

# Anomalous absorption in $c\text{-C}_3\text{H}$ and $c\text{-C}_3\text{D}$ radicals<sup>★</sup> (Research Note)

S. Chandra<sup>1,2</sup>, S. V. Shinde<sup>2</sup>, W. H. Kegel<sup>1</sup>, and E. Sedlmayr<sup>1</sup>

<sup>1</sup> Zentrum für Astronomie und Astrophysik, Technische Universität Berlin, Hardenbergstrasse 36, 10623 Berlin, Germany  
e-mail: [kegel;sedlmayr]@astro.physik.tu-berlin.de

<sup>2</sup> School of Physical Sciences, S. R. T. M. University, Nanded 431 606, India  
e-mail: suresh492000@yahoo.co.in

Received 14 June 2006 / Accepted 9 February 2007

## ABSTRACT

**Context.** The  $c\text{-C}_3\text{H}$  radical was first detected in TMC-1 by Yamamoto et al. (1987, ApJ, 322, L55), who observed the  $2_{12} \rightarrow 1_{11}$  transition at 91.5 GHz in emission. Mangum & Wootten (1990, A&A, 239, 319) observed the  $1_{10} \rightarrow 1_{11}$  transition at 14.8 GHz in emission in 12 additional galactic objects.

**Aims.** The aim of this investigation is a quantitative estimate of relative line intensities under NLTE conditions.

**Methods.** For wide ranges of physical parameters, where these molecules may be found, we have solved a set of statistical equilibrium equations coupled with the equations of radiative transfer in an on-the-spot approximation. For  $c\text{-C}_3\text{H}$ , we accounted for 51 energy levels connected by 207 radiative transitions, and for  $c\text{-C}_3\text{D}$ , we accounted for 51 energy levels connected by 205 radiative transitions.

**Results.** Our results show that the  $3_{31} \rightarrow 3_{30}$  transition of  $c\text{-C}_3\text{H}$  and  $c\text{-C}_3\text{D}$  may be found in absorption against the cosmic microwave background (CMB). Furthermore, we found population inversion for the  $1_{10} \rightarrow 1_{11}$  transition. These findings may be useful in identifying these molecules in other cosmic objects, as well as for the determination of physical parameters in these objects.

**Key words.** ISM: molecules – molecular data

## 1. Introduction

Cyclopropynylidyne,  $c\text{-C}_3\text{H}$ , was the second hydrocarbon ring molecule found in interstellar space (after  $c\text{-C}_3\text{H}_2$ ). It was first detected by Yamamoto et al. (1987) in TMC-1, who observed the  $2_{12} \rightarrow 1_{11}$  transition at 91.5 GHz in emission. Later, Mangum & Wootten (1990) observed the transition  $1_{10} \rightarrow 1_{11}$  at 14.8 GHz in emission in 12 additional galactic objects. The linear isotopomer  $l\text{-C}_3\text{H}$  has also been identified in TMC-1 and IRC +10216 (Thaddeus et al. 1985). The most probable production mechanism of  $\text{C}_3\text{H}$  and  $\text{C}_3\text{H}_2$  in dark clouds is a common dissociation reaction of the  $\text{C}_3\text{H}_3^+$  ion (Adams & Smith 1987). Although  $c\text{-C}_3\text{H}$  is 0.8 eV less stable than  $l\text{-C}_3\text{H}$ , the column density of  $c\text{-C}_3\text{H}$  is found almost comparable to that of  $l\text{-C}_3\text{H}$  in TMC-1.

We have performed NLTE radiative transfer calculations for  $c\text{-C}_3\text{H}$  and  $c\text{-C}_3\text{D}$ . Our results indicate that the  $3_{31} \rightarrow 3_{30}$  transition will show up in absorption against the CMB over a wide range of physical parameters. In addition, we found population inversion in a certain parameter range for the  $1_{10} \rightarrow 1_{11}$  transition. These findings may be useful for identifying these molecules and for determination of the physical conditions within the objects where they are found. In this investigation we did not account for fine and hyperfine splittings of the levels.

## 2. Basic formulation

The  $c\text{-C}_3\text{H}$  and  $c\text{-C}_3\text{D}$  both are  $a$ -type asymmetric top molecules with equal electric dipole moment  $\mu = 2.4$  Debye. Rotational levels in an asymmetric top molecule are specified as  $J_{k_a, k_c}$ , where  $J$

is the rotational quantum number and  $k_a$  and  $k_c$  the projections of  $J$  on the axis of symmetry in case of prolate and oblate symmetric tops, respectively. In an  $a$ -type asymmetric top molecule, rotational radiative transitions are governed by the selection rules:

$$\begin{aligned} J : & \quad \Delta J = 0 \pm 1 \\ k_a, k_c : & \quad \text{even, odd} \longleftrightarrow \text{even, even} \\ & \quad \text{odd, even} \longleftrightarrow \text{odd, odd.} \end{aligned}$$

Owing to the nuclear symmetry in  $c\text{-C}_3\text{H}$  and  $c\text{-C}_3\text{D}$ , the rotational levels with even value of  $k_a$  are missing in their spectra. In the present investigation we accounted for the rotational levels up to  $106 \text{ cm}^{-1}$  for  $c\text{-C}_3\text{H}$  (Fig. 1, Table 1) and up to  $94 \text{ cm}^{-1}$  for  $c\text{-C}_3\text{D}$  (Table 2).

In our investigation, NLTE occupation numbers are calculated in an on-the-spot approximation based on an escape probability method. For this we used a VLG (large velocity gradient) code (Goldreich & Kwan 1974) in the form as described by Rausch et al. (1996), where the external radiation field is assumed to be the CMB only. In this approximation, the escape probabilities depend on the local occupation numbers only. Inserting these into the rate equations, the latter become nonlinear. For  $c\text{-C}_3\text{H}$ , a set of 51 nonlinear rate equations involving 207 radiative transitions and for  $c\text{-C}_3\text{D}$ , a set of 51 nonlinear rate equations involving 205 radiative transitions are solved by iteration for the given values of the hydrogen density  $n_{\text{H}_2}$  and of the parameter  $\gamma \equiv n_{\text{mol}}/(dv_r/dr)$ ,  $n_{\text{mol}}$  being the density of the molecule and  $(dv_r/dr)$  the velocity gradient in the object. The input data required in the present investigation are the radiative transition probabilities (Einstein  $A$ -coefficients) and the collisional rate coefficients.

<sup>★</sup> Tables 1–3 and Figs. 4, 5 are only available in electronic form at <http://www.aanda.org>

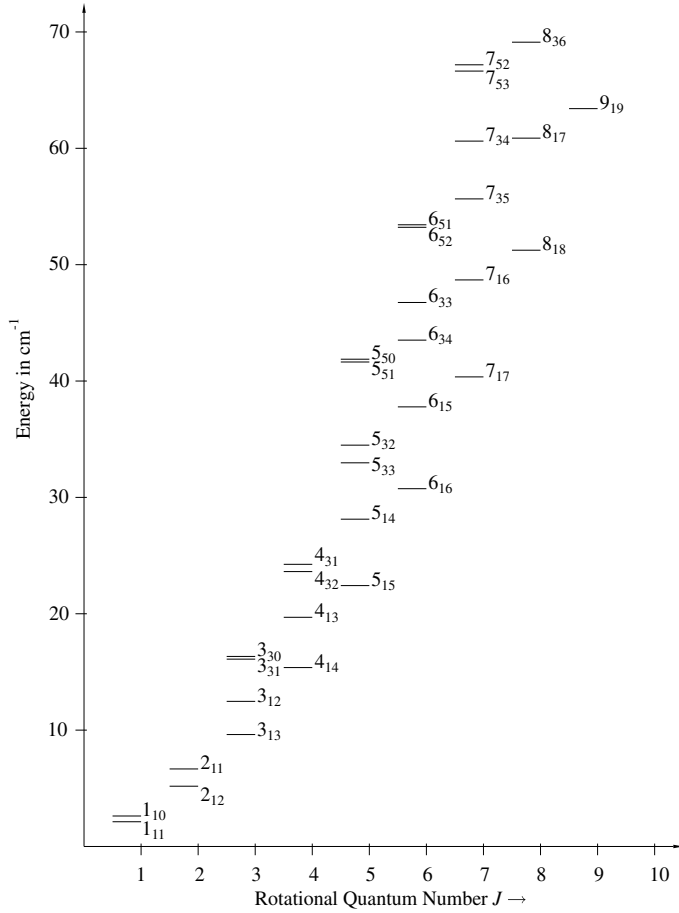


Fig. 1. Energy level diagram for  $c\text{-C}_3\text{H}$ .

### 2.1. Einstein A-coefficients

The energy levels and Einstein A-coefficients of radiatively allowed transitions are calculated as described by Chandra et al. (1984) and Chandra & Rashmi (1998). The required rotational and distortion constants are taken from Yamamoto & Saito (1994). The numerical values of the energy levels and the Einstein A-coefficients are given in Tables 1–3.

### 2.2. Collisional rate coefficients

Since rate coefficients for collisions with  $\text{H}_2$  molecules are not available in the literature, we assumed for the downward transitions  $J'k'_ak'_c \rightarrow Jk_ak_c$  at a kinetic temperature  $T_k$ :

$$C(J'k'_ak'_c \rightarrow Jk_ak_c) = 1 \times 10^{-11} (2J + 1) \sqrt{\frac{T_k}{30}}. \quad (1)$$

The upward collisional rate coefficients were determined using the detailed balancing equations.

## 3. Anomalous absorption and population inversion

The observation of a spectral line in absorption against the CMB is a phenomenon as unusual as the occurrence of population inversion (maser emission) as discussed e.g. by ter Haar & Pelling (1974). Both phenomena are a sign of extreme deviation from LTE. They do not show up if one models the molecules as two level systems, i.e. for any theoretical explanation, three or more energy levels have to be accounted for.

The intensity,  $I_\nu$ , of a line formed in an interstellar cloud, with homogeneous excitation conditions, is given by

$$I_\nu - I_{\nu,\text{bg}} = (S_\nu - I_{\nu,\text{bg}}) (1 - e^{-\tau_\nu}) \quad (2)$$

where  $I_{\nu,\text{bg}}$  is the intensity of the continuum against which the line is observed,  $\tau_\nu$  being the optical depth of the line, and  $S_\nu$  the source function. Expressing the intensity  $I_\nu$  in terms of the brightness temperature  $T_B$ , and the source function  $S_\nu$  in terms of the excitation temperature  $T_{\text{ex}}$ , Eq. (2) may also be written in the form (Rayleigh Jeans approximation):

$$T_B - T_{\nu,\text{bg}} = (T_{\text{ex}} - T_{\nu,\text{bg}}) (1 - e^{-\tau_\nu}). \quad (3)$$

Whether a given line appears in absorption or in emission, depends in general on the physical conditions within the cloud where it is formed and on the background against which it is observed. In the case of anomalous absorption, i.e. if we have  $0 < T_{\text{ex}} < T_{\text{CMB}}$ , where  $T_{\text{CMB}}$  is 2.73 K, however, the line will appear in absorption against any background, even against the CMB. Similarly, in case we have an occupation inversion, in which case  $T_{\text{ex}}$  and  $\tau_\nu$  both are negative, the line will appear in emission against any background.

## 4. Results and discussion

In order to include a large number of cosmic objects where  $c\text{-C}_3\text{H}$  and  $c\text{-C}_3\text{D}$  may be found, numerical calculations are carried out for wide ranges of physical parameters. The molecular hydrogen density  $n_{\text{H}_2}$  has been varied over the range from  $10^3 \text{ cm}^{-3}$  to  $10^7 \text{ cm}^{-3}$  and calculations are performed for kinetic temperatures from 10 K to 70 K and for two values of  $\gamma$ ,  $10^{-5} \text{ cm}^{-3}(\text{km/s})^{-1}\text{pc}$  and  $10^{-6} \text{ cm}^{-3}(\text{km/s})^{-1}\text{pc}$ . In the calculations, the molecular hydrogen density  $n_{\text{H}_2}$ , the kinetic temperature  $T_k$  and  $\gamma$  are free parameters. As we have used scaled values of collisional rates, our results are qualitative in nature.

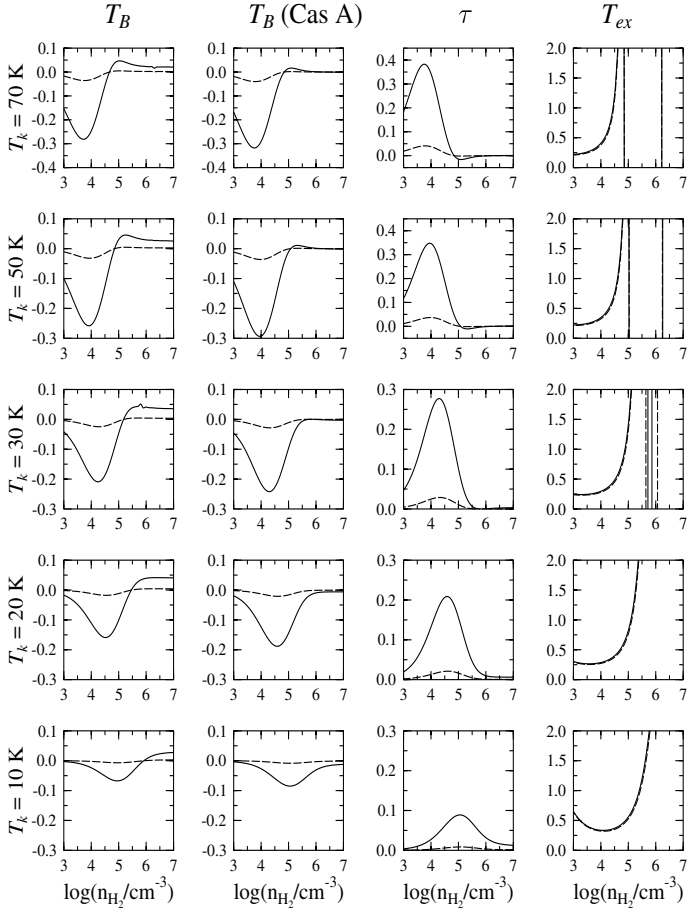
### 4.1. The $c\text{-C}_3\text{H}$ radical

Figures 2–4 show the results of our numerical computations for three particular lines of the  $c\text{-C}_3\text{H}$  radical, the  $3_{31} \rightarrow 3_{30}$  transition at 3.4 GHz, the  $1_{10} \rightarrow 1_{11}$  transition at 14.8 GHz, and the  $2_{12} \rightarrow 1_{11}$  transition at 91.5 GHz.

The figures give the relative brightness temperatures – i.e. the quantity  $(T_B - T_{\nu,\text{bg}})/T_{\nu,\text{bg}}$  – for two different backgrounds, the first being the CMB and the second a non-thermal source corresponding to Cas A. The latter background was determined by taking the brightness temperature at 14.5 GHz from the map of Batrla et al. (1983) at the position where Yamamoto et al. searched for the 91.5 GHz line and then scaling it to the frequencies of our lines according to the frequency dependence as determined by Baars et al. (1977) and adding the CMB. In addition, the figures show the optical depth, and in case of the 3.4 GHz line the excitation temperature. All these quantities are shown as functions of  $n_{\text{H}_2}$  for given values of  $T_k$  and  $\gamma$ .

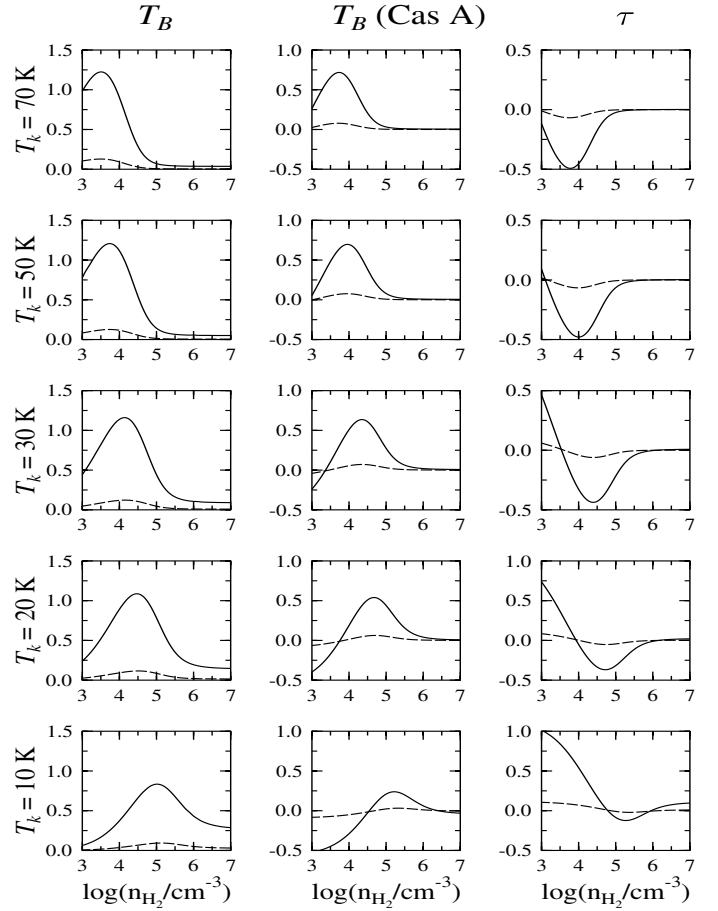
We found a number of lines of  $c\text{-C}_3\text{H}$  to occur in absorption against the CMB, the transition  $3_{31} \rightarrow 3_{30}$  at 3.4 GHz (Fig. 2) being the strongest one.

We note in particular, that for a given kinetic temperature, the excitation temperature  $T_{\text{ex}}$  first decreases with increasing values of  $n_{\text{H}_2}$ , then goes through a wide minimum and finally increases again. This general behavior can be understood by qualitative arguments: since in the limit  $n_{\text{H}_2} \rightarrow 0$  the radiative transition rates dominate over the collisional rates, we have  $T_{\text{ex}} \rightarrow T_{\text{CMB}}$ .



**Fig. 2.** Results for the transition  $3_{31} \rightarrow 3_{30}$  of  $c\text{-C}_3\text{H}$ .  $T_B$ : relative brightness temperature with respect to the CMB,  $T_B$  (Cas A): with respect to a non-thermal source like Cas A (see text),  $\tau$ : optical depth and  $T_{\text{ex}}$ : excitation temperature. All quantities are given as function of hydrogen density  $n_{\text{H}_2}$  for given values of kinetic temperatures  $T_k$  (10, 20, 30, 50, 70 K). Solid line is for  $\gamma = 10^{-5} \text{ cm}^{-3} (\text{km/s})^{-1} \text{ pc}$  and dashed line for  $\gamma = 10^{-6} \text{ cm}^{-3} (\text{km/s})^{-1} \text{ pc}$ .

In the other limit,  $n_{\text{H}_2} \rightarrow \infty$ , the collisional transition rates dominate and we have  $T_{\text{ex}} \rightarrow T_k$ . In the range in between the two limits, the excitation of most of the levels is dominated by the collisions, while in the optically thin limit the deexcitation rate for most of the levels is determined by spontaneous emission. In this range, one finds as a crude estimate for a line for which the energy difference between the upper and the lower level is small compared to  $kT$  and small as compared to the excitation energy of both levels – conditions satisfied for the  $3_{31} \rightarrow 3_{30}$  transition of  $c\text{-C}_3\text{H}$  – that the ratio of the occupation numbers of the upper and the lower level is roughly given by the inverse of the radiative life times of the levels. For the  $3_{31} \rightarrow 3_{30}$  transition, this value is 0.53 (see Table 1). Since  $T_{\text{ex}}$  is defined through the ratio of the occupation numbers, this explains why the minimum of  $T_{\text{ex}}$  as function of  $n_{\text{H}_2}$  is so wide. The variation of  $T_{\text{ex}}$  with the kinetic temperature  $T_k$  is essentially a consequence of the variation of the upward collision rates with  $T_k$ . (The excitation energies of  $3_{31}$  and  $3_{30}$  levels are of the order of 20 K, see Fig. 1) The collision rates increase with  $T_k$ . The most important effect of this is that the above discussed limits are shifted to lower densities. It is noteworthy that in the density range for which we have  $T_{\text{ex}} < T_{\text{CMB}}$ ,  $T_{\text{ex}}$  varies considerably less with  $T_k$  than  $T_B$  and  $\tau$ . This is due to the fact that  $T_{\text{ex}}$  depends only on the ratio of the occupation numbers, while  $\tau$  depends on the difference.



**Fig. 3.** Same as Fig. 2 but for the transition  $1_{10} \rightarrow 1_{11}$  of  $c\text{-C}_3\text{H}$ .

From Fig. 2 we also see that at higher temperatures ( $T_k \geq 30 \text{ K}$ ) there is a small range of densities in which we have population inversion.

The curves for  $T_{\text{ex}}$  calculated for the two values of  $\gamma$  hardly differ from each other. This indicates that optical thickness effects are unimportant in this parameter range.

From our numerical results, we also find population inversion for several transitions, the most important one being the transition  $1_{10} \rightarrow 1_{11}$  (Fig. 3). The inversion indicated in Fig. 3 by negative values of  $\tau$ , is a consequence of the interplay between collisional and radiative transitions. Collisions provide in our model the basic excitation. Therefore at low densities, the inversion increases with increasing density, goes through a maximum (minimum of  $\tau$ ) and then decreases again. In this context we note that the minimum of  $\tau$  occurs approximately when the collisional and the radiative lifetime of the upper level become equal. The strong dependence on the kinetic temperature shows that the higher lying energy levels are involved. The higher levels are excited by collisions and then are deexcited mostly due to radiative transitions. In this context we note, that the radiative lifetime of the  $1_{10}$  level is larger than that of all other levels by more than one order of magnitude (see Tables 1 and 2).

Figure 4 shows the results for the  $2_{12} \rightarrow 1_{11}$  transition at 91.5 GHz, which was observed by Yamamoto et al. (1987) in TMC-1 and searched for in the direction of Cas A. At this high frequency, the non-thermal background is very low and therefore has only little effect on the appearance of the line.

#### 4.2. The $c\text{-C}_3\text{D}$ radical

The results we obtained for the  $c\text{-C}_3\text{D}$  radical are qualitatively very similar to those found for  $c\text{-C}_3\text{H}$ . In particular, we found for the  $3_{31} \rightarrow 3_{30}$  transition at 0.9 GHz anomalous absorption, and for the  $1_{10} \rightarrow 1_{11}$  transition at 10.8 GHz population inversion over a wide range of parameters. We shall not go into a detailed discussion, but just refer to Fig. 5 showing the results for the  $3_{31} \rightarrow 3_{30}$  transition of  $c\text{-C}_3\text{D}$ .

### 5. Conclusions

Our numerical results show that in a range of physical parameters typical for interstellar molecular clouds, strong deviations from LTE of the relative occupation numbers of the molecules  $c\text{-C}_3\text{H}$  and  $c\text{-C}_3\text{D}$  are to be expected. For certain transitions these may imply anomalous absorption or population inversion. Therefore, it is difficult to derive the total column density of a given molecule by observing just one line. For a quantitative estimate of the column density and other physical parameters one should measure several lines, if possible. In this context the observation of anomalous absorption may be of particular use, since this phenomenon is restricted to a certain range of densities. If observations are made against a non-thermal background source, the  $1_{10} \rightarrow 1_{11}$  transition may be found in emission or

absorption, depending on the physical parameters within the cloud in which the line is formed.

*Acknowledgements.* Financial supports from the Department of Science & Technology, New Delhi and the Indian Space Research Organisation, Bangalore in the form of research projects are thankfully acknowledged. Suresh Chandra is grateful to Prof. Dr. W. H. Kegel and Prof. Dr. E. Sedlmayr for their kind invitation and hospitality under scientific exchange programme between the Indian National Science Academy (INSA), New Delhi, India and the Deutsche Forschungsgemeinschaft (DFG), Bonn, Germany.

### References

- Adams, N. G., & Smith, D. 1987, *ApJ*, 317, L25  
 Baars, J. W. M., Genzel, R., Pauliny-Toth, I. I. K., & Witzel, A. 1977, *A&A*, 61, 99  
 Batrla, W., Wilson, T. L., & Martin-Pintado, J. 1983, *A&A*, 119, 139  
 Chandra, S., & Rashmi 1998, *A&AS*, 131, 137  
 Chandra, S. Varshalovich, D. A., & Kegel, W. H 1984, *A&AS*, 55, 51  
 Goldreich, P., & Kwan, J. 1974, *ApJ*, 189, 144  
 Mangum, J. G., & Wootten, A. 1990, *A&A*, 239, 319  
 Rausch, E., Kegel, W. H., Tsuji, T., & Piehler, G. 1996, *A&A*, 315, 533  
 Thaddeus, P., Gottlieb, C. A., Hjalmarsen, A., et al. 1985a, *ApJ*, 294, L49  
 ter Haar, D., & Pelling, M. A. 1974, *Rep. Prog. Phys.*, 37, 481  
 Yamamoto, S., & Saito, S. 1994, *J. Chem. Phys.*, 101, 5484  
 Yamamoto, S., Saito, S., Ohishi, M., et al. 1987, *ApJ*, 322, L55

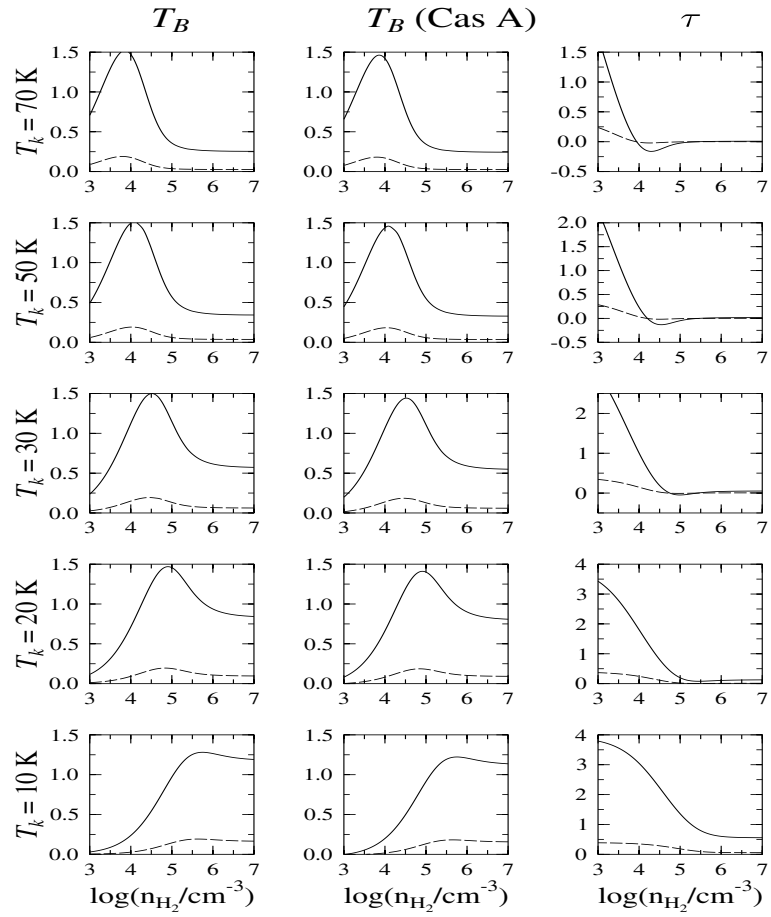
# Online Material

**Table 1.** Energy (in  $\text{cm}^{-1}$ ) and radiative lifetime of rotational levels for  $c\text{-C}_3\text{H}$ .

$c\text{-C}_3\text{H}$					
Level	Energy	Radiative lifetime	Level	Energy	Radiative lifetime
1 <sub>11</sub>	2.126	–	6 <sub>51</sub>	53.378	$1.366 \times 10^3$
1 <sub>10</sub>	2.620	$9.154 \times 10^6$	7 <sub>35</sub>	55.653	$7.619 \times 10^2$
2 <sub>12</sub>	5.180	$6.473 \times 10^4$	7 <sub>34</sub>	60.619	$4.920 \times 10^2$
2 <sub>11</sub>	6.664	$2.716 \times 10^4$	8 <sub>17</sub>	60.874	$7.136 \times 10^2$
3 <sub>13</sub>	9.620	$1.698 \times 10^4$	9 <sub>19</sub>	63.410	$6.772 \times 10^2$
3 <sub>12</sub>	12.472	$7.708 \times 10^3$	7 <sub>53</sub>	66.637	$6.905 \times 10^2$
4 <sub>14</sub>	15.376	$7.202 \times 10^3$	7 <sub>52</sub>	67.177	$5.496 \times 10^2$
3 <sub>31</sub>	16.174	$4.354 \times 10^4$	8 <sub>36</sub>	69.120	$5.407 \times 10^2$
3 <sub>30</sub>	16.289	$2.341 \times 10^4$	9 <sub>18</sub>	74.336	$5.187 \times 10^2$
4 <sub>13</sub>	19.696	$3.930 \times 10^3$	8 <sub>35</sub>	75.712	$3.860 \times 10^2$
5 <sub>15</sub>	22.421	$3.762 \times 10^3$	10 <sub>1,10</sub>	76.856	$4.969 \times 10^2$
4 <sub>32</sub>	23.625	$4.990 \times 10^3$	7 <sub>71</sub>	79.382	$4.208 \times 10^3$
4 <sub>31</sub>	24.249	$3.139 \times 10^3$	7 <sub>70</sub>	79.383	$4.184 \times 10^3$
5 <sub>14</sub>	28.128	$2.389 \times 10^3$	8 <sub>54</sub>	81.887	$4.069 \times 10^2$
6 <sub>16</sub>	30.748	$2.215 \times 10^3$	8 <sub>53</sub>	83.447	$2.937 \times 10^2$
5 <sub>33</sub>	32.792	$2.059 \times 10^3$	9 <sub>37</sub>	83.878	$4.015 \times 10^2$
5 <sub>32</sub>	34.490	$1.247 \times 10^3$	10 <sub>19</sub>	89.075	$3.890 \times 10^2$
6 <sub>15</sub>	37.780	$1.525 \times 10^3$	11 <sub>1,11</sub>	91.582	$3.753 \times 10^2$
7 <sub>17</sub>	40.356	$1.413 \times 10^3$	9 <sub>36</sub>	91.921	$3.078 \times 10^2$
5 <sub>51</sub>	41.864	$9.225 \times 10^3$	8 <sub>72</sub>	94.551	$7.785 \times 10^2$
5 <sub>50</sub>	41.876	$8.606 \times 10^3$	8 <sub>71</sub>	94.564	$7.669 \times 10^2$
6 <sub>34</sub>	43.515	$1.162 \times 10^3$	9 <sub>55</sub>	98.849	$2.774 \times 10^2$
6 <sub>33</sub>	46.751	$7.036 \times 10^2$	10 <sub>38</sub>	99.916	$3.073 \times 10^2$
7 <sub>16</sub>	48.690	$1.020 \times 10^3$	9 <sub>54</sub>	102.033	$1.915 \times 10^2$
8 <sub>18</sub>	51.243	$9.565 \times 10^2$	11 <sub>1,10</sub>	105.094	$2.993 \times 10^2$
6 <sub>52</sub>	53.262	$1.535 \times 10^3$			

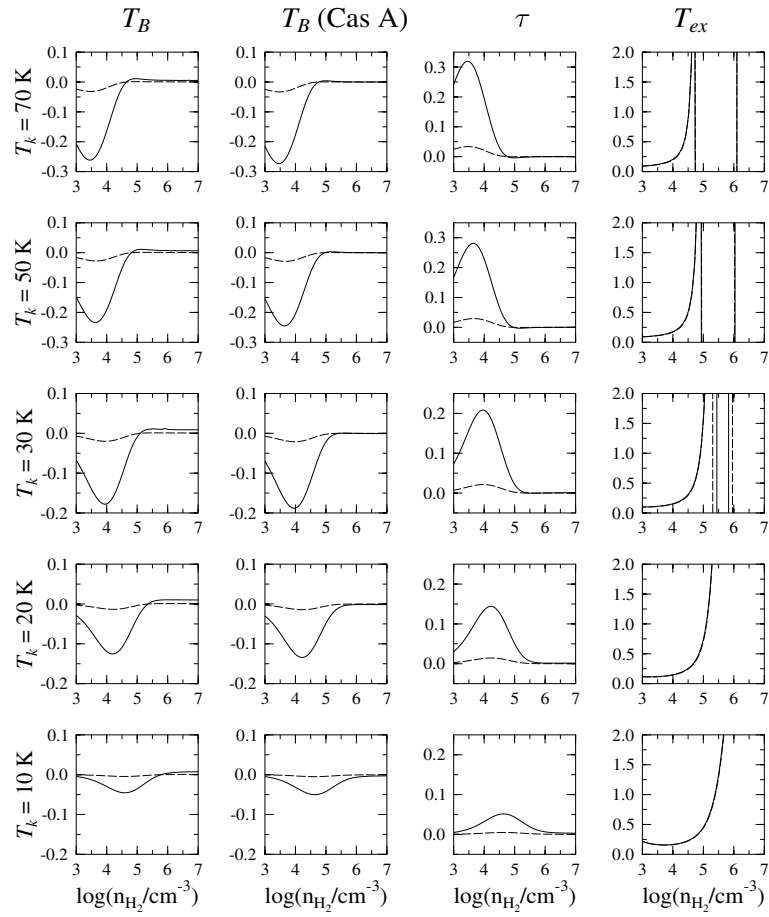
**Table 2.** Energy (in  $\text{cm}^{-1}$ ) and radiative lifetime of rotational levels for  $c\text{-C}_3\text{D}$ .

$c\text{-C}_3\text{D}$					
Level	Energy	Radiative lifetime	Level	Energy	Radiative lifetime
1 <sub>11</sub>	2.055	–	6 <sub>52</sub>	50.330	$3.149 \times 10^3$
1 <sub>10</sub>	2.416	$2.345 \times 10^7$	6 <sub>51</sub>	50.341	$3.094 \times 10^3$
2 <sub>12</sub>	4.695	$1.002 \times 10^5$	7 <sub>34</sub>	52.136	$7.462 \times 10^2$
2 <sub>11</sub>	5.779	$4.762 \times 10^4$	8 <sub>17</sub>	54.024	$9.862 \times 10^2$
3 <sub>13</sub>	8.586	$2.491 \times 10^4$	9 <sub>19</sub>	56.415	$9.523 \times 10^2$
3 <sub>12</sub>	10.724	$1.204 \times 10^4$	8 <sub>36</sub>	61.196	$7.746 \times 10^2$
4 <sub>14</sub>	13.678	$1.024 \times 10^4$	7 <sub>53</sub>	61.292	$1.350 \times 10^3$
3 <sub>31</sub>	15.684	$9.167 \times 10^4$	7 <sub>52</sub>	61.354	$1.291 \times 10^3$
3 <sub>30</sub>	15.715	$6.745 \times 10^4$	8 <sub>35</sub>	65.525	$5.118 \times 10^2$
4 <sub>13</sub>	17.096	$5.301 \times 10^3$	9 <sub>18</sub>	66.018	$7.221 \times 10^2$
5 <sub>15</sub>	19.936	$5.290 \times 10^3$	10 <sub>1,10</sub>	68.385	$6.994 \times 10^2$
4 <sub>32</sub>	21.876	$9.020 \times 10^3$	8 <sub>54</sub>	73.889	$7.498 \times 10^2$
4 <sub>31</sub>	22.072	$7.098 \times 10^3$	8 <sub>53</sub>	74.128	$6.798 \times 10^2$
5 <sub>14</sub>	24.694	$3.075 \times 10^3$	9 <sub>37</sub>	74.279	$5.643 \times 10^2$
6 <sub>16</sub>	27.344	$3.107 \times 10^3$	7 <sub>71</sub>	78.161	$1.042 \times 10^4$
5 <sub>33</sub>	29.596	$3.478 \times 10^3$	7 <sub>70</sub>	78.161	$1.042 \times 10^4$
5 <sub>32</sub>	30.257	$2.513 \times 10^3$	10 <sub>19</sub>	79.144	$5.436 \times 10^2$
6 <sub>15</sub>	33.385	$2.017 \times 10^3$	9 <sub>36</sub>	80.234	$3.884 \times 10^2$
7 <sub>17</sub>	35.894	$1.983 \times 10^3$	11 <sub>1,11</sub>	81.493	$5.287 \times 10^2$
6 <sub>34</sub>	38.778	$1.833 \times 10^3$	9 <sub>55</sub>	88.099	$4.763 \times 10^2$
6 <sub>33</sub>	40.314	$1.243 \times 10^3$	10 <sub>38</sub>	88.540	$4.289 \times 10^2$
5 <sub>51</sub>	40.995	$2.248 \times 10^4$	9 <sub>54</sub>	88.796	$4.004 \times 10^2$
5 <sub>50</sub>	40.996	$2.224 \times 10^4$	8 <sub>72</sub>	90.575	$1.649 \times 10^3$
7 <sub>16</sub>	43.153	$1.388 \times 10^3$	8 <sub>71</sub>	90.575	$1.648 \times 10^3$
8 <sub>18</sub>	45.585	$1.344 \times 10^3$	11 <sub>1,10</sub>	93.406	$4.192 \times 10^2$
7 <sub>35</sub>	49.339	$1.134 \times 10^3$			



**Fig. 4.** Same as Fig. 2 but for the transition  $2_{12} \rightarrow 1_{11}$  of  $c\text{-C}_3\text{H}$ .





**Fig. 5.** Same as Fig. 2 but for the transition  $3_{31} \rightarrow 3_{30}$  of  $c\text{-C}_3\text{D}$ .

Table 3: Einstein coefficients for  $\text{C}_3\text{H}$  and  $\text{C}_3\text{D}$ 

			For $\text{C}_3\text{H}$					
1(1, 0) → 1(1, 1)	$1.092 \times 10^{-7}$		2(1, 2) → 1(1, 1)	$1.545 \times 10^{-5}$		2(1, 1) → 1(1, 0)	$3.584 \times 10^{-5}$	
2(1, 1) → 2(1, 2)	$9.831 \times 10^{-7}$		3(1, 3) → 2(1, 2)	$5.889 \times 10^{-5}$		3(1, 2) → 2(1, 1)	$1.257 \times 10^{-4}$	
3(1, 2) → 3(1, 3)	$4.002 \times 10^{-6}$		4(1, 4) → 3(1, 3)	$1.389 \times 10^{-4}$		3(3, 1) → 2(1, 2)	$1.995 \times 10^{-5}$	
3(3, 1) → 3(1, 2)	$3.014 \times 10^{-6}$		3(3, 1) → 4(1, 4)	$2.508 \times 10^{-10}$		3(3, 0) → 2(1, 1)	$4.142 \times 10^{-5}$	
3(3, 0) → 3(1, 3)	$1.305 \times 10^{-6}$		3(3, 0) → 3(3, 1)	$1.908 \times 10^{-9}$		4(1, 3) → 3(1, 2)	$2.435 \times 10^{-4}$	
4(1, 3) → 4(1, 4)	$1.061 \times 10^{-5}$		4(1, 3) → 3(3, 0)	$3.081 \times 10^{-7}$		5(1, 5) → 4(1, 4)	$2.658 \times 10^{-4}$	
4(3, 2) → 3(1, 3)	$4.337 \times 10^{-5}$		4(3, 2) → 3(3, 1)	$1.487 \times 10^{-4}$		4(3, 2) → 4(1, 3)	$8.287 \times 10^{-6}$	
4(3, 2) → 5(1, 5)	$8.248 \times 10^{-10}$		4(3, 1) → 3(1, 2)	$1.299 \times 10^{-4}$		4(3, 1) → 4(1, 4)	$1.576 \times 10^{-6}$	
4(3, 1) → 3(3, 0)	$1.869 \times 10^{-4}$		4(3, 1) → 4(3, 2)	$1.543 \times 10^{-7}$		5(1, 4) → 4(1, 3)	$3.977 \times 10^{-4}$	
5(1, 4) → 5(1, 5)	$2.052 \times 10^{-5}$		5(1, 4) → 4(3, 1)	$3.912 \times 10^{-7}$		6(1, 6) → 5(1, 5)	$4.515 \times 10^{-4}$	
5(3, 3) → 4(1, 4)	$6.497 \times 10^{-5}$		5(3, 3) → 4(3, 2)	$4.038 \times 10^{-4}$		5(3, 3) → 5(1, 4)	$1.684 \times 10^{-5}$	
5(3, 3) → 6(1, 6)	$2.854 \times 10^{-9}$		5(3, 2) → 4(1, 3)	$1.927 \times 10^{-4}$		5(3, 2) → 5(1, 5)	$1.218 \times 10^{-6}$	
5(3, 2) → 4(3, 1)	$6.062 \times 10^{-4}$		5(3, 2) → 5(3, 3)	$1.710 \times 10^{-6}$		6(1, 5) → 5(1, 4)	$6.226 \times 10^{-4}$	
6(1, 5) → 6(1, 6)	$3.318 \times 10^{-5}$		6(1, 5) → 5(3, 2)	$9.906 \times 10^{-8}$		7(1, 7) → 6(1, 6)	$7.076 \times 10^{-4}$	
5(5, 1) → 4(1, 4)	$4.361 \times 10^{-7}$		5(5, 1) → 4(3, 2)	$1.024 \times 10^{-4}$		5(5, 1) → 5(1, 4)	$9.009 \times 10^{-7}$	
5(5, 1) → 6(1, 6)	$8.847 \times 10^{-10}$		5(5, 1) → 5(3, 2)	$4.653 \times 10^{-6}$		5(5, 0) → 4(1, 3)	$6.517 \times 10^{-6}$	
5(5, 0) → 5(1, 5)	$5.816 \times 10^{-8}$		5(5, 0) → 4(3, 1)	$1.045 \times 10^{-4}$		5(5, 0) → 5(3, 3)	$5.144 \times 10^{-6}$	
5(5, 0) → 6(1, 5)	$2.562 \times 10^{-9}$		5(5, 0) → 5(1, 5)	$2.567 \times 10^{-12}$		6(3, 4) → 5(1, 5)	$8.492 \times 10^{-5}$	
6(3, 4) → 5(3, 3)	$7.452 \times 10^{-4}$		6(3, 4) → 6(1, 5)	$3.042 \times 10^{-5}$		6(3, 4) → 7(1, 7)	$7.213 \times 10^{-9}$	
6(3, 4) → 5(5, 1)	$3.216 \times 10^{-9}$		6(3, 3) → 5(1, 4)	$2.179 \times 10^{-4}$		6(3, 3) → 6(1, 6)	$9.835 \times 10^{-7}$	
6(3, 3) → 5(3, 2)	$1.194 \times 10^{-3}$		6(3, 3) → 5(5, 0)	$2.716 \times 10^{-7}$		6(3, 3) → 6(3, 4)	$7.956 \times 10^{-6}$	
7(1, 6) → 6(1, 5)	$9.319 \times 10^{-4}$		7(1, 6) → 7(1, 7)	$4.864 \times 10^{-5}$		7(1, 6) → 6(3, 3)	$8.744 \times 10^{-9}$	
8(1, 8) → 7(1, 7)	$1.046 \times 10^{-3}$		6(5, 2) → 5(1, 5)	$9.224 \times 10^{-7}$		6(5, 2) → 5(3, 3)	$2.426 \times 10^{-4}$	
6(5, 2) → 6(1, 5)	$2.250 \times 10^{-6}$		6(5, 2) → 7(1, 7)	$1.720 \times 10^{-9}$		6(5, 2) → 5(5, 1)	$3.939 \times 10^{-4}$	
6(5, 2) → 6(3, 3)	$1.187 \times 10^{-5}$		6(5, 1) → 5(1, 4)	$1.914 \times 10^{-5}$		6(5, 1) → 6(1, 6)	$1.042 \times 10^{-7}$	
6(5, 1) → 5(3, 2)	$2.965 \times 10^{-4}$		6(5, 1) → 5(5, 0)	$4.052 \times 10^{-4}$		6(5, 1) → 6(3, 4)	$1.122 \times 10^{-5}$	
6(5, 1) → 7(1, 6)	$5.325 \times 10^{-9}$		6(5, 1) → 6(5, 2)	$1.486 \times 10^{-9}$		7(3, 5) → 6(1, 6)	$1.046 \times 10^{-4}$	
7(3, 5) → 6(3, 4)	$1.158 \times 10^{-3}$		7(3, 5) → 7(1, 6)	$5.006 \times 10^{-5}$		7(3, 5) → 8(1, 8)	$1.393 \times 10^{-8}$	
7(3, 5) → 6(5, 2)	$1.920 \times 10^{-8}$		7(3, 4) → 6(1, 5)	$2.583 \times 10^{-4}$		7(3, 4) → 7(1, 7)	$9.389 \times 10^{-7}$	
7(3, 4) → 6(3, 3)	$1.746 \times 10^{-3}$		7(3, 4) → 6(5, 1)	$3.482 \times 10^{-6}$		7(3, 4) → 7(3, 5)	$2.375 \times 10^{-5}$	
8(1, 7) → 7(1, 6)	$1.334 \times 10^{-3}$		8(1, 7) → 8(1, 8)	$6.700 \times 10^{-5}$		8(1, 7) → 7(3, 4)	$1.112 \times 10^{-11}$	
9(1, 9) → 8(1, 8)	$1.477 \times 10^{-3}$		7(5, 3) → 6(1, 6)	$1.093 \times 10^{-6}$		7(5, 3) → 6(3, 4)	$3.978 \times 10^{-4}$	
7(5, 3) → 7(1, 6)	$3.192 \times 10^{-6}$		7(5, 3) → 8(1, 8)	$2.079 \times 10^{-9}$		7(5, 3) → 6(5, 2)	$1.024 \times 10^{-3}$	
7(5, 3) → 7(3, 4)	$2.205 \times 10^{-5}$		7(5, 2) → 6(1, 5)	$2.142 \times 10^{-5}$		7(5, 2) → 7(1, 7)	$8.708 \times 10^{-8}$	
7(5, 2) → 6(3, 3)	$6.463 \times 10^{-4}$		7(5, 2) → 6(5, 1)	$1.138 \times 10^{-3}$		7(5, 2) → 7(3, 5)	$1.388 \times 10^{-5}$	
7(5, 2) → 8(1, 7)	$7.628 \times 10^{-9}$		7(5, 2) → 7(5, 3)	$9.674 \times 10^{-8}$		8(3, 6) → 7(1, 7)	$1.244 \times 10^{-4}$	
8(3, 6) → 7(3, 5)	$1.649 \times 10^{-3}$		8(3, 6) → 8(1, 7)	$7.592 \times 10^{-5}$		8(3, 6) → 9(1, 9)	$2.244 \times 10^{-8}$	
8(3, 6) → 7(5, 3)	$2.314 \times 10^{-8}$		9(1, 8) → 8(1, 7)	$1.840 \times 10^{-3}$		9(1, 8) → 9(1, 9)	$8.831 \times 10^{-5}$	
8(3, 5) → 7(1, 6)	$3.172 \times 10^{-4}$		8(3, 5) → 8(1, 8)	$9.671 \times 10^{-7}$		8(3, 5) → 7(3, 4)	$2.214 \times 10^{-3}$	
8(3, 5) → 7(5, 2)	$7.796 \times 10^{-6}$		8(3, 5) → 8(3, 6)	$5.098 \times 10^{-5}$		8(3, 5) → 9(1, 8)	$1.332 \times 10^{-9}$	
10(1,10) → 9(1, 9)	$2.012 \times 10^{-3}$		7(7, 1) → 6(1, 6)	$3.640 \times 10^{-9}$		7(7, 1) → 6(3, 4)	$1.910 \times 10^{-6}$	
7(7, 1) → 7(1, 6)	$2.136 \times 10^{-8}$		7(7, 1) → 8(1, 8)	$1.698 \times 10^{-11}$		7(7, 1) → 6(5, 2)	$2.268 \times 10^{-4}$	
7(7, 1) → 7(3, 4)	$9.831 \times 10^{-7}$		7(7, 1) → 7(5, 2)	$7.964 \times 10^{-6}$		7(7, 1) → 8(3, 6)	$2.473 \times 10^{-9}$	
7(7, 0) → 6(1, 5)	$1.342 \times 10^{-7}$		7(7, 0) → 7(1, 7)	$5.939 \times 10^{-10}$		7(7, 0) → 6(3, 3)	$4.792 \times 10^{-6}$	
7(7, 0) → 6(5, 1)	$2.251 \times 10^{-4}$		7(7, 0) → 7(3, 5)	$2.700 \times 10^{-7}$		7(7, 0) → 8(1, 7)	$4.275 \times 10^{-10}$	
7(7, 0) → 7(5, 3)	$8.660 \times 10^{-6}$		7(7, 0) → 8(3, 5)	$1.522 \times 10^{-9}$		8(5, 4) → 7(1, 7)	$1.023 \times 10^{-6}$	
8(5, 4) → 7(3, 5)	$5.404 \times 10^{-4}$		8(5, 4) → 8(1, 7)	$3.616 \times 10^{-6}$		8(5, 4) → 9(1, 9)	$2.083 \times 10^{-9}$	
8(5, 4) → 7(5, 3)	$1.876 \times 10^{-3}$		8(5, 4) → 8(3, 5)	$3.686 \times 10^{-5}$		8(5, 4) → 7(7, 1)	$6.608 \times 10^{-9}$	
8(5, 3) → 7(1, 6)	$1.384 \times 10^{-5}$		8(5, 3) → 8(1, 8)	$4.594 \times 10^{-8}$		8(5, 3) → 7(3, 4)	$9.875 \times 10^{-4}$	
8(5, 3) → 7(5, 2)	$2.390 \times 10^{-3}$		8(5, 3) → 8(3, 6)	$1.210 \times 10^{-5}$		8(5, 3) → 9(1, 8)	$8.049 \times 10^{-9}$	
8(5, 3) → 7(7, 0)	$3.512 \times 10^{-8}$		8(5, 3) → 8(5, 4)	$1.448 \times 10^{-6}$		9(3, 7) → 8(1, 8)	$1.443 \times 10^{-4}$	
9(3, 7) → 8(3, 6)	$2.238 \times 10^{-3}$		9(3, 7) → 9(1, 8)	$1.078 \times 10^{-4}$		9(3, 7) → 10(1,10)	$3.216 \times 10^{-8}$	
9(3, 7) → 8(5, 4)	$9.861 \times 10^{-9}$		10(1, 9) → 9(1, 8)	$2.458 \times 10^{-3}$		10(1, 9) → 10(1,10)	$1.126 \times 10^{-4}$	
11(1,11) → 10(1,10)	$2.664 \times 10^{-3}$		9(3, 6) → 8(1, 7)	$3.788 \times 10^{-4}$		9(3, 6) → 9(1, 9)	$9.924 \times 10^{-7}$	
9(3, 6) → 8(3, 5)	$2.778 \times 10^{-3}$		9(3, 6) → 8(5, 3)	$4.698 \times 10^{-6}$		9(3, 6) → 9(3, 7)	$8.670 \times 10^{-5}$	
9(3, 6) → 10(1, 9)	$8.612 \times 10^{-9}$		8(7, 2) → 7(1, 7)	$8.928 \times 10^{-9}$		8(7, 2) → 7(3, 5)	$6.688 \times 10^{-6}$	
8(7, 2) → 8(1, 7)	$5.863 \times 10^{-8}$		8(7, 2) → 9(1, 9)	$3.926 \times 10^{-11}$		8(7, 2) → 7(5, 3)	$5.264 \times 10^{-4}$	
8(7, 2) → 8(3, 5)	$4.679 \times 10^{-6}$		8(7, 2) → 7(7, 1)	$7.284 \times 10^{-4}$		8(7, 2) → 8(5, 3)	$1.826 \times 10^{-5}$	
8(7, 2) → 9(3, 7)	$6.677 \times 10^{-9}$		8(7, 1) → 7(1, 6)	$3.958 \times 10^{-7}$		8(7, 1) → 8(1, 8)	$1.393 \times 10^{-9}$	
8(7, 1) → 7(3, 4)	$3.236 \times 10^{-5}$		8(7, 1) → 7(5, 2)	$5.185 \times 10^{-4}$		8(7, 1) → 8(3, 6)	$8.468 \times 10^{-7}$	
8(7, 1) → 9(1, 8)	$1.097 \times 10^{-9}$		8(7, 1) → 7(7, 0)	$7.301 \times 10^{-4}$		8(7, 1) → 8(5, 4)	$2.168 \times 10^{-5}$	
8(7, 1) → 9(3, 6)	$1.950 \times 10^{-9}$		8(7, 1) → 8(7, 2)	$2.457 \times 10^{-12}$		9(5, 5) → 8(1, 8)	$8.843 \times 10^{-7}$	
9(5, 5) → 8(3, 6)	$6.649 \times 10^{-4}$		9(5, 5) → 9(1, 8)	$3.769 \times 10^{-6}$		9(5, 5) → 10(1,10)	$1.954 \times 10^{-9}$	
9(5, 5) → 8(5, 4)	$2.876 \times 10^{-3}$		9(5, 5) → 9(3, 6)	$5.973 \times 10^{-5}$		9(5, 5) → 8(7, 2)	$1.028 \times 10^{-7}$	
10(3, 8) → 9(1, 9)	$1.643 \times 10^{-4}$		10(3, 8) → 9(3, 7)	$2.945 \times 10^{-3}$		10(3, 8) → 10(1, 9)	$1.456 \times 10^{-4}$	

Table 3: Continued.

10(3, 8) $\rightarrow$ 11(1,11) $4.269 \times 10^{-8}$	10(3, 8) $\rightarrow$ 9(5, 5) $1.147 \times 10^{-9}$	9(5, 4) $\rightarrow$ 8(1, 7) $7.853 \times 10^{-6}$
9(5, 4) $\rightarrow$ 9(1, 9) $2.218 \times 10^{-8}$	9(5, 4) $\rightarrow$ 8(3, 5) $1.091 \times 10^{-3}$	9(5, 4) $\rightarrow$ 8(5, 3) $4.103 \times 10^{-3}$
9(5, 4) $\rightarrow$ 9(3, 7) $9.715 \times 10^{-6}$	9(5, 4) $\rightarrow$ 10(1, 9) $7.308 \times 10^{-9}$	9(5, 4) $\rightarrow$ 8(7, 1) $1.025 \times 10^{-6}$
9(5, 4) $\rightarrow$ 9(5, 5) $8.376 \times 10^{-6}$	11(1,10) $\rightarrow$ 10(1, 9) $3.201 \times 10^{-3}$	11(1,10) $\rightarrow$ 11(1,11) $1.398 \times 10^{-4}$
	For $\text{C}_3\text{D}$	
1(1, 0) $\rightarrow$ 1(1, 1) $4.264 \times 10^{-8}$	2(1, 2) $\rightarrow$ 1(1, 1) $9.976 \times 10^{-6}$	2(1, 1) $\rightarrow$ 1(1, 0) $2.062 \times 10^{-5}$
2(1, 1) $\rightarrow$ 2(1, 2) $3.838 \times 10^{-7}$	3(1, 3) $\rightarrow$ 2(1, 2) $4.014 \times 10^{-5}$	3(1, 2) $\rightarrow$ 2(1, 1) $8.153 \times 10^{-5}$
3(1, 2) $\rightarrow$ 3(1, 3) $1.552 \times 10^{-6}$	4(1, 4) $\rightarrow$ 3(1, 3) $9.761 \times 10^{-5}$	3(3, 1) $\rightarrow$ 2(1, 2) $8.966 \times 10^{-6}$
3(3, 1) $\rightarrow$ 3(1, 2) $1.938 \times 10^{-6}$	3(3, 1) $\rightarrow$ 4(1, 4) $4.254 \times 10^{-9}$	3(3, 0) $\rightarrow$ 2(1, 1) $1.354 \times 10^{-5}$
3(3, 0) $\rightarrow$ 3(1, 3) $1.281 \times 10^{-6}$	3(3, 0) $\rightarrow$ 3(3, 1) $3.744 \times 10^{-11}$	4(1, 3) $\rightarrow$ 3(1, 2) $1.843 \times 10^{-4}$
4(1, 3) $\rightarrow$ 4(1, 4) $4.359 \times 10^{-6}$	4(1, 3) $\rightarrow$ 3(3, 0) $9.545 \times 10^{-9}$	5(1, 5) $\rightarrow$ 4(1, 4) $1.890 \times 10^{-4}$
4(3, 2) $\rightarrow$ 3(1, 3) $2.148 \times 10^{-5}$	4(3, 2) $\rightarrow$ 3(3, 1) $8.422 \times 10^{-5}$	4(3, 2) $\rightarrow$ 4(1, 3) $5.159 \times 10^{-6}$
4(3, 2) $\rightarrow$ 5(1, 5) $5.430 \times 10^{-9}$	4(3, 1) $\rightarrow$ 3(1, 2) $4.713 \times 10^{-5}$	4(3, 1) $\rightarrow$ 4(1, 4) $2.374 \times 10^{-6}$
4(3, 1) $\rightarrow$ 3(3, 0) $9.138 \times 10^{-5}$	4(3, 1) $\rightarrow$ 4(3, 2) $5.569 \times 10^{-9}$	5(1, 4) $\rightarrow$ 4(1, 3) $3.156 \times 10^{-4}$
5(1, 4) $\rightarrow$ 5(1, 5) $9.474 \times 10^{-6}$	5(1, 4) $\rightarrow$ 4(3, 1) $1.528 \times 10^{-7}$	6(1, 6) $\rightarrow$ 5(1, 5) $3.219 \times 10^{-4}$
5(3, 3) $\rightarrow$ 4(1, 4) $3.520 \times 10^{-5}$	5(3, 3) $\rightarrow$ 4(3, 2) $2.426 \times 10^{-4}$	5(3, 3) $\rightarrow$ 5(1, 4) $9.727 \times 10^{-6}$
5(3, 3) $\rightarrow$ 6(1, 6) $7.749 \times 10^{-9}$	5(3, 2) $\rightarrow$ 4(1, 3) $1.017 \times 10^{-4}$	5(3, 2) $\rightarrow$ 5(1, 5) $2.611 \times 10^{-6}$
5(3, 2) $\rightarrow$ 4(3, 1) $2.935 \times 10^{-4}$	5(3, 2) $\rightarrow$ 5(3, 3) $1.266 \times 10^{-7}$	6(1, 5) $\rightarrow$ 5(1, 4) $4.788 \times 10^{-4}$
6(1, 5) $\rightarrow$ 6(1, 6) $1.673 \times 10^{-5}$	6(1, 5) $\rightarrow$ 5(3, 2) $2.448 \times 10^{-7}$	7(1, 7) $\rightarrow$ 6(1, 6) $5.042 \times 10^{-4}$
6(3, 4) $\rightarrow$ 5(1, 5) $4.835 \times 10^{-5}$	6(3, 4) $\rightarrow$ 5(3, 3) $4.810 \times 10^{-4}$	6(3, 4) $\rightarrow$ 6(1, 5) $1.613 \times 10^{-5}$
6(3, 4) $\rightarrow$ 7(1, 7) $1.275 \times 10^{-8}$	6(3, 3) $\rightarrow$ 5(1, 4) $1.465 \times 10^{-4}$	6(3, 3) $\rightarrow$ 6(1, 6) $2.258 \times 10^{-6}$
6(3, 3) $\rightarrow$ 5(3, 2) $6.547 \times 10^{-4}$	6(3, 3) $\rightarrow$ 6(3, 4) $9.829 \times 10^{-7}$	5(5, 1) $\rightarrow$ 4(1, 4) $1.829 \times 10^{-7}$
5(5, 1) $\rightarrow$ 4(3, 2) $4.033 \times 10^{-5}$	5(5, 1) $\rightarrow$ 5(1, 4) $4.054 \times 10^{-7}$	5(5, 1) $\rightarrow$ 6(1, 6) $1.796 \times 10^{-9}$
5(5, 1) $\rightarrow$ 5(3, 2) $3.557 \times 10^{-6}$	5(5, 1) $\rightarrow$ 6(3, 4) $4.170 \times 10^{-9}$	5(5, 0) $\rightarrow$ 4(1, 3) $1.142 \times 10^{-6}$
5(5, 0) $\rightarrow$ 5(1, 5) $4.823 \times 10^{-8}$	5(5, 0) $\rightarrow$ 4(3, 1) $3.990 \times 10^{-5}$	5(5, 0) $\rightarrow$ 5(3, 3) $3.869 \times 10^{-6}$
5(5, 0) $\rightarrow$ 6(1, 5) $8.863 \times 10^{-9}$	5(5, 0) $\rightarrow$ 6(3, 3) $1.641 \times 10^{-10}$	7(1, 6) $\rightarrow$ 6(1, 5) $6.946 \times 10^{-4}$
7(1, 6) $\rightarrow$ 7(1, 7) $2.567 \times 10^{-5}$	7(1, 6) $\rightarrow$ 6(3, 3) $1.067 \times 10^{-7}$	8(1, 8) $\rightarrow$ 7(1, 7) $7.442 \times 10^{-4}$
7(3, 5) $\rightarrow$ 6(1, 6) $6.070 \times 10^{-5}$	7(3, 5) $\rightarrow$ 6(3, 4) $7.958 \times 10^{-4}$	7(3, 5) $\rightarrow$ 7(1, 6) $2.525 \times 10^{-5}$
7(3, 5) $\rightarrow$ 8(1, 8) $2.118 \times 10^{-8}$	6(5, 2) $\rightarrow$ 5(1, 5) $5.917 \times 10^{-7}$	6(5, 2) $\rightarrow$ 5(3, 3) $9.650 \times 10^{-5}$
6(5, 2) $\rightarrow$ 6(1, 5) $1.630 \times 10^{-6}$	6(5, 2) $\rightarrow$ 7(1, 7) $4.714 \times 10^{-9}$	6(5, 2) $\rightarrow$ 6(3, 3) $8.558 \times 10^{-6}$
6(5, 2) $\rightarrow$ 5(5, 1) $2.103 \times 10^{-4}$	6(5, 2) $\rightarrow$ 7(3, 5) $1.035 \times 10^{-9}$	6(5, 1) $\rightarrow$ 5(1, 4) $6.365 \times 10^{-6}$
6(5, 1) $\rightarrow$ 6(1, 6) $1.413 \times 10^{-7}$	6(5, 1) $\rightarrow$ 5(3, 2) $9.601 \times 10^{-5}$	6(5, 1) $\rightarrow$ 6(3, 4) $9.686 \times 10^{-6}$
6(5, 1) $\rightarrow$ 5(5, 0) $2.109 \times 10^{-4}$	6(5, 1) $\rightarrow$ 7(1, 6) $2.208 \times 10^{-8}$	6(5, 1) $\rightarrow$ 6(5, 2) $1.402 \times 10^{-12}$
7(3, 4) $\rightarrow$ 6(1, 5) $1.670 \times 10^{-4}$	7(3, 4) $\rightarrow$ 7(1, 7) $1.886 \times 10^{-6}$	7(3, 4) $\rightarrow$ 6(3, 3) $1.167 \times 10^{-3}$
7(3, 4) $\rightarrow$ 7(3, 5) $4.062 \times 10^{-6}$	7(3, 4) $\rightarrow$ 6(5, 1) $1.055 \times 10^{-8}$	8(1, 7) $\rightarrow$ 7(1, 6) $9.778 \times 10^{-4}$
8(1, 7) $\rightarrow$ 8(1, 8) $3.616 \times 10^{-5}$	8(1, 7) $\rightarrow$ 7(3, 4) $1.634 \times 10^{-8}$	9(1, 9) $\rightarrow$ 8(1, 8) $1.050 \times 10^{-3}$
8(3, 6) $\rightarrow$ 7(1, 7) $7.266 \times 10^{-5}$	8(3, 6) $\rightarrow$ 7(3, 5) $1.180 \times 10^{-3}$	8(3, 6) $\rightarrow$ 8(1, 7) $3.782 \times 10^{-5}$
8(3, 6) $\rightarrow$ 9(1, 9) $3.312 \times 10^{-8}$	7(5, 3) $\rightarrow$ 6(1, 6) $1.051 \times 10^{-6}$	7(5, 3) $\rightarrow$ 6(3, 4) $1.701 \times 10^{-4}$
7(5, 3) $\rightarrow$ 7(1, 6) $3.332 \times 10^{-6}$	7(5, 3) $\rightarrow$ 8(1, 8) $7.366 \times 10^{-9}$	7(5, 3) $\rightarrow$ 6(5, 2) $5.515 \times 10^{-4}$
7(5, 3) $\rightarrow$ 7(3, 4) $1.466 \times 10^{-5}$	7(5, 3) $\rightarrow$ 8(3, 6) $1.521 \times 10^{-12}$	7(5, 2) $\rightarrow$ 6(1, 5) $1.588 \times 10^{-5}$
7(5, 2) $\rightarrow$ 7(1, 7) $2.273 \times 10^{-7}$	7(5, 2) $\rightarrow$ 6(3, 3) $1.831 \times 10^{-4}$	7(5, 2) $\rightarrow$ 7(3, 5) $1.609 \times 10^{-5}$
7(5, 2) $\rightarrow$ 6(5, 1) $5.593 \times 10^{-4}$	7(5, 2) $\rightarrow$ 8(1, 7) $3.362 \times 10^{-8}$	7(5, 2) $\rightarrow$ 7(5, 3) $1.764 \times 10^{-10}$
8(3, 5) $\rightarrow$ 7(1, 6) $1.834 \times 10^{-4}$	8(3, 5) $\rightarrow$ 8(1, 8) $1.700 \times 10^{-6}$	8(3, 5) $\rightarrow$ 7(3, 4) $1.757 \times 10^{-3}$
8(3, 5) $\rightarrow$ 8(3, 6) $1.171 \times 10^{-5}$	8(3, 5) $\rightarrow$ 7(5, 2) $3.631 \times 10^{-7}$	9(1, 8) $\rightarrow$ 8(1, 7) $1.337 \times 10^{-3}$
9(1, 8) $\rightarrow$ 9(1, 9) $4.827 \times 10^{-5}$	9(1, 8) $\rightarrow$ 8(3, 5) $1.661 \times 10^{-10}$	10(1,10) $\rightarrow$ 9(1, 9) $1.430 \times 10^{-3}$
8(5, 4) $\rightarrow$ 7(1, 7) $1.358 \times 10^{-6}$	8(5, 4) $\rightarrow$ 7(3, 5) $2.568 \times 10^{-4}$	8(5, 4) $\rightarrow$ 8(1, 7) $4.865 \times 10^{-6}$
8(5, 4) $\rightarrow$ 9(1, 9) $8.978 \times 10^{-9}$	8(5, 4) $\rightarrow$ 7(5, 3) $1.048 \times 10^{-3}$	8(5, 4) $\rightarrow$ 8(3, 5) $2.229 \times 10^{-5}$
8(5, 3) $\rightarrow$ 7(1, 6) $2.353 \times 10^{-5}$	8(5, 3) $\rightarrow$ 8(1, 8) $2.532 \times 10^{-7}$	8(5, 3) $\rightarrow$ 7(3, 4) $3.313 \times 10^{-4}$
8(5, 3) $\rightarrow$ 8(3, 6) $2.125 \times 10^{-5}$	8(5, 3) $\rightarrow$ 7(5, 2) $1.095 \times 10^{-3}$	8(5, 3) $\rightarrow$ 9(1, 8) $4.186 \times 10^{-8}$
8(5, 3) $\rightarrow$ 8(5, 4) $7.366 \times 10^{-9}$	9(3, 7) $\rightarrow$ 8(1, 8) $8.458 \times 10^{-5}$	9(3, 7) $\rightarrow$ 8(3, 6) $1.633 \times 10^{-3}$
9(3, 7) $\rightarrow$ 9(1, 8) $5.417 \times 10^{-5}$	9(3, 7) $\rightarrow$ 10(1,10) $4.809 \times 10^{-8}$	9(3, 7) $\rightarrow$ 8(5, 4) $1.112 \times 10^{-10}$
7(7, 1) $\rightarrow$ 6(1, 6) $1.521 \times 10^{-9}$	7(7, 1) $\rightarrow$ 6(3, 4) $3.832 \times 10^{-7}$	7(7, 1) $\rightarrow$ 7(1, 6) $1.033 \times 10^{-8}$
7(7, 1) $\rightarrow$ 8(1, 8) $2.834 \times 10^{-11}$	7(7, 1) $\rightarrow$ 6(5, 2) $8.913 \times 10^{-5}$	7(7, 1) $\rightarrow$ 7(3, 4) $1.494 \times 10^{-7}$
7(7, 1) $\rightarrow$ 8(3, 6) $3.646 \times 10^{-9}$	7(7, 1) $\rightarrow$ 7(5, 2) $6.289 \times 10^{-6}$	7(7, 1) $\rightarrow$ 8(5, 4) $1.153 \times 10^{-8}$
7(7, 0) $\rightarrow$ 6(1, 5) $2.965 \times 10^{-8}$	7(7, 0) $\rightarrow$ 7(1, 7) $4.738 \times 10^{-10}$	7(7, 0) $\rightarrow$ 6(3, 3) $4.595 \times 10^{-7}$
7(7, 0) $\rightarrow$ 7(3, 5) $1.019 \times 10^{-7}$	7(7, 0) $\rightarrow$ 6(5, 1) $8.905 \times 10^{-5}$	7(7, 0) $\rightarrow$ 8(1, 7) $5.469 \times 10^{-10}$
7(7, 0) $\rightarrow$ 7(5, 3) $6.345 \times 10^{-6}$	7(7, 0) $\rightarrow$ 8(3, 5) $5.131 \times 10^{-9}$	7(7, 0) $\rightarrow$ 8(5, 3) $9.803 \times 10^{-9}$
10(1, 9) $\rightarrow$ 9(1, 8) $1.778 \times 10^{-3}$	10(1, 9) $\rightarrow$ 10(1,10) $6.203 \times 10^{-5}$	9(3, 6) $\rightarrow$ 8(1, 7) $2.104 \times 10^{-4}$
9(3, 6) $\rightarrow$ 9(1, 9) $1.678 \times 10^{-6}$	9(3, 6) $\rightarrow$ 8(3, 5) $2.334 \times 10^{-3}$	9(3, 6) $\rightarrow$ 8(5, 3) $2.196 \times 10^{-6}$
9(3, 6) $\rightarrow$ 9(3, 7) $2.658 \times 10^{-5}$	9(3, 6) $\rightarrow$ 10(1, 9) $1.320 \times 10^{-9}$	11(1,11) $\rightarrow$ 10(1,10) $1.892 \times 10^{-3}$
9(5, 5) $\rightarrow$ 8(1, 8) $1.450 \times 10^{-6}$	9(5, 5) $\rightarrow$ 8(3, 6) $3.469 \times 10^{-4}$	9(5, 5) $\rightarrow$ 9(1, 8) $5.905 \times 10^{-6}$
9(5, 5) $\rightarrow$ 10(1,10) $9.524 \times 10^{-9}$	9(5, 5) $\rightarrow$ 8(5, 4) $1.713 \times 10^{-3}$	9(5, 5) $\rightarrow$ 9(3, 6) $3.200 \times 10^{-5}$
10(3, 8) $\rightarrow$ 9(1, 9) $9.661 \times 10^{-5}$	10(3, 8) $\rightarrow$ 9(3, 7) $2.161 \times 10^{-3}$	10(3, 8) $\rightarrow$ 10(1, 9) $7.431 \times 10^{-5}$
10(3, 8) $\rightarrow$ 11(1,11) $6.532 \times 10^{-8}$	10(3, 8) $\rightarrow$ 9(5, 5) $1.606 \times 10^{-10}$	9(5, 4) $\rightarrow$ 8(1, 7) $2.363 \times 10^{-5}$
9(5, 4) $\rightarrow$ 9(1, 9) $2.085 \times 10^{-7}$	9(5, 4) $\rightarrow$ 8(3, 5) $5.526 \times 10^{-4}$	9(5, 4) $\rightarrow$ 8(5, 3) $1.898 \times 10^{-3}$

Table 3: Continued.

$9(5, 4) \rightarrow 9(3, 7)$	$2.306 \times 10^{-5}$	$9(5, 4) \rightarrow 10(1, 9)$	$4.519 \times 10^{-8}$	$9(5, 4) \rightarrow 9(5, 5)$	$1.298 \times 10^{-7}$
$8(7, 2) \rightarrow 7(1, 7)$	$6.134 \times 10^{-9}$	$8(7, 2) \rightarrow 7(3, 5)$	$1.786 \times 10^{-6}$	$8(7, 2) \rightarrow 8(1, 7)$	$4.599 \times 10^{-8}$
$8(7, 2) \rightarrow 9(1, 9)$	$1.017 \times 10^{-10}$	$8(7, 2) \rightarrow 7(5, 3)$	$2.005 \times 10^{-4}$	$8(7, 2) \rightarrow 8(3, 5)$	$9.681 \times 10^{-7}$
$8(7, 2) \rightarrow 8(5, 3)$	$1.605 \times 10^{-5}$	$8(7, 2) \rightarrow 9(3, 7)$	$1.368 \times 10^{-8}$	$8(7, 2) \rightarrow 7(7, 1)$	$3.870 \times 10^{-4}$
$8(7, 2) \rightarrow 9(5, 5)$	$7.633 \times 10^{-9}$	$8(7, 1) \rightarrow 7(1, 6)$	$1.536 \times 10^{-7}$	$8(7, 1) \rightarrow 8(1, 8)$	$1.808 \times 10^{-9}$
$8(7, 1) \rightarrow 7(3, 4)$	$2.880 \times 10^{-6}$	$8(7, 1) \rightarrow 8(3, 6)$	$4.585 \times 10^{-7}$	$8(7, 1) \rightarrow 7(5, 2)$	$1.996 \times 10^{-4}$
$8(7, 1) \rightarrow 9(1, 8)$	$2.118 \times 10^{-9}$	$8(7, 1) \rightarrow 8(5, 4)$	$1.659 \times 10^{-5}$	$8(7, 1) \rightarrow 7(7, 0)$	$3.870 \times 10^{-4}$
$8(7, 1) \rightarrow 9(3, 6)$	$2.109 \times 10^{-8}$	$8(7, 1) \rightarrow 9(5, 4)$	$2.957 \times 10^{-9}$	$11(1,10) \rightarrow 10(1, 9)$	$2.308 \times 10^{-3}$
$11(1,10) \rightarrow 11(1,11)$	$7.748 \times 10^{-5}$				