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Incompatibility of long-period neutron star precession with creeping neutron vortices *(Research Note)*

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ABSTRACT

Aims. To determine whether "vortex creep" in neutron stars, the slow motion of neutron vortices with respect to pinning sites in the core or inner crust, is consistent with observations of long-period precession.

Methods. Using the concept of vortex drag, I discuss the precession dynamics of a star with imperfectly-pinned (i.e., "creeping") vortices.

Results. The precession frequency is far too high to be consistent with observations, indicating that the standard picture of the outer core (superfluid neutrons in co-existence with type II, superconducting protons) should be reconsidered. There is a slow precession mode, but it is highly over-damped and cannot complete even a single cycle. Moreover, the vortices of the inner crust must be able to move with little dissipation with respect to the solid.

Key words. stars: neutron – stars: pulsars: general – dense matter – stars: rotation

There is mounting evidence that some isolated neutron stars undergo long-period precession (nutation). The strongest evidence is found in PSR 1828-11, which shows highly-periodic variations in pulse phase over a period of ∼500 d, accompanied by correlated changes in beam width (Stairs et al. 2000). PSR 1642-03 also shows periodic changes in pulse phase, though correlated changes in the beam have not been detected (Shabanov et al. 2001). RX J0720.9-3125 is the first X-ray pulsar to show evidence for precession; the precession period is ∼7 yr, with correlated changes in line depth (Haberl et al. 2006). Low-level timing "noise", seen in all pulsars, is quasi-periodic in many cases, and could represent precession at low amplitude (for examples see, e.g., Downs & Reichley 1983; and D'Alessandro et al. 1993). Physically-motivated models of precession provide good fits to the data of PSR 1828-11 (Link & Epstein 2001; Akgün et al. 2006) and RX J0720.9-3125 (Haberl et al. 2006), supporting a precession interpretation.

The manner in which a neutron star precesses depends on the dynamics of its interior, and so observations of precession can be used to study the stellar interior. The purpose of this Note is to discuss the role that *quantized neutron vortices*, which are expected to occupy most of the neutron star interior, play in the dynamics of neutron star precession. I focus on recent work on this question and emphasize that pinning of vortices to magnetic flux tubes in the core, or to nuclei in the inner crust, is incompatible with observations of long-period precession.

The first part of this Note is aimed at constraining the vortex dynamics and state of the outer core. To make this argument, I assume a priori that there is no pinning or significant dissipation associated with the dynamics of vortices in the *inner crust*. At the end of the Note, I argue that the assumption of weak dissipation in the inner crust is required by observations, and obtain an upper bound on how strong the dissipation can be.

Nucleon pairing calculations predict that the outer core of a neutron star consists of a neutron superfluid in coexistence with superconducting protons (for a review, see, Dean & Hjorth-Jensen 2003). The superconductor is expected to be in a type II state (Baym et al. 1969a), so that the magnetic field that penetrates the core is organized in flux tubes (Baym et al. 1969b), long structures of microscopic cross section. The flux tubes probably have a very complicated arrangement that froze to the core medium when it became a superconductor shortly after the star's formation (Ruderman et al. 1998; Jones 2006). The superconducting protons, other charges (e.g., electrons and muons) and the crust are all coupled together through magnetic stresses over time scales of several seconds, nearly corotating as a rigid body (Easson 1979). By contrast, the neutron superfluid rotates by establishing a nearly rectilinear array of vortex lines, whose arrangement determines the angular momentum of the neutron fluid. The neutron vortices of the outer core, which are themselves magnetized through Fermi liquid effects (Alpar et al. 1984a), *pin* to the flux tubes (Sauls 1989). The origin of the pinning is that bringing a (magnetized) vortex close to a flux tube raises (or lowers, depending on orientation) the magnetic energy by ∼5 MeV per intersection. For typical neutron star rotation rates and magnetic fields, there are $\gtrsim 10^{16}$ vortex lines and $\sim 10^{31}$ flux tubes. The vortices are thus tangled in the flux tube array, with an intersection spacing of $\sim 10^{-10}$ cm along a vortex. This entanglement prevents the neutron fluid from corotating with the rest of the star.

From the standpoint of precession dynamics, the star can be regarded as consisting of two components: 1) the core protons,

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other charges and crust, which rotate as a single "body"; and, 2) a neutron superfluid which rotates according to the distribution and movement of its vortices. The angular momentum vector of the superfluid (most of the star) is fixed to the body to the extent that the vortices remain pinned against flux tubes, while, in a precessing body, the angular velocity vector changes its orientation with respect to the body's symmetry axes. As I argue in this Note, this immobilization of vortices with respect to the body gives a precession period that is far to fast to reconcile with observations.

Precession is a rotational mode of a rigid body in which the body's angular velocity is not aligned with any principal axis. Recall that for a rigid, biaxial body of oblateness ϵ, the precession frequency ϖ, in units of the spin frequency ω, is $\varpi = \epsilon$. If a neutron star precesses approximately as a rigid body, the observed precession period of PSR 1828-11 implies $\epsilon \simeq 10^{-8}$; the other precession candidates imply similar values of ϵ.

Pinning of vortices anywhere dramatically increases the precession frequency. Shaham (1977) showed that if vortex pinning is perfectly effective then precession would occur at very high frequency:

$$\varpi = \epsilon + \frac{L_{\rm p}}{I_{\rm b}\omega}, \tag{1}$$

where $L_{\rm p}$ is the angular momentum contained in the pinned superfluid and $I_{\rm b}$ is the moment of inertia of the "body" (crust plus charges). The pinned fluid effectively acts as a huge additional contribution to the star's oblateness, since $L_{\rm p} \simeq I_{\rm p}\omega$ and $L_{\rm p}/I_{\rm b}\omega \simeq I_{\rm p}/I_{\rm b} \gg \epsilon$. The precession frequency is thus determined by how much of the neutron vorticity is pinned. If most of the neutron superfluid of the outer core is pinned to the flux tubes, the precession frequency will be $\varpi \simeq I_{\rm p}/I_{\rm b} \simeq 10$, compared to 10^{-8} observed for PSR 1828-11. Hence, perfect pinning in even a small fraction of the core gives a precession frequency that is far too fast to be consistent with observations. This result is a consequence of the fact that the vortex array carries enormous vorticity; with the added constraint that there is vorticity fixed to the body, the star must precess at a frequency higher than ϵ in order to conserve angular momentum. For precession to occur with long period, the vortices must be able to closely follow the instantaneous spin vector of the rest of the star, and so cannot be perfectly pinned in any sizeable region of the star; the picture of vortex lines entangled in flux tubes appears to be incompatible with observations of long-period precession (Link 2003).

The assumption of perfect pinning is, however, an idealization. Some vortex motion with respect to pinning sites could occur by a process of *creep* through thermal activation (Alpar et al. 1984b; Link et al. 1993; Chau & Cheng 1993) or quantum tunneling (Link et al. 1993). Alpar (2005) has suggested that vortex creep could resolve the problem that precession is too fast with perfectly-pinned vortices. I now show that even if core vortices creep with respect to the flux tubes against which they are pinned, the precession frequency is still too fast, by a factor of $\sim 10^9$, to explain the observations.

The problem at hand is how the creep of vortices with respect to defects affects precession. The defects I will first consider are flux tube/vortex junctions in the outer core. I will then consider the nuclei of the inner crust. The formulation of Sedrakian et al. (1999, hereafter SWC), who studied precession of a star with neutron vortices using a description of vortex motion based on the concept of vortex drag, is particularly useful for addressing this question. They introduced a drag force per unit length of vortex of the form (SWC, Eq. (29), with notational changes)

$$\boldsymbol{f}_{\rm d} = -\eta(\boldsymbol{v}_v - \boldsymbol{v}_{\rm b}) - \eta' \boldsymbol{n}_v \times (\boldsymbol{v}_v - \boldsymbol{v}_{\rm b}), \tag{2}$$

where $\boldsymbol{v}_v$ is the velocity of a vortex line, $\boldsymbol{v}_{\rm b}$ is the velocity of the background body against which the vortices are dragged, η and η' are drag coefficients, and $\boldsymbol{n}_v$ is a unit vector in the direction of the angular velocity of the superfluid (coincident with the vorticity axis and also parallel to the vortex array). In the outer core, the background against which the vortices are dragged is the fence of flux tubes. The first term refers to (dissipative) drag anti-parallel to the vortex motion, while the second term describes a (possible) non-dissipative force transverse to the vortex motion. It is convenient to define the dimensionless drag coefficients

$$\beta = \frac{\eta\rho_{\rm s}\kappa}{(\rho_{\rm s}\kappa - \eta')^2 + \eta^2}, \tag{3}$$

$$\beta' = 1 - \frac{\rho_{\rm s}\kappa(\rho_{\rm s}\kappa - \eta')}{(\rho_{\rm s}\kappa - \eta')^2 + \eta^2}, \tag{4}$$

where $\rho_{\rm s}$ is the superfluid mass density and κ ($\equiv h/2m_{\rm n}$; $m_{\rm n}$ is the neutron mass) is the quantum of vorticity in a neutron superfluid. The drag coefficients determine how the vortex moves with respect to the background (SWC, Eq. (33)):

$$\boldsymbol{v}_v - \boldsymbol{v}_{\rm b} = (\beta' - 1)(\boldsymbol{v}_{\rm b} - \boldsymbol{v}_{\rm s}) + \beta\boldsymbol{n}_v \times (\boldsymbol{v}_{\rm b} - \boldsymbol{v}_{\rm s}), \tag{5}$$

where $\boldsymbol{v}_{\rm s}$ is the velocity of the neutron superfluid.

Vortex creep, by definition, means that $\boldsymbol{v}_v \simeq \boldsymbol{v}_{\rm b}$, which from Eqs. (3)–(5), implies that $\eta/\rho_{\rm s}\kappa \gg 1$. *Hence, vortex creep is in the high-drag limit.* To calculate the precession dynamics, however, we must know the relative magnitudes of η and η'. The transverse force (second term of Eq. (2)) has not been shown to exist in any calculation of drag on vortices by any process in a neutron star. The existence of this term is, in fact, a controversial issue in laboratory superfluids. On the one hand, Thouless et al. (1996) and Wexler (1997) have argued that this term is not present at all for an isolated vortex moving in an infinite, uniform superfluid (but see the comments by Hall & Hook 1998 and Sonin 1998 and the replies therein). On the other hand, mutual friction experiments with superfluid He (for which the system is, of course, neither infinite nor uniform) do demonstrate the existence of the transverse force (e.g., Bevan et al. 1997). Whether a transverse force exists for a vortex moving with respect to defects in a neutron star is unknown, so I will consider the possibility that it exists to maintain generality and appeal to laboratory experiments to obtain a bound. All mutual friction experiments in superfluid He that measure η' find that this coefficient goes to zero for temperatures well below the superfluid critical temperature (e.g., Bevan et al. 1997), the appropriate temperature regime for a neutron star. As the temperature is increased, $\eta'/\rho_{\rm s}\kappa$ usually increases, but never exceeds unity. Unless η' has very different behavior in a neutron star, then $\eta \gg \eta'$ in the limit of high drag. I henceforth assume this, but will return to the unlikely possibility of $\eta' \gg \eta$ at the end of this Note. Equations (3) and (4) become

$$\beta \simeq \frac{\eta\rho_{\rm s}\kappa}{\eta^2 + (\rho_{\rm s}\kappa)^2} \qquad \beta' \simeq \frac{\eta^2}{\eta^2 + (\rho_{\rm s}\kappa)^2}. \tag{6}$$

If there is no drag ($\eta = \beta = \beta' = 0$, with $\eta' = 0$), then $\boldsymbol{v}_v = \boldsymbol{v}_{\rm s}$ and the vortices corotate with the superfluid. When the vortices are dragged, $\boldsymbol{v}_{\rm s}$ and $\boldsymbol{v}_v$ differ. Perfect pinning corresponds to $\eta \to \infty$, $\beta = 0, \beta' = 1$. In the high-drag limit ($\eta/\rho_{\rm s}\kappa \gg 1$),

$$\beta \simeq \frac{\rho_{\rm s}\kappa}{\eta} \ll 1 \qquad\qquad 1-\beta' \simeq \beta^2. \tag{7}$$

SWC considered a two-component biaxial star with vortices moving according to the drag force of Eq. (2), and obtained the modes of the system. In the limit of high drag, the limit appropriate to vortex creep, the modes take simple forms. There is a high-frequency mode of complex frequency (SWC, Eq. (81), with $\varpi_f \equiv -{\rm i}p_{\rm p}$)

$$\varpi_f = \frac{I_{\rm p}}{I_{\rm b}} + {\rm i}\beta\left(1+\frac{I_{\rm p}}{I_{\rm b}}\right)\cdot \tag{8}$$

The real part gives the precession frequency, and the imaginary part gives the damping rate. This mode corresponds to precession at very high frequency $(I_{\rm p}/I_{\rm b})\omega$, as found by Shaham (1977), with little damping (since $\beta \ll 1$). But there is also a new slow mode, of frequency (SWC, Eq. (80), with $\varpi_{\rm s} \equiv -{\rm i}p_{\rm d}$)

$$\varpi_{\rm s} = -\epsilon\frac{I_{\rm b}}{I_{\rm p}}\beta^2 + {\rm i}\epsilon\frac{I_{\rm b}}{I_{\rm p}}\beta. \tag{9}$$

This mode can have a very low frequency compared to ω. The mode is absent (has zero frequency) in the case of perfect pinning; it arises only if the vortices can move with respect to the body. Since $\beta \ll 1$ in the high-drag limit, as relevant to vortex creep, the damping rate of the mode is faster than the oscillation frequency by a factor of $\beta^{-1} \gg 1$. Hence, this mode is not oscillatory at all; *it is highly over-damped, and never completes even one cycle*. The problem becomes even more serious the more effective the pinning is (smaller β). *Hence even if the vortices move through creep*, the fast precession mode of Eq. (8) is the only persistent and dynamically relevant one. The effects of triaxiality do not change the basic picture (SWC).

Alpar (2005) concluded that the slow mode is under-damped based on an assumed torque between the crust and the superfluid of the following linear form:

$$\boldsymbol{N}_{\rm drag} = -(\boldsymbol{\omega}_{\rm s} - \boldsymbol{\omega}_{\rm b})/\tau \tag{10}$$

where τ is a damping time, $\boldsymbol{\omega}_{\rm s}$ is the angular velocity of the superfluid and $\boldsymbol{\omega}_{\rm b}$ is the angular velocity of the body. This drag torque does not correctly describe the dynamics of a dragged vortex array; the correct form, obtained by integrating $\boldsymbol{r} \times \boldsymbol{f}_{\rm d}$ from Eq. (2) throughout the star, takes a more complicated form (see SWC, Eqs. (50) and (51)), and is much larger than that of Eq. (10). Use of Eq. (10) greatly underestimates the strength of the damping that arises from dissipative vortex motion, and leads to the incorrect conclusion that the slow precession mode is under-damped.

Provided the assumption that the vortices of the crust are not significantly dragged remains valid, a simple and general conclusion follows from this analysis: long-period precession demands that the vortices of the outer core move with little drag, that is, they do not creep, but *flow* essentially everywhere. Hence, in the outer core, *vortices and flux tubes cannot co-exist* (Link 2003). The possibilities that might be consistent with long-period precession are: 1) normal neutrons, superconducting protons; 2) superfluid neutrons, type I protons; 3) superfluid neutrons, normal protons; 4) normal neutrons and protons; and, 5) the core magnetic field has somehow been almost completely expelled. The existence of proton superconductivity is on rather firm footing, though the *type* of superconductivity is less clear. In a type I core, the magnetic flux could be organized in slabs or other geometries of mesoscopic dimensions, rather than the flux tubes found in a type II superconductor. It might be possible for vortices to move through a type I superconductor with sufficiently low drag to allow long-period, under-damped precession; Sedrakian (2005) showed that this scenario works for two specific geometries for the magnetic field. The pairing state of neutrons is far less certain. Most pairing calculations show pairing, but at least one calculation does not (Schwenk & Friman 2004). Lacking a mechanism for complete expulsion of the core field, possibilities 1) and 2) seem the most likely within existing uncertainties regarding nucleon pairing. Until the issue of neutron pairing in the outer core is settled by first-principles calculations, astrophysical arguments of the type presented here are useful for constraining the various possibilities for the hadronic ground states.

I now turn to the inner crust, where vortices could pin to lattice nuclei (Alpar 1977; Epstein & Baym 1988; Pizzochero et al. 1997; Avogadro et al. 2006; Donati & Pizzochero 2006). At the relatively low densities of the inner crust, neutron pairing is well-understood and neutron superfluidity there is theoretically well-established (see Dean & Hjorth-Jensen 2003 for a review). Assuming, for example, that almost all of the inner-crust superfluid is pinned to nuclei and that the rest of the star rotates as a rigid body over time scales less than the precession period, the precession frequency becomes $\varpi \simeq 0.01$ (Shaham 1977), again far too fast. There is a slow mode if the vortices creep, but as before it is highly over-damped. This result does not necessarily rule out vortex pinning in the inner crust; it just cannot happen in stars that are slowly precessing. In fact, the hydrodynamic forces acting on *pinned vortices* in a star with a "wobble angle" θ between the star's symmetry axis and the angular momentum of $\sim 3°$, as inferred for PSR 1828-11, would be sufficient to unpin the vortices in the inner crust (Link & Cutler 2002). The question then becomes one of how strong these unpinned vortices are dragged. For the precession to be of long period, the vorticity axis must be able to closely follow the instantaneous rotation axis of the body. To do this, the vortices must move at a speed

$$|\boldsymbol{v}_v - \boldsymbol{v}_{\rm b}| \simeq R\omega\,\theta \simeq 10^{-2}\ {\rm cm\ s^{-1}}\ \text{(PSR 1828-11)}. \tag{11}$$

Here R is the stellar radius and θ is the wobble angle. The motion of vortices past the nuclei with which they interact excites waves on the vortices (Kelvin modes), an inherently dissipative process. Epstein & Baym (1992) and Jones (1992) studied this problem in the context of explaining the short spin-up time scale of pulsar glitches. They found Kelvin mode excitation to be highly dissipative for $|\boldsymbol{v}_v - \boldsymbol{v}_{\rm b}| \gtrsim 10^6\ {\rm cm\,s^{-1}}$, a characteristic velocity difference that could develop as a consequence of vortex pinning. These treatments were done in a perturbative approximation, cannot be extended below $|\boldsymbol{v}_v - \boldsymbol{v}_{\rm b}| \sim 10^5\ {\rm cm\,s^{-1}}$, and so cannot be applied to the precession problem; instead we require a description of vortex drag at much lower velocities, in which the vortex response to motion past a nucleus is highly nonlinear. Jones (1998) showed that an infinitely long vortex moving slowly past a single nucleus tends to be temporarily trapped by the nucleus, and argued that vortices will pin, making it theoretically impossible (at least, within our current understanding of the inner crust) for a neutron star to precess slowly (Jones 2004). The treatment of Jones (1998), however, gives an unphysical divergence in the dissipation rate at low velocities when the rate should instead vanish (Link 2004). For vortex dynamics in the inner crust to allow long-period precession, assuming no difficulties with the core of the sort described above, the drag coefficient must satisfy $\eta/\rho_{\rm s}\kappa \ll I_{\rm b}/I_{\rm s}$ (Link 2004; Sedrakian 2005); here $I_{\rm s}$ is the moment of inertia of the inner crust superfluid and $I_{\rm b}$ is now the moment of inertia of the crust plus all other components tightly coupled to it over time scales less than the precession period. If I_b comprises most of the star, then $I_b/I_s \simeq 100$, which allows some room for vortex motion at moderately strong drag, $\eta/\rho_s\kappa \sim 1-10$ for example. Whether or not η can satisfy this upper bound for $|\boldsymbol{v}_v - \boldsymbol{v}_b| \simeq 10^{-2}$ is an interesting, unsolved problem.

Based on the above arguments, I conclude that in slowly precessing neutron stars:

1. vortices do not pin essentially anywhere in the star, so that,
2. the standard picture of the neutron star outer core in which vortices coexist with flux tubes should be reconsidered, and,
3. inner crust vortices must move with little dissipation ($\eta/\rho_s\kappa \lesssim 10$) with respect to the solid.

These results are robust provided that precession is real; they are not compromised by the numerous uncertainties regarding the composition and dynamics of the inner core. The only possible loophole for vortex creep to be compatible with long-period precession seems to be that η' is not only finite, but for some reason satisfies $\eta'/\rho_s\kappa \gg \eta/\rho_s\kappa \gg 1$ everywhere that vortices are pinned. Only under these unlikely circumstances can vortex creep admit a long-period, under-damped precession mode (SWC).

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