Multicomponent radiatively driven stellar winds

IV. On the helium decoupling in the wind of \( \sigma \) Orionis \( \text{E} \)★

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ABSTRACT

We study the possibility of the helium decoupling in the stellar wind of \( \sigma \) Ori E. To obtain reliable wind parameters for this star we first calculate NLTE wind model and derive wind mass-loss rate and terminal velocity. Using corresponding force multipliers we study the possibility of helium decoupling. We find that helium decoupling is not possible for realistic values of helium charge (calculated from NLTE wind models). Helium decoupling only seems possible for a very low helium charge. The reason for this behavior is the strong coupling between helium and hydrogen. We also find that frictional heating becomes important in the outer parts of the wind of \( \sigma \) Ori E due to the collisions between some heavier elements and the passive components – hydrogen and helium. For a metallicity ten times lower than the solar one, both hydrogen and helium decouple from the metals and may fall back onto the stellar surface. However, this does not explain the observed chemical peculiarity since both these components decouple together from the absorbing ions. Although we do not include the effects of the magnetic field into our models, we argue that the presence of a magnetic field will likely not significantly modify the derived results because in such case model equations describe the motion parallel to the magnetic field.

Key words. stars: winds, outflows – stars: mass-loss – stars: early-type – hydrodynamics – stars: chemically peculiar

1. Introduction

The radiative force plays an important role in the atmospheres of early-type stars. In luminous early-type stars it may be greater than the gravitational force. In such a case, the outer stellar layers can hardly be in hydrostatic equilibrium. Consequently, the radiatively driven wind is blowing from these stars (see, e.g., Kudritzki & Puls 2000; Owocki 2004; or Krtička & Kubát 2006, for a review). For many cooler early-type stars the radiative force is not able to expel the material into the circumstellar environment; however, it is able to cause elemental drift and, consequently, abundance stratification (e.g., Vauclair 2003; Michaud 2005) and becomes manifested as chemical peculiarity.

Hot stars with radiatively driven stellar winds and hot chemically peculiar stars seem to form two diverse groups of stars. While the flow in the atmospheres of stars with strong winds would eliminate any possible abundance stratification and chemical peculiarity, the atmospheres of chemically peculiar stars are supposed to be very quiet (likely due to magnetic fields) to enable elemental diffusion (e.g., Vaclář 1975).

However, there exists a small group of hot stars for which both effects, stellar wind and chemical peculiarity, seem to be important. The sparsely populated group of helium-rich stars probably exhibits both effects. In some cases, chemical peculiarity can be even used as a test of wind existence (Landstreet et al. 1998; Dworetsky & Budaj 2000). Some of the stars with parameters corresponding to hot chemically peculiar stars may have also purely metallic winds, as proposed by Babel (1996). One of the most enigmatic stars of the group of helium-rich stars is the star denoted as \( \sigma \) Ori E or HD 37479. To our knowledge, this star is the only known Bp star with hydrogen lines in emission. Its photometrical and emission line profile variability can be explained assuming the circumstellar cloud model (Smith & Groote 2001; Townsend et al. 2005). Similarly to the star HD 37776 (Mikulášek et al. 2006), this star may exhibit rotational braking, probably due to the magnetically controlled stellar wind (Oksala & Townsend 2006).

Stellar winds of hot stars are accelerated mainly due to the light scattering in the lines of elements like carbon, nitrogen, oxygen, or iron. Because these heavier elements have much lower density than the bulk of the flow (composed of hydrogen and helium), hot star winds have a multicomponent nature (Castor et al. 1976; Springmann & Pauldrach 1992; Curé 1992; Babel 1995; Krtička & Kubát 2000, 2001, hereafter KKII). Momentum is transferred from low-density heavier elements to the high-density component composed of hydrogen and helium due to Coulomb collisions of charged particles. The corresponding frictional force depends on the velocity difference between wind components. For very low velocity differences, lower than the mean thermal velocity (basically for stars with dense winds), the flow is well coupled. For higher velocity differences (basically for stars with low density winds) the Coulomb collisions are not able to efficiently transfer momentum from heavier ions to hydrogen and helium, and frictional heating becomes

★ Appendix is only available in electronic form at http://www.aanda.org
2. NLTE wind model

2.1. Basic assumptions

Since there is no accurate determination of \( \sigma \) Ori E wind parameters from observation, we decided to calculate wind mass-loss rate and terminal velocity using our NLTE wind models (Krtička & Kubát 2004) first. These models make it possible to solve (generally multicomponent) hydrodynamical equations for radiatively driven winds and enable prediction of mass-loss rates and terminal velocities. The radiative force and the radiative heating/cooling term are calculated with NLTE level populations obtained using atomic data based on an extended set of the TLUSTY input files (Hubeny 1988; Hubeny & Lanz 1992, 1995; Lanz & Hubeny 2003). The data are based on the Opacity Project (Seaton 1987; Luo & Pradhan 1989; Sawey & Berrington 1992; Seaton et al. 1992; Butler et al. 1993; Nahar & Pradhan 1993), the Iron Project (Hummer et al. 1993; Bautista 1996; Nahar & Pradhan 1996; Zhang 1996; Bautista & Pradhan 1997; Zhang & Pradhan 1997; Chen & Pradhan 1999), and the VALD database (Piskunov et al. 1995; Kupka et al. 1999). The most important simplification of our code compared to other models available in the literature (Vink et al. 1999; Pauldrach et al. 2001; Gräfener & Hamann 2005) is the simplified treatment of the radiative transfer equation. We use the Sobolev approximation for the solution of the radiative transfer equation in lines (Sobolev 1947; Rybicki & Hummer 1978), and we neglect UV line blocking as well as line overlaps.

For the calculation of wind models we assume spherical symmetry. Since it is well-known that \( \sigma \) Ori E has a magnetic field (Landstreet & Borra 1978) that generally breaks down the assumption of spherical symmetry, our models, at least approximately, describe the flow along individual field lines. Note that to obtain a more realistic description of the multicomponent wind in the presence of a stellar magnetic field, it would be necessary to solve the hydrodynamical equations along the field lines with an inclusion of the proper projections of individual forces acting on the wind or to use the magnetohydrodynamical simulations. In the case of a one-component wind model, the former was done in a simplified form by Friend & MacGregor (1984) and MacGregor & Friend (1987), and more recently, the latter, using magnetohydrodynamical simulations by ud-Doula & Owocki (2002).

Stellar parameters appropriate for \( \sigma \) Ori E (Hungler et al. 1989) are given in Table 1. The stellar radiative flux (the lower boundary of the wind solution) of \( \sigma \) Ori E is calculated using the spherically symmetric NLTE model atmosphere code described in Kubát (2003).

Since we intend to study self-initiation of the chemical peculiarity of \( \sigma \) Ori E, we also assume solar helium abundance. If not explicitly stated, we assume solar chemical composition for heavier elements.

### Table 1. Adopted stellar (Hungler et al. 1989) and calculated wind parameters of \( \sigma \) Ori E.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Effective temperature ( T_{\text{eff}} )</td>
<td>22 500 K</td>
</tr>
<tr>
<td>Stellar mass ( M )</td>
<td>8.9 ( M_\odot )</td>
</tr>
<tr>
<td>Stellar radius ( R_\odot )</td>
<td>5.3 ( R_\odot )</td>
</tr>
<tr>
<td>Log ( g ) [CGS]</td>
<td>3.94</td>
</tr>
<tr>
<td>Mass-loss rate ( M )</td>
<td>( 2.4 \times 10^{-9} M_\odot ) yr(^{-1} )</td>
</tr>
<tr>
<td>Terminal velocity ( v_\infty )</td>
<td>1460 km s(^{-1} )</td>
</tr>
</tbody>
</table>

![Fig. 1. The radial variation of helium ionization fractions in the wind calculated using the NLTE model. He II is the dominant ionization stage throughout the wind. Close to the star (above the photosphere) the wind density decreases; consequently, the wind becomes more ionized and the ionization fraction of He I decreases with the increasing radius, while the fraction of He III increases with the radius. In the outermost regions the wind becomes slightly less ionized again, due to the decrease of the mean intensity.](image-url)
Fig. 2. Calculated relative velocity differences (see Eq. (1)) between individual heavier elements and the passive component. Velocity differences for elements with low $x_{hp}$ values are not plotted in this graph. Note that here the effects of frictional heating are neglected.

Since the radiative acceleration acting on individual heavier elements is different, each element moves with slightly different velocity. The dimensionless velocity difference between a given individual heavier element $h$ and hydrogen and helium,

$$x_{hp} = \frac{v_h - v_p}{\alpha_{hp}},$$  \hspace{1cm} (1)

is roughly equal to (Krtička 2006)

$$x_{hp} \approx g_h^{\text{rad}} \frac{m_h}{m_p} \frac{3kT}{8\sqrt{\pi}q_h^2q_p^2} \ln \Lambda.$$  \hspace{1cm} (2)

We use the subscript $h$ to denote individual heavier elements, i.e., $h$ stands for N, Si, S, etc. For this purpose we do not use the subscript $i$ that denotes all heavier elements described together. The subscript $p$ denotes the passive component (hydrogen and helium). Here $v_h$ and $v_p$ are the radial velocities of the components, $m_h$ and $m_p$ are their atomic masses, $q_h$ and $q_p$ their charges, $g_h^{\text{rad}}$ is the radiative acceleration acting on a given element $h$, $n_p$ is the number density of hydrogen and helium, $T$ is the temperature (assuming that the temperatures of all wind components are the same), $\ln \Lambda$ is the Coulomb logarithm, and finally,

$$\alpha_{hp}^2 = \frac{2kT(m_p + m_h)}{m_p m_h}.$$  \hspace{1cm} (3)

Using Eq. (2) we calculated the approximate velocity differences between hydrogen and helium and individual heavier ions (see Fig. 2). Apparently, the relative velocity differences between silicon or nitrogen and the hydrogen-helium component are so high that frictional heating may be enabled. To test this possibility we calculated four-component wind models. Despite the fact that the relative velocity difference between silicon and the passive component is slightly higher than that of nitrogen, we selected the nitrogen component for this test because nitrogen has higher abundance and, consequently, heating effects caused by friction are expected to be higher for this element. Resulting four-component wind model (with wind components hydrogen plus helium, free electrons, nitrogen, and the remaining heavier ions) is given in Fig. 3. Apparently, the stellar wind temperature significantly increases in the outer parts due to frictional heating. However, no decoupling occurs in this case (since $x_{np}$ is too low to enable the decoupling). The wind terminal velocity significantly increases compared to the case when the frictional heating is neglected. This is caused by the change in the ionization and excitation equilibrium due to a higher wind temperature. Since this effect occurs for velocities higher than that at the critical point, the mass-loss rate is not essentially modified in the four-component models. A similar effect, but to a lesser extent, was already reported by Krtička (2006).

Because the effects of frictional heating between heavier elements (in our case nitrogen) and the passive component are important basically only for velocities higher than the escape velocity (which is $v_{\text{esc}} = 790 \text{ km s}^{-1}$ at the stellar surface), they
The velocity structure of a calculated four-component wind
model for a realistic value of helium charge $\alpha = 1$. Hydrogen and helium velocities are nearly identical and helium does not decouple.

To test the possibility of helium decoupling we calculated four-component wind models with the four following explicitly included wind components: hydrogen, helium, heavier ions, and free electrons. For this purpose we did not use our NLTE models, but simpler (and faster) models with the radiative force in the CAK approximation (Castor et al. 1975; see KKII for the description of these models). The parameterized description of the radiative force in these models enables us to vary both wind parameters and wind ionization state independently to discuss the parameter space for which helium decoupling seems possible.

To model the mass and charge of heavier ions, we selected parameters corresponding to oxygen (because elements like C, N, O, Si, and S are important for radiative driving). The selection of oxygen does not have a significant impact on derived results. The CAK force parameters

$$k = 0.38, \quad \alpha = 0.50, \quad \delta = 0.03$$

(CAK, Abbott 1982; Puls et al. 2000) are chosen to obtain the same mass-loss rate and a similar velocity profile as derived from our NLTE models. Note that the terminal velocity calculated using these parameters is slightly higher than calculated from NLTE models. However, this does not significantly influence our analysis.

3.2. Realistic helium charge

We calculated four-component wind models with the realistic value of the helium charge taken from the previously discussed NLTE model. Since helium is mostly singly ionized in the wind region (see Fig. 1), we simply assume $z_\alpha = 1$, where $z_\alpha$ is the relative charge in units of the elementary charge $e$. $q_\alpha = e z_\alpha$.

The velocity profile of a calculated four-component wind model is shown in Fig. 4. Evidently, the radial velocities of all wind components are nearly the same and the wind components are well coupled. Consequently, helium decoupling cannot be found in this case.

3.3. Low helium charge

For a very low helium charge, significantly lower than that calculated from NLTE equations, helium decoupling is possible. As an example, in Fig. 5 we plot a comparable wind model, but for helium charge $z_\alpha = 0.01$. Apparently, the wind density at the wind base is high enough to accelerate helium from the stellar surface to supersonic velocities. However, due to its very low charge, helium decouplings from the mean flow at velocities lower than the escape velocity. Consequently, helium remains bound to the star and may fall back to the stellar surface. Hydrodynamical simulations are necessary to study this process in detail (cf. Porter & Skouza 1999; Votruba et al. 2006).

3.4. Helium coupling

The main result of this paper – that for realistic values of its charge helium does not decouple from the wind flow for parameters suitable for a star like $\sigma$ Ori E – seems to be in contradiction to the result of Hunger & Groote (1999), who argued on the basis of the work of Babel (1995, 1996) for the decoupling hypothesis that explains the observational results. To find the reason for this difference, we study the hydrodynamical equations analytically. The equation of motion for helium (neglecting the radiative force acting on helium, the gas-pressure term, and the charge separation electric field) has the form of (Burgers 1969)

$$\frac{d v_\alpha}{d r} = -g + \frac{1}{\rho_\alpha} \sum_{a=H,e,i} K_{ab} g_{\mathrm{Ch}}(x_{ab}) \frac{v_\alpha - v_b}{|v_\alpha - v_b|},$$

where $v_\alpha$ is the helium radial velocity, $\rho_\alpha$ is the helium mass density, $g_{\mathrm{Ch}}(x)$ is the Chandrasekhar function, and $g$ is the gravitational acceleration, $g = GM/r^2$. The summation of the frictional force in Eq. (5) takes into account the friction due to hydrogen “H”, free electrons “e”, and all heavier ions “i” (the latter are described as one component). The frictional coefficient $K_{ab}$ due to the Coulomb collisions between any component $a$ and $b$ is given by

$$K_{ab} = n_\alpha n_b \frac{4\pi q_a^2 q_b^2}{kT_{ab}} \ln \Lambda,$$
where \( n_a \) and \( n_b \) are the number densities of individual components and \( q_a, q_b \) are charges of these components. The mean temperature is

\[
T_{ab} = \frac{m_a T_a + m_b T_b}{m_a + m_b} \tag{7}
\]

In the case of the helium decoupling, helium has to decouple from both hydrogen and heavier ions. The maximum possible value of the Chandrasekhar function is in both cases the same; however, the value of the frictional coefficient is different, \( K_{ab} \gg K_{ai} \), since \( n_H \gg n_i \) (assuming an ionized medium). This means that hydrogen-helium collisions are extremely important for the dynamics of helium component. Consequently, hydrogen may be (depending on its ionization state) much more important for the helium acceleration than the heavier ions. The neglected strong coupling between hydrogen and helium is the main reason why Hunger & Groote (1999) concluded that the helium decoupling from flow of hydrogen and passive plasma is possible (even for realistic parameters of wind plasma, like the helium ionization state).

However, the situation may differ from the extreme case described above. The contributions of hydrogen and heavier ions to helium acceleration may be equally important due to different values of the relative velocity differences \( x_{ab} \) and \( x_{ai} \) that are arguments of the Chandrasekhar function (see Appendix A for a more detailed approximate description of the flow). However, in the case where the velocity difference between helium and heavier ions is large, \( x_{ai} \approx 1 \), the frictional force due to hydrogen dominates. Note that after the decoupling of helium and hydrogen (in the particular case displayed in Fig. 5 for \( r/R \approx 1.06 \)), helium still remains coupled to electrons. This is due to the fact that \( x_{ai} \gg x_{ab} \). The coupling of helium to electrons, however, modifies the helium velocity only slightly.

The only possibility of how to enable helium to fall back onto the stellar surface is to break the strong coupling between helium and hydrogen while keeping the coupling between hydrogen and heavier ions. The only way to secure this in our self-consistent models is to lower the helium charge. This is why for very low helium charges, helium decoupling is possible.

We may derive an approximate condition for the helium decoupling in the stellar wind. If helium decouples from the stellar wind and subsequently falls back, there is a point where the helium velocity gradient is zero (see Fig. 5). The helium momentum Eq. (5) has, at this point, the form of

\[
g = \frac{1}{\rho_0} K_{He} G_{Ch}(x_{He}) \tag{8}
\]

(neglecting friction with heavier ions and electrons). The Chandrasekhar function is limited by its maximum value \( G_{Ch}(x_{He}) \leq G_{\text{max}} \). In the case of decoupling, the frictional acceleration on the right-hand side of Eq. (8) is lower than the gravitational acceleration (inequality occurs in Eq. (8)). With \( g = GM/r^2 \), the hydrogen mass-fraction \( X \approx 0.71 \), and using Eq. (6), the condition for helium decoupling for the radial velocity \( v \) is

\[
\frac{m_\text{He} m_\text{H} k GM}{e^2 X \ln A_{\text{max}}} \geq \frac{M^2 \dot{a}_{\text{He}}}{v_0^2} \tag{9}
\]

where \( e \) is the elementary charge. Note that the wind temperature \( T \) (or, explicitly, the mean temperature \( T_{\text{He}} \), see Eq. (7))) is in our case usually influenced by the frictional heating. Using scaled quantities, expression (9) can be rewritten as

\[
\dot{a}_{\text{He}}^2 \left( \frac{M}{10^{12} M_\odot/\text{year}} \right) \leq \frac{3}{x_{\text{He}}} \left( \frac{M}{1 M_\odot} \right) \left( \frac{T}{10^5 \text{ K}} \right) \left( \frac{v}{100 \text{ km s}^{-1}} \right). \tag{10}
\]

A similar condition was obtained by Owocki & Puls (2002, Eq. (23) therein), but for coupling between hydrogen and heavier ions. The inequality in expression (10) gives a condition for the helium charge that enables helium decoupling. For example, for values taken from the model displayed in Fig. 5, we obtain the relative helium charge necessary for decoupling \( z_{\text{He}} \approx 0.01 \), consistent with our previous detailed calculations.

3.5. The possibility of the decoupling of neutral helium

Our conclusion that helium decoupling is unlikely in the case of a realistic helium charge is based on the assumption that all helium ionization stages can be treated as a single wind component. This means physically that the processes of ionization and recombination are so rapid that the typical time of recombination or ionization is much shorter than the characteristic time of change in flow properties (i.e., velocity and density). While the violation of this condition would not affect the results in the case of doubly ionized helium (because it has higher charge than the singly ionized helium and, consequently, is more tightly coupled to hydrogen, see Eq. (6)), the violation of this condition in the case of neutral helium may offer an interesting possibility of a helium decoupling mechanism. The reason for this is that the cross-section of collisions of protons with neutral helium is much lower than the effective cross-section for collisions between charged particles (Kristić & Shultz 1999).

To test this, we compare the characteristic time of change in flow properties (roughly corresponding to the time during which the velocity increases to its actual value)

\[
\tau_{\text{flow}} = \left( \frac{dr}{dR} \right)^{-1}, \tag{11}
\]

with the typical time of ionization of He I

\[
\tau_{\text{ioniz}} = \frac{1}{P_{\text{He I}}-\text{He II}}, \tag{12}
\]

where \( P_{\text{He I}}-\text{He II} \) is the ionization rate of He I (Mihalas 1978). This value was taken from our NLTE models. Since excited levels of neutral helium have very low occupation numbers, we neglected their contribution to \( P_{\text{He I}}-\text{He II} \).

Comparison of characteristic times \( \tau_{\text{flow}} \) and \( \tau_{\text{ioniz}} \) is given in Fig. 6. The characteristic time \( \tau_{\text{flow}} \) first slightly decreases with radius and then increases again. This marks the region of fast wind acceleration. Also, the He I ionization time increases...
with radius. However, the characteristic time $\tau_{\text{flow}}$ is much larger than the He I ionization time $\tau_{\text{ioniz}}$. Consequently, helium may be treated as one component and no decoupling between different helium ionization stages occurs.

4. Models with lower metallicity

Since the metallicity of individual surface elements of $\sigma$ Ori E varies with location on the stellar surface, we also studied the influence of different metallicity on the possibility of helium decoupling. In particular, we selected the metallicity to be $Z/Z_\odot = 0.1$, as derived by Smith & Groote (2001) for $\sigma$ Ori E.

4.1. One-component NLTE wind model

Our NLTE wind model for metallicity $Z/Z_\odot = 0.1$ (not shown here) predicts a very low mass-loss rate of $\dot{M} = 7.4 \times 10^{-11} \, M_\odot \, \text{yr}^{-1}$. This is much lower than that which can be inferred from the simple relation $M \sim Z^{0.67}$ derived by Krtička (2006) for SMC O stars. This indicates that for lower wind densities the dependence of mass-loss rate on metallicity is more pronounced.

Steeper metallicity dependence of mass-loss rates for small metallicities may seem unexpected. To explain this behavior, let us first discuss the contribution of lines of different elements to the radiative force. This is connected with the value of the line optical depth $\tau$, which is in the Sobolev approximation given by (e.g., Abbott 1982, Eq. (1), neglecting the finite-disk correction)

$$\tau = \frac{\pi e^2}{m_e v_i j} \left( n_i - n_j \right) g_i f_i \left( \frac{\text{d}v}{\text{d}r} \right)^{-1},$$

where $n_i$, $n_j$, $g_i$, and $g_j$ are the number densities and statistical weights of individual states $i$ and $j$ that give rise to the studied line, $g_i f_i$ is the line oscillator strength, and $v_i j$ is the line frequency. The optically thick lines ($\tau > 1$) are important for the radiative acceleration. Note that the radiative force due to the optically thick lines does not depend on the number densities of involved atomic states.

Very close to the star, the Sobolev optical depth, Eq. (13), is large for many lines both of iron group elements and of lighter elements (like carbon, nitrogen, etc.), mainly due to the very large wind density. Since iron group elements have effectively larger number of lines than lighter elements, the former dominate the radiative force close to the star. Farther out in the wind, the line optical depth, Eq. (13), is lower (mainly due to decreasing wind density). Since iron group elements have lower abundances than carbon and nitrogen (i.e., effectively lower $n_i$), many lines, e.g. iron ones, become optically thin ($\tau < 1$), and these lines do not significantly contribute to the radiative force at higher wind velocities. Consequently, the relative contribution of iron group elements to the radiative force decreases with increasing wind velocity, and lighter elements become more important for the radiative acceleration (see also Vink et al. 2001). This behavior can also be interpreted as the result of the fact that lighter elements and iron group elements dominate different parts of the line-strength distribution function (Puls et al. 2000). In our case (even for the solar metallicity model) the iron lines already become inefficient close to the critical point due to a very low wind density.

However, for low metallicities and densities, many lines of lighter elements that are optically thick for higher metallicities, also become optically thin. This leads to a significant decrease of the mass-loss rate. In our specific case there is only one optically thick line (the C III resonance line at $\lambda = 977$ Å) that significantly contributes to the radiative force at the critical point. From the contribution of this optically thick line we can calculate the value of the $\hat{a}$ parameter (cf. Puls et al. 2000, Eq. (26)) as $\hat{a} = 0.46$. Using Eq. (87) of Puls et al. (2000), we can roughly estimate the dependence of the mass-loss rate on the metallicity as $M \sim Z^{1.2}$. This is more consistent with the decrease of the mass-loss rate with metallicity obtained in our case than the dependence $M \sim Z^{0.67}$ derived by Krtička (2006) for the denser wind of SMC stars. Note however that the value of $\hat{a}$ may itself depend on the metallicity. In terms of the line-strength distribution functions (Puls et al. 2000), the steepening of the dependence of mass-loss rate on metallicity may be explained as a consequence of the steepening of the line-strength distribution function for high line-strengths. Our result supports the findings of Kudritzki (2002), see Fig. 9 therein, who also found more pronounced dependence of mass-loss rate on metallicity for a very low-metallicity environment.

The calculated wind model is indeed very close to the wind existence limit. As discussed above, iron lines become inefficient for wind driving and the stellar wind itself is accelerated mainly due to carbon and nitrogen lines. CAK critical point is located closer to the stellar surface as a result of the decrease of a number of optically thick lines (cf. Babel 1995; Vink 2006).

For the calculation of NLTE models we neglected the multicomponent effects; however, from the values of relative velocity differences, Eq. (2), it is clear, that the stellar wind would decouple relatively close to the stellar surface. We have tested this possibility in the three-component model where the metallic ions are described as one component and concluded that already in this model the decoupling occurs close to the stellar surface.

The wind terminal velocity $v_\infty = 1120 \, \text{km} \, \text{s}^{-1}$ is lower than in the case of a solar-metallicity model (cf. Fig. 4). The lowering of the terminal velocity with respect to the solar metallicity model is an example of the dependence of the terminal velocity on metallicity.

The most important parameter that influences the possibility of helium decoupling is the helium charge. However, due to the lower wind density, helium is even more ionized than in the case of the solar metallicity model.

4.2. Four-component CAK model

We have calculated low-metallicity four-component wind models with helium as the fourth component again with the simpler models using the CAK radiative force. Although the helium ionization is slightly higher than that in the solar metallicity model, we assumed $\zeta_0 = 1$. To obtain the lower mass-loss rate inferred from NLTE calculations, we used $k = 0.23$. The other force multipliers are the same as in Eq. (4). It may seem that the adopted value for $k$ was not adequately lowered compared to the value adopted for the solar metallicity model (where $k = 0.38$ was used), because $k$ scales with metallicity as $k \sim Z^{1-\alpha}$ (e.g., Puls et al. 2000). However, the value $k = 0.23$ is appropriate for a four-component wind model of a star with low metallicity (i.e., $Z/Z_\odot = 0.1$). Because the radiative force in the four-component models is calculated using metal density, the decrease of wind metallicity causes lower radiative force and lower mass-loss rate. Consequently, with decreasing metallicity the value of $k$ in the four-component models varies only slightly.

The results from the calculated four-component wind model can be seen in Fig. 7. Apparently, the friction between metallic
ions and the passive component is low and is not able to accelerate the passive component to velocities higher than the escape velocity. Consequently, the wind components decouple and the passive component may start to fall back to the stellar surface. However, hydrogen and helium are still coupled together.

From the first point of view it could be expected that there would be a very high velocity gradient of heavier elements downstream from the point where the decoupling occurs. However, this is not the case; the velocity increase of heavier elements is only moderate after the decoupling. The reason for this behavior is the same as that which causes non-decoupling described for the two-component flow by Krtička & Kubát (2000; see the arguments in Sect. 3.2 therein). According to the momentum equation of heavier ions, the radiative force is approximately equal to the frictional force. Since there is a deficit of the frictional force in the region where decoupling starts to be effective, the radiative force is lower there. Consequently, due to the dependence of the radiative force on the velocity gradient in the Sobolev approximation, the velocity gradient of heavier ions decreases. Because the velocity of the passive component (hydrogen and helium) is lower than the escape velocity, this component decelerates and may fall back onto the stellar surface (Porter & Skouza 1999). The wind in this region is likely to be unstable for the perturbations of a multicomponent flow (Owocki & Puls 2002; Krtička & Kubát 2002).

The derived terminal velocity for the absorbing ions is very low in this case, roughly 130 km s\(^{-1}\). Consequently, due to low wind velocities there would be basically no X-rays emitted in this case by the possible wind shocks (provided that the absorbing ion component does not accelerate as a result of other instabilities). This model corresponds to models discussed by Groote & Schmitt (2004), who argued that the possible explanation of missing periodic X-rays coming from \(\sigma\) Ori E may be connected with too low wind velocities.

5. Discussion

Although we have used the best models available to us, there are many simplifications that may slightly modify the results obtained (see also KKII; Krtička 2006).

**Mass-loss rate:** the mass-loss rate of \(\sigma\) Ori E may be actually lower than that obtained from NLTE wind models, as was similarly deduced from observations of O stars with low luminosities (e.g., Bouret et al. 2003; Martins et al. 2004). To obtain helium decoupling for \(\sigma\) Ori E due to a low mass-loss rate, mass-loss rates of the order of \(10^{-13} M_\odot\) year\(^{-1}\) are necessary. However, in such a case, hydrogen also decouples from the flow (e.g., KKII). Therefore, we conclude that the change of the wind mass-loss rate likely does not influence our basic results.

Helium-rich stars could also eventually have a purely metallic wind, as proposed by Babel (1996). However, it is not clear how this could lead to a helium chemical peculiarity.

**Clumping:** winds of OB stars may be clumped (see Martins et al. 2005; or Fullerton et al. 2006, for recent observational support of the existence of wind clumping in O stars). It is not clear how this effect influences the multicomponent flow. Whereas the physical state of the interclump medium may significantly change due to the processes connected with the decoupling, it is possible that this effect is not very significant inside the clumps. This is caused by the fact that the frictional force depends on the wind density as \(\rho^2\), but on the other hand the mass-loss rate derived from observations may be overestimated due to the clumping. Possibly, both effects may relatively cancel and the net effect of wind clumping may be marginal. This picture however significantly depends on the physical properties of clumps and of surrounding media.

**Magnetic field:** we have also neglected the influence of the magnetic field, although it is known that \(\sigma\) Ori E has a strong one. The self-consistent treatment of this problem is very complicated and involves the solution of magnetohydrodynamic equations (e.g., ud-Doula & Owocki 2002). It should be noted that for such strong field as that of \(\sigma\) Ori E this has not yet been done, even in the one-component approximation, due to extremely low Courant time steps. An approximate treatment can be done by solving the hydrodynamical equations along corresponding field lines (e.g., Mestel 1968). Since this approach applies merely a geometric projection of corresponding terms in hydrodynamical equations, it is not likely that this would change the possibility of helium decoupling significantly.

Although MacGregor (1988) showed that strong magnetic fields may significantly influence the wind terminal velocity, Owocki & ud-Doula (2004) concluded that the change of the terminal velocity is only moderate. The latter argument supports our basic results to be valid also in the case of a strong magnetic field.

The magnetic field, however, may also change the frictional coefficients due to the particle gyration induced by the magnetic field or may also alter the velocity distribution function of individual species. The modification of a frictional coefficient due to the magnetic field could explain the observational finding of Groote & Hunger (1997) that the helium-enrichment in \(\sigma\) Ori E is found where the wind flow in the non-magnetic case will be mostly parallel to the magnetic field lines of this star. Similar findings for other helium strong stars were reported by Groote (2003).

The importance of the magnetic field for the collision frequency of helium atoms is given by the value of the product of the mean collision time between helium ions and protons \(\tau_{\text{coll}}\) and gyration frequency of helium ions \(\omega_\alpha\) (Mestel 2003,
where the frictional coefficient $K_{\text{eff}}$ is given by Eq. (6), and $\alpha_{\text{eff}}$ is given by Eq. (3). The gyration frequency is

$$\omega_{\alpha} = \frac{q_{\alpha} B}{m_{\alpha} c}$$  \hspace{1cm} (15)

where $B$ is the magnetic field intensity. For collision-dominated plasmas $\tau_{\text{coll}} \ll 1$, the influence of the magnetic field on the momentum transport between individual plasma components can be neglected. In the opposite case, $\tau_{\text{coll}} \gg 1$, a strong anisotropy of microscopic plasma properties may occur (in the directions parallel and perpendicular to the magnetic field). However, the frictional coefficients parallel to the magnetic field remain the same as in the case without a magnetic field. Consequently, the change of frictional coefficients due to the magnetic field is not important in the case when the magnetic energy density dominates over the gas kinetic energy density, i.e., in the case when plasma moves along the magnetic field lines.

The application of Eqs. (14) and (15) to our derived wind model has shown that for a typical magnetic field with intensities of the order of $10^3$ G, which corresponds to the surface magnetic field of $\sigma$ Ori E derived by Landstreet & Borra (1978), in the outer parts of the $\sigma$ Ori E wind, the inequality $\tau_{\text{eff}} \omega_{\alpha} \approx 1$ holds. This inequality holds even for lower magnetic field intensities of the order of $10^2$ G (the lower value of the magnetic field intensity roughly takes into the account its decrease with radius and its latitudinal variations). Since wind flows along magnetic field lines due to the strong wind confinement by the magnetic field, the above-mentioned variation of the frictional coefficient with the magnetic field is likely not very important in our case.

**Ionization:** Our results are sensitive to the ionization state of helium atoms. However, because the derived helium charge is higher by two orders of magnitude than the value necessary for helium decoupling, it seems that our results are also robust with respect to errors in the calculation of statistical equilibrium equations (note that helium is already dominated by its first ionization stage in the stellar atmosphere).

Even in the case when doubly ionized helium comprises a significant fraction of all helium atoms, some of them remain neutral (Fig. 1). This means that each helium atom for some (however very small) fraction of time is neutral. Recall that in the case of $\sigma$ Ori E, some wind material may remain trapped in the minima of gravitational and centrifugal potential along magnetic field lines (Townsend & Owocki 2005). The non-zero fraction of time spent by each helium atom in the neutral state may then induce the drift of helium atoms across magnetic field lines. Let us at least approximatively derive the magnitude of such an effect. Recall that the typical time of neutral helium ionization is $t_{\text{ioniz}}$ (see Eq. (12)) and the typical time of neutral helium recombination is introduced similarly as $t_{\text{rec}}$. During the time $t_{\text{ioniz}}$ the helium atom is not supported by the magnetic field (since it is neutral) and its absolute value of the radial velocity increases by (neglecting the centrifugal force) $g_{\text{ioniz}}$ where $g \equiv GM/\tau^2$. During the time $t_{\text{rec}}$, the helium atom is supported by the magnetic field and has zero radial velocity. Assuming that helium is mostly ionized, $t_{\text{rec}} \gg t_{\text{ioniz}}$, this means that the average velocity is roughly $g_{\text{ioniz}}/(t_{\text{rec}} + t_{\text{ioniz}}) \approx g_{\text{ioniz}}^2/t_{\text{rec}}$. The typical time necessary to travel the distance $R$, is $t_{\text{diff}} = R^2 t_{\text{rec}}/(GM^2 t_{\text{ioniz}}^2)$. For typical values for the outer wind parts $t_{\text{rec}} \approx 10^5$ s and $t_{\text{ioniz}} \approx 1$ s, the typical diffusion time $t_{\text{diff}} \approx 10^5$ years is too large compared to the typical time for the emptying of the magnetosphere, which is of the order of a hundred years (Townsend & Owocki 2005). Moreover, the drift may occur in the outward direction, since the total force acting on neutral helium atoms (i.e., the centrifugal force plus the gravitational force) may be directed outwards, and in such a case no fall back will occur. Note that the rotation of the magnetosphere containing charged particles may be roughly approximated as a rigid body rotation, at least near to the stellar surface.

**6. Conclusions**

We have studied whether the helium decoupling can occur in the stellar wind of $\sigma$ Ori E and, consequently, we tested whether this process may consistently explain enhanced helium abundance in the atmospheres of Bp stars (as proposed by Hunger & Groote 1999).

For this purpose we first calculated an NLTE wind model for $\sigma$ Ori E and derived its mass-loss rate, terminal velocity, and helium ionization. These parameters were consequently used for the calculation of four-component wind models to test whether helium decoupling may occur. As a by-product of our study, we have found that the frictional heating due to collisions between individual heavier elements (for example nitrogen) and the passive component (hydrogen and helium) is important in the outer wind regions of $\sigma$ Ori E.

Our four-component wind models showed that for realistic values of helium charge, the decoupling of helium from the stellar wind of $\sigma$ Ori E is unlikely. Helium decoupling is possible only for a very low helium charge, much lower than that derived from NLTE equations. We discussed the possibilities that may change this result, however, we were not able to find any consistent model that could lead to helium decoupling in the wind under such conditions.

The main result of our paper, that for realistic values of helium charge helium will not decouple from the wind flow for parameters suitable for a star like $\sigma$ Ori E, is in contradiction to the result of Hunger & Groote (1999) who obtained the opposite behavior. The reason for this is that helium is strongly coupled to hydrogen and this coupling cannot be neglected in model calculations.

We have shown that for lower metallicities the wind mass-loss rate is significantly lower. This is caused by a significant decrease of the number of optically thick lines. For such low mass-loss rates the decoupling of wind components and a subsequent fall back of the passive component may occur. However, helium and hydrogen remain coupled in this case; consequently, this does not explain the chemical peculiarity.

We studied the multicomponent flow analytically and derived a simple condition for helium decoupling in the stellar wind. This analysis supports our basic result that helium decoupling is possible only for a very low helium charge (i.e., when helium is basically neutral).

We have also studied the possibility of the decoupling of neutral helium. This could be an interesting way of helium decoupling (at least partially), since the cross-section of collisions of protons with neutral helium is much lower than the effective cross-section for collisions between charged particles. However, the neutral helium atoms do not remain neutral for a sufficiently long time since the characteristic helium ionization time is much
shorter than the characteristic time of change in flow properties.

As observations seem to favor the hypothesis of helium decoupling (Groote & Hunger 1997; Hunger & Groote 1999), our result leaves us in the very unpleasant situation that we do not have any self-consistent explanation for the chemical peculiarity of He-strong stars. While the scenario proposed by Hunger & Groote (1999) requires artificial lowering of the helium charge, models for helium diffusion moderated by the stellar wind (Vaucouleurs 1975; Michaud et al. 1987) require artificial lowering of the wind mass-loss rate. The only remaining influence on the helium decoupling, which is very difficult to calculate, is that of a strong magnetic field. Nevertheless, it seems clear that the decoupling of helium in normal (non-magnetic) stars with comparable stellar parameters can be excluded as a mechanism of creating the helium overabundance in the stellar atmosphere. Moreover, we were not able to find any self-consistent mechanism that would enable the decoupling in the case of magnetic stars.

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Online Material
Appendix A: Approximate description of H-He-i flow

To obtain an approximate description of a well coupled multicomponent flow of hydrogen, helium, and heavier ions\(^1\) (neglecting electrons), we can write the momentum equations of hydrogen (H) and helium (\(\alpha\)) as (Burgers 1969; neglecting the gas-pressure term and polarization electric field)

\[
\begin{align*}
\frac{d\nu_H}{dr} &= -g - \frac{K_{\alpha H}}{\rho_H} G_{\text{Ch}}(x_{\alpha H}) \frac{x_{\alpha H}}{|x_{\alpha H}|} + \frac{K_{H H}}{\rho_H} G_{\text{Ch}}(x_{H H}) \frac{x_{H H}}{|x_{H H}|}, \\
\frac{d\nu_{\alpha}}{dr} &= -g + \frac{K_{\alpha H}}{\rho_\alpha} G_{\text{Ch}}(x_{\alpha H}) \frac{x_{\alpha H}}{|x_{\alpha H}|} + \frac{K_{\alpha \alpha}}{\rho_\alpha} G_{\text{Ch}}(x_{\alpha \alpha}) \frac{x_{\alpha \alpha}}{|x_{\alpha \alpha}|}. \quad (A.1a)
\end{align*}
\]

We assume that the flow is well coupled, \(\nu_H \approx \nu_{\alpha} \approx \nu_i\), and we use only the linear term of the Taylor expansion of the Chandrasekhar function \(G_{\text{Ch}}(x) \approx \frac{2i}{x^2}\). Then we can rewrite Eqs. (A.1a) as

\[
\begin{align*}
\frac{d\nu_H}{dr} &= -g - \frac{k_{\alpha H}}{\rho_H} \Delta v_{\alpha H} + \frac{k_{H H}}{\rho_H} \Delta v_{H H}, \\
\frac{d\nu_{\alpha}}{dr} &= -g + \frac{k_{\alpha H}}{\rho_\alpha} \Delta v_{\alpha H} + \frac{k_{\alpha \alpha}}{\rho_\alpha} \Delta v_{\alpha \alpha} + \Delta v_{\alpha H}. \quad (A.2a)
\end{align*}
\]

where we used \(x_{ab} = \Delta v_{ab}/\alpha_{ab}\). Here \(k_{ab} = 2K_{ab}/(3\sqrt{\pi})\) and the velocity difference \(\Delta v_{ab} = \nu_a - \nu_b\) for all \(a, b\), which stand for \(H, \alpha\), and \(i\). Subtracting Eqs. (A.2b) and (A.2a) we obtain

\[
\Delta v_{\alpha H} \left( \frac{k_{H H}}{\rho_H \alpha H} - \frac{k_{\alpha H}}{\rho_\alpha \alpha H} \right) = \Delta v_{\alpha H} \left( \frac{k_{\alpha H}}{\rho_\alpha \alpha H} + \frac{k_{\alpha \alpha}}{\rho_\alpha \alpha \alpha} + \frac{k_{\alpha \alpha}}{\rho_\alpha \alpha \alpha} \right). \quad (A.3)
\]

Due to the very low number density of heavier ions \(n_\alpha\), we can neglect the second right-hand side term and derive (assuming that the temperatures of all wind components are equal; note that the similar term on the left-hand side cannot be neglected because all terms on the left-hand side are of the same order)

\[
\Delta v_{\alpha H} \left( \frac{n_\alpha}{n_\alpha} \frac{\alpha_H}{\alpha \alpha} \left( 1 - \frac{m_{\alpha H} \alpha_H^2}{m_\alpha \alpha \alpha \alpha_H^2} \right) \right) \left( 1 + \frac{\rho_\alpha}{\rho_\alpha} \right)^{-1} = \Delta v_{\alpha H}. \quad (A.4)
\]

The sign of \(\Delta v_{\alpha H}\) depends on the sign of the first bracket in Eq. (A.4).

\[
1 - \frac{m_\alpha \alpha \alpha_H^2}{m_\alpha \alpha \alpha_H^2} > 0, \quad (A.5)
\]

i.e., for a low helium charge, the hydrogen velocity is higher than the helium velocity. Because \(n_\alpha < n_i\), we derive \(\Delta v_{\alpha H} \gg \Delta v_{\alpha H}\) for a well coupled flow and realistic values of the other parameters. Helium decoupling may occur in the case \(\Delta v_{\alpha H} \gg \Delta v_{\alpha H}\). Apparently, this may occur for a very low helium charge \(z_\alpha\), as was demonstrated in the paper.

Finally, let us compare the frictional force acting on helium due to heavier ions and hydrogen in the case of a well coupled flow. The frictional acceleration after Eq. (5), assuming \(\nu_\alpha > \nu_H > \nu_i\) and \(\Delta v_{\alpha H} \gg \Delta v_{\alpha H}\), is

\[
\frac{1}{\rho_\alpha} \frac{K_{\alpha H} G_{\text{Ch}}(x_{\alpha H})}{\alpha_{\alpha H}} + \frac{1}{\rho_i} \frac{K_{H H} G_{\text{Ch}}(x_{H H})}{\alpha_{H H}} \approx \frac{k_{\alpha H} \Delta v_{\alpha H}}{\rho_\alpha \alpha_{\alpha H} \Delta v_{\alpha H}} \left( 1 + \frac{k_{\alpha H} \alpha_{\alpha H} \Delta v_{\alpha H}}{k_{\alpha H} \alpha_{\alpha H} \Delta v_{\alpha H}} \right). \quad (A.6)
\]

Inserting Eq. (A.4) for the fraction of velocity differences and performing just an order-of-magnitude estimate, we may assume \(\alpha_{\alpha H} \approx \alpha_{\alpha H}, z_\alpha \approx z_\alpha, \) and \(\alpha_i \alpha_H \approx \alpha_i H_{\alpha H}\) and conclude that both terms in

\[
1 + \frac{k_{\alpha H} \alpha_{\alpha H} \Delta v_{\alpha H}}{k_{\alpha H} \alpha_{\alpha H} \Delta v_{\alpha H}} \approx (A.7)
\]

are of the same order. This means that the contribution of heavier ions and hydrogen to the helium acceleration are of the same order in the case of a well coupled flow (e.g., for O supergiants). However, in the case when heavier ions are not able to effectively accelerate helium, the friction with hydrogen dominates.

\(^1\) Note that in this Appendix all heavier ions are described as one component.