A&A 452, 423–430 (2006)
DOI: 10.1051/0004-6361:20054527

Astronomy & Astrophysics

The extragalactic Cepheid bias: a new test using the period-luminosity-color relation

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Received 16 November 2005 / Accepted 9 February 2006

ABSTRACT

We use the Period-Luminosity-Color relation (PLC) for Cepheids to test for the existence of a bias in extragalactic distances derived from the classical Period-Luminosity (PL) relation.
We calculate the parameters of the PLC using several galaxies observed with the Hubble Space Telescope and show that this calculation must be conducted with a PLC written in a form where the parameters are independent. The coefficients thus obtained are similar to those derived from theoretical models.
Calibrating with a few unbiased galaxies, we apply this PLC to all galaxies of the Hubble Space Telescope Key Program (HSTKP) and compare the distance moduli with those published by the HSTKP team. The new distance moduli are larger (more exactly, the larger the distance the larger the difference), consistent with a bias.
Further, the bias trend that is observed is the same previously obtained from two independent methods based either on the local Hubble law or on a theoretical model of the bias.
The results are quite stable but when we force the PLC relation closer to the classical PL relation by using unrealistic parameters, the agreement with HSTKP distance moduli is retrieved. This also suggests that the PL relation leads to biased distance moduli.
The new distance moduli reduce the scatter in the calibration of the absolute magnitude of supernovae SNIa at their maximum. This may also suggest that the relation between the amplitude at maximum and the decay of the light curve Δm_{15} may not be as strong as believed.

Key words. Cepheids – distance scale – methods: statistical

1. Introduction

We suggested that a strong statistical bias may still exist in the Period Luminosity method (PL) for Cepheids, resulting in too short extragalactic distances (Teerikorpi & Paturel 2002). We found this new bias by comparing the extragalactic distance moduli from The Key Project of the Hubble Space Telescope (HSTKP – Freedman et al. 2001) with relative unbiased distances given by the Hubble law applied within the local universe. It has been shown that the Hubble law works at short distances (e.g., Sandage et al. 1972; Ekholm et al. 1999; Ekholm et al. 2001; Karachentsev & Makarov 2001) when the velocities are corrected for known peculiar velocities. Then, by using numerical simulations (Paturel & Teerikorpi 2004) in which a statistical selection was applied to simulated data, as could be present in real observations, we confirmed the possibility of such a bias. We then analyzed the effect of this bias on the derived Hubble law and found that the corrected distances lead to a more linear Hubble diagram and that the different long range distance criteria are in better agreement (Paturel & Teerikorpi 2005, PT05).

Here, we propose an independent way to confirm this bias by using the Period-Luminosity-Color relation (PLC) without refering to Hubble distances. In previous papers we used the standard calibrated PL relation. Here, we derive the PLC coefficients (see Eq. (1)) and check for the bias in a differential manner, using as "calibrators" those galaxies that were previously found to be unbiased. The reason for using the PLC relation instead of the PL relation is that the PL relation is prone to a statistical bias, as we explained in our previous papers (see also Sect. 2 of the present paper). On the contrary, the PLC relation is a physical relation which is less affected by selection effects. One problem is that it is difficult to disentangle the reddening effect from the intrinsic variation of Cepheid colors. This question will be addressed in Sect. 3.

Our main result is that the bias is confirmed by this method in a direct manner, without having to use the Hubble law as a relative distance indicator and without the need to correct for any major bias.

2. PL versus PLC

The physical relation between period and mass density ($P \propto 1/\sqrt{\rho_m}$) for a pulsating star leads to a relation between luminosity, period and intrinsic color (Sandage 1994). It can be written as:

$$M_I = a \cdot \log P + b \cdot (V - I)_0 + c, \quad (1)$$

where M_I is the absolute I-band magnitude, P is the period of the luminosity variation and $(V - I)_0$ the intrinsic color. The parameters a, b and c are considered as slowly varying. We adopt the V and I bands because most accurate observations are made

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with them, essentially those from the HSTKP team. Some additional parameters like metallicity, pulsation mode, velocity rotation, etc., may also be present as additional effect, that could affect the values of the PLC parameters. In a first approximation, they can be gathered in $c = c_p$, making a and b true numerical constants.

2.1. The PL relation

The precepts behind the use of a simple PL relation are the following: if one assumes that the calibrating sample of Cepheids has the same properties as the distant samples to which one applies the relation, the color term, $(V - I)_0$, in Rel.1, may be considered as constant and the same for all galaxies. The mean intrinsic color of all samples is assumed to be constant. The physically relevant PLC relation is thus transformed into a simpler PL relation. The absolute magnitude being known through this PL relation, the distance modulus can be derived from the mean apparent magnitude, properly corrected for the total extinction, i.e. the sum of the extinction $a_{\rm g}$ in our own Galaxy and the extinction $a_{\rm h}$ in the host galaxy. Note that in this paper $a_{\rm g}$ and $a_{\rm h}$ will be expressed in the B-band photometric system.

As shown by Madore & Freedman (1991) it is possible to correct for the total extinction by using two different photometric bands (V and I). However, the assumption that the distributions of intrinsic colors must be the same for all samples prevents us from applying the PL relation to a single Cepheid. Only a large, unbiased sample guarantees that the mean intrinsic color is the same for all considered galaxies. Even then, the scatter in the PL relation, including the amplitude of variation, can lead to biased distance moduli of a galaxy.

2.2. The PLC relation

On the contrary, if one could obtain the parameters needed to use Rel.1, it would be possible to derive the absolute magnitude of each individual Cepheid. As said by Caputo, Marconi & Ripepi (1999a): "as beautifully introduced in the pioneering work by Sandage (1994), only the PLC relation is able to reproduce the tight correlation among the parameters of individual Cepheids". Di Benedetto (1996) noted that the PLC relation is less sensitive to metallicity variations than the PL relation[1].

As the color term allows one to pinpoint the position of a Cepheid within the M-distribution at a fixed $\log P$, the kind of bias that we have discussed in our previous papers is expected to be significantly reduced.

This does not completely solve the problem of distance determination, because the extinctions $a_{\rm g}$ and $a_{\rm h}$ must be determined in order to correct the apparent magnitude for each Cepheid. A major difficulty is to derive the coefficient b of the $(V - I)_0$ term. Considering this problem, Sandage and many authors after him have suggested the use of the coefficient derived from theoretical models. Here we use both an empirical solution and theoretical coefficients. As our main purpose is to test for the bias considered in PT05, we do not need to know the zero-point c in the PLC relation, and we compare, in a differential manner, more distant galaxies with closer, unbiased calibrator galaxies.

[1] The PLC relation might diminish the problem caused by the galaxy-to-galaxy PL variation emphasized by Sandage & Tammann (2006, ARA&A, in press).

3. Calculation of the form of the PLC relation

Here we recall some basic relations in order to introduce the notations we use. We express $(V - I)_0$ in Rel.1, by using the color excess E_{B-V} and the observed color $(V - I)$.

$$(V - I)_0 = (V - I) - (R_V - R_I) \cdot E_{B-V}, \tag{2}$$

with

$$E_{B-V} = \frac{(a_{\rm g} + a_{\rm h})}{R_B}. \tag{3}$$

R_X denotes the ratio of the total (A_X) to the differential extinction E_{B-V} for each photometric band X, as tabulated by Cardelli et al. (1989) or Caldwell & Coulson (1987) or Laney & Stobie (1994) or Gieren et al. (1998).

R_X depends on the extinction law, assumed to have the same wavelength dependence for each considered galaxy. The adopted values are the following:

$$R_B = 4.07 \qquad R_V = 3.07 \qquad R_I = 1.82. \tag{4}$$

Equation (1) may be developed into the following expression containing observed I, P and $(V - I)$:

$$I = a \cdot \log P + c_p + b \cdot (V - I) + K, \tag{5}$$

with

$$K = \mu + \rho(a_{\rm g} + a_{\rm h}), \tag{6}$$

and

$$\rho = \frac{R_I - b(R_V - R_I)}{R_B}. \tag{7}$$

The extinction $a_{\rm g}$ in the direction of a galaxy is constant (this may not be true for a large galaxy like LMC) and can be known from the map of the galactic extinction. The extinction $a_{\rm h}$ is different and unknown for each Cepheid, and the average extinction for the observed Cepheids in a given host galaxy has an unknown value $\langle a_{\rm h} \rangle$. Thus, for each galaxy, the quantity K is constant, though unknown. Further, the extinction $a_{\rm h}$ makes I and V dependent. This effect is discussed below (Sect. 3.3). Some additional terms (metallicity, pulsation mode, velocity rotation, etc.), may also be present. We assume that they do not severely affect the Cepheid luminosity or at least that the change is the same for all Cepheids of the same galaxy.

The conclusion is that it is possible to find a and b from Eq. (5) by considering each galaxy independently, provided that the number of observed Cepheids is large enough to ensure an accurate determination.

An additional precaution must be taken to solve Eq. (5). There is a correlation of errors because I appears on both sides of the equation. So, we write it as:

$$I = A \cdot \log P + B \cdot V + C, \tag{8}$$

with

$$A = \frac{a}{1+b} \qquad B = \frac{b}{1+b} \qquad C = \frac{K + c_p}{1+b}. \tag{9}$$

Note that this precaution for solving Eq. (5) is compulsory here. It is generally assumed that the error on a color is smaller than the error on each individual magnitude if these magnitudes are measured simultaneously because any global change in the zero-point is canceled. But, in our case, this rule does not apply because V and I are calculated separately from the light curves.

Table 1. The galaxy sample. Note that the distance moduli of this table are calculated assuming μ(LMC) = 18.5.

Galaxy	Class	a_g	V_c	$\langle I \rangle$	$\langle \log P \rangle$	$\langle V \rangle$	$A \pm \sigma_A$	$B \pm \sigma_B$	n	μ_{HSTKP}	μ_{PLC}
IC 4182	C	0.06	303.	23.079	1.124	23.778	−1.111 ± 0.243	0.706 ± 0.090	52	28.26	28.06
LMCogle	s	0.26	85.	14.987	0.638	15.797	−1.245 ± 0.012	0.625 ± 0.004	718	18.5	18.5
NGC 1326A	h	0.07	1370.	25.315	1.351	26.196	−0.407 ± 0.356	0.884 ± 0.126	17	31.04	31.68
NGC 1365	h	0.09	1370.	25.169	1.501	26.153	−0.945 ± 0.210	0.758 ± 0.061	40	31.27	32.09
NGC 1425	h	0.06	1370.	25.454	1.490	26.389	−1.144 ± 0.153	0.572 ± 0.061	29	31.70	32.24
NGC 2090	h	0.17	807.	24.526	1.380	25.456	−1.013 ± 0.181	0.709 ± 0.067	31	30.35	31.01
NGC 2541	h	0.22	802.	24.551	1.373	25.507	−0.268 ± 0.586	0.934 ± 0.185	28	30.25	31.05
NGC 3031	C	0.35	254.	22.045	1.334	22.982	−1.024 ± 0.249	0.722 ± 0.093	22	27.80	28.39
NGC 3198	h	0.05	857.	24.708	1.408	25.611	−1.136 ± 0.214	0.674 ± 0.062	77	30.70	31.19
NGC 3319	h	0.06	929.	24.885	1.342	25.718	−0.740 ± 0.275	0.832 ± 0.098	33	30.62	30.97
NGC 3351	c	0.12	588.	24.442	1.278	25.373	−1.044 ± 0.297	0.773 ± 0.098	30	30.00	30.51
NGC 3368	h	0.11	588.	24.282	1.381	25.225	−0.761 ± 0.198	0.724 ± 0.083	18	30.11	30.77
NGC 3621	c	0.35	493.	23.598	1.372	24.650	−1.201 ± 0.157	0.640 ± 0.046	49	29.11	30.40
NGC 3627	h	0.14	504.	24.130	1.406	25.217	−0.531 ± 0.121	0.796 ± 0.038	104	30.01	30.96
NGC 4258	c	0.07	512.	24.236	1.203	25.123	−1.031 ± 0.372	0.639 ± 0.119	15	29.51	29.81
NGC 4321	h	0.11	1200.	24.710	1.502	25.691	−0.823 ± 0.200	0.734 ± 0.077	42	30.91	31.65
NGC 4496A	h	0.11	1200.	24.741	1.462	25.648	−1.047 ± 0.089	0.740 ± 0.029	142	30.86	31.37
NGC 4535	h	0.08	1200.	24.827	1.479	25.760	−0.792 ± 0.221	0.815 ± 0.067	27	30.99	31.60
NGC 4536	h	0.08	1200.	24.792	1.452	25.750	−0.998 ± 0.184	0.717 ± 0.065	75	30.87	31.60
NGC 4548	h	0.16	1200.	25.224	1.348	26.132	−1.456 ± 0.214	0.553 ± 0.069	24	31.05	31.39
NGC 4639	h	0.11	1200.	25.229	1.594	26.193	−0.927 ± 0.167	0.707 ± 0.049	25	31.71	32.31
NGC 4725	h	0.05	696.	24.607	1.427	25.662	−0.660 ± 0.180	0.772 ± 0.064	20	30.46	31.38
NGC 5253	C	0.24	161.	23.137	0.936	23.994	−0.774 ± 0.729	0.773 ± 0.239	13	27.49	27.79
NGC 5457	c	0.04	388.	23.117	1.461	23.972	−1.614 ± 0.265	0.558 ± 0.090	31	29.13	29.59
NGC 7331	h	0.39	1099.	25.162	1.373	26.175	−1.390 ± 0.402	0.571 ± 0.134	13	30.84	31.80
NGC 925	c	0.33	775.	24.380	1.283	25.331	−1.030 ± 0.188	0.727 ± 0.058	60	29.81	30.71

The error in $V - I$ cannot be considered as independent and negligible with respect to the error in I. We verified numerically that the result would be very severely affected without this precaution (see next section, after Eq. (10)).

We can now derive A and B (and then a and b) from the HSTKP Cepheid sample.

3.1. The Cepheid sample

We utilize the sample of extragalactic Cepheids used in previous studies (Lanoix et al. 1999; Paturel et al. 2002b). The corresponding files were made accessible through the Centre de Données de Strasbourg[2] when we published the previous papers. The main parameters of the galaxies are listed in Table 1. The input distance moduli are from the HSTKP. The galactic extinction is from Schlegel et al. (1999), with a mean constant value (a_g = 0.26) for LMC.

The galaxy sample is given in Table 1 with the mean characteristics of the Cepheid samples. Each galaxy is classified as follows:

1. **C**- For presumably unbiased HST observations according to Fig. 8 in PT05.
2. **c**- For slightly biased HST observations from the same reference.
3. **h**- For all other HST observations.
4. **s**- For LMC, (with a distance modulus of 18.5, following HSTKP). The Cepheid observations for LMC are from the OGLE program (Udalski et al. 1999).

[2] CDS. The compilation of raw data is available in electronic form at CDS `http://cdsweb.u-strasbg.fr/`. Here we use the catalog known in CDS as `table3.txt` from Paturel et al. (2002a).

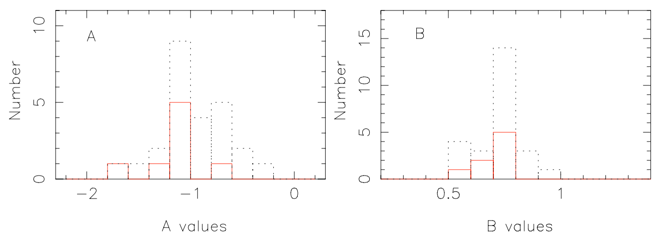

Fig. 1. Histograms of the coefficients A and B calculated for each class of the sample. The dotted lines correspond to classes "C+c+h", while the full lines correspond to the "C+c" class.

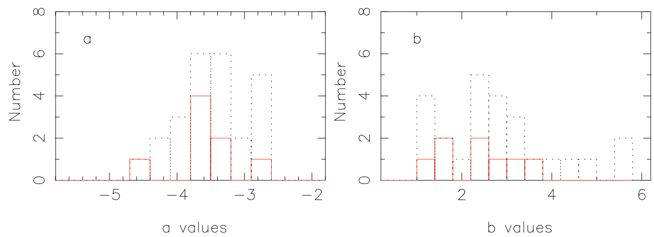

Fig. 2. Histograms of the coefficients a and b calculated for each class of the sample. The dotted lines correspond to classes "C+c+h", while the full lines correspond to the "C+c" class.

3.2. Numerical results for the PLC relation

Equation (8) is solved for all individual galaxies from their Cepheids. A total of 1735 Cepheids is used. In Figs. 1 and 2 the histograms of the distribution of A, B, a, b are shown. A and B are better defined than a and b. Thus, we will calculate the weighted means of A and B, and deduce the means for the coefficients a and b. The classes of calibrators (C) and slightly biased galaxies (c) are shown with full or dashed lines, respectively. These do not show any departures from the total sample (black dotted lines).

Table 2. Coefficients of the PLC relation.

#	$\langle A\rangle$	$\langle B\rangle$	$\langle b\rangle$	$\langle a\rangle$
1	−1.053 ± 0.079	0.717 ± 0.017	2.539 ± 0.024	−3.726 ± 0.082
2†	−1.149 ± 0.179	0.677 ± 0.060	2.097 ± 0.084	−3.558 ± 0.197
3	−0.965 ± 0.247	0.718 ± 0.073	2.549 ± 0.104	−3.425 ± 0.268
4	−1.220 ± 0.108	0.634 ± 0.034	1.730 ± 0.049	−3.332 ± 0.118

†Adopted solution.

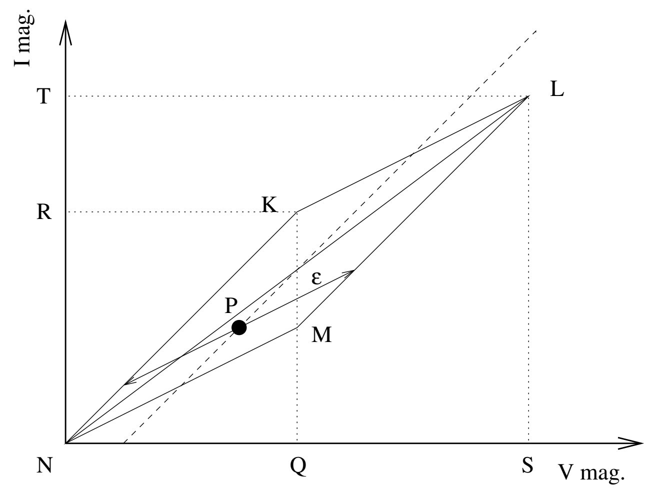

Fig. 3. Effect of the correlation induced by a_h between I and V magnitudes. The dashed line is the true relation between I and V magnitudes. Each point P of this relation is moved following either the left or right arrow, with a maximum amplitude $\epsilon = a_\mathrm{h}$. The slope of these arrows is $R_I/R_V \approx 0.60$. Because of this effect, the actual distribution of points is spread within the KLMN quadrilateral. The observed slope is given by the slope of the NL line.

Weighted means of A and B (and the corresponding $\langle a\rangle$ and $\langle b\rangle$) are calculated in four cases : #1 for calibrators (C), #2 for calibrators and secondary calibrators (C+c), #3 for all HST observations (C+c+h), #4 for all galaxies. The numerical results are summarized in Table 2. We will adopt the result #2 which gives the best compromise between the different results and because it corresponds to galaxies closer to us and thus well observed. In Sect. 6, we analyze the effect of a different choice. The adopted a and b coefficients are:

$$a = -3.56 \pm 0.20 \qquad b = 2.10 \pm 0.08. \tag{10}$$

Without the precaution explained in previous section (after Rel.9), the result would have been very different ($a \approx -2.7$ and b with two modes $b \approx -0.3$ and $b \approx 0.5$).

3.3. A remaining effect due to a_h

The variable extinction a_h inside the host galaxy makes I and V dependent and thus influences the derived value of the slope B because of a regression effect similar to a correlation of errors (See Fig. 3). The effect may be small because the slope $R_I/R_V \approx 0.60$ is close to the range of theoretically expected B (0.6–0.75).

We show here how the true slope can be estimated from the observed one. The intrinsic slope B_int (dashed line in Fig. 3) is given also by the slope of NK, i.e. NR/NQ. The observed slope B is given by the slope of NL, i.e. NT/NS . The maximum scatter introduced by a_h is $2\epsilon = KL$. Because of the correlation on both axes, the slope of "extinction line" KL is $R_I/R_V \approx 0.60$. Then $QS = 1.715\epsilon$ and $RT = 1.029\epsilon$. Finally, the observed slope can be written as ($\delta = NQ$ being the intrinsic range of V magnitudes):

$$B = \frac{B_\mathrm{int} + 1.029(\epsilon/\delta)}{1 + 1.715(\epsilon/\delta)}. \tag{11}$$

From this expression it is possible to estimate the intrinsic value B_int by using a reasonable value of the "extinction range/V range" ϵ/δ. From Fig. 2 of our previous paper (PT05) one can roughly see that $NS = 2$ mag. and that $\Delta(\log P) \approx 0.6$. Thus, from $\delta = 2.77\Delta(\log P)$ (2.77 is the slope of the PL relation in V) and $NS = \delta + 1.715\epsilon$, we derive that $\epsilon/\delta \approx 0.25$. If $B = 0.677$ then the intrinsic value would be $B_\mathrm{int} = 0.71$. The corresponding b is $b_\mathrm{int} = 2.45$ instead of $b = 2.10$. The value of B_int is larger than the observed $B = 0.677$ (and consequently $b_\mathrm{int} > b_\mathrm{observed}$).

The true effect is probably smaller than the calculated one. Indeed, if the internal extinction a_h moves the Cepheid above the limiting observational magnitude, the Cepheid will simply disappear from the sample. In other terms, the upper right corner (L) of the quadrilateral KLMN may be truncated in I and V, and then the observed slope B can be closer to the true slope B_int. For this reason we will use the observed values from Rel.10 but will consider the influence of a possible increase of B in the discussion (Sect. 6).

3.4. Comparison with theoretical values

The values we obtained for a and b can be compared with those predicted from models (Chiosi et al. 1993; Caputo al. 1999b; Baraffe & Alibert 2001). Although the PLC relation is rarely expressed as we do here, it is easy to derive these parameters. From Chiosi et al., one derives $a = -3.71$ and $b = 2.94$ for a metallicity similar to the galactic metallicity ($Y = 0.27$; $Z = 0.016$ with the original notation of the paper). From Caputo et al., one finds directly $a = -3.54$ and $b = 2.74$, by rewriting their PLC relation in the form of our Rel.1. From Baraffe & Alibert (2001) one can find $a = -3.10$ and $b = 1.55$ from the relation between W_I (Wesenheit function) and $\log P$ calculated for the LMC. A rough mean of these theoretical values gives $a \approx -3.5$ and $b \approx 2.4$, in good agreement with our adopted values. There is also good agreement with the K-band PLC relation found in LMC by Persson et al. (2004).

The practical use of the PLC relation for distance determination will now be considered.

4. Practical use of the PLC relation

Let L be the function:

$$L = I - a\cdot\log P - b\cdot(V - I) - \rho\cdot a_\mathrm{g}, \tag{12}$$

It can be calculated for each Cepheid from known quantities. From Eq. (5) one sees that, for a given galaxy with the true distance modulus μ, the function L is a linear function of a_h

$$L = c_p + \rho a_\mathrm{h} + \mu. \tag{13}$$

The intrinsic distribution of L for the Cepheids in a galaxy should reflect the distribution of a_h but also the dispersion of secondary parameters (metallicity, oscillation mode, velocity rotation, etc.)[3]. More precisely: here c_p is the parameter seen in

[3] When one calculates L from observed quantities (Rel.12), its distribution also reflects observational errors.

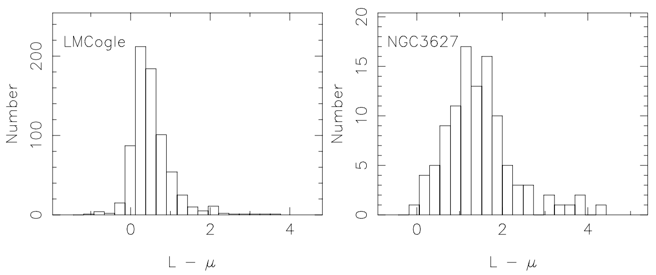

Fig. 4. Histograms of the function $L - \mu$, for two galaxies: LMC (*left*) and NGC 3627 (*right*). To draw these histograms we used previously known distance moduli.

Eq. (1) and $a_{\rm h}$ is the B-band extinction inside the host galaxy. The numerical value of ρ, derived from R_X and b, is:

$$\rho = -0.1897. \quad (14)$$

The small value of ρ means that the method is not very sensitive to errors in the host B-extinction $a_{\rm h}$. Our first idea was to select the Cepheids having the largest $L - \mu$ and thus the smallest $a_{\rm h}$. By doing the same for calibrators, we hoped to minimize the internal extinction term (the difference of two small quantities being, hopefully, negligibly small). However, the range of $L - \mu$ values seems too large (about 2–4 mag in I-band) to result from internal extinction only (see Fig. 4). The uncertainties on a and b and observational errors cannot account for this large dispersion. Some secondary effects included in c_p may thus affect the value of $L - \mu$. Further, the tail of the right side distribution is very difficult to represent accurately. Nevertheless, by measuring the shift S between the L distributions of calibrating and non-calibrating galaxies, we can cancel unknown terms. So, the basic assumption is that these secondary terms are, on the mean, the same for each subsample with the same $L - \mu$. Applying this assumption to the Cepheids of a calibrator and to the Cepheids of another galaxy, we obtain (with obvious notations):

$$\mu = \mu_{\rm calib} + S(L - L_{\rm calib}). \quad (15)$$

Note that the shift $S(L - L_{\rm calib})$ contains the extinction terms: $\rho\Delta a_{\rm g} + \rho\Delta a_{\rm h}$ where the coefficient ρ is small ($\approx$0.2). $\Delta a_{\rm g}$ is known from the galactic extinction map and $\Delta a_{\rm h}$ is close to zero, on average.

How to easily measure the shift $S(L - L_{\rm calib})$ between the two distributions? Due to the long tails on both sides of distributions, the simple mean may be unsuitable to determine a characteristic position. As a preliminary method, we used the mean of a percentile on the right side to minimize $a_{\rm h}$ as explained above. The result was acceptable but unstable for galaxies with a small Cepheid sample. Extending the percentile to 100% we retrieved the classical mean value of the distribution. The result was better. However, it is known that the median[4] is a very stable criterion when the distribution has extended tails. Another more sophisticated method would be to minimize the χ^2. For our present purpose the median is the most convenient criterion. Thus the shift between the distributions is measured as:

$$S(L - L_{\rm calib}) = {\rm median}(L) - {\rm median}(L_{\rm calib}). \quad (16)$$

In the Appendix we describe how, in practice, a distance modulus is calculated.

[4] Position where we have the same number of objects on the right and on the left of the distribution.

5. The PLC distance moduli

When we started this calculation, to compare the newly derived PLC distance moduli with the old HSTKP PL distance moduli, we could not predict if the result would confirm the bias or not. We suspected that the method used here might give results equivalent to the classical PL relation, i.e. again producing the HSTKP distance moduli. Further, if the agreement with the HSTKP distance moduli would not be satisfactory we were not sure that the result would support the bias. An opposite result would be equally probable. However, we will see that the bias is confirmed.

We first apply the method with the LMC as a calibrator, even though this galaxy may not be a good calibrator (even if its distance is known with a reliable confidence interval) because it is a dwarf, metal-poor galaxy, very different from most of the galaxies in the HSTKP sample. The result is shown in Fig. 5a. The first bisector passes, by definition, through the point representing the LMC and reasonably well through the barycenter of the three points representing the class C galaxies, assumed to be not biased. This means that the method is able to produce unbiased distances over a range of 10 mag (from 18.5 to 28.5) and perhaps more. Beyond magnitude 28.5 there is a clear disagreement with the HSTKP distance moduli. This may be exaggerated if the calibration with the LMC is not satisfactory.

We then apply the method using the three "class C" calibrators (Fig. 5b). The departure from the HSTKP distance moduli is similar and looks like the one we published previously (Fig. 6 in PT05).

To test the validity of the present PLC method, we compare the PLC distance moduli with the HSTKP PL distance moduli corrected for the bias as explained in PT05. The correction was calculated from a theoretical bias curve (Teerikorpi 1987; Bottinelli et al. 1987). This model, originally made to correct the incompleteness bias in clusters of galaxies, assumes that the amplitude of the Cepheid variation must be taken into account as part of the actual scatter of the PL relation. Indeed, a Cepheid will be present in the sample only if it has been observed over the full range of variation (otherwise, the mean magnitude cannot be calculated without distortion). The result of the comparison is given in Fig. 5c. The agreement is not perfect but there is no systematic departure from the calibrators ($\mu \approx 28.5$) up to the most distant galaxies ($\mu \approx 32$) of the sample. Thus, the new PLC distance estimates may be regarded as supporting our previous results.

To show that the PLC distance moduli do not show any systematic effect, we test them directly against relative distance moduli derived from the corrected velocity $V_{\rm c}$ (see Table 1). No attempt is made to derive the local Hubble constant from this plot. We simply adopt the local value found in PT05: $H = 56\ {\rm km\,s^{-1}\,Mpc^{-1}}$. The agreement is reasonably good (Fig. 6).

6. Discussion

To carefully assess the possible bias in the HSTKP program we checked the stability of the present results by changing the coefficients adopted for the PLC relation. We used values found for different samples in Table 2: from −3.3 to −3.7 and from 1.7 to 2.5, for a and b, respectively. We also used different theoretical a and b, without any changes. We also used several values for the R_B, R_V and R_I that characterize the extinction law. In all cases the results were very similar to those found in Fig. 5.

How far can we go in changing a and b? We did find values that give a good agreement with the HSTKP distance moduli.

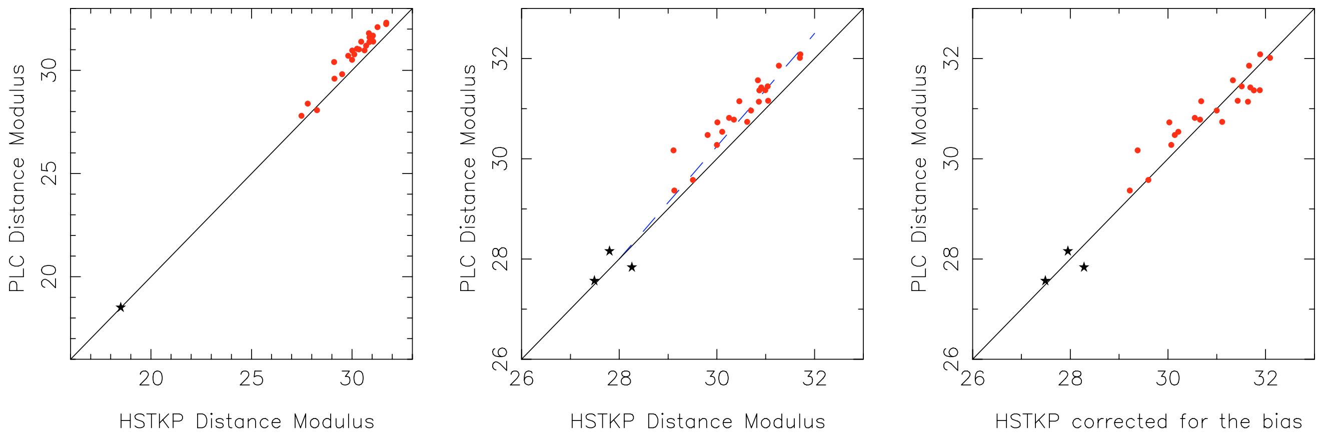

Fig. 5. a) *Left panel*: comparison of new PLC distance moduli with HSTKP ones when the calibration is made with LMC (black star on the left). **b)** *Middle panel*: comparison of new distance moduli with the PL distance moduli from HSTKP when the calibration is made with class C galaxies (star in black). The dashed line gives the bias from Fig. 6 in PT05. **c)** *Right panel*: comparison of new PLC distance moduli with the HSTKP ones after the latter are corrected for the bias using a theoretical model of the incompleteness bias, as was done in PT05.

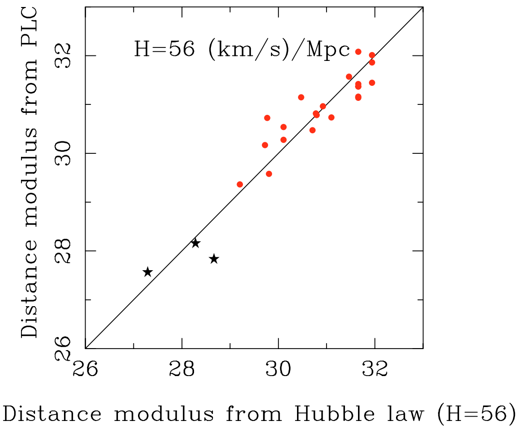

Fig. 6. Comparison of the new distance moduli with those deduced from the Hubble law, using the local Hubble constant found in PT05: $H = 56\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$.

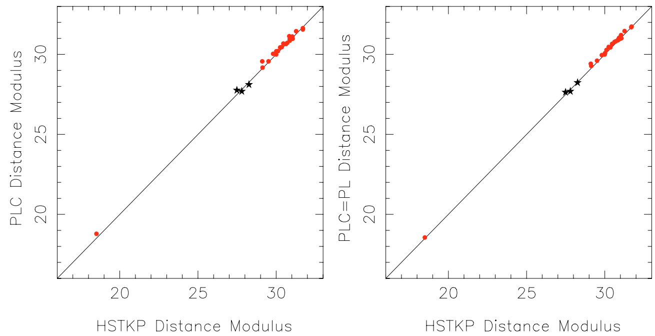

Fig. 7. Comparison of PLC distance moduli with HSTKP ones, when one uses different PLC coefficients: $b = 1.0$ (*left*) or $b = 0.0$ (*right panel*). Although unacceptable in view of direct determination and theoretical models, these parameters give closer agreement with HSTKP distance moduli. The calibration is made with class C galaxies (star in black).

For instance, using values (see Fig. 2) $a = -2.6$ and $b = 1.0$ leads to Fig. 7 (left) which shows an excellent agreement between PLC and HSTKP distance moduli. However, these values are incompatible with the solution found for Eq. (1), although it is based on a simple and well-known procedure. These values also disagree with the known theoretical models. By using still more extreme values we obtained even better results. The scatter in the diagram is much smaller than before. Is this an indication that the distance moduli are better? Unfortunately, it is not possible to conclude so, because we use almost the same Cepheid sample as the HSTKP team. When the coefficient b is reduced, the solution is closer to the previous PL relation results (i.e. HSTKP distance moduli).

This suggests to set $b = 0$ and $a = -3.05$ (the I-band slope of the PL relation) to see if one closely retrieves the result of HSTKP (some differences exist in the way we correct for the extinction). The result conforms with the expectation (Fig. 7 right).

When one plots the corrected velocity V_c versus our PL distances (Fig. 8), one sees a signature of the bias in the sense that a Hubble law with $H = 56$ can fit the HSTKP data up to 15 Mpc and that beyond this limit, the distances are underestimated, showing an increase of H. If one fits all the points one obtains a biased value $H \approx 70$.

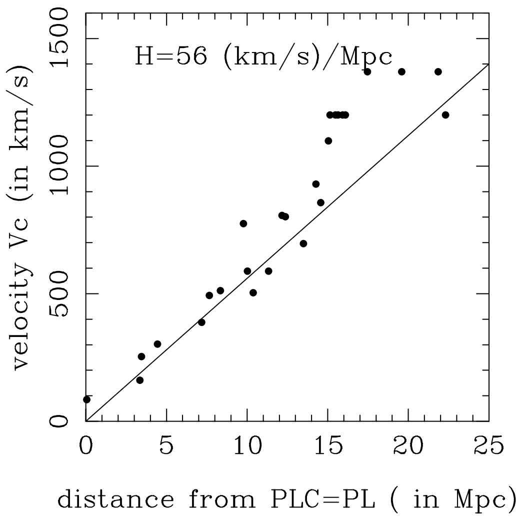

Fig. 8. Hubble diagram based on distances derived from the PLC relation transformed into a PL relation by setting $b = 0$ in Eq. (1).

We have seen in Sect. 3.3 that the calculation of b can be improved by taking into account the correlation between I and V magnitudes introduced by the internal extinction a_h. The result was that the corrected b is slightly larger than the observed one.

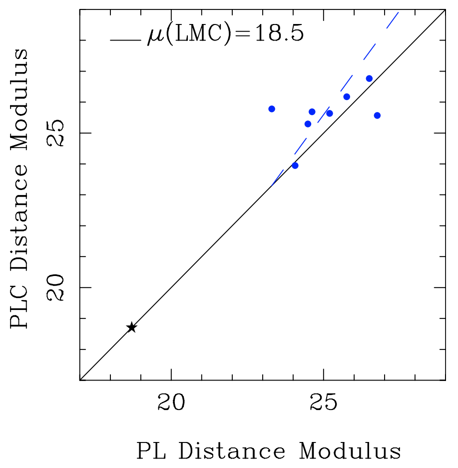

Fig. 9. Comparison of PLC distance moduli with those deduced from classical PL. line shows the unbiased line when μ(LMC) = 18.5. The dashed line shows the bias trend observed from an independent method in PT05.

In the light of the above results, a larger b would introduce a slightly larger departure from the HSTKP distance moduli, thus strenghtening our conclusion about the bias.

We have shown in PT05 that, like the HSTKP observations, the galaxies observed from ground suffer from the same bias and that this bias begins at a smaller distance because the Cepheid limiting magnitude is brighter. Thus, we have performed the calculation using the PLC relation with ground-based observations. The result, although slightly scattered due to small numbers of Cepheids and less accurate photometry, does not contradict the previous results (Fig. 9). The fit of the bias used in PT05 is superimposed (dashed line). Again, this agrees with the results we obtained from HSTKP data: the same bias exists starting at a shorter distance.

Why is the present statistical PLC method expected to be better than the classical PL relation or, in other words, why it does not suffer from the same bias? First, the use of the PLC relation diminishes the selection bias due to the dispersion. Second, the hidden parameters do not affect the Cepheid sample with a strong distance dependency. Thus calibrating and non-calibrating samples are affected in the same way and on average, the calibration removes the unknown terms. On the contrary, when an effect is dependent on the limiting magnitude, the selection effect on distant objects is much larger than the selection on nearby ones.

We tested whether the new distance moduli[5] improve the constancy of the absolute magnitude of supernovae (SNIa) at their maximum (Gibson et al. 2000). It appears that the scatter is strongly reduced with our new PLC distances (from $\sigma_B = 0.65$ to $\sigma_B = 0.44$ in the B-band and from $\sigma_V = 0.54$ to $\sigma_V = 0.36$ in the V-band).

When the residuals are plotted against the decay factor Δm_{15}, the increase of $M_{\rm max}$ with Δm_{15} (Phillips 1993) is visible when the HSTKP distances are used but not when one uses the PLC distances. This may suggest that the maximum vs. decay time relation has also been influenced by a bias. This question will be addressed in more detail in another paper. In Fig. 10 we show the plot of the V-band absolute magnitude at maximum $M_{\rm max}$ versus Δm_{15}.

[5] Distance moduli derived from our statistical PLC relation and calibrated with the HSTKP distance moduli of three unbiased galaxies: NGC 3031, NGC 5253 and IC 4182. The observed magnitude at maximum is corrected for the galactic extinction. The extinction within the host galaxy was provisionally neglected.

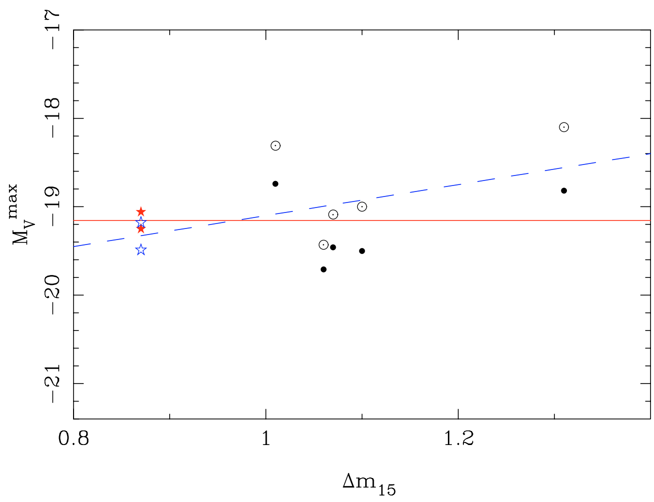

Fig. 10. Maxima of supernovae SNIa in V-band, $M_{\rm max}$, versus the decay factor Δm_{15}. The stars are two calibrating galaxies (NGC 5253 and IC 4182). The circles are non calibrating galaxies. Open symbols refer to the calculations based on the PL relation. Filled symbols refer to the calculations based on our present statistical PLC relation. The data from the PLC relation do not show any trend (full line), while the data from the PL relation show a small trend (dashed line) and in general a larger scatter of $M_V^{\rm max}$.

7. Conclusions

Several conclusions can be drawn from this study.

- The bias that we proposed in our previous papers and that affects the PL relation for extragalactic Cepheids is confirmed from the PLC relation in an independent way (no use of relative distances from the Hubble law, no use of a limiting apparent magnitude in the Cepheid magnitudes).
- The PLC relation works without bias on a distance modulus range of 13.5 mag (18.5 to 32), provided that the galactic extinction is corrected using extinction maps and that the internal host extinctions and the remaining secondary parameters are canceled in our differential method which compares galaxies with calibrators.
- The PLC seems to be less precise than the conventional PL relation, but it may be said to be more correct in the sense of having much smaller statistical selection bias.
- The parameters of the PLC relation can be computed from each individual galaxy with a large enough Cepheid sample provided that it is written in a form where all parameters are independent (i.e. without the correlation of errors of the involved quantities).
- When the PLC relation is used with existing theoretical values of its parameters it leads to the same result, i.e. that the distance moduli based on the PL relation are biased.
- The new distance moduli reduce the scatter of the calibration of the magnitude of supernovae at their maximum. This suggests that the relation between the amplitude at maximum and the decay of the light curve Δm_{15} may not be as strong as believed.

We recommend that independent research teams test whether our results are confirmed by other distance criteria (TRGB, SBF, PN) with the premise that the latter ones are unbiased, which is not necessarily the case.

Note added in proofs: The PLC relation might diminish the problem of non-uniqueness of the PL relation recently emphasized by Sandage & Tamman (ARA&A, in press).

Appendix A: Practical steps for calculations with the statistical PLC relation

We explained the principles of the use of the statistical PLC relation in Sect. 4. Here, we give a practical method for the calculation.

1. For each Cepheid of each calibrating galaxies we calculate the quantity:

$$L - \mu^{\rm calib} = I - a \cdot \log P - b \cdot (V - I) - \rho \cdot a_{\rm g} - \mu^{\rm calib}. \quad (A.1)$$

Here $\mu^{\rm calib}$ is the adopted distance modulus of the calibrators.

2. For each galaxy, we calculate the median $\mathrm{med}(L - \mu^{\rm calib})$ of the distribution of L. If there are several calibrators, we adopt as the zero-point the mean value of these medians

$$ZP = \langle \mathrm{med}(L - \mu^{\rm calib}) \rangle_{\rm calib}. \quad (A.2)$$

3. For a non calibrating galaxy we calculate for each Cepheid the quantity:

$$\mu_i = I - a \cdot \log P - b \cdot (V - I) - \rho \cdot a_{\rm g} - ZP. \quad (A.3)$$

4. We obtain an estimate of the distance of this non calibrating galaxy from the median of its distribution of individual μ_i

$$\mu = \mathrm{med}(\mu_i). \quad (A.4)$$

Acknowledgements. This work has been supported by the Turku University Foundation (the project "Cosmology in the local galaxy universe"). G.P. thanks Tuorla Observatory for hospitality during his stay (Oct. 17th–Nov. 17th, 2005) and for excellent working conditions. We thank the referee for very interesting and useful remarks.

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