

Physical models of ten asteroids from an observers' collaboration network[★]

J. Āurech^{1,2}, M. Kaasalainen², A. Marciniak³, W. H. Allen²³, R. Behrend²², C. Bembrick⁴, T. Bennett¹¹, L. Bernasconi⁵, J. Berthier²⁵, G. Bolt⁶, S. Boroumand²⁸, L. Crespo da Silva²⁸, R. Crippa²¹, M. Crow⁷, R. Durkee⁸, R. Dymock⁹, M. Fagas³, M. Fauerbach¹¹, S. Fauvaud^{10,29}, M. Frey¹², R. Gonalves²⁰, R. Hirsch³, D. Jardine¹⁵, K. Kamiński³, R. Koff¹³, T. Kwiatkowski³, A. Lopez¹⁴, F. Manzini²¹, T. Michałowski³, R. Pacheco¹⁴, M. Pan²⁸, F. Pilcher¹⁵, R. Poncy¹⁹, D. Pray¹⁶, W. Pych²⁴, R. Roy¹⁷, G. Santacana¹⁰, S. Slivan^{12,28}, S. Sposetti²⁷, R. Stephens¹⁸, B. Warner²⁶, and M. Wolf¹

(Affiliations can be found after the references)

Received 5 September 2006 / Accepted 14 December 2006

ABSTRACT

Aims. We present physical models of ten asteroids obtained by means of lightcurve inversion. A substantial part of the photometric data was observed by amateur astronomers. We emphasize the importance of a coordinated network of observers that will be of extreme importance for future all-sky asteroid photometric surveys.

Methods. The lightcurve inversion method was used to derive spin states and shape models of the asteroids.

Results. We derived spin states and shape model for ten new asteroids: (110) Lydia, (125) Liberatrix, (130) Elektra, (165) Loreley, (196) Philomela, (218) Bianca, (306) Unitas, (423) Diotima, (776) Berbericia, and (944) Hidalgo. This increases the number of asteroid models up to nearly one hundred.

Key words. minor planets, asteroids

1. Introduction

The lightcurve inversion method has become a standard tool for asteroid shape and spin state determination (Kaasalainen & Torppa 2001; Kaasalainen et al. 2001, 2002a). Convex models are a good representation of real shapes of asteroids, as has been proven by ground truths from, e.g., Kaasalainen et al. (2001, 2005), and Marchis et al. (2006). Slightly fewer than one hundred asteroid models have been derived so far (Kaasalainen et al. 2002c, 2004; Torppa et al. 2003). However, the number of asteroid models increases slowly, mainly due to the fact that at least three well-covered apparitions are necessary for a main-belt asteroid to be modeled. The Uppsala Asteroid Photometric Catalogue (UAPC, Lagerkvist et al. 2001) – the main source of asteroid photometric data – has been already exploited and the global morphology of all well-observed asteroids has been determined. The UAPC still contains valuable photometric data of many asteroids, but the amount of the data is not sufficient for a unique physical model. For many such targets, observations from only one more apparition are sufficient for a model, and many of those targets are within the reach of amateur astronomers. We present new observations and physical models of asteroids (110) Lydia, (125) Liberatrix, (130) Elektra, (165) Loreley, (196) Philomela, (218) Bianca, (306) Unitas, (423) Diotima, (776) Berbericia, and (944) Hidalgo. In the last section, we discuss the possibility of combining the ordinary lightcurves with the sparse photometric data that will be available from all-sky photometric surveys in the near future.

2. Observations

To derive unique spin state solutions and shape models, we combined photometric data published in the UAPC with the new observations that were carried out by a large number of both amateur and professional observers. Some lightcurves from the UAPC that were too noisy, consisted of only a few points, or were clearly wrong were not included in the analysis. Information about observers and telescopes is listed in Table 1. All the new observations are listed in Table 2. For each lightcurve, there is the date of observation, the aspect data, the asteroid's ecliptic coordinates, and the code number of the observatory referring to Table 1. All photometric observations were treated as relative and we used a combination of the Lommel-Seeliger and Lambert light-scattering laws (Kaasalainen et al. 2002a) as our scattering model.

We do not present all the lightcurves in a graphical form, but only a selection of three representative lightcurves for each asteroid. Most of the lightcurves from Table 2 can be found at the Collaborative Asteroid Lightcurve Link (<http://www.MinorPlanetObserver.com/astlc/default.htm>) or at http://obswww.unige.ch/~behrend/page_cou.html. The complete lightcurve datasets that were used for deriving shape models in the following section can be downloaded from <http://cdsweb.u-strasbg.fr/cgi-bin/qcat?J/A+A/465/331>.

3. Models

The spin solutions and shape models were derived using the lightcurve inversion method developed by Kaasalainen & Torppa (2001), and Kaasalainen et al. (2001). The spin axis

[★] Tables 1–3 are only available in electronic form at <http://www.aanda.org>



Fig. 1. The shape model of (110) Lydia. There are two equatorial views 90° apart (the first two figures) and a pole-on view (the third plot).

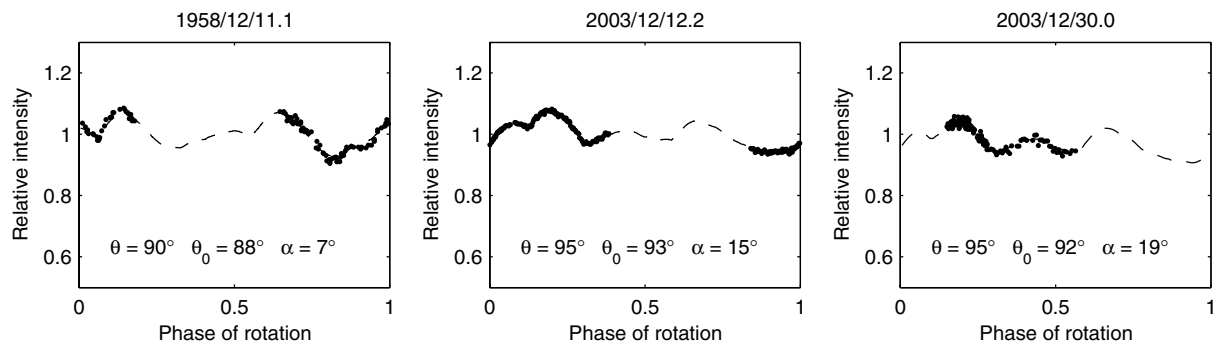


Fig. 2. Observed data (points) and the modeled brightness (dashed curve) for three representative lightcurves of (110) Lydia. The plots cover one rotation cycle, the brightness is given in relative intensity units. The viewing/illumination geometry is given by the aspect angle of the Earth θ and of the Sun θ_0 and by the solar phase angle α .

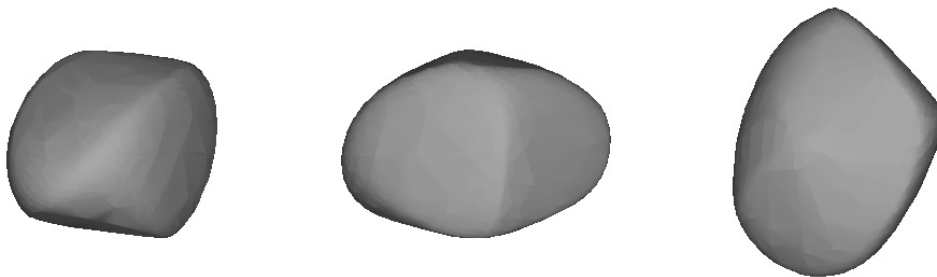


Fig. 3. The shape model of (125) Liberatrix. The viewing geometry is the same as in Fig. 1.

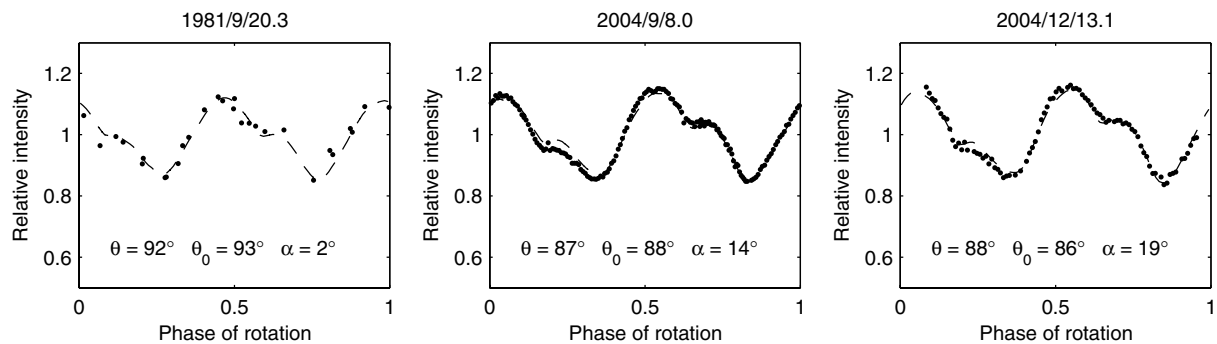


Fig. 4. Lightcurves and the corresponding fits for (125) Liberatrix.

direction in J2000 ecliptic coordinates λ_p, β_p and the rotation period P for each asteroid are listed in Table 3. In the case of the lightcurve inversion, the systematic errors in lightcurves and model errors dominate over the observational noise. Thus formal errors for pole positions derived from confidence limits based on χ^2 distribution would be underestimated. A good estimation of a typical error in the pole direction obtained by comparison of lightcurve inversion results with ground truths from space probes and laboratory experiments (Kaasalainen et al. 2005) is about 5°

of arc. The accuracy of the period determination is of the order of the last unit digit of the period value given in Table 3. For more detailed discussion of error estimation see Torppa et al. (2003), and Kaasalainen & Ďurech (2007).

In Figs. 1 to 20, we plot the the shape model of each asteroid viewed from the plane of its equator (two views 90° apart) and pole-on, and the corresponding lightcurve fit. In some cases, there are two possible pole solutions with the ecliptic longitudes λ about 180° apart and with similar values



Fig. 5. The shape model of (130) Elektra. The viewing geometry is the same as in Fig. 1.

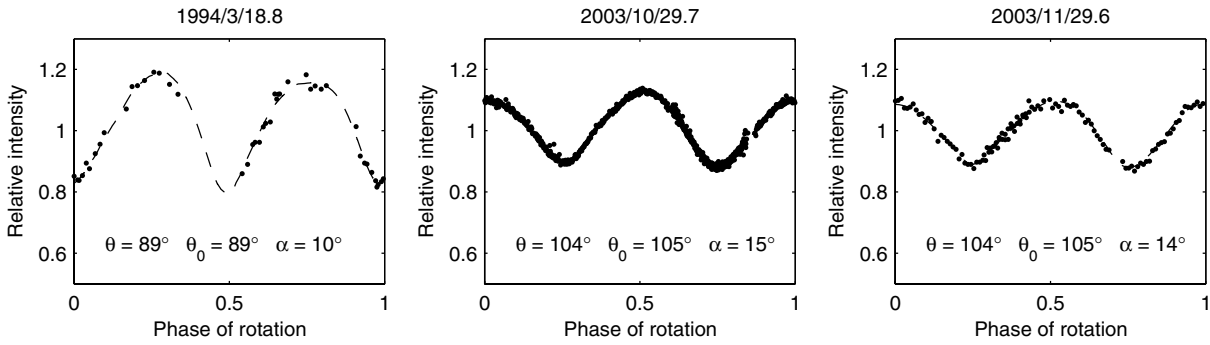


Fig. 6. Lightcurves and the corresponding fits for (130) Elektra.



Fig. 7. The shape model of (165) Loreley. The viewing geometry is the same as in Fig. 1.

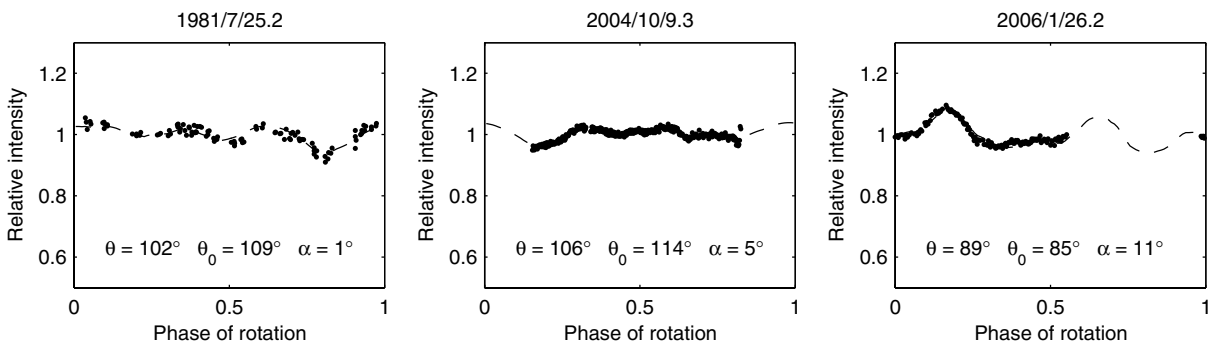


Fig. 8. Lightcurves and the corresponding fits for (165) Loreley.



Fig. 9. The shape model of (196) Philomela. The viewing geometry is the same as in Fig. 1.

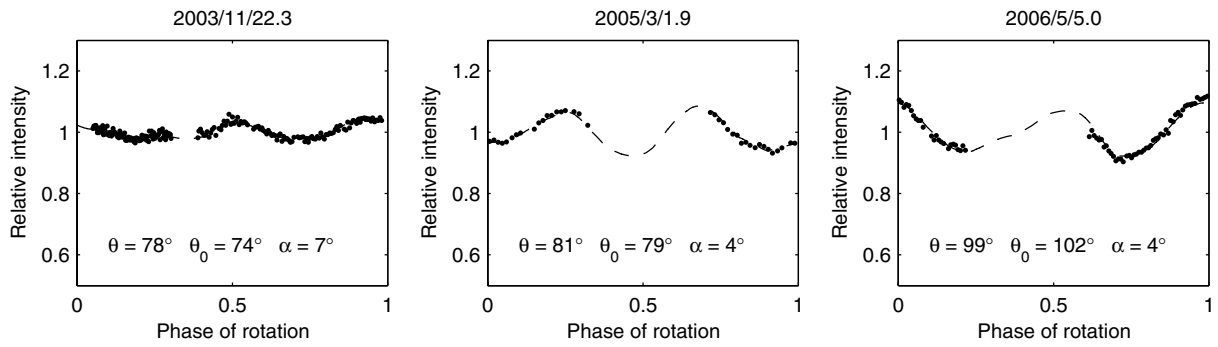


Fig. 10. Lightcurves and the corresponding fits for (196) Philomela.



Fig. 11. The shape model of (218) Bianca. The viewing geometry is the same as in Fig. 1.

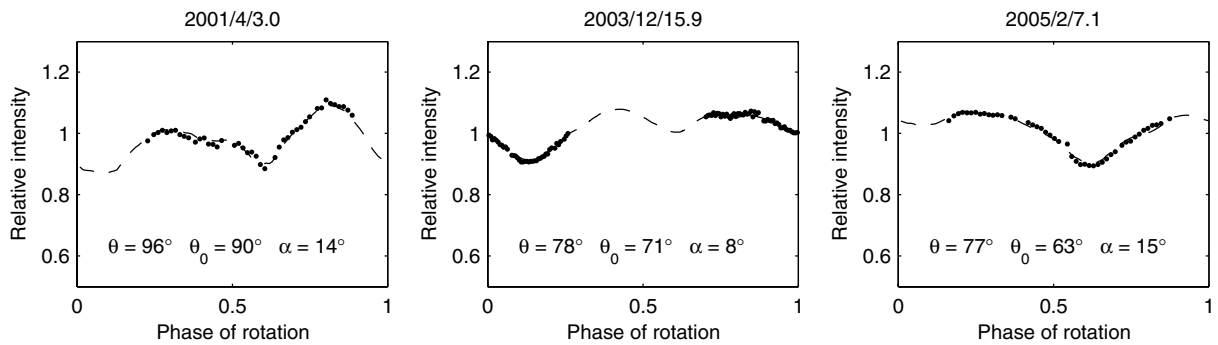


Fig. 12. Lightcurves and the corresponding fits for (218) Bianca.



Fig. 13. The shape model of (306) Unitas. The viewing geometry is the same as in Fig. 1.

of the pole ecliptic latitude β . This ambiguity is inevitable for disk-integrated measurements of objects orbiting near the plane of ecliptic (see Kaasalainen & Lamberg 2006). Due to the fact that we used only relative photometry, the dimensions along the rotation axis are not well constrained. The pole-on silhouettes are very good approximations of asteroids' real shapes, whereas the silhouettes viewed from the equatorial plane can be significantly stretched or squeezed along the rotation axis. The principal axis of the inertia tensor (assuming uniform density) corresponding to the maximum moment of inertia is very close to the rotation axis for every model. The

models together with the spin vector solutions are available at <http://astro.troja.mff.cuni.cz/projects/asteroids3D>.

(110) Lydia. Lightcurve amplitudes do not exceed 0.2 mag. The shape is flat with a regular pole-on silhouette. There are two pole solutions.

(125) Liberatrix. The rotation axis is almost perpendicular to the ecliptic plane and the orbit is close to the ecliptic (for most

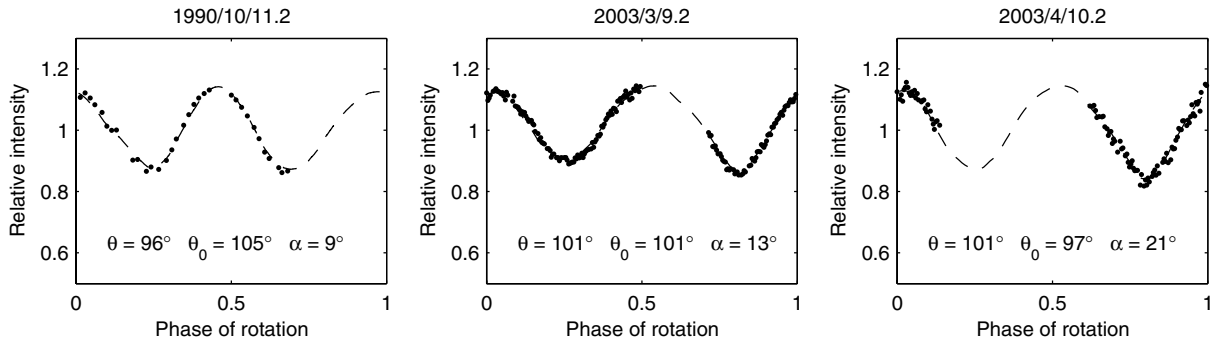


Fig. 14. Lightcurves and the corresponding fits for (306) Unitas.



Fig. 15. The shape model of (423) Diotima. The viewing geometry is the same as in Fig. 1.

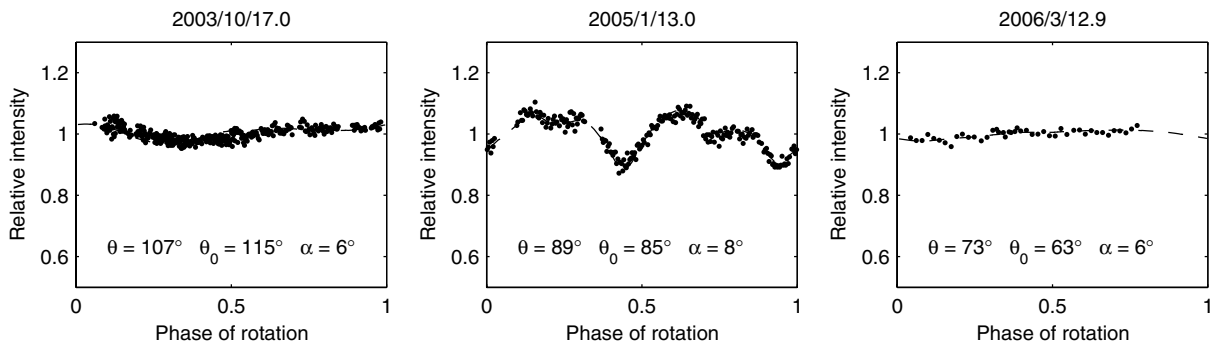


Fig. 16. Lightcurves and the corresponding fits for (423) Diotima.



Fig. 17. The shape model of (776) Berbericia. The viewing geometry is the same as in Fig. 1.

lightcurves $|\beta| \leq 5^\circ$). Liberatrix has been seen equator-on all the time and the lightcurves hardly change from one opposition to another, having the same amplitude 0.4 mag. Relative lightcurves and the restricted geometry do not allow us to constrain the dimension along the rotation axis accurately – the shape model can be more or less stretched along this axis and the lightcurve fits remain almost the same.

(130) Elektra. Lightcurves of Elektra are typical double sinusoidal, the shape model is regular and elongated. Although

the rotation axis is perpendicular to the plain of ecliptic, the viewing/illumination geometry is not restricted to the equatorial view/illumination (contrary to the previous case of Liberatrix), due to the high ecliptic latitude Elektra reached ($-35^\circ < \beta < 25^\circ$).

(165) Loreley. The shape model has many flat areas, the lightcurves have small amplitudes of 0.2 mag at most and a complicated structure. The pole direction solution ($346^\circ, +29^\circ$) is

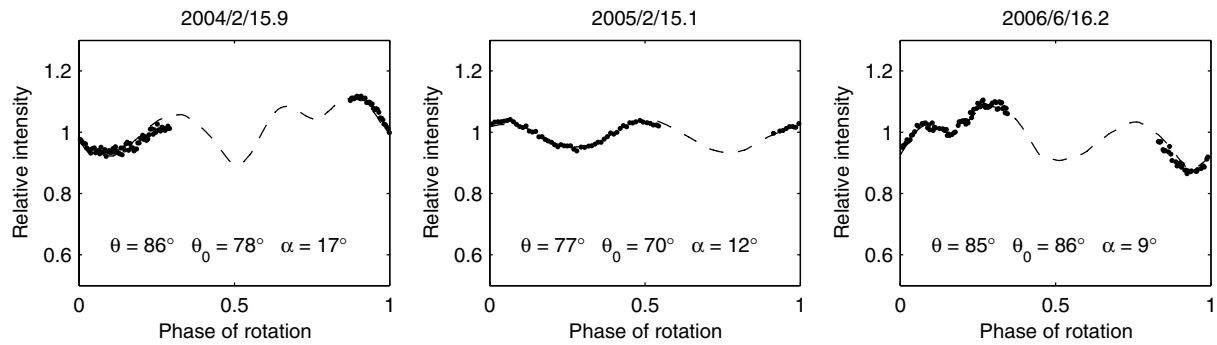


Fig. 18. Lightcurves and the corresponding fits for (776) Berbericia.

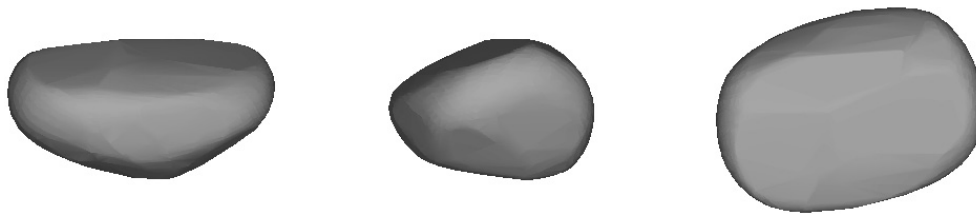


Fig. 19. The shape model of (944) Hidalgo. The viewing geometry is the same as in Fig. 1.

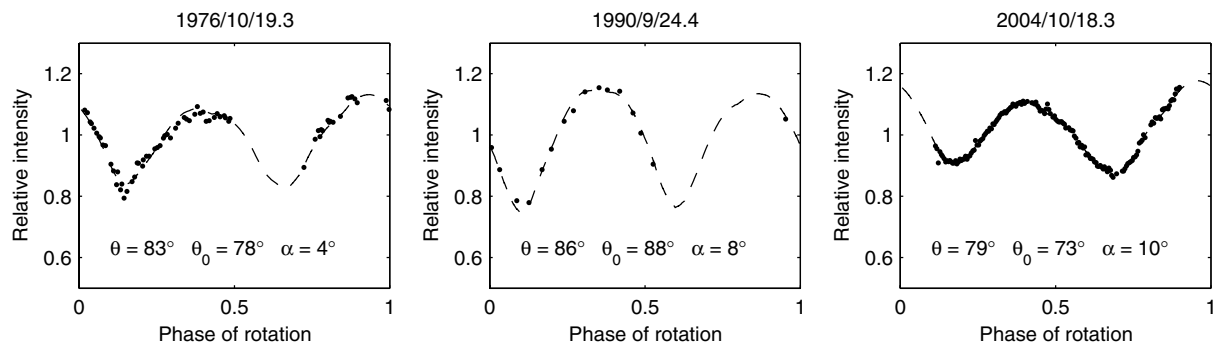


Fig. 20. Lightcurves and the corresponding fits for (944) Hidalgo.

clearly the best one, but there is the second solution ($165^\circ, +15^\circ$) giving only a slightly worse fit.

(196) Philomela. The shape model is asymmetric and smooth, the geometry varies a lot, lightcurves vary from almost flat to those with amplitudes up to 0.4 mag.

(218) Bianca. There are two pole solutions corresponding to almost the same spin axis with both a prograde and a retrograde sense of rotation. The shape is asymmetric.

(306) Uitas. The shape is regular, lightcurves typically exhibit two extrema per rotation.

(423) Diotima. The lightcurves vary a lot – some are almost flat and others exhibit 0.2 mag deep minima. From the photometric data we obtained two solutions for the pole direction: ($351^\circ, +4^\circ$) and ($176^\circ, +33^\circ$), but only the first one is consistent with the adaptive optics image obtained by Marchis et al. (2006).

(776) Berbericia. Lightcurves are very different for different apparitions – sometimes there is only one maximum per period. The corresponding shape model is asymmetric with sharp edges.

(944) Hidalgo. Although our model is based on only 14 lightcurves from four oppositions, the pole and period solution is unique. The shape model has very large flat areas and a “rectangular” pole-on silhouette, which are strong indications of a highly nonconvex shape (Kaasalainen et al. 2002b; Ďurech & Kaasalainen 2003). Also, the sharp minima of some lightcurves support the idea of a two-lobed shape.

4. Future work

The number of asteroid models available so far is very small when compared with the whole asteroid population. The classical approach of observing a selected target (or a few targets) during the night to densely cover the lightcurve in the rotation phase is time consuming. The number of asteroids with enough observations to derive a model increases only slowly. The situation is going to change in the near future with the asteroid photometric surveys (for example Pan-STARRS). It has been shown (see Kaasalainen 2004; Ďurech et al. 2005) that asteroid models can be derived from calibrated photometric measurements that are

sparse in time. This kind of data will be provided by future photometric surveys – instead of tens of lightcurves covering several apparitions we will have typically a hundred or more individual brightness measurements spread over several years. A difficulty that appears when analyzing sparse data is that the rotation period of an asteroid is not “visible” from the data as it is in the case of an ordinary well-covered lightcurve. Thus a very wide interval of all possible periods must be densely scanned for the correct value. The time-consuming process of period search can be sped up dramatically by adding just one ordinary lightcurve that constrains the search to a narrow interval of periods. A network of observers who will carry out photometric follow-up observations of selected asteroids will be very important for the next generation of all-sky surveys.

Acknowledgements. This work was supported, in part, by CIMO and the Academy of Finland. The observations carried out at the Borowiec Station were supported by Polish Grant 1 P03D 020 27.

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- ⁴ Mt Tarana Observatory, PO Box 1537, Bathurst, NSW 2795, Australia
- ⁵ Observatoire des Engarouines, 84570 Mallemort-du-Comtat, France
- ⁶ 295 Camberwarra Drive, Craigie, WA 6025, Australia
- ⁷ The WW Crow Observatory, 118 Mill Road, Hawley, Kent, DA2 7RT, England
- ⁸ Shed of Science Observatory, 5213 Washburn Ave. S., Minneapolis, MN 55410, USA
- ⁹ 67 Haslar Crescent, Waterlooville, Hampshire, PO7 6DD, England
- ¹⁰ Association AstroQueyras, Le Bois de Bardou, 16110 Taponnat, France
- ¹¹ Florida Gulf Coast University, 10501 FGCU Boulevard South, Fort Myers, FL 33965, USA
- ¹² Department of Astronomy, Whitin Observatory, Wellesley College, 106 Central Street, Wellesley, MA 02481, USA
- ¹³ Antelope Hills Observatory, 980 Antelope Drive West, Bennett, CO 80102, USA
- ¹⁴ Observatori Astronomic de Consell, Mallorca, Spain
- ¹⁵ Illinois College, 1101 West College Avenue, Jacksonville, IL 62650, USA
- ¹⁶ Caruncle Hill Observatory, PO Box 946, Coventry, RI 02816, USA
- ¹⁷ Observatoire de Blauvac, 84570 St-Estève, France
- ¹⁸ Goat Mountain Astronomical Research Station, 11355 Mount Johnson Court, Rancho Cucamonga, CA 91737, USA
- ¹⁹ 2 rue des Écoles 34920, Le Crès, France
- ²⁰ Linhaceira Observatory, Instituto Politécnico de Tomar, 2300-313, Tomar, Portugal
- ²¹ Stazione Astronomica di Sozzago, 28060 Sozzago, Italy
- ²² Geneva Observatory, 1290 Sauverny, Switzerland
- ²³ Vintage Lane Observatory, 83 Vintage Lane, RD3, Blenheim, New Zealand
- ²⁴ Copernicus Astronomical Center, Bartycka 18, 00-716 Warszawa, Poland
- ²⁵ Institut de mécanique céleste et de calcul des éphémérides, Observatoire de Paris, 77 Av. Denfert Rochereau, 75014 Paris, France
- ²⁶ Palmer Divide Observatory, 17995 Bakers Farm Rd., Colorado Springs, CO 80908, USA
- ²⁷ Observatorio di Gnosca, 6525 Gnosca, Switzerland
- ²⁸ Dept. of Earth, Atmospheric, and Planetary Sciences, Massachusetts Institute of Technology, 77 Massachusetts Ave., Cambridge, MA 02139, USA
- ²⁹ Groupe Européen d’Observations Stellaires (GEOS), 23 Parc de Levesville, 28300 Bailleau l’Évêque, France

¹ Astronomical Institute, Charles University in Prague V Holešovičkách 2, 18000 Prague, Czech Republic
e-mail: durech@sirrah.troja.mff.cuni.cz

² Department of Mathematics and Statistics, Rolf Nevanlinna Institute, University of Helsinki, PO Box 68, 00014 Helsinki, Finland

³ Astronomical Observatory, A. Mickiewicz University, Słoneczna 36, 60-286 Poznań, Poland

Online Material

Table 1. The list of observatories and telescopes, D is the telescope aperture diameter.

| Code | Observing site | D [cm] | Observers |
|------|---|----------|---|
| 1 | Blauvac Observatory, France | 31 | R. Roy |
| 2 | Borowiec Station, Poznań Observatory, Poland | 40 | A. Marciniak, R. Hirsch, K. Kamiński, M. Fagas, T. Michałowski, T. Kwiatkowski |
| 3 | Carbuncle Hill Observatory, Rhode Island, USA | 35 | D. Pray |
| 4 | Egan Observatory, Florida, USA | 40 | M. Fauerbach, T. Bennett |
| 5 | Observatori Astronomic de Consell, Mallorca, Spain | 40 | A. Lopez, R. Pacheco |
| 7 | Ondřejov Observatory, Czech Republic | 65 | M. Wolf, J. Ďurech |
| 8 | Ostrowik, Poland | 60 | W. Pych |
| 9 | Pic du Midi, France | 105 | T. Michałowski, J. Berthier |
| 11 | Santana Observatory, CA, USA | 35 | R. Stephens |
| 12 | Shed of Science Observatory, Minneapolis, USA | 25 | R. Durkee |
| 13 | Pic de Chateau-Renard Observatory, France | 62 | S. Fauvaud, G. Santacana |
| 14 | Waterlooville, Hampshire, England | 25 | R. Dymock |
| 15 | Whitin Observatory, Massachusetts, USA | 61 | S. Slivan, M. Frey |
| 16 | Mt Tarana Observatory, Bathurst, Australia | 40 | C. Bembrick |
| 17 | Craigie, Australia | 25 | G. Bolt |
| 18 | Pleasant Plains, Illinois, USA | 35 | F. Pilcher, D. Jardine |
| 19 | Le Crès, France | 25 | R. Poncy |
| 20 | Linhaceira Observatory, Portugal | 25 | R. Gonçalves |
| 21 | Stazione Astronomica di Sozzago, Italy | 40 | R. Crippa, F. Manzini |
| 22 | Vintage Lane Observatory, Blenheim, New Zealand | 30 | W. H. Allen |
| 23 | The WW Crow Observatory, England | 20 | M. Crow |
| 24 | Antelope Hills Observatory, Colorado, USA | 25 | R. Koff |
| 25 | Les Engarouines Observatory, France | 21 | L. Bernasconi |
| 26 | Goat Mountain Astronomical Research Station, USA | 35 | R. Stephens |
| 27 | Gnosca Observatory, Switzerland | 40 | S. Sposetti |
| 28 | Wallace Astrophysical Observatory, Massachusetts, USA | 61 | S. Slivan |
| 29 | Wallace Astrophysical Observatory, Massachusetts, USA | 35 | M. Pan, L. Crespo da Silva, S. Boroumand |

Table 2. Aspect data for new observation of the modeled asteroids. The table lists asteroid distance from the Sun r , from the Earth Δ , the solar phase angle α , and the ecliptical coordinates of the asteroid (λ , β).

| Date | r [AU] | Δ [AU] | α [deg] | λ [deg] | β [deg] | Obs. code |
|------------------|-------------|------------------|-------------------|--------------------|------------------|--------------|
| (110) Lydia | | | | | | |
| 2003 12 02.2 | 2.642 | 1.757 | 11.6 | 37.0 | -1.3 | 3 |
| 2003 12 13.1 | 2.651 | 1.857 | 15.1 | 35.9 | -0.9 | 3 |
| 2003 12 21.2 | 2.657 | 1.947 | 17.2 | 35.7 | -0.6 | 3 |
| 2003 12 22.0 | 2.658 | 1.957 | 17.4 | 35.7 | -0.6 | 3 |
| 2003 12 23.1 | 2.659 | 1.969 | 17.6 | 35.7 | -0.5 | 3 |
| 2003 12 26.1 | 2.661 | 2.007 | 18.3 | 35.8 | -0.4 | 3 |
| 2003 12 27.1 | 2.662 | 2.018 | 18.5 | 35.8 | -0.4 | 3 |
| 2003 12 28.0 | 2.663 | 2.029 | 18.6 | 35.8 | -0.4 | 3 |
| 2003 12 29.1 | 2.663 | 2.042 | 18.8 | 35.9 | -0.3 | 3 |
| 2003 12 30.0 | 2.664 | 2.055 | 19.0 | 35.9 | -0.3 | 3 |
| 2003 12 12.2 | 2.650 | 1.848 | 14.9 | 36.0 | -0.9 | 11 |
| 2003 12 17.2 | 2.654 | 1.901 | 16.3 | 35.8 | -0.7 | 11 |
| 2003 12 19.2 | 2.656 | 1.923 | 16.8 | 35.7 | -0.7 | 11 |
| 2003 12 22.2 | 2.658 | 1.958 | 17.5 | 35.7 | -0.6 | 11 |
| (125) Liberatrix | | | | | | |
| 2004 12 13.0 | 2.818 | 2.278 | 18.7 | 14.7 | -4.0 | 4 |
| 2004 12 13.1 | 2.818 | 2.279 | 18.7 | 14.7 | -4.0 | 4 |
| 2004 12 15.0 | 2.820 | 2.305 | 18.9 | 14.9 | -4.0 | 4 |
| 2004 12 28.0 | 2.830 | 2.489 | 20.0 | 16.4 | -3.9 | 4 |
| 2004 12 28.1 | 2.830 | 2.490 | 20.0 | 16.4 | -3.9 | 4 |
| 2004 09 08.0 | 2.740 | 1.907 | 14.2 | 27.5 | -2.7 | 7 |
| 2004 10 08.0 | 2.765 | 1.773 | 3.0 | 22.3 | -3.7 | 7 |
| 2004 12 30.7 | 2.832 | 2.529 | 20.1 | 16.9 | -3.9 | 7 |
| 2005 01 13.8 | 2.842 | 2.734 | 20.2 | 19.6 | -3.7 | 7 |
| 2004 11 10.1 | 2.792 | 1.905 | 11.0 | 15.7 | -4.2 | 15 |
| (130) Elektra | | | | | | |
| 2001 04 22.9 | 3.705 | 2.817 | 8.4 | 226.1 | 29.9 | 27 |
| 2001 04 23.0 | 3.705 | 2.817 | 8.4 | 226.1 | 29.9 | 27 |
| 2003 10 21.8 | 2.503 | 1.701 | 16.4 | 59.4 | -34.4 | 17 |
| 2003 10 23.7 | 2.505 | 1.694 | 16.1 | 59.1 | -34.7 | 17 |
| 2003 10 24.7 | 2.506 | 1.690 | 15.9 | 58.9 | -34.8 | 17 |
| 2003 10 29.7 | 2.511 | 1.676 | 15.0 | 58.0 | -35.4 | 17 |
| 2003 11 14.5 | 2.528 | 1.662 | 13.4 | 54.5 | -36.2 | 16 |
| 2003 11 15.6 | 2.530 | 1.663 | 13.4 | 54.2 | -36.2 | 16 |
| 2003 11 19.5 | 2.534 | 1.669 | 13.4 | 53.3 | -36.2 | 16 |
| 2003 11 27.6 | 2.544 | 1.692 | 13.8 | 51.5 | -35.8 | 16 |
| 2003 11 29.6 | 2.547 | 1.700 | 14.0 | 51.0 | -35.6 | 16 |
| (165) Loreley | | | | | | |
| 2004 09 28.3 | 3.064 | 2.102 | 6.3 | 17.7 | 15.6 | 12 |
| 2004 10 09.2 | 3.073 | 2.100 | 5.1 | 15.4 | 15.8 | 12 |
| 2004 10 09.3 | 3.073 | 2.100 | 5.1 | 15.4 | 15.8 | 12 |
| 2004 10 23.9 | 2.778 | 1.799 | 4.5 | 18.8 | -4.0 | 1 |
| 2004 11 05.9 | 2.789 | 1.872 | 9.5 | 16.4 | -4.1 | 1 |
| 2004 11 06.9 | 2.790 | 1.880 | 9.9 | 16.2 | -4.1 | 1 |
| 2004 11 10.0 | 2.792 | 1.904 | 10.9 | 15.8 | -4.2 | 1 |
| 2004 11 10.9 | 2.793 | 1.912 | 11.3 | 15.6 | -4.2 | 1 |
| 2006 01 15.2 | 3.365 | 2.482 | 8.6 | 84.8 | 7.6 | 18 |
| 2006 01 26.3 | 3.368 | 2.581 | 11.5 | 83.5 | 6.9 | 18 |
| 2006 01 26.2 | 3.368 | 2.579 | 11.5 | 83.5 | 6.9 | 18 |
| (196) Philomela | | | | | | |
| 2003 11 16.3 | 3.182 | 2.279 | 8.6 | 82.1 | 0.2 | 3 |
| 2003 11 22.3 | 3.183 | 2.244 | 6.6 | 81.1 | 0.3 | 3 |
| 2003 11 23.4 | 3.183 | 2.238 | 6.2 | 80.9 | 0.4 | 3 |
| 2005 03 01.9 | 3.152 | 2.182 | 4.5 | 151.5 | 10.4 | 19 |
| 2005 03 02.8 | 3.152 | 2.184 | 4.7 | 151.3 | 10.4 | 19 |
| 2005 03 04.9 | 3.151 | 2.189 | 5.3 | 150.9 | 10.4 | 19 |
| 2005 03 06.9 | 3.151 | 2.195 | 5.9 | 150.5 | 10.4 | 19 |
| 2006 05 05.0 | 3.061 | 2.068 | 4.1 | 236.2 | 3.7 | 20 |
| 2006 05 07.0 | 3.060 | 2.062 | 3.4 | 235.8 | 3.7 | 20 |
| 2006 05 05.1 | 3.061 | 2.068 | 4.0 | 236.1 | 3.7 | 20 |
| 2006 05 19.0 | 3.059 | 2.050 | 1.8 | 233.4 | 3.3 | 20 |
| 2006 05 19.0 | 3.059 | 2.050 | 1.8 | 233.4 | 3.3 | 20 |
| 2006 05 12.0 | 3.060 | 2.052 | 1.7 | 234.8 | 3.5 | 21 |

Table 2. continued.

| Date | r [AU] | Δ [AU] | α [deg] | λ [deg] | β [deg] | Obs. code |
|---------------|-------------|------------------|-------------------|--------------------|------------------|--------------|
| (218) Bianca | | | | | | |
| 1996 01 19.1 | 2.676 | 1.776 | 10.4 | 144.5 | -13.7 | 9 |
| 1996 03 18.9 | 2.605 | 1.818 | 16.2 | 132.4 | -8.5 | 8 |
| 1997 05 14.0 | 2.396 | 1.632 | 19.2 | 280.7 | 22.7 | 2 |
| 1997 05 17.0 | 2.398 | 1.609 | 18.5 | 280.6 | 23.0 | 2 |
| 1997 05 19.0 | 2.399 | 1.595 | 18.1 | 280.5 | 23.2 | 2 |
| 1997 06 06.0 | 2.411 | 1.498 | 13.4 | 278.4 | 24.8 | 2 |
| 1997 06 07.0 | 2.412 | 1.494 | 13.1 | 278.2 | 24.8 | 2 |
| 1997 06 07.9 | 2.412 | 1.491 | 12.9 | 278.0 | 24.9 | 2 |
| 2000 03 22.8 | 2.756 | 2.467 | 21.1 | 98.6 | -13.3 | 2 |
| 2000 03 23.8 | 2.755 | 2.479 | 21.1 | 98.8 | -13.2 | 2 |
| 2001 03 02.1 | 2.384 | 1.779 | 22.0 | 225.2 | 11.1 | 2 |
| 2001 03 06.1 | 2.382 | 1.734 | 21.3 | 225.6 | 11.7 | 2 |
| 2001 03 07.1 | 2.381 | 1.723 | 21.1 | 225.6 | 11.9 | 2 |
| 2001 04 01.1 | 2.369 | 1.494 | 14.7 | 225.3 | 16.0 | 2 |
| 2001 04 03.0 | 2.368 | 1.481 | 14.1 | 225.1 | 16.4 | 2 |
| 2003 12 13.9 | 2.949 | 2.026 | 8.0 | 70.5 | -22.3 | 5 |
| 2003 12 15.9 | 2.949 | 2.031 | 8.4 | 70.1 | -22.3 | 5 |
| 2003 12 16.9 | 2.948 | 2.034 | 8.5 | 69.8 | -22.3 | 5 |
| 2003 12 17.8 | 2.948 | 2.037 | 8.7 | 69.6 | -22.2 | 5 |
| 2003 12 20.8 | 2.946 | 2.048 | 9.4 | 68.9 | -22.1 | 5 |
| 2004 01 10.8 | 2.935 | 2.183 | 14.4 | 65.6 | -20.7 | 5 |
| 2004 01 14.8 | 2.932 | 2.219 | 15.3 | 65.2 | -20.3 | 5 |
| 2004 01 17.8 | 2.931 | 2.248 | 15.9 | 65.1 | -20.0 | 5 |
| 2005 01 17.1 | 2.546 | 1.925 | 19.8 | 178.4 | -4.4 | 2 |
| 2005 02 02.0 | 2.527 | 1.736 | 16.1 | 178.5 | -3.4 | 2 |
| 2005 02 07.1 | 2.521 | 1.684 | 14.5 | 178.1 | -3.0 | 2 |
| 2005 02 09.1 | 2.519 | 1.665 | 13.8 | 177.9 | -2.8 | 2 |
| 2005 03 11.6 | 2.485 | 1.491 | 0.4 | 172.0 | 0.3 | 16 |
| 2005 03 12.6 | 2.484 | 1.490 | 0.2 | 171.8 | 0.4 | 16 |
| 2005 03 13.6 | 2.483 | 1.489 | 0.6 | 171.5 | 0.5 | 22 |
| 2005 03 13.6 | 2.482 | 1.489 | 0.7 | 171.5 | 0.5 | 16 |
| 2005 03 26.5 | 2.469 | 1.499 | 7.0 | 168.4 | 2.0 | 16 |
| 2005 03 31.5 | 2.464 | 1.515 | 9.4 | 167.3 | 2.5 | 16 |
| 2005 04 03.4 | 2.461 | 1.527 | 10.7 | 166.8 | 2.8 | 22 |
| 2005 04 16.9 | 2.447 | 1.609 | 16.0 | 164.9 | 4.1 | 23 |
| 2005 04 18.9 | 2.445 | 1.624 | 16.7 | 164.7 | 4.2 | 23 |
| 2005 04 19.9 | 2.444 | 1.632 | 17.0 | 164.6 | 4.3 | 23 |
| 2005 04 06.9 | 2.457 | 1.545 | 12.1 | 166.1 | 3.2 | 23 |
| 2005 04 10.9 | 2.453 | 1.568 | 13.8 | 165.5 | 3.5 | 23 |
| 2005 02 08.4 | 2.520 | 1.671 | 14.1 | 178.0 | -2.9 | 4 |
| 2005 02 11.4 | 2.516 | 1.643 | 13.0 | 177.6 | -2.6 | 4 |
| 2005 02 12.2 | 2.515 | 1.636 | 12.7 | 177.5 | -2.5 | 4 |
| 2005 03 30.1 | 2.465 | 1.510 | 8.7 | 167.6 | 2.4 | 4 |
| 2005 04 05.1 | 2.459 | 1.535 | 11.4 | 166.5 | 3.0 | 4 |
| 2005 04 06.1 | 2.458 | 1.540 | 11.8 | 166.3 | 3.1 | 4 |
| (306) Unitas | | | | | | |
| 2003 03 09.2 | 2.700 | 1.849 | 13.2 | 129.8 | 0.2 | 24 |
| 2003 03 10.2 | 2.699 | 1.857 | 13.5 | 129.6 | 0.2 | 24 |
| 2003 04 01.1 | 2.689 | 2.077 | 19.3 | 128.4 | 0.9 | 24 |
| 2003 04 10.2 | 2.684 | 2.184 | 20.7 | 128.8 | 1.2 | 24 |
| (423) Diotima | | | | | | |
| 1998 09 24.3 | 3.114 | 2.365 | 14.1 | 49.6 | -8.4 | 28 |
| 1998 09 25.3 | 3.115 | 2.354 | 13.9 | 49.5 | -8.4 | 28 |
| 1998 09 26.3 | 3.115 | 2.344 | 13.6 | 49.5 | -8.4 | 28 |
| 1998 09 28.3 | 3.116 | 2.326 | 13.1 | 49.3 | -8.4 | 28 |
| 1998 09 29.2 | 3.116 | 2.317 | 12.9 | 49.2 | -8.4 | 28 |
| 1998 10 16.2 | 3.122 | 2.194 | 7.9 | 47.0 | -8.1 | 29 |
| 1998 10 28.2 | 3.126 | 2.150 | 4.0 | 44.7 | -7.8 | 29 |
| 1998 10 28.2 | 3.126 | 2.150 | 4.0 | 44.7 | -7.8 | 28 |
| 1998 10 30.1 | 3.127 | 2.146 | 3.5 | 44.3 | -7.7 | 29 |
| 1998 10 30.2 | 3.127 | 2.146 | 3.5 | 44.3 | -7.7 | 28 |
| 1998 11 10.2 | 3.131 | 2.149 | 2.9 | 42.0 | -7.2 | 28 |
| 1998 11 12.2 | 3.131 | 2.153 | 3.4 | 41.6 | -7.1 | 28 |
| 2003 10 16.1 | 3.078 | 2.111 | 5.5 | 11.6 | -13.5 | 13 |
| 2003 10 17.0 | 3.078 | 2.113 | 5.7 | 11.5 | -13.4 | 13 |

Table 2. continued.

| Date | r [AU] | Δ [AU] | α [deg] | λ [deg] | β [deg] | Obs. code |
|------------------|-------------|------------------|-------------------|--------------------|------------------|--------------|
| 2004 12 07.1 | 3.189 | 2.237 | 5.4 | 92.4 | 4.9 | 4 |
| 2004 12 07.3 | 3.189 | 2.236 | 5.3 | 92.3 | 4.9 | 4 |
| 2004 12 13.2 | 3.190 | 2.218 | 3.4 | 91.1 | 5.2 | 4 |
| 2004 12 13.4 | 3.190 | 2.218 | 3.3 | 91.1 | 5.2 | 4 |
| 2005 01 12.8 | 3.191 | 2.291 | 8.4 | 85.0 | 6.4 | 14 |
| 2005 01 13.0 | 3.191 | 2.291 | 8.5 | 85.0 | 6.4 | 14 |
| 2005 01 31.8 | 3.191 | 2.461 | 13.6 | 82.9 | 6.7 | 2 |
| 2005 02 21.9 | 3.191 | 2.720 | 17.0 | 82.9 | 6.7 | 2 |
| 2005 03 20.8 | 3.190 | 3.099 | 18.1 | 86.2 | 6.7 | 2 |
| 2006 01 17.1 | 3.124 | 2.477 | 15.3 | 172.4 | 14.2 | 2 |
| 2006 02 28.1 | 3.109 | 2.152 | 5.7 | 167.2 | 16.3 | 2 |
| 2006 03 09.1 | 3.106 | 2.141 | 5.2 | 165.3 | 16.3 | 2 |
| 2006 03 13.0 | 3.105 | 2.144 | 5.7 | 164.5 | 16.3 | 2 |
| (776) Berbericia | | | | | | |
| 2003 11 24.3 | 2.673 | 1.863 | 14.5 | 104.0 | 4.4 | 3 |
| 2003 11 26.2 | 2.676 | 1.848 | 13.9 | 103.8 | 4.7 | 3 |
| 2004 02 10.8 | 2.796 | 2.078 | 16.1 | 90.2 | 11.2 | 5 |
| 2004 02 12.8 | 2.799 | 2.102 | 16.6 | 90.2 | 11.2 | 5 |
| 2004 02 13.8 | 2.801 | 2.115 | 16.8 | 90.2 | 11.2 | 5 |
| 2004 02 15.9 | 2.804 | 2.141 | 17.2 | 90.2 | 11.3 | 5 |
| 2004 02 19.9 | 2.811 | 2.194 | 17.9 | 90.2 | 11.3 | 5 |
| 2004 02 23.9 | 2.817 | 2.249 | 18.5 | 90.4 | 11.4 | 5 |
| 2004 02 28.9 | 2.826 | 2.319 | 19.1 | 90.7 | 11.4 | 5 |
| 2005 02 15.1 | 3.309 | 2.529 | 12.0 | 184.5 | 24.1 | 21 |
| 2005 02 19.1 | 3.313 | 2.500 | 11.2 | 184.0 | 24.4 | 21 |
| 2005 03 12.0 | 3.330 | 2.412 | 7.7 | 180.0 | 25.3 | 25 |
| 2005 03 13.0 | 3.331 | 2.411 | 7.6 | 179.8 | 25.4 | 25 |
| 2005 05 09.9 | 3.370 | 2.759 | 15.2 | 171.0 | 21.6 | 2 |
| 2006 06 14.2 | 3.276 | 2.335 | 8.0 | 237.3 | 6.8 | 11 |
| 2006 06 15.2 | 3.275 | 2.340 | 8.3 | 237.1 | 6.7 | 11 |
| 2006 06 16.2 | 3.274 | 2.345 | 8.6 | 237.0 | 6.6 | 11 |
| 2006 06 18.2 | 3.272 | 2.357 | 9.1 | 236.6 | 6.4 | 26 |
| 2006 06 19.2 | 3.271 | 2.363 | 9.4 | 236.5 | 6.4 | 11 |
| 2006 06 20.2 | 3.270 | 2.369 | 9.7 | 236.3 | 6.3 | 11 |
| (944) Hidalgo | | | | | | |
| 2004 10 14.3 | 2.184 | 1.240 | 11.2 | 40.6 | 16.3 | 24 |
| 2004 10 17.3 | 2.171 | 1.221 | 10.6 | 39.8 | 17.8 | 24 |
| 2004 10 18.3 | 2.166 | 1.215 | 10.4 | 39.5 | 18.4 | 24 |
| 2004 10 21.4 | 2.153 | 1.200 | 10.2 | 38.7 | 20.0 | 24 |

Table 3. The table lists the ecliptic coordinates of the asteroid's spin axis direction (λ_p , β_p), its sidereal rotation period P , the span of observations in years, the number of oppositions N_{opp} , the number of lightcurves N_{lc} , and the rms residual of the fit.

| Asteroid | λ_p [deg] | β_p [deg] | P [hr] | Years of obs. | N_{opp} | N_{lc} | rms [mag] |
|------------------|----------------------|--------------------|-------------|---------------|------------------|-----------------|--------------|
| (110) Lydia | 331 | -61 | 10.92580 | 1958–2003 | 4 | 26 | 0.011 |
| | 149 | -55 | | | | | |
| (125) Liberatrix | 280 | +74 | 3.968199 | 1981–2005 | 7 | 34 | 0.024 |
| | 95 | +68 | | | | | |
| (130) Elektra | 65 | -88 | 5.224664 | 1980–2003 | 11 | 54 | 0.013 |
| (165) Loreley | 346 | +29 | 7.22667 | 1981–2006 | 6 | 29 | 0.015 |
| (196) Philomela | 276 | -49 | 8.332827 | 1964–2006 | 8 | 27 | 0.013 |
| | 111 | -41 | | | | | |
| (218) Bianca | 305 | +17 | 6.33717 | 1979–2005 | 10 | 50 | 0.015 |
| | 121 | -10 | | | | | |
| (306) Unitas | 79 | -35 | 8.73875 | 1979–2003 | 5 | 15 | 0.015 |
| | 254 | -18 | | | | | |
| (423) Diotima | 351 | +4 | 4.775377 | 1981–2006 | 10 | 48 | 0.017 |
| (776) Berbericia | 347 | +12 | 7.66701 | 1977–2006 | 8 | 36 | 0.012 |
| (944) Hidalgo | 281 | +5 | 10.058634 | 1976–2004 | 4 | 14 | 0.013 |