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Letter to the Editor

Rotation at 1122 Hz and the neutron star structure

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ABSTRACT

Aims. Recent observations of XTE J1739-285 suggest that it contains a neutron star rotating at 1122 Hz. Such rotation imposes bounds on the structure of neutron star in XTE J1739-285. These bounds may be used to constrain poorly known equation of state of dense matter at densities $\gtrsim 10^{15}$ g cm^{-3}.

Methods. One-parameter families of stationary configurations rotating rigidly at 1122 Hz are constructed, using a precise 2D code solving Einstein equations. Hydrostatic equilibrium solutions are tested for stability with respect to axi-symmetric perturbations. A set of ten diverse EOSs of neutron stars is considered. Hypothetical strange stars are also studied.

Results. For each EOS, the family of possible neutron star models is limited by the mass shedding limit, corresponding to maximum allowed equatorial radius, $R_{\rm max}$, and by the instability with respect to the axi-symmetric perturbations, reached at the minimum allowed equatorial radius, $R_{\rm min}$. We get $R_{\rm min} \simeq 10-13$ km, and $R_{\rm max} \simeq 16-18$ km, with allowed mass $1.4-2.3\ M_\odot$. Allowed stars with hyperonic or exotic-phase core are supramassive and have a very narrow mass range. Quark star with accreted crust might be allowed, provided such a model is able to reproduce X-ray bursts from XTE J1739-285.

Key words. dense matter – equation of state – stars: neutron – stars: rotation

1. Introduction

Because of their strong gravity, neutron stars can be very rapid rotators; theoretical studies indicate that they could rotate at submillisecond periods, i.e., at frequency $f = 1/\text{period} > 1000$ Hz (e.g., Cook et al. 1994a; Salgado et al. 1994). During 24 years after its detection, the first millisecond pulsar B1937+21, rotating at $f = 641$ Hz (Backer et al. 1982), remained the most rapid one. In 2006, a more rapid pulsar J1748-2446ad, rotating at $f = 716$ Hz, was detected (Hessels et al. 2006). However, such frequencies are too low to significantly affect structure of neutron stars with $M > 1\ M_\odot$ (Shapiro et al. 1983). They belong to a *slow rotation* regime, because f is significantly smaller than the mass shedding (Keplerian) frequency $f_{\rm K}$. Under the slow rotation regime, effects of rotation on neutron star structure are $\propto (f/f_{\rm K})^2 \ll 1$. To enter the rapid rotation regime for $M > 1\ M_\odot$, one needs submillisecond pulsars with $f > 1000$ Hz.

Very recently Kaaret et al. (2006) reported a discovery of oscillation frequency $f = 1122$ Hz in an X-ray burst from the X-ray transient, XTE J1739-285. According to Kaaret et al. (2006) "this oscillation frequency suggests that XTE J1739-285 contains the fastest rotating neutron star yet found". The very fact that stable rotation of neutron star at $f > 1000$ Hz exists means that instabilities that could set in at lower f (r-mode instability, other gravitational-radiation-reaction instabilities) are effectively damped. In the present Letter we derive, using precise 2D calculations of neutron stars with different EOSs, constraints on neutron star in XTE J1739-285, which result from the condition of stable rotation at $f = 1122$ Hz.

Neutron star models at $f = 1122$ Hz are analyzed in Sect. 2, where we also derive constraints on neutron star parameters resulting from the condition of stable rotation. Possibility of strange (quark) star in XTE J1739-285 is briefly discussed in Sect. 3. Section 4 contains discussion of our results and conclusions.

2. Neutron stars at 1122 Hz

The stationary configurations of rigidly rotating neutron stars have been computed in the framework of general relativity by solving the Einstein equations for stationary axi-symmetric spacetime (see Bonazzola et al. 1993, or Gourgoulhon et al. 1999). The numerical computations have been performed using the `Lorene/Codes/Rot_star/rotstar` code from the LORENE library (`http://www.lorene.obspm.fr`). One-parameter families of stationary 2D configurations were calculated for ten EOSs of neutron-star matter, listed in Table 1. Stability with respect to the mass-shedding from the equator implies that at a given gravitational mass M the circumferential equatorial radius $R_{\rm eq}$ should be smaller than $R_{\rm max}$ which corresponds to the mass shedding (Keplerian) limit. The value of $R_{\rm max}$ results from the condition that the frequency of a test particle at circular equatorial orbit of radius $R_{\rm max}$ just above the equator of the *actual rotating star* is equal to 1122 Hz. It is interesting that the relation between M and $R_{\rm eq}$ at this "mass shedding point" is extremely well approximated by the formula for the orbital frequency for a test particle orbiting at $r = R_{\rm eq}$ in the Schwarzschild space-time created by a *spherical mass* M (which can be replaced by a point mass M at $r = 0$). We denote the orbital frequency of such a test particle by $f_{\rm orb}^{\rm Schw.}(M, R_{\rm eq})$. The formula giving the locus of points satisfying $f_{\rm orb}^{\rm Schw.}(M, R_{\rm eq}) = 1122$ Hz, represented by a dash line in Fig. 1, is

$$\frac{1}{2\pi}\left(\frac{GM}{R_{\rm eq}{}^3}\right)^{1/2} = 1122\ \text{Hz}. \qquad (1)$$

Table 1. Parameters of neutron stars rotating at $f = 1122$ Hz at the $R_{\rm min}$ termination (instability with respect to axi-symmetric perturbations) and the $R_{\rm max}$ termination (mass shedding instability) of the $M - R_{\rm eq}$ line, for ten EOSs of neutron star matter.

EOS	$M(R_{\rm min})$ ($M_\odot$)	$R_{\rm min}$ (km)	$r_{\rm pole}/r_{\rm eq}$	$T/\vert W\vert$
GN3[a]	2.309	12.77	0.813	0.057
APR[b]	2.274	10.47	0.896	0.038
DH[c]	2.138	10.50	0.887	0.038
GNH3[a]	2.119	12.67	0.804	0.055
BBB2[d]	1.996	9.928	0.896	0.034
FPS[e]	1.872	9.75	0.894	0.033
BG1H1[f]	1.699	9.99	0.876	0.032
BPAL12[g]	1.521	9.64	0.875	0.032
GMHSm[h]	1.481	9.65	0.874	0.029
GMGSp[i]	1.473	9.60	0.875	0.028
EOS	$M(R_{\rm max})$ ($M_\odot$)	$R_{\rm max}$ (km)	$r_{\rm pole}/r_{\rm eq}$	$T/\vert W\vert$
GN3[a]	2.298	18.25	0.564	0.117
APR[b]	1.498	15.93	0.563	0.108
DH[c]	1.648	16.45	0.560	0.111
GNH3[a]	2.163	17.54	0.583	0.103
BBB2[d]	1.473	15.85	0.563	0.107
FPS[e]	1.393	15.56	0.566	0.104
BG1H1[f]	1.818	16.92	0.568	0.103
BPAL12[g]	1.422	15.61	0.582	0.089
GMHSm[h]	1.594	16.19	0.582	0.086
GMGSp[i]	1.606	16.23	0.581	0.086

References for the EOSs: [a] Glendenning (1985); [b] Akmal et al. (1998); [c] Douchin & Haensel (2001); [d] Baldo et al. (1997), for Paris nucleon-nucleon potential; [e] Pandharipande & Ravenhall (1989); [f] Balberg & Gal (1997); [g] Bombaci (1995); [h] Pons et al. (2000), GM+GS model with $U_K^{\rm lin} = -130$ MeV and pure normal and kaon condensed phases; [i] Pons et al. (2000), GM+GS model with $U_K^{\rm lin} = -130$ MeV and mixed-phase state between the normal and kaon-condensed phases.

This formula for the Schwarzschild metric coincides with that obtained in Newtonian gravity for a point mass M. It passes through (or extremely close to) the open circles denoting the actual mass shedding (Keplerian) configurations. This is quite surprising in view of rapid rotation and strong flattening of neutron star at the mass-shedding point (see Fig. 2). Equation (1) implies

$$R_{\rm max} = 15.52 \left(\frac{M}{1.4\ M_\odot}\right)^{1/3}\ {\rm km}. \qquad (2)$$

Out of ten EOSs of neutron star matter, two were chosen to represent a soft (BPAL12) and stiff (GN3) extremes of the set. These two limiting EOSs should not be considered as "realistic", but they are used just to "bound" the neutron star models from the soft and the stiff side. The curves representing allowed configurations rotating at 1122 Hz are plotted in the $M - R_{\rm eq}$ plane in Fig. 1.

The EOSs based on most realistic models involving only nucleons (FPS, BBB2, DH, APR) result in monotonic $M(R_{\rm eq})$ of very characteristic "tilda-like" shape. For these EOSs we get $R_{\rm min} \approx 10$ km and $R_{\rm max} \approx 16$ km. The value of $M(R_{\rm min})$ is significantly larger than $M(R_{\rm max})$, the difference ranging from 0.5 $M_\odot$ to 0.8 $M_\odot$. The ratio of the kinetic to the gravitational energy $T/|W|$ is about 0.09–0.11 at the mass shedding limit at $R_{\rm max}$ and is as low as 0.03–0.04 at $R_{\rm min}$.

Four EOSs are softened at high density either by the appearance of hyperons (GNH3, BGN1H1), or a phase transition

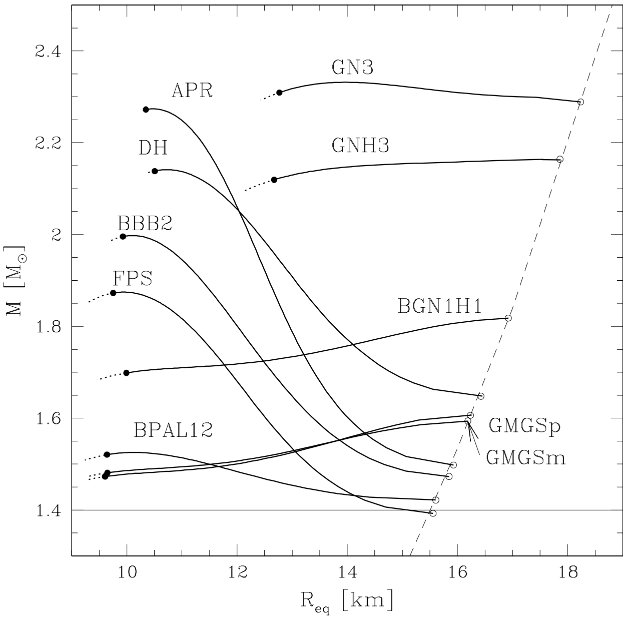

Fig. 1. Gravitational mass, M, vs. circumferential equatorial radius, $R_{\rm eq}$, for neutron stars stably rotating at $f = 1122$ Hz, for ten EOSs labeled as in Table 1. Small-radius termination by filled circle: setting-in of instability with respect to the axi-symmetric perturbations. Dotted segments to the left of the filled circles: configurations unstable with respect to those perturbations. Large-radius termination by an open circle: the mass-shedding instability. The mass-shedding points are very well fitted by the dashed curve $R_{\rm min} = 15.52\ (M/1.4\ M_\odot)^{1/3}$ km. For further explanation see the text.

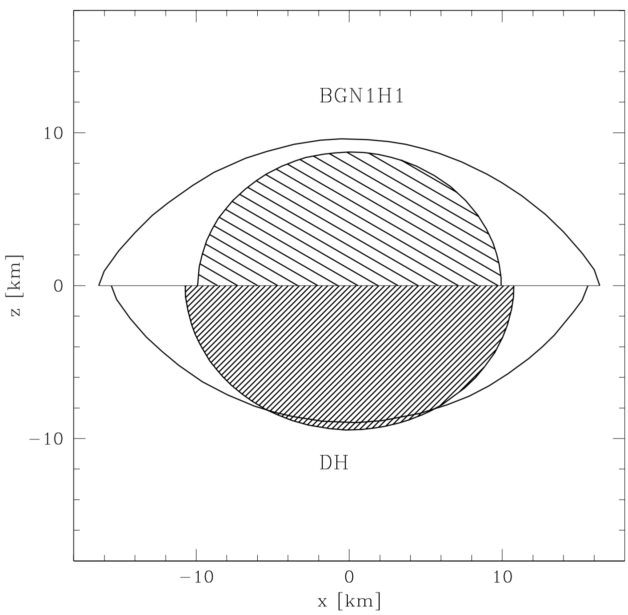

Fig. 2. Cross section in the plane passing through the rotation axis of neutron stars rotating at 1122 Hz, for the BGN1H1 EOS ($z > 0$) and DH EOS ($z < 0$). Blank area with solid line contour – $R_{\rm eq} = R_{\rm max}$; hatched area – $R_{\rm eq} = R_{\rm min}$. The coordinates x and z are defined as $x = r \sin\theta \cos\phi$, $z = r\cos\theta$, where r is radial coordinate in the space-time metric.

(GMGSm, GMGSp). For them, the range of allowed masses is very narrow, ~0.1 $M_\odot$, and moreover $M(R_{\rm min}) < M(R_{\rm max})$. In spite of $(\partial M/\partial \rho_{\rm c})_{f=1122\ {\rm Hz}} < 0$, configurations between $R_{\rm min}$

and $R_{\rm max}$ are stable with respect to small axi-symmetric perturbations, because they satisfy stability condition

$$\left(\frac{\partial M}{\partial \rho_{\rm c}}\right)_J > 0, \qquad (3)$$

where the derivative is calculated along a sequence with a constant stellar angular momentum J (Friedman et al. 1988). Peculiar $M(R_{\rm eq})$ dependence is related to the fact that configurations belonging to the $R_{\rm min} < R_{\rm eq} < R_{\rm max}$ are *supramassive*, i.e., their baryon mass $M_{\rm b}$ is larger than the maximum allowable baryon mass for non-rotating stars, $M_{\rm b} > M^{\rm stat}_{\rm b,max}$. The range of allowed values of $R_{\rm eq}$ is not much different from that for purely nucleon EOSs. EOSs GMGSp and GMGSm describe nucleon matter with a first order phase transition due to kaon condensation. The hadronic Lagrangian is the same in both cases. However, to get GMGSp we assumed that the phase transition takes place between two pure phases and is accompanied by a density jump. On the contrary, assuming that the transition occurs via a mixed state of two phases, we get EOS GMGSm. This last situation prevails when the surface tension between the two phases is below a certain critical value. The actual value of the surface tension is very poorly known, and therefore it is comforting to find that both curves are nearly identical.

3. Quark stars at 1122 Hz

The possibility that the true ground state of hadronic matter is a self-bound plasma of u, d and s quarks was contemplated since 1970s (Bodmer 1971; Witten 1984). If such a state, called *strange matter*, is the true ground state of hadronic matter, quark stars built entirely of strange quark matter could exist (Witten 1984; Haensel et al. 1986; Alcock et al. 1986). Could compact star in XTE J1739-285 be actually a quark star? An accreting quark star would have a crust of normal matter and due to instabilities of thermonuclear burning of accreted plasma it could be an X-ray burster (Haensel & Zdunik 1991; Page & Cumming 2005). The mass of the crust on quark star is at most $10^{-5}\ M_\odot$, because the density at the crust bottom cannot exceed neutron drip density (Alcock et al. 1986).

As we see in Fig. 3, the $M(R_{\rm eq})$ curves for quark stars terminate at $R_{\rm max}$ which is rather well approximated by Eq. (2) (within 2%). The parameters of non-rotating strange stars are subject to the scaling laws with the value of the bag constant B ($M \propto B^{-1/2}$, $R \propto B^{-1/2}$) if the strange quark mass is scaled as $\propto B^{1/4}$ (Zdunik 2000). The same scaling rules are valid also for strange stars with crust provided that the mass of the crust is much smaller than the mass of the strange core (see Eq. (7) in Zdunik 2002, which results in $R \propto B^{-1/2}$). For rotating stars the application of these scaling laws requires also scaling of the rotational frequency with $f \propto B^{1/2}$ (Gourgoulhon et al. 1999). Thus these scaling laws cannot be directly applied to the situation presented if Fig. 3 since the models SQM1, SQM2, SQM3 rotate with the same frequency 1122 Hz, and the condition $f_2 = f_1\sqrt{B_2/B_1}$ is not fulfilled.

In contrast to hadronic EOSs, minimum mass allowed for accreting quark star can be very low. However, if we add the constraint of $M^{\rm stat}_{\rm max} > 2.1 \pm 0.2\ M_\odot$ resulting from the existence of a massive slowly rotating pulsar (Nice et al. 2005), we get $M(R_{\rm max}) > 1.4\ M_\odot$. Let us notice that the values of $T/|W|$ at $R_{\rm max}$ are much larger than for neutron stars, 0.174 and 0.188 for SQM1 and SQM2, which makes these configurations secularly unstable with respect to the triaxial instability (Gondek-Rosińska et al. 2003), so that the actual value of $R_{\rm max}$ could be smaller and $M(R_{\rm max})$ could be larger.

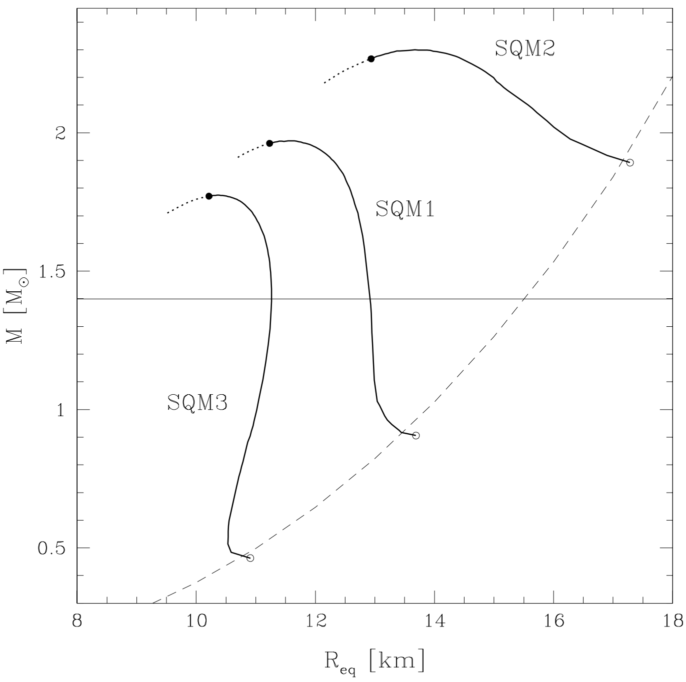

Fig. 3. Gravitational mass, M, vs. circumferential equatorial radius, $R_{\rm eq}$, for strange stars with crust stably rotating at f = 1122 Hz, for several EOS of strange matter. Notation as in Fig. 1. EOSs calculated using MIT bag model, with $\alpha_{\rm s}$ = 0.2 for all models and $m_{\rm s}c^2$ = 200, 185, 205 MeV, B = 56, 45, 67 MeV fm^{-3} for SQM1, 2, 3, respectively (notation as in Haensel et al. 1986).

4. Discussion and conclusions

We constructed neutron star models rotating at 1122 Hz, suggested to be the rotation frequency of neutron star in XTE J1739-285. At such frequency, rotation significantly affects neutron star structure. The neutron star mass is larger then $1.4\ M_\odot$. The equatorial (circumferential) radius is contained between $R_{\rm min}$ = 10−13 km and $R_{\rm max} = 15.52\ (M/1.4\ M_\odot)^{1/3}$ km = 16−18 km. The formula for $R_{\rm max}(M)$ is very precise, despite very strong deformation of neutron star at the mass-shedding limit.

The mass of neutron star in XTE J1739-285 is between $M(R_{\rm max})$ and $M(R_{\rm min})$. For EOSs assuming purely nucleon composition of matter, $M(R_{\rm min})$ is maximum allowable mass at f = 1122 Hz and $M(R_{\rm min}) - M(R_{\rm max}) \approx 0.5$–$0.7\ M_\odot$.

The situation is very different for EOSs with high-density softening due to hyperons or due to a phase transitions. For such EOSs $M(R_{\rm min})$ is slightly *smaller* than $M(R_{\rm max})$, and the range of allowed masses is very narrow, $|M(R_{\rm min}) - M(R_{\rm max})| \approx 0.1\ M_\odot$. Moreover, rotating neutron stars are then supramassive, with baryon mass greater than the maximum allowable baryon mass for nonrotating stars, $M_{\rm b} > M^{\rm stat}_{\rm b,max}$. Their existence requires rapid rotation and they collapse into black holes after spinning down. However, because of a very narrow range of allowed masses, possible presence of such a star in XTE J1739-285 would be a result of "fine-tuning" in accretion process. This could explain the rarity of submillisecond pulsars. However, by turning the argument around, this implies that we are very lucky to detect an extremely rare system! It should be stressed that a measurement of the mass $2.1 \pm 0.2\ M_\odot$ of a *slowly rotating* neutron star (Nice et al. 2005) would anyway rule out soft-core neutron stars not only in XTE J1739-285, but generally.

Concerning neutron stars with a phase transition, as models of XTE J1739-285, we found that their $M(R_{\rm eq})$ curves are nearly independent on whether phase transition occurs between pure phases, or via a mixed-phase state.

Could compact star in XTE J1739-285 be actually a strange star? If so, the strange star should have an accreted crust of normal matter. It remains to be studied, whether the observed X-ray bursts characteristics could then be reproduced with such an exotic burster model. In contrast to neutron stars, rotation at 1122 Hz alone would not require a high M. However, if we additionally require $M_{\rm max}^{\rm stat} > 2.1 \pm 0.2\ M_{\odot}$, then the range of masses is narrower, 1.4–2.3 $M_{\odot}$.

Constraints on the dense matter EOS, derived in the present Letter, are *necessary* conditions resulting from the existence of neutron star rotating at 1122 Hz, but they are not *sufficient*. Actually, neutron star in XTE J1739-285 acquired its rapid rotation by accretion. Spin-up by accretion to 1122 Hz leads to a stronger constraint on the EOS than just stable rotation at this frequency. This problem was studied for the 641 Hz pulsar by Cook et al. (1994b). Accretion associated with spin can be followed as a line in the mass – equatorial radius plane, and 1122 Hz should be reached before the star becomes unstable either with respect to the mass shedding or the axi-symmetric instability. As an example, we studied the accretion-driven spin up for the DH EOS. We found that to spin up the star by accretion from a Keplerian disk to $f = 1122$ Hz, one needs to accrete 0.37 $M_{\odot}$ for an initial mass $M_{\rm i} = 1.4\ M_{\odot}$. For $M_{\rm i} = 1.93\ M_{\odot}$, accretion of 0.25 $M_{\odot}$ is needed. For higher masses of initially nonrotating star, $1.93\ M_{\odot} < M_{\rm i} < M_{\rm max} = 2.05\ M_{\odot}$, accretion leads to the stellar collapse into a black hole before reaching rotation frequency of 1122 Hz. Our complete results on the constraints resulting from the spin-up by accretion will be presented in a separate paper.

When this Letter was being completed, the preprint of Lavagetto et al. (2006) appeared on the `astro-ph` server, in which neutron star rotating at 1122 Hz is used to derive one of several constraints on the dense matter EOS. In contrast to the present Letter, Lavagetto et al. (2006) use approximate "empirical formula" for the mass-shedding frequency and get constraints in the mass-radius plane of non-rotating neutron stars and strange stars.

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