

New electron-proton Bremsstrahlung rates for a hot plasma where the electron temperature is much smaller than the proton temperature

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ABSTRACT

Context. Observations of X-Ray sources harbouring a black hole and an accretion disc show the presence of at least two spectral components. One component is black-body radiation from an optically thick standard accretion disc. The other is produced in an optically thin corona and usually shows a powerlaw behaviour. Electron-proton (ep) bremsstrahlung is one of the contributing radiation mechanisms in the corona. Soft photons from the optically thick disc can Compton cool the electrons in the corona and therefore lead to a two-temperature plasma, where electrons and ions have different temperatures.

Aims. We qualitatively discuss effects on ep-bremsstrahlung in the presence of such a two-temperature plasma.

Methods. We use the classical dipole approximation allowing for non-relativistic electrons and protons and apply quantum corrections through high-precision Gaunt factors.

Results. In the two-temperature case ($T_e < T_p$) the protons cause a significant fraction of the ep-bremsstrahlung if their speed is high compared to the electrons. We give accurate values for ep-bremsstrahlung including quantum-mechanical corrections in the non-relativistic limit and give some approximations in the relativistic limit.

Conclusions. The formulae presented in this paper can be used in models of black hole accretion discs where an optically thin corona can comprise a two-temperature plasma. This work could be extended to include the fully relativistic case if required.

Key words. radiation mechanisms: general – X-rays: general

1. Introduction

Models of black hole accretion discs are widely used to explain spectral characteristics of X-Ray observations of such objects. These models usually consist of an optically thin and hot corona and a cool, optically thick standard accretion disc. The optically thin corona consists of a two-temperature plasma. Electrons cool by bremsstrahlung, mainly by electron-proton bremsstrahlung. The electron-proton system has got a dipole moment while the electron-electron and proton-proton system can only emit radiation through the (much weaker) quadrupole moment.

Eardley et al. (1975) presented a model for Cyg X-1 incorporating a two-temperature plasma in the inner parts of the accretion flow. This kind of model has been subsequently developed (e.g. Shapiro et al. 1976; Zycki et al. 1995) and applied to different objects (Haardt & Maraschi 1991, 1993) and different geometries (Wandel & Liang 1991).

The calculation of the rate of bremsstrahlung coming from a hot gas has received much attention over a long period of time (e.g. Brussaard & van de Hulst 1962; Blumenthal & Gould 1970; Gould 1980, 1981). Today the bremsstrahlung rate is known with a very high accuracy. All these calculations however assume electron and proton temperature to be equal.

In the following we limit ourselves to a purely classical and non-relativistic treatment of electron-proton bremsstrahlung. We do this in order to highlight the elementary physical processes involved. Since we apply Gaunt factors to account for quantum mechanical effects, our results are strictly accurate only in the

non-relativistic case. An expansion to the fully relativistic case is feasible but beyond the scope of this paper.

In this contribution we first highlight the physical mechanism in Sect. 2 and then present a recalculation of the electron-proton Bremsstrahlung rate for a two-temperature plasma in the non-relativistic limit in Sect. 3. While we limit ourselves to an electron-proton plasma, the formulae can easily be modified to consider mixtures of ions with different mass and charge. We give a discussion of the results and our conclusions in Sect. 4.

2. The mechanism

Electron-proton Bremsstrahlung in the standard picture is created through the acceleration of electrons in the field of a proton. This acceleration leads to the emission of electromagnetic radiation resulting in the loss of kinetic electron energy. We show this situation in Fig. 1.

Consider an electron and a proton travelling at a speed u_e and u_p , respectively, have an encounter at a minimal distance d and at a relative speed of $u_{\text{rel}} = u_e - u_p$. The Coulomb field leads to a mutual attraction with the corresponding Coulomb force $F_p = -F_e$ with $F_e \propto d^{-2}$ and subsequently to an acceleration $a_p = F_p/m_p$ and $a_e = F_p/m_e$. This interaction happens on a characteristic timescale $\tau \approx d/u_{\text{rel}}$ and the change in the speed of the particles is small compared to their velocity (small-angle scattering).

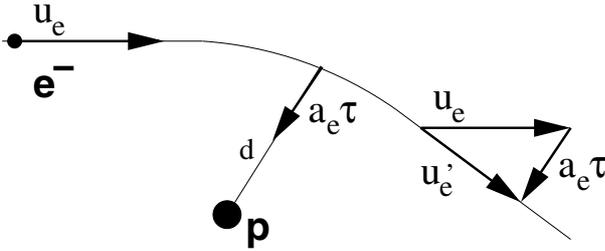


Fig. 1. Trajectory of an electron (e^-) at speed u_e . For simplicity the proton (p) is assumed to be at rest. At the closest approach, the acceleration $a_e \tau$ moves the electron closer to the proton and the electron loses kinetic energy.

After the interaction, the speed of the electron is $u'_e = u_e + a_e \tau$ and that of the proton is $u'_p = u_p + a_p \tau$. The change in kinetic energy for the electron and proton can be written as

$$\Delta E_{\text{kin},e} = m_e \tau a_e \cdot u_e, \quad (1)$$

$$\Delta E_{\text{kin},p} = -m_e \tau a_e \cdot u_p, \quad (2)$$

where we have made use of the fact that $a_p = -m_e/m_p a_e$ and the change in speed $|\Delta u| \ll |a\tau|$. Thus we can neglect terms of the order τ^2 . Both the electrons and the proton lose kinetic energy (a_e and u_e form an angle larger than $\pi/2$ and thus their scalar product is negative).

It is evident from Eqs. (1) and (2) that for a one-temperature plasma, where the mean speed of the electron is faster by a factor of $(m_p/m_e)^{1/2} \approx 43$ than the speed of the protons, most of the energy of the electron-proton bremsstrahlung arises from the kinetic energy change of the electrons.

However for a two-temperature plasma the ratio for the mean speed of protons and electrons is given by

$$\frac{u_e}{u_p} = \sqrt{\frac{T_e m_p}{T_p m_e}}. \quad (3)$$

For $T_p/T_e > m_p/m_e \approx 1836$ the mean speed of the protons exceeds the mean speed of the electrons and then the change in kinetic energy is greater for the protons. In this case it is the protons which are responsible for providing the energy to create a photon leading to electron-proton bremsstrahlung.

We now explore this mechanism in a classical and non-relativistic treatment of electron-proton bremsstrahlung in the dipole-approximation.

3. Non-relativistic electron-proton Bremsstrahlung

The emission per unit time t , volume V and photon energy $E_\nu = h\nu$ for a single proton can be expressed as (e.g. Rybicki & Lightman 1979)

$$\frac{dW}{dE_\nu dV dt} = \frac{16\alpha_f r_e^2 c^2}{3u_{\text{rel}}} n_e n_p \log\left(\frac{b_{\text{max}}}{b_{\text{min}}}\right), \quad (4)$$

where α_f is the fine-structure constant, r_e is the classical electron radius, c the speed of light, u_{rel} the relative velocity of the ion with respect to the electron, and n_e and n_p are the electron and ion number densities, respectively.

b_{max} and b_{min} are the maximum and minimum value of the impact parameter. Through the definition of the Gaunt factor

$$g_{\text{ff}}(u_{\text{rel}}, \omega) = \sqrt{3}/\pi \log(b_{\text{max}}/b_{\text{min}}), \quad (5)$$

where $\omega = 2\pi\nu$ and ν is the frequency of the bremsstrahlung photon, and subsequent use of quantum mechanical calculations of $g_{\text{ff}}(u_{\text{rel}}, \omega)$ we do not need to specify the maximum and minimum impact parameters here and instead can use Gaunt factors from the literature which already account for quantum mechanical effects.

Substituting the Gaunt factor into (4), we get

$$\frac{dW}{dE_\nu dV dt} = \frac{16\pi\alpha_f r_e^2 c^2}{3\sqrt{3}u_{\text{rel}}} n_e n_p g_{\text{ff}}(u_{\text{rel}}, \omega). \quad (6)$$

For a one temperature plasma, the electrons are much faster (a factor of $(m_p/m_e)^{1/2} \approx 43$). Thus the protons can be considered to be at rest and the relative velocity is dominated by the speed of the electron. To get the electron-proton bremsstrahlung rate for this type of plasma, it is sufficient to average the single proton emission rate over the (assumed Maxwellian) velocity distribution of the electrons. Then the energy of the photon is extracted from the kinetic energy of the electron which slows, i.e. cools down.

For a two-temperature plasma, however, the proton speed can become comparable or even exceed the electron speed. Hence we need to consider that and write the relative speed as

$$u_{\text{rel}} = \sqrt{u_e^2 + u_p^2 - 2u_e u_p \cos \theta}, \quad (7)$$

where u_e and u_p are the electron and proton speed, respectively, and θ the angle between the two speed vectors.

For an isotropic Maxwellian distribution for the electrons and protons we have the probability dP_e for an electron to have a speed between u_e and $u_e + du_e$

$$dP_e = \sqrt{\frac{2}{\pi}} \left(\frac{m_e}{kT_e}\right)^{3/2} u_e^2 \exp\left(-\frac{m_e u_e^2}{2kT_e}\right) du_e, \quad (8)$$

and the corresponding probability dP_p for a proton to have a speed between u_p and $u_p + du_p$

$$dP_p = \sqrt{\frac{2}{\pi}} \left(\frac{m_p}{kT_p}\right)^{3/2} u_p^2 \exp\left(-\frac{m_p u_p^2}{2kT_p}\right) du_p, \quad (9)$$

where k is the Boltzmann constant. Note that $\int dP_e$ and $\int dP_p$ are normalised to unity.

For the total emissivity of the electron-proton bremsstrahlung we have to average over dP_e and dP_p , respectively. We have to calculate, using Eqs. (6)–(9)

$$\epsilon_\nu = \int_{u_{\text{min},p}}^{\infty} \int_{u_{\text{min},e}}^{\infty} \frac{dW}{dE_\nu dV dt} dP_e dP_p, \quad (10)$$

where $u_{\text{min},e} = (2E_\nu/m_e)^{1/2}$ and $u_{\text{min},p} = (2E_\nu/m_p)^{1/2}$ accounts for the photon discreteness effect. The result then still depends on the angle θ to lie between the velocities u_e and u_p . We then need to integrate over the probability for an angle θ to occur. For an assumed isotropic distribution of angles, the probability for an angle between θ and $\theta + d\theta$ is

$$dP_\theta = \frac{1}{2} \sin \theta d\theta. \quad (11)$$

The angle averaged integral can be written in the form

$$\epsilon_\nu = \frac{16\pi\alpha_f r_e^2 c^2}{3\sqrt{3}} n_e n_p \bar{g}_{\text{ff}} \left(\frac{m_e m_p}{k^2 T_e T_p}\right)^{3/2} \quad (12)$$

$$\times \int_0^\pi \int_{u_{\text{min},p}}^{\infty} \int_{u_{\text{min},e}}^{\infty} \frac{u_e^2 u_p^2}{u_{\text{rel}}} \exp\left(-\frac{m_e u_e^2}{2kT_e} - \frac{m_p u_p^2}{2kT_p}\right) \sin \theta du_e du_p d\theta. \quad (13)$$

This finally has to be integrated over the photon energy to get the bremsstrahlung rate. For the energy and temperature dependent Gaunt factor, \bar{g}_{ff} , we use Sutherland (1998). Thus we do not longer need to justify our choice for the maximum and minimum impact parameter. Higher precision values are now hidden in the tabulated values of \bar{g}_{ff} , taken from Sutherland (1998).

The integral over the electron and proton velocities has to be taken carefully. As long as $u_e > u_p$, the electron-proton bremsstrahlung is mainly caused by the kinetic energy change of the electron. For $u_e < u_p$, the electron-proton bremsstrahlung is caused by the kinetic energy change of the proton, as then the electrons do not have much energy to put in the bremsstrahlung. The electron-proton bremsstrahlung rate, however, is then reduced by a factor m_e/m_p , correcting for centre of mass effects.

In our calculations, we divided the velocity integrals accordingly and get the contributions of protons and electrons to the electron-proton bremsstrahlung rate.

We give the corresponding analytic results for two limiting cases,

$$\epsilon_v \left(T_e \gg T_p \frac{m_e}{m_p} \right) = \frac{16}{3} \alpha_f r_e^2 c^2 n_e n_p \bar{g}_{\text{ff}} \sqrt{\frac{2\pi m_e}{3kT_e}} \exp\left(-\frac{E_v}{kT_e}\right)$$

and

$$\epsilon_v \left(T_e \ll T_p \frac{m_e}{m_p} \right) = \frac{16}{3} \frac{m_e}{m_p} \alpha_f r_e^2 c^2 n_e n_p \bar{g}_{\text{ff}} \sqrt{\frac{2\pi m_p}{3kT_p}} \exp\left(-\frac{E_v}{kT_p}\right),$$

where the first result includes a one-temperature plasma, where the protons are virtually at rest and the second corresponds to a two-temperature plasma, where only the protons contribute to the electron-proton bremsstrahlung and the electrons are virtually at rest.

Energy integrated this leads to

$$\epsilon \left(T_e \gg T_p \frac{m_e}{m_p} \right) = \frac{16}{3} \alpha_f r_e^2 c^2 n_e n_p \bar{g}_{\text{ff}} \sqrt{\frac{2\pi}{3} m_e k T_e} \quad (14)$$

and

$$\epsilon \left(T_e \ll T_p \frac{m_e}{m_p} \right) = \frac{16}{3} \frac{m_e}{m_p} \alpha_f r_e^2 c^2 n_e n_p \bar{g}_{\text{ff}} \sqrt{\frac{2\pi}{3} m_p k T_p}. \quad (15)$$

Here, \bar{g}_{ff} is the frequency, i.e. photon energy, averaged Gaunt factor (approximately 1.2...1.5)

The results for the one-temperature plasma reproduces the well known result from the literature (e.g. Rybicki & Lightman 1979). For the two-temperature plasma the electron-proton bremsstrahlung rate depends on the temperature of the protons, accordingly. In either case the formulae only depend on the temperature of the species which mainly cause bremsstrahlung. The general dependence on the temperature is the same in both limiting cases (proportional to $T^{-1/2} \exp(-E_v/(kT))$ in the energy-dependent and $T^{1/2}$ in the energy integrated case).

We show sample electron-proton bremsstrahlung spectra for a gas at different electron and proton temperatures in Fig. 2. Note that the electron component of the electron-proton bremsstrahlung shows a high-energy cut-off at lower energies than the proton component. Although the magnitude of electron-proton bremsstrahlung caused by the kinetic energy of protons is always lower than that of bremsstrahlung created by the kinetic energy of electrons for the cases considered here, the total, energy integrated bremsstrahlung contribution of protons can be dominant due to the higher high energy cut-off. This effect is

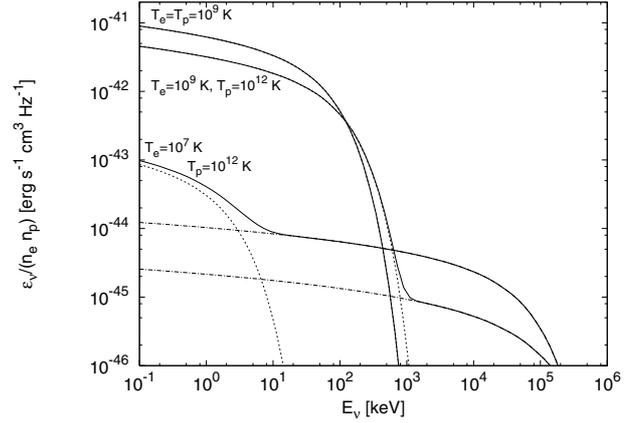


Fig. 2. Electron-proton Bremsstrahlung emissivity ϵ_v of an optically thin gas for different combinations of electron and proton temperatures. The total emissivity is plotted as a solid line, the contribution of electrons as a dotted line and the proton contribution as a dash-dotted line. Note the different relative contributions of proton and electron bremsstrahlung for the same proton temperature, but different electron temperature.

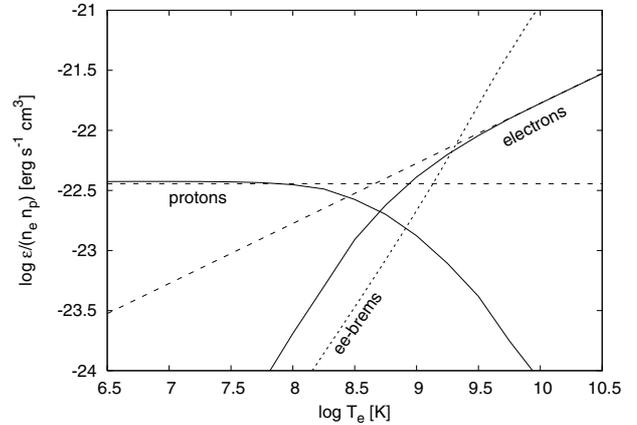


Fig. 3. Electron-proton Bremsstrahlung emissivity of an optically thin gas for different electron temperatures at a proton temperature of $T_p = 10^{12}$ K. For electron temperatures lower than $m_e/m_p T_p \approx 5.4 \times 10^8$ K the contributions of the protons dominate (then the electron-proton bremsstrahlung rate is essentially independent of the electron temperature), while for larger electron temperatures the classical result is reproduced. The long-dashed lines indicate the limiting cases calculated in Eqs. (14) and (15), respectively, while the short-dashed line gives the electron-electron Bremsstrahlung rate of Svensson (1982). Note that for electron temperatures in excess of approx. 3×10^9 K the electrons are relativistic and the plotted bremsstrahlung rate may be a very crude approximation in this regime.

shown in Fig. 3 where we plot the energy integrated electron-proton bremsstrahlung rate for protons and electron as a function of electron temperature. We also plot the corresponding electron-electron bremsstrahlung rate, taken from Svensson (1982).

The results for the electron-proton bremsstrahlung are only valid for non-relativistic protons and electrons. However a first-order estimate of the relativistic correction can be made by multiplying the energy integrated bremsstrahlung rate with $\theta_p^{1/2}$ and $\theta_e^{1/2}$ in the relativistic case ($\theta_e = kT_e/(m_e c^2)$ and $\theta_p = kT_p/(m_p c^2)$). This leads to a steepening of the temperature dependence from $T^{1/2}$ in the non-relativistic to T in the relativistic energy integrated case as indicated by Svensson (1982) for $\theta_e \gg 1$. Gould (1980, 1981) finds a first-order correction factor of $(1 + 11/24\theta_e)$ for $\theta_e \ll 1$.

Figure 3 shows that for electron temperatures in excess of 10^9 K electron-electron bremsstrahlung is the dominant emission process. Hence a relativistic treatment of electron-proton bremsstrahlung may not be necessary. Haug (1975) however shows that although electron-electron bremsstrahlung dominates the emission the electron-proton bremsstrahlung still contributes half of the electron-electron bremsstrahlung rate.

4. Conclusions

We have recalculated electron-proton bremsstrahlung for a two-temperature plasma. Owing to the increasing importance of the proton speed relative to the electron speed for $T_e < T_p$, the protons can contribute significantly to the electron-proton bremsstrahlung. They dominate the electron-proton Bremsstrahlung losses for electron temperatures lower than $m_e/m_p \approx 1/1836$ times the proton temperature.

While our results are strictly valid only in the non-relativistic regime, we give a crude extrapolation for the relativistic case. The work presented here should only be considered as exploratory work to outline the influence of a two-temperature plasma on the electron-proton bremsstrahlung. If calculated for both the non-relativistic and relativistic regime, one needs to account for the relativistic kinematics and consider the Bethe & Heitler (1934) cross section, adjusted for the effect presented here. While it is certainly needed and desirable, such a treatment is beyond the scope of this paper. The apparent dominance of electron-electron bremsstrahlung for electron temperatures in excess of 10^9 K, however, does not make it that necessary.

The mechanism presented here, seen in the context of Coulomb collisions between electrons and protons, is already well known in plasma physics. The NRL Plasma Formulary (Huba 2006), a standard reference in plasma physics for more than 25 years, gives a formula for the Coulomb collision rate which depends only on the electron temperature for $T_e > m_e/m_p T_p$. This is the classical Spitzer (1962) result. For higher proton temperatures, the electron temperature dependence weakens and the collision rate only depends on the proton temperature owing to their larger speed compared to the electrons. This effect is seen as well

in the work by Stepney (1983) who derives the Coulomb collision rate in a fully relativistic framework. Similar to our extrapolation for the electron-proton bremsstrahlung rate, they find a change in the exponent of the temperature dependence of $+1/2$, i.e. from $T^{-3/2}$ to T^{-1} .

The importance of the effect presented in this paper needs to be examined by comparing the characteristic timescales for Coulomb collisions to equilibrate electron and proton temperature and the Bremsstrahlung timescale. While it may not be of that strong influence for the energetics of the two-temperature plasma, it could have an observable signature due to the higher high-energy cut-off for the Bremsstrahlung created by the kinetic energy of the protons, if it is not hidden behind some more important emission mechanisms at these energies.

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