2) = 0, \end{aligned} \quad (3)$$

which must be solved numerically. The spin dependence of the humpy radius and the related humpy frequency is illustrated in Fig. 1. The humpy radius r_h falls monotonically with increasing spin a , while the humpy frequency ν_h has a maximum for

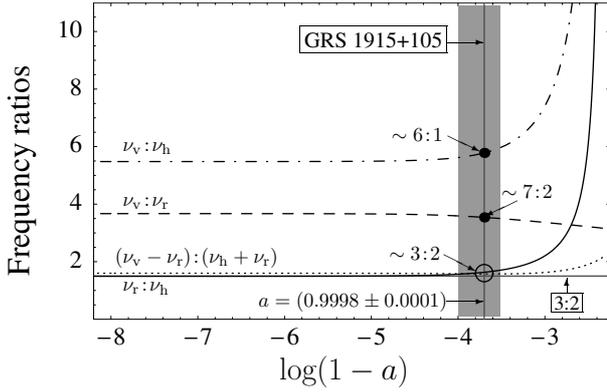


Fig. 2. Spin dependence of frequency ratios including the radial (ν_r) and vertical (ν_v) epicyclic frequencies, and the humpy frequency (ν_h) evaluated at the same radius r_h where the humpy frequency is defined. The range of the spin relevant for GRS 1915+105 Kerr black hole is shaded. For the mean value $a = 0.9998$, the frequency ratios are close to the ratios of integer numbers, suggesting a possibility of resonances between hump-induced and epicyclic oscillations in GRS 1915+105.

$a = 0.9998$, where $\nu_{h(\max)} = 607 (M_\odot/M)$ Hz, and it tends to $\nu_{h(a \rightarrow 1)} = 588 (M_\odot/M)$ Hz.

When particles following a Keplerian circular orbit are perturbed, they begin to follow, in the first approximation, an epicyclic motion around the equilibrium Keplerian orbit, generally characterized by the frequencies of the radial and vertical epicyclic oscillations ν_r, ν_v (Aliev & Galtsov 1981; Nowak & Lehr 1998):

$$\nu_r^2 = \nu_K^2 (1 - 6r^{-1} + 8ar^{-3/2} - 3a^2r^{-2}), \quad (4)$$

$$\nu_v^2 \equiv \nu_\theta^2 = \nu_K^2 (1 - 4ar^{-3/2} + 3a^2r^{-2}), \quad (5)$$

where ν_K is the Keplerian orbital frequency

$$\nu_K = \frac{1}{2\pi(r^{3/2} + a)}. \quad (6)$$

The ratios of the humpy frequency and the epicyclic frequencies at the humpy radius were determined in Stuchlík et al. (2007) revealing almost spin-independent asymptotic behavior for $a \rightarrow 1$ represented closely by the ratios of integer numbers, $\nu_v:\nu_r:\nu_h \sim 11:3:2$, which imply a possibility of resonant phenomena between the hump-induced and epicyclic oscillations predicted by Aschenbach (2004). The ratios of the epicyclic frequencies and the humpy frequency are given in the dependence on the black-hole spin in Fig. 2.

3. Application of the hump-induced resonance model to high-frequency QPOs in GRS 1915+105

Primarily concentrating on the lower pair of frequencies, we assume that the lowest frequency is directly the humpy frequency,

$$\nu_h \equiv \nu_1 = (41 \pm 1) \text{ Hz}, \quad (7)$$

while the second lowest frequency corresponds directly to the radial epicyclic frequency at the same radius r_h ,

$$\nu_r \equiv \nu_2 = (67 \pm 1) \text{ Hz}. \quad (8)$$

These frequencies are close to a 3:2 ratio, therefore the forced non-linear resonance can be relevant in such a situation. The

ratio of $\nu_r/\nu_h = (1.63 \pm 0.06)$ gives the black hole spin $a = (0.9998 \pm 0.0001)$ (the uncertainty of the spin is implied by uncertainties of the lower pair of frequencies being ~ 1 Hz); see Fig. 2. Notice that this spin corresponds to the maximal possible value of the humpy frequency $\nu_{h(\max)}$ (Fig. 1). Since the humpy frequency is $1/M$ -scaled, the absolute value of ν_h implies the black hole mass $M = (14.8 \pm 0.4) M_\odot$. The corresponding humpy radius is $r_h = 1.29^{+0.01}_{-0.02}$ (Fig. 1). At such a radius, the vertical epicyclic frequency of a particle orbiting the Kerr black hole with the mass and spin inferred above reaches the value $\nu_v = (0.23 \pm 0.01)$ kHz. Then the upper pair of observed frequencies can be explained, within the range of observational errors ± 5 Hz, by combinational frequencies at the humpy radius r_h in the following way:

$$\nu_3 \sim (\nu_r + \nu_h) = (108 \pm 2) \text{ Hz} \quad (9)$$

$$\nu_4 \sim (\nu_v - \nu_r) = (0.17 \pm 0.01) \text{ kHz}. \quad (10)$$

4. Conclusions

The idea of epicyclic oscillations induced by the LNRf-velocity hump in the region where the positive part of the velocity gradient reaches its maximum is able to address all four high-frequency QPOs observed in the X-ray source GRS 1915+105. The model implies a near-extreme spin of the central black hole ($a \sim 0.9998$), which agrees well with results from the spectral continuum fits, and the black-hole mass $M \sim 14.8 M_\odot$ being well inside the interval given by other observational methods. Note that the orbital resonance model of Kluźniak & Abramowicz, assuming the parametric resonance between the vertical and radial epicyclic oscillations in frequency ratio 3:2 represented by the upper pair of observed frequencies, also gives the spin $a > 0.99$ but for $M \approx 18 M_\odot$ (Török et al. 2005). On the other hand, the ‘‘Relativistic Precession Model’’ gives a substantially lower value for the spin: $a \sim 0.3$ (Stella et al. 1999).

In the presented model, we assume that all four observed frequencies arise due to forced non-linear oscillations of the Keplerian disc at the same radius r_h , excited by the hump-induced oscillations characterized by the humpy frequency ν_h . The black-hole parameters a, M are fixed by the requirement that the lower pair of observed frequencies is identified with the humpy frequency and the radial epicyclic frequency, $\nu_1 \equiv \nu_h$, $\nu_2 \equiv \nu_r$. Assuming non-linear resonant phenomena enabling the existence of combinational frequencies and the possibility of observing them, the upper pair of observed frequencies can be explained as the combinational ones of the humpy frequency and both epicyclic frequencies, $\nu_3 \sim (\nu_r + \nu_h)$, $\nu_4 \sim (\nu_v - \nu_r)$. Moreover, both frequency ratios $\nu_r:\nu_h$, and $(\nu_v - \nu_r):(\nu_r + \nu_h)$ are close to 3:2 ratio (Fig. 2), in which the resonant phenomena can be strong enough. On the other hand, as $4\nu_h = (164 \pm 4)$ Hz, which is also close to the uppermost frequency, there is another possibility of explaining ν_4 through a sub-harmonic resonance forced by the humpy oscillations as well. Finally, note that Strohmayer (2001) also reports another relatively weak QPO at frequency of (56 ± 2) Hz. If this is the case (which, according to our knowledge, has not been confirmed by other observations yet), it could be related to the second harmonic of the combinational frequency⁴ $(\nu_r - \nu_h) = (26 \pm 2)$ Hz.

Generally, other harmonics and combinational frequencies may occur in a non-linear oscillating system corresponding to

⁴ Combinational frequency $(\nu_r - \nu_h)$ corresponds to the same order of nonlinearity as $(\nu_r + \nu_h)$.

Note added in the manuscript: After the paper was accepted we obtained an information that a weak QPO at frequency 27 Hz is referenced in Belloni et al. (2001).

higher approximations, when the equation of motion describing the non-linear oscillations is solved by the method of successive approximations. The statement by Landau & Lifshitz (1976) that “As the degree of approximation increases, however, the strength of the resonances, and the widths of the frequency ranges in which they occur, decrease so rapidly that in practice only the resonances at frequencies⁵ $\nu \approx p\nu_0/q$ with small p and q can be observed” can explain why a QPO near the frequency 237 Hz, corresponding to the vertical epicyclic frequency ν_v at the same radius r_h as the previously mentioned humpy and radial epicyclic frequencies ν_h , ν_r , is not directly observed, despite the commensurability of these frequencies represented by the frequency ratios $\nu_v:\nu_h \sim 6:1$ and $\nu_v:\nu_r \sim 7:2$ (Fig. 2).

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⁵ p, q are integers.