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Letter to the Editor

The impact of magnetic field on the cluster $M-T$ relation

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ABSTRACT

We discuss the impact of magnetic field on the mass–temperature relation for groups and clusters of galaxies based on the derivation of the general Magnetic Virial Theorem. The presence of a magnetic field B yields a decrease of the virial temperature T for a fixed mass M: such a decrease in T is stronger for low-mass systems than for high-mass systems. We outline several implications of the presence of B-field and of its mass scaling for the structure and evolution of groups and clusters.

Key words. cosmology: theory – galaxies: clusters: general – magnetic field

1. Introduction

Magnetic fields fill intracluster and interstellar space, affect the evolution of galaxies, contribute significantly to the total pressure of interstellar gas, are essential for the onset of star formation, and control the diffusion, the confinement and the evolution of cosmic rays in the interstellar and intracluster medium (ICM). In clusters of galaxies, magnetic fields may play also a critical role in regulating heat conduction (e.g., Chandran et al. 1998; Narayan & Medvedev 2001), and may also govern and trace cluster formation and evolution.

We know that magnetic fields exist in clusters of galaxies for several reasons. First, in many galaxy clusters we observe the synchrotron radio-halo emission produced by relativistic electrons spiraling along magnetic field lines. Second, the Faraday rotation of linearly polarized radio emission traversing the ICM proves directly and independently the existence of intracluster magnetic fields (see, e.g., Carilli & Taylor 2002; Govoni & Feretti 2004 for recent reviews). The Rotation Measure (RM) data throughout the inner (∼0.5 Mpc) cluster region support magnetic field strengths of the order of several to tens of μG (see Carilli & Taylor 2002; Govoni & Feretti 2004). The high local values of B observed in the central, cool region of clusters are likely related, however, to quite special conditions (such as turbulent amplification of the local B-field driven by radio bubbles or AGN jets, see e.g. Ensslin & Vogt 2005) and thus are probably not representative of the overall system (see, e.g. Carilli & Taylor 2002). Other estimates of the magnetic field strength on the cluster wide scale come from the combination of synchrotron radio and inverse Compton detections in the hard X-rays (e.g., Colafrancesco et al. 2005), from the study of cold fronts and from numerical simulations (see, e.g., Govoni & Feretti 2004). This evidence provides indication on the wide-scale B-field which is at the level of a few tens up to several μG (and in some cases up to ∼10 μG, as in Coma) with the larger values being attained by the most massive systems.

Numerical simulations (e.g., Dolag et al. 2001a) have shown that the wide-scale magnetic fields in massive clusters produce variations of the cluster mass at the level of ∼5−10% of their unmagnetized value. Such mass variations induce a comparable variation on the IC gas temperature T for virialized systems. Such variations are not expected to produce strong variations in the relative $M-T$ relation for massive clusters.

The $M-T$ relation predicted in a pure CDM model for $B = 0$ follows the self-similar scaling $M \propto T^{\eta}$ with $\eta = 3/2$ (see, e.g., Colafrancesco et al. 1997; Arnaud 2005). A Chandra study (Allen et al. 2001) of five hot clusters (with $k_{\rm B}T > 5.5$ keV) derived a $M-T$ relation slope of $\eta = 1.51 \pm 0.27$, consistent with the self-similar model. However, due to the relatively small Chandra field of view, the $M-T$ relation was established at R_{2500}, i.e., about $0.3R_{200}$ (here R_{δ} and M_{δ} are the radius and mass at which the density contrast of the system is δ). More recently, the $M-T$ relation was established down to lower density contrasts ($\delta = 200$) from a sample of ten nearby relaxed galaxy clusters covering a wider temperature range, $k_{\rm B}T \approx 2$−9 keV (Arnaud et al. 2005). The masses were derived from mass profiles measured with XMM-Newton at least down to R_{1000} and extrapolated beyond that radius using the NFW (Navarro et al. 1997) model. The $M_{2500} - T$ for hot clusters is consistent with the Chandra results. The slope of the $M-T$ relation is the same at all δ values, reflecting the self-similarity of the mass profiles. At $\delta = 500$ the slope of the relation for the sub-sample of hot clusters ($k_{\rm B}T > 3.5$ keV) is $\eta = 1.49 \pm 0.15$ consistent with the standard CDM self-similar expectation. The relation, however, steepens when the whole sample of clusters is considered, providing a slope $\eta = 1.71 \pm 0.09$. The normalisation of the $M-T$ relation differs, at all density contrasts from the prediction of pure gravitation based models by ∼30% (see Arnaud 2005 for a discussion).

In this Letter we will explore the effect of wide-scale magnetic fields on the $M-T$ relation over a large range of masses and temperatures by using the predictions of the magnetic virial theorem. We will discuss its implications for the evolution and the scaling relations of magnetized clusters. The relevant physical quantities are calculated using $H_0 = 71$ km s^{-1} Mpc^{-1} and a flat, vacuum-dominated CDM ($\Omega_{\rm m} = 0.3, \Omega_{\Lambda} = 0.7$) cosmological model.

2. The magnetic virial theorem for galaxy clusters

Under the assumption of a ICM in hydrostatic equilibrium with the potential well of a spherically-symmetric, virialized and magnetized cluster, the general relation between the ICM temperature T and the cluster virial mass M is obtained by applying the magnetic virial theorem (MVT):

$$\frac{1}{2}\frac{\mathrm{d}^2 I_{ik}}{\mathrm{d}t^2} = 2K_{ik} + \frac{2}{3}U\delta_{ik} + \int_V F_{ik}\mathrm{d}^3x + W_{ik}, \tag{1}$$

where I_{ik} is the inertia momentum tensor, K_{ik} is the kinetic energy tensor, U is the thermal energy of the intra-cluster gas, F_{ik} is the Maxwell tensor associated to the magnetic field and W_{ik} is the potential energy tensor. The full derivation of the MVT is reported in the Appendix. For a static and isothermal galaxy cluster the trace of Eq. (1) yields the condition

$$2U + U_B + W = 0, \tag{2}$$

where U_B is the magnetic energy of the system (see Appendix for details). For the general case of a cluster which is immersed in a Inter Galactic Medium (IGM) or external medium which exerts an external pressure $P_{\rm ext}$, Eq. (2) yields the formula for the temperature of the gas in virial equilibrium

$$\frac{k_{\rm B}T_{\rm g}}{\mu m_{\rm p}} = \frac{\alpha G}{3}\frac{M_{\rm vir}}{r_{\rm vir}}\left(1 - \frac{M_\phi^2}{M_{\rm vir}^2} + \frac{4\pi}{\alpha G}\frac{r_{\rm vir}^4}{M_{\rm vir}^2}P_{\rm ext}\right), \tag{3}$$

where usually $\alpha \simeq 1$ and we defined the quantity

$$M_\phi \simeq 1.32\times 10^{13} M_\odot \left[\frac{I(c)}{c^3}\right]^{1/2}\left(\frac{B_*}{\mu\mathrm{G}}\right)\left(\frac{r_{\rm vir}}{\mathrm{Mpc}}\right)^2, \tag{4}$$

where $I(c) = \int_0^c (\rho_{\rm g}(r=0)/\bar{\rho}(z=0))^{2\alpha} x^2 y_{\rm g}^{2\alpha}(x, B=0)\mathrm{d}x$. Here $c = r_{\rm vir}/r_{\rm s}$ (we assume a NFW Dark Matter density profile with scale radius $r_{\rm s}$) and $y_{\rm g}(x, B=0) = \rho_{\rm g}(x)/\rho_{\rm g}(x=0)$ is the gas density profile normalized to the central gas density (i.e. the solution of the hydrostatic equilibrium equation in the absence of magnetic field, see Colafrancesco & Giordano 2006a for details). The radial profile of the magnetic field has been assumed as $B(r) = B_*[\rho_{\rm g}(r,B)/10^4\bar{\rho}(z=0)]^\alpha$ with $\alpha = 0.9$ (see, e.g., Dolag et al. 2001b).

For the case $B = 0$, the quantity $M_\phi = 0$ and the well-known relation (for $P_{\rm ext} = 0$)

$$T_{\rm g}(B=0) = -\mu m_{\rm p} W/(3k_{\rm B}M_{\rm vir}) \tag{5}$$

re-obtains (here $\mu = 0.63$ is the mean molecular weight, corresponding to a hydrogen mass fraction of 0.69, $m_{\rm p}$ is the proton mass and $k_{\rm B}$ is the Boltzmann constant).

For $B > 0$, the quantity $M_\phi > 0$ and the gas temperature at fixed $M_{\rm vir}$, as obtained from Eq. (3), is

$$kT_{\rm g} = kT_{\rm g}(B=0)\left(1 - \frac{M_\phi^2}{M_{\rm vir}^2} + \frac{4\pi}{\alpha G}\frac{r_{\rm vir}^4}{M_{\rm vir}^2}P_{\rm ext}\right), \tag{6}$$

and is lower than that given by Eq. (5) because the additional magnetic field energy term U_B adds to the MVT. The presence of an external pressure $P_{\rm ext}$ tends to compensate the decrease of $T_{\rm g}$ induced by the magnetic field. For values of the temperature and density of the IGM (as estimated by the WHIM structure around large-scale overdensities, see, e.g., Fang & Bryan 2001), $P_{\rm ext} \sim 1.7\times 10^{-3}$ eV cm$^{-3}(n_{\rm IGM}/10^{-5}$ cm$^{-3})(T_{\rm IGM}/2\times 10^6$ K). However, in the outer regions of massive clusters (at $r \gtrsim r_{\rm vir}$) the external gas pressure can reach values $P_{\rm ext} \sim 0.2$ eV cm$^{-3}(n/10^{-4}$ cm$^{-3})(T_{\rm g}/1.7\times 10^7$ K) [here we considered the mean projected temperature profile for the cluster sample studied by Piffaretti et al. (2005, see their Fig. 4) and a typical cluster with $T_{\rm X} = 10$ keV]. In such a case, the value of $P_{\rm ext}$ is a significant fraction ~4% of the central ICM pressure and ~50% of the ICM pressure at the virial radius for a typical cluster. Thus, it cannot be neglected in the $T_{\rm g}$ estimate from Eq. (6). A value $P_{\rm ext} \sim 0.2$ eV cm^{-3}, as estimated at the outskirts ($r \gtrsim r_{\rm vir}$) of rich clusters, can be considered as an upper bound to $P_{\rm ext}$, since an exact determination of the total cluster mass (which is subject to various systematic uncertainties, see, e.g., Rasia et al. 2006) certainly requires to go beyond $r_{\rm vir}$. We thus consider in the following this value of $P_{\rm ext}$ as a reference upper bound to be used in our temperature estimate from Eq. (6) in the presence of a B-field. Lower values of $P_{\rm ext}$, down to its value in the WHIM, have progressively minor importance.

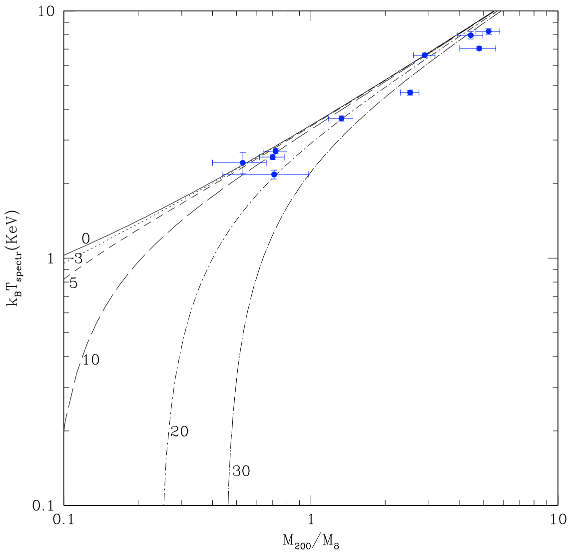

Fig. 1. We show the $T_{\rm spectr} - M_{200}$ relation at $z = 0$ for clusters which contain a magnetic field B_* in the illustrative range 0–30 μG (as labelled) in the case of $P_{\rm ext} = 0.2$ eV cm^{-3}. Here $M_8 \simeq 2\times 10^{14}\ M_\odot h_{71}^{-1}$. Data are taken from Arnaud (2005).

For reasonable values of $B_* \gtrsim$ a few μG, the quantity $M_\phi > \frac{4\pi r_{\rm vir}^4}{\alpha G}P_{\rm ext}$ and the main effect is a reduction of the cluster temperature which is more pronounced for less massive systems where M_ϕ becomes comparable to $M_{\rm vir}$. The effect of the magnetic field and of the external pressure are larger for low-M clusters (see Fig. 2).

3. The magnetized $M-T$ relation

The $T-M$ relation for magnetized clusters is shown in Fig. 1. We use here the relation $M_{200} \simeq 0.77\ M_{\rm vir}$ and we adopt the scaling $k_{\rm B}T_{\rm spectr} \simeq 1.8k_{\rm B}T_{\rm g}$ to compare our predictions with the available data. This last scaling, $k_{\rm B}T_{\rm spectr} \simeq 1.8k_{\rm B}T_{\rm g}$, is required to recover the normalization of the $M-T$ relation indicated by the available data and is, nonetheless, valid in the inner regions of the clusters (see, e.g., Pratt & Arnaud 2005). Small variations of temperatures with respect to their unmagnetized values are found for massive clusters since the quantity $M_\phi \ll M_{\rm vir}$ in this mass range and the value of $P_{\rm ext}$ has little or negligible effect (see Fig. 2). This is in agreement with the results of numerical simulations (Dolag et al. 2001a). However,

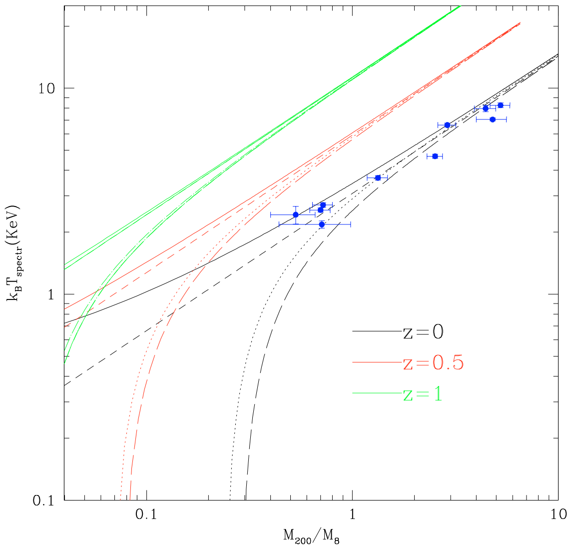

Fig. 2. Same as Fig. 1 but for a $P_{\rm ext} = 0$ with $B_* = 0$ (solid curves) and $B_* = 20\,\mu$G (long-dashes curves) and $P_{\rm ext} = 0.2$ eV cm^{-3} for the same values of $B_* = 0$ (short-dashes curves) and $B_* = 20\,\mu$G (dotted curves) The evolution of the $T_{\rm spectr} - M_{200}$ relation for magnetized clusters is shown at $z = 0$ (black), $z = 0.5$ (red) and $z = 1$ (green).

when M_ϕ becomes comparable to $M_{\rm vir}$, the IC gas temperature becomes lower than its unmagnetized value and the $T - M$ relation steepens in the range of less massive systems like groups and poor clusters. The temperature $T_{\rm g}$ formally tends to zero when $M_\phi \rightarrow M_{\rm vir}(1 + P_{\rm ext}/P_{\rm vir})$, where $P_{\rm vir} \equiv \left(\frac{4\pi}{\alpha G}\frac{r_{\rm vir}^4}{M_{\rm vir}^2}\right)^{-1}$. However, this limit is unphysical since it corresponds to an unstable system in which the magnetic pressure overcomes the gravitational pull. Thus, any physical configuration of magnetized virialized structures must have $M_\phi < M_{\rm vir}(1 + P_{\rm ext}/P_{\rm vir})$. The effect of $P_{\rm ext}$ counterbalances the effect of the B-field on the $T - M$ relation, increases for low-M systems and decreases with increasing redshift (see Fig. 2) because $P_{\rm vir}$ increases with increasing redshift.

4. Discussion and conclusions

We have derived here, for the first time, a relation between the temperature of the IC gas from the general MVT in the presence of magnetic field and external pressure. The result of the MVT for clusters bring relevant modifications to the gas temperature for virialized and magnetized clusters. As a consequence, the observed $T - M$ relation is steeper than the simple predictions of a ΛCDM structure formation scenario and its effective slope increases in the low-M region. However, since the masses of the observed clusters have been derived under the assumption of absence of B field, the slope indicated by the data of the observed $T - M$ relation could be not completely representative. In this context we also stress that the $T - M$ relation might be affected by other systematic uncertainties in the mass derived by X-ray observations (e.g., Rasia et al. 2006) which would change the slope of the $T - M$ relation especially in the low-M range. A robust analysis of the cluster mass estimate should require the use of a detailed hydrostatic equilibrium condition in combination with reliable temperature profiles. In both these aspects the effect of the B-field is relevant and should be taken into account. Furthermore, the predictions at low-T, where the effects of the B-field are stronger, are rendered uncertain by the absence of a clear definition of a spectroscopic temperature (e.g., Mazzotta et al. 2004). Thus, a complete analysis of the $T - M$ relation relies on a very detailed understanding of the physical properties of the IC gas in the presence of B-field with the input of a precise total mass reconstruction and temperature determination.

The results we derived here have a broad range of implications on cluster structure and evolution: flattening of the entropy – temperature relation and higher entropies in cluster cores are expected in the presence of magnetic fields. Further effects on the X-ray luminosity – temperature relation are also expected as well as modifications of the thermal Sunyaev-Zel'dovich effect. Since these studies are far beyond the scope of this paper, we refer the interested reader to much more detailed analysis which are presented elsewhere (Colafrancesco & Giordano 2006a,b).

To conclude, we notice that a full description of the structure and evolution of the population of groups and clusters of galaxies which considers also the role of magnetic fields will definitely shed light on several, still unclear aspects of the interference between gravitational and non-gravitational mechanisms in the evolution of these systems, and calls for a more refined physical description to use galaxy clusters as appropriate cosmological probes.

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Appendix A: The MVT for galaxy clusters

Let us introduce the following quantities:

$$\varphi(r) = -G\int_V \frac{\rho(x'_j)}{|x_j - x_{j'}|}\mathrm{d}^3x' \tag{A.1}$$

$$W = -\frac{1}{2}G\int_V\int_V \frac{\rho(x_j)\rho(x'_j)}{|x_j - x_{j'}|}\mathrm{d}^3x\mathrm{d}^3x' \tag{A.2}$$

$$F_{ij} = \frac{B^2}{8\pi}\delta_{ij} - \frac{B_iB_j}{4\pi} \tag{A.3}$$

where φ is the gravitational potential, W is the gravitational energy and F_{ij} is the Maxwell tensor associated to the magnetic field. Then, the equation of motion for the systems given by the Euler equation writes as

$$\rho\left(\frac{\partial}{\partial t} + (\boldsymbol{v}\cdot\boldsymbol{\nabla})\right)v_i = -\frac{\partial p}{\partial x_i} - \frac{\partial F_{ij}}{\partial x_i} - \rho\frac{\partial\varphi}{\partial x_i} \tag{A.4}$$

where v_i is the ith component of the velocity and p is the pressure (we consider a fluid with no viscosity for which $P_{ij} = p\,\delta_{ij}$). Multiplying by x_k and integrating over the cluster volume we obtain:

$$\begin{aligned}&\int_V x_k\frac{\partial}{\partial t}(\rho v_i)\mathrm{d}^3x\\ &+\int_V x_k\frac{\partial}{\partial x_j}(\rho v_iv_j)\mathrm{d}^3x = -\int_V x_k\frac{\partial p}{\partial x_i}\mathrm{d}^3x\\ &-\int_V x_k\rho\frac{\partial\varphi}{\partial x_i}\mathrm{d}^3x - \int_V x_k\frac{\partial F_{ij}}{\partial x_j}\mathrm{d}^3x.\end{aligned} \tag{A.5}$$

Using the continuity equation and the standard integral theorems, we convert the first member of this equation in the form

$$\begin{aligned}&\int_V x_k\frac{\partial}{\partial t}(\rho v_i)\mathrm{d}^3x + \int_V x_k\frac{\partial}{\partial x_j}(\rho v_iv_j)\mathrm{d}^3x\\ &\qquad= \frac{\mathrm{d}}{\mathrm{d}t}\int_V \rho v_iv_j\mathrm{d}^3x - 2K_{ik} + \oint x_k\rho v_iv_j\mathrm{d}S_j\end{aligned} \tag{A.6}$$

where $K_{ij} = 1/2 \int \rho v_i v_j \mathrm{d}^3x$ indicates the kinetic energy tensor and $U = 3/2 \int p\ \mathrm{d}^3x$ is the IC gas thermal energy. The second member of Eq. (A.5) writes as

$$-\int_V x_k \frac{\partial p}{\partial x_i} \mathrm{d}^3x = \frac{2}{3} U\delta_{ik} - \oint \mathrm{d}S_j x_k p \tag{A.7}$$

$$-\int_V x_k \frac{\partial F_{ij}}{\partial x_j} \mathrm{d}^3x = \int_V F_{ik} \mathrm{d}^3x - \oint \mathrm{d}S_j F_{ij} x_k \tag{A.8}$$

and one can show that

$$-\int_V x_k \rho \frac{\partial \phi}{\partial x_i} \mathrm{d}^3x = \frac{1}{2} W_{ik}. \tag{A.9}$$

Neglecting (in our case) the surface integrals (these physical quantities are negligible when the integration surface is chosen far from the cluster center) one obtains:

$$\frac{1}{2}\frac{\mathrm{d}^2 I_{ik}}{\mathrm{d}t^2} = 2K_{ik} + \frac{2}{3} U\delta_{ik} + \int_V F_{ik} \mathrm{d}^3x + W_{ik}, \tag{A.10}$$

where I_{ij} is defined as

$$I_{ij} = \int_V \rho x_i x_j \mathrm{d}^3x. \tag{A.11}$$

Using the trace of Eq. (A.10), we obtain the equation:

$$\frac{1}{2}\frac{\mathrm{d}^2 I}{\mathrm{d}t^2} = 2K + 2U + U_B + W \tag{A.12}$$

where $U_B \equiv F$, and $F \equiv \int \frac{B^2(r)}{2\mu_0} \mathrm{d}^3x$. For a cluster in a static configuration (quite a good approximation for real systems) one has

$$\frac{1}{2}\frac{\mathrm{d}^2 I}{\mathrm{d}t^2} = K = 0, \tag{A.13}$$

from which Eq. (2) derives.

The magnetic energy writes as

$$U_B - \oint x_k F_{ij} \mathrm{d}S_j = \frac{\phi^2}{r}, \tag{A.14}$$

where $\phi \equiv \pi(B_*/\mu G) r_{\mathrm{vir}}^2$ is the magnetic flux through the equatorial section of the system. The trace of Eq. (A.7) writes as

$$P_{\mathrm{ext}} \oint (\boldsymbol{r} \cdot \mathrm{d}\boldsymbol{S}) = P_{\mathrm{ext}} 4\pi r_{\mathrm{vir}}^3, \tag{A.15}$$

and is usually considered in the analysis of the standard Virial Theorem without the influence of a B-field (see, e.g., Carlberg et al. 1997). In the case of isothermal systems:

$$2U = 3\int_V p\ \mathrm{d}^3x = 3\int_V c_{\mathrm{s}}^2 \rho \mathrm{d}^3x \simeq 3c_{\mathrm{s}}^2 M \tag{A.16}$$

where

$$c_{\mathrm{s}}^2 = \frac{k_{\mathrm{B}} T_{\mathrm{g}}}{\mu m_{\mathrm{p}}}. \tag{A.17}$$

Using Eqs. (2), (A.14), (A.15) and Eq. (A.16), the Eq. (3) can be derived.

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