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The cyclo-synchrotron process and particle heating through the absorption of photons

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ABSTRACT

Aims. We propose a new approximation for the cyclo-synchrotron emissivity of a single electron. In the second part of this work, we discuss a simple application for our approximation, and investigate the heating of electrons through the self-absorption process. Finally, we investigate the self-absorbed part of the spectrum produced by a power-law population of electrons.

Methods. In comparison to earlier approximations, our formula provides a few significant advantages. Integration of the emissivity over the whole frequency range, starting from the proper minimal emitting frequency, gives the correct cooling rate for any energy particle. Further, the spectrum of the emission is well-approximated over the whole frequency range, even for relatively low particle energies ($\beta \ll 0.1$), where most of the power is emitted in the first harmonic. In order to test our continuous approximation, we compare it with a recently derived approximation of the first ten harmonics. Finally, our formula connects relatively smooth to the synchrotron emission at $\beta = 0.9$.

Results. We show that the self-absorption is a very efficient heating mechanism for low energy particles, independent of the shape of the particle distribution responsible for the self-absorbed synchrotron emission. We find that the energy gains for low energy particles are always higher than energy losses by cyclo-synchrotron emission. We show also that the spectral index of the self-absorbed part of the spectrum at very low frequencies differs significantly from the well-known standard relation $I(\nu) \sim \nu^{5/2}$.

Key words. radiation mechanisms: non-thermal – radiation mechanisms: thermal – radiative transfer

1. Introduction

Synchrotron emission is well understood (see e.g. reviews by Ginzburg & Syrovatskii 1965, 1969; Pacholczyk 1970) and is thought to be responsible for a significant part of the radiation we receive from a variety of cosmic objects, such as supernova remnants, radio jets, compact radio sources, active galactic nuclei, and gamma-ray bursts. The reverse process, synchrotron absorption, has only recently disclosed some novel features, when the attention of researchers shifted from what is the amount of absorbed flux of photons to what happens to the absorbing electrons. In fact, they can absorb the *energy* of the photons and thereby change their initial distribution (Ghisellini et al. 1988, hereafter GGS88), and/or absorb the *momentum* of the photons, with the possibility of bulk motion acceleration (Ghisellini et al. 1990).

These authors demonstrate that the only stable equilibrium solution of particles emitting and absorbing synchrotron radiation is a relativistic or quasi-relativistic Maxwellian distribution. This paper ended a long debate about the existence of another equilibrium solution: a power law of slope 3, i.e. $N(\gamma) \propto \gamma^{-3}$, which was the main result of the so-called "Plasma Turbulent Reactor" (PTR) theory, as introduced in a series of papers in the 1970s (Norman 1977; Norman & ter Haar 1975; Kaplan & Tsytovich 1973). Note also that the stability of this $N(\gamma) \propto \gamma^{-3}$ solution was already questioned by Rees (1967), stating that this power law solution would evolve away from γ^{-3}, if slightly perturbed (see also the numerical results by McCray 1969, demonstrating this instability).

One of the aims of this paper is to explicitly demonstrate, that the γ^{-3} distribution is not only unstable, but is not even an equilibrium solution. To do so in an accurate way, however, it is necessary to also consider the trans–relativistic regime, namely, the cyclo-synchrotron emissivity and absorption coefficient. This, of course, is the more complex regime, because the emitted power is not concentrated at all in the first harmonic but also not at the typical synchrotron frequency (i.e. $\sim\gamma^2\nu_B$, where ν_B is the Larmor frequency). Recently, a useful approximation has been proposed by Marcowith & Malzac (2003), introducing polynomial expressions for the first 10 harmonics for a range of particle energies. They compare their results with an existing *analytical* formula that tried to approximate the emission (and the absorption) with smooth functions (i.e. not as sums of harmonics), as proposed by Ghisellini et al. (1998). From this comparison it appears that there might still be room for improvement in this smooth, approximated function, which is the second main aim of our paper here.

We present our new approximation in Sect. 2, and compare it with the Marcowith & Malzac (2003) results. We show that our approximation works well for particles with $\beta < 0.9$, where βc is the particle velocity, corresponding to $\gamma < 2.3$. For slightly higher energies, the standard synchrotron formulae describe the shape of the emission well, but the frequency–integrated emissivity must still be corrected to become equal to the cooling rate (which is known exactly for any particle energy). Since we are interested in the *total* amount of energy absorbed and lost by a

⋆ Deceased.

single particle, in Sect. 3 we introduce a correction to the standard synchrotron formula, which is important for $\gamma \gtrsim 2.3$, but automatically negligible for ultra-relativistic energies.

We then consider the rate of energy gains and losses suffered by an electron of a given energy as a consequence of cyclo-synchrotron emission and absorption (Sect. 4), showing that only particles at a single energy can be in equilibrium, where gains equal losses, independently of the slope of the particle distribution that produces the cyclo-synchrotron intensity. Studying the low frequency part of the synchrotron intensity in detail, we show that there are novel features below the Larmor frequencies that appear to have been overlooked in the past. These are presented in Sect. 5. Finally, we draw our conclusions in Sect. 6.

2. Approximation of the cyclo-synchrotron power spectrum

The single particle cyclo-synchrotron power spectrum can be approximated relatively well by a simple analytical formula. One of the best approximations was proposed by Ghisellini et al. (1998)

$$P_{\rm c}(\nu, p) = \frac{4}{3}\frac{\sigma_{\rm T} c U_B}{\nu_B} p^2 f(p) \exp\left[f(p)\left(1-\frac{\nu}{\nu_B}\right)\right], \quad (1)$$

$$f(p) = \frac{2}{1+ap^2}, \quad (2)$$

where $a = 3$, $\nu_B = eB/(2\pi m_{\rm e} c)$ is the Larmor frequency, $U_B = B^2/(8\pi)$ the magnetic field energy density, B the magnetic field strength, $p = \gamma\beta = \sqrt{\gamma^2-1}$ the dimensionless particle momentum, γ the particle Lorentz factor related to the total energy by $E = \gamma m_{\rm e} c^2$, β the particle velocity in units of c, and $\sigma_{\rm T}$, $m_{\rm e}$, c are constant Thomson cross-section, electron rest mass, and the velocity of light, respectively. Equation (1) describes the power, integrated over the emission angles, of an "average" electron: i.e. the power has been averaged over the pitch angle, which is assumed to be distributed isotropically. In this case the emissivity of a single electron is equal to the emitted power divided by the solid angle factor 4π. We use the term "power spectrum" to indicate the emitted power as a function of frequency of an "average" electron, in the sense specified above. This phenomenological formula has three advantages:

+ can be integrated easily over the frequency range,
+ integration from $\nu = \nu_B$ up to infinity gives the correct cooling rate

$$\dot{\gamma}_{\rm c} m_{\rm e} c^2 = \frac{4}{3}\sigma_{\rm T} c U_B p^2; \quad (3)$$

+ has the correct frequency dependence $[\exp(-\nu/\nu_B)]$ at large harmonics ($\nu \gg \nu_B$).

On the other hand, the formula also has three significant disadvantages:

– gives the correct cooling rate only if integrated from $\nu = \nu_B$ therefore cannot correctly describe the emission level below the frequency ν_B;
– does not approximate the emission spectrum well for $\beta < 0.5$, as we will show;
– does not join smoothly to the synchrotron power spectrum.

Therefore, in order to improve this formula, we introduce three important modifications.

First, the problem with the lower integration limit, ($\nu = \nu_B$) is solved by multiplying the formula with the term

$$g(\nu, p) = \frac{\nu - \nu_{\rm min}(p)}{\nu}, \quad (4)$$

where $\nu_{\rm min}$ indicates the minimal emitting frequency of the first harmonic

$$\nu_{\rm min}(p) = \frac{\nu_B}{\gamma(1+\beta)}. \quad (5)$$

This modification significantly reduces the power emitted below the frequency ν_B in comparison to the original formula and provides an automatic cut-off at the limiting frequency ($\nu_{\rm min}$). Note that this additional term becomes unity for $\nu \gg \nu_{\rm min}$, thus maintaining the original formula in this regime. However, we introduce a ν-dependent term in front of the exponential function ($\exp[f(1-\nu/\nu_B)]$), which also depends on the frequency. Therefore, the integral over the frequency range of the new formula must be expressed by the exponential integral.

Second, to improve the shape of the spectrum for $\beta < 0.5$, we replace the term $f(p)$ by a modified expression $f'(p)$ reading as

$$f'(p) = \frac{2}{1+ap^2}\frac{p^2+b}{p^2}, \quad (6)$$

where $a = 3.65$ and $b = 0.02$. The term $(p^2+b)/p^2$, which makes the difference between $f(p)$ and $f'(p)$, becomes equal to unity for $p \gg b$, and therefore our approximation becomes equivalent to the original formula in this regime. The necessary change to the constant $a = 3 \to 3.65$ is discussed in the next section.

Third, the new expression is normalized in order to yield the correct cooling rate (Eq. (3)) when integrating over the frequency range. This normalization is done by multiplying with a factor

$$c(p) = \left\{\exp\left[f'(p)\left(1-\frac{\nu_{\rm min}}{\nu_B}\right)\right] - f'(p)\frac{\nu_{\rm min}}{\nu_B}\exp[f'(p)]{\rm Ei}_1\left[f'(p)\frac{\nu_{\rm min}}{\nu_B}\right]\right\}^{-1}, \quad (7)$$

where Ei is the exponential integral (e.g. Press et al. 1989).

Finally, the improved approximation for the cyclo-synchrotron power spectrum of a single particle is given by

$$P_{\rm c}(\nu, p) = \frac{4}{3}\frac{\sigma_{\rm T} c U_B}{\nu_B} p^2 c(p) g(\nu, p) f'(p) \exp\left[f'(p)\left(1-\frac{\nu}{\nu_B}\right)\right]. \quad (8)$$

In comparison to the original relation this formula has three significant advantages:

+ integrated from $\nu = \nu_{\rm min}$ gives the correct cooling rate for any energy particle;
+ describes the emission spectrum well, starting from the minimal frequency for a wide energy range ($\beta = 0.01 \to 0.9$);
+ provides a relatively smooth connection to the synchrotron power spectrum at $\beta = 0.9$.

The price we pay for these improvements is that the integral over frequency of the new formula must be expressed in terms of an exponential integral. Moreover, in some sense our formula averages over the harmonics, providing a continuous emission spectrum. Therefore, it cannot be used for modeling the emission where the cyclo-synchrotron lines are observed directly (e.g. Pottschmidt et al. 2005). On the other hand, our approximation may have a wide range of applications in sources where the particle energy extends at least over one order of magnitude. In such a case the emission by particles with different energies may produce a continuous spectrum, where the spectral lines are barely visible or completely negligible.

The approximation of the cyclo-synchrotron emission provided by Marcowith & Malzac (2003) has been compared with the results of the precise numerical computations showing discrepancies that are less than 20%. Of course, the discrepancy between our continuous approximation and the precise calculation of (discrete) harmonics can be very large, if we compare our formula with the emission level between two well-separated harmonics. On the other hand, our main goal is to derive a formula that always provides the correct value of the total emitted energy. This is achieved through the normalization term (Eq. (7)) that independent of the values of the constants a and b, always provides the correct cooling ratio. This construction of the formula introduces a freedom in manipulating of the spectral shape. Therefore, by choosing appropriate values for the parameters a and b we can approximate the spectra at different energies rather well.

In Fig. 1 we compare our new formula with the old relation and the approximation of the first ten harmonics provided by Marcowith & Malzac (2003). A few general conclusions can be drawn from this comparison:

- For very low particle energies ($\beta \ll 0.1$), most of the power is emitted in the first harmonic, i.e. in a very narrow frequency range. For this energy range, our approximation shows the biggest disagreement in comparison to the approximation of the first harmonic. However, in compared to the old formula, our approach gives significantly better results.
- In the intermediate energy range ($0.1 \lesssim \beta \lesssim 0.5$) the contribution of high-order harmonics to the total emission becomes more important. Moreover, with increasing particle energy, the emission from each harmonic spreads over a wider frequency range. The spectrum transforms from a set of discrete lines into a continuous emission. Therefore, our approximation as a continuous function becomes more and more accurate with increasing particle energy.
- Finally, in the range of relatively high energy ($0.5 \lesssim \beta \lesssim 0.9$), the emission is dominated by the high-order harmonics. Our approximation gives the best agreement with the approximation for the first ten harmonics. The old formula also provides a relatively good approximation in this energy range. However, the total radiated power calculated from this formula is correct only if integrated starting from frequency ν_B. Therefore, it cannot describe the spectrum below ν_B, since it overestimates the total emitted energy if integrated from $\nu_{\rm min}$ The approximation of the first ten harmonics does not provide a correct value of the total emitted energy, either due to the fact that the ten harmonics considered provide a good description only up to $\nu \sim 4\nu_B$, while there is still considerable power above, too. For this particular particle energy range, many more harmonics are needed to provide the correct value of the total emitted energy.

Above $\beta = 0.9$, the synchrotron power spectrum can be used; however, this coefficient also requires some correction, which we describe in the next section.

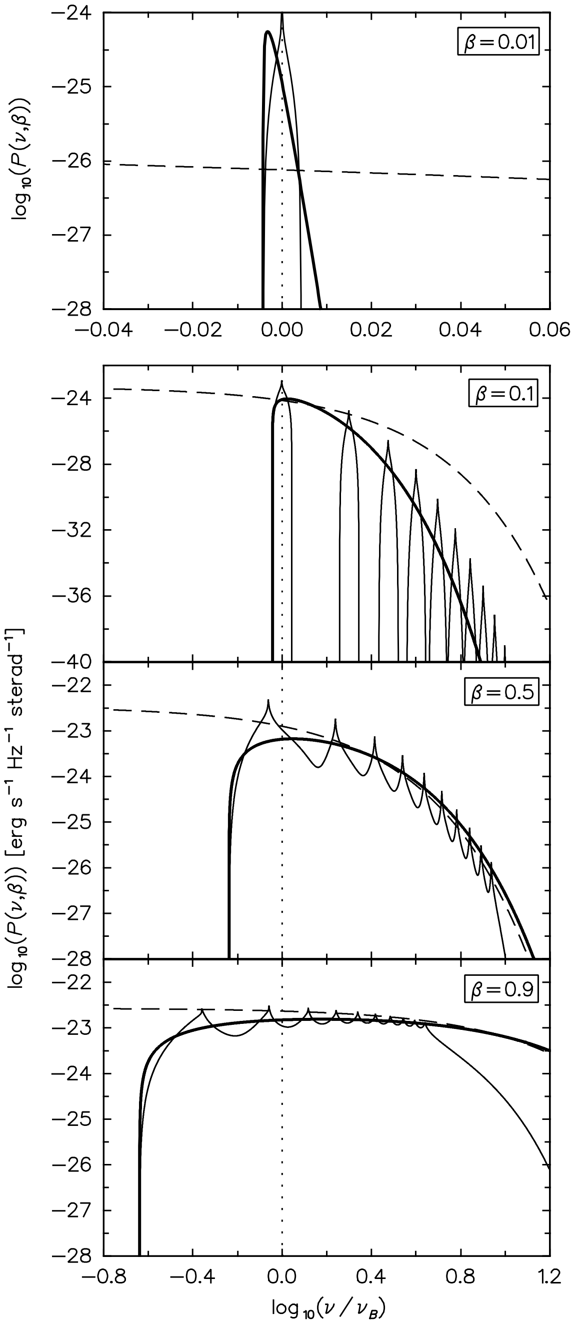

Fig. 1. The comparison between different approximations of the cyclo-synchrotron power spectrum of a single particle at four different energies. The thin solid line in each panel shows the polynomial approximation of the first ten harmonics provided by Marcowith & Malzac (2003). The continuous approximation derived in this work is shown by the bold solid line. The simple approximation proposed by Ghisellini et al. (1998) is shown by the thin dashed line. Note that the last approximation gives the correct cooling rate only if integrated from frequency ν_B (indicated by the vertical dotted line).

3. Correction of the synchrotron power spectrum

The synchrotron power spectrum from a single particle in a random magnetic field, integrated over an isotropic distribution of pitch angles, has been derived by Crusius & Schlickeiser (1986) and GGS88:

$$P_{\rm s}(\nu,\gamma) = \frac{3\sqrt{3}}{\pi}\frac{\sigma_{\rm T} c U_B}{\nu_B}\, x^2 \times \left\{ K_{4/3}(x)K_{1/3}(x) - \frac{3}{5}x\left[K^2_{4/3}(x) - K^2_{1/3}(x)\right]\right\}, \quad (9)$$

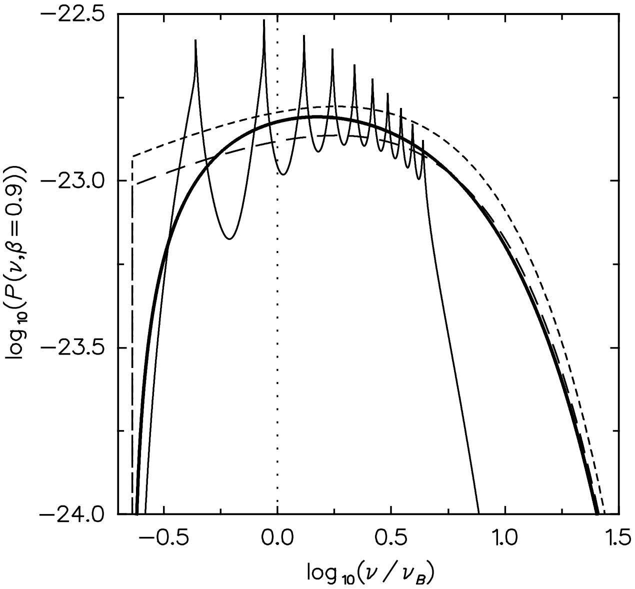

Fig. 2. The approximated cyclo-synchrotron power spectrum and the corrected synchrotron spectrum for $\beta = 0.9$. The thin solid line shows the approximation of the first ten harmonics provided by Marcowith & Malzac (2003), the continuous approximation derived in this paper is shown by the bold line, the synchrotron power spectrum derived by Crusius & Schlickeiser (1986) and GGS88 is shown by the short dashed line and the long dashed line shows the corrected synchrotron spectrum.

where $x = \nu/(3\gamma^2\nu_B)$ and $K_y(x)$ is the modified Bessel function of order y. This formula does not provide the correct cooling rate for $\gamma \lesssim 15$, where it overestimates the total emitted energy. Since the spectral shape, however, is approximately correct even for low particle energies ($\gamma < 15$), we multiply the original formula by a simple correction term that only depends on particle energy

$$s(\gamma) = \frac{\dot{\gamma}_c(\gamma)}{\int_{\nu_{\min}}^{\infty} P_s(\nu,\gamma)\mathrm{d}\nu}. \tag{10}$$

The difference between the cooling rate ($\dot{\gamma}_c$) and the frequency integrated synchrotron power disappears for $\gamma \gg 15$; therefore, the correction term reduces to unity for high particle energies.

In Fig. 2 we compare our approximated cyclo-synchrotron power spectrum with the corrected synchrotron spectrum at $\beta = 0.9$. Since the correction term for the synchrotron formula only depends on the particle energy, the correction only affects the normalization of the synchrotron spectrum. The figure also shows, that our approximation joins relatively smoothly to the corrected synchrotron spectrum at $\nu \gg \nu_B$. However, in order to achieve this smooth connection, we had to modify the constant a (Eqs. (2) and (7)), that controls the spectrum shape at $\nu \gg \nu_B$. The constant b in our formula controls the spectral shape for $\beta \lesssim 0.5$. Note that any modification of the parameters a or b changes the spectral shape and thus, in principle, also the total emitted energy. This problem has been solved through the normalization term (Eq. (7)), which also contains the parameters a and b, and thus changes the level of the spectrum in order to keep the correct value of the total emission.

4. Particle heating through the absorption of photons

We present a simple application for our approximation of the cyclo-synchrotron power spectrum and the corrected synchrotron emission coefficient. We analyze the amount of energy gain corresponding to the cyclo-synchrotron absorption process. This process may lead to an efficient exchange of the energy, and may therefore provide a very powerful heating mechanism for low energy particles. This kind of heating is a stochastic process. This process of competing radiative cooling and radiative heating, through emission and re-absorption, respectively, leads to the accumulation of most particles around the equilibrium energy (γ_e), where heating and cooling are in balance. In an ideal case, such a competition would transform any initial particle distribution into a thermal Maxwellian distribution with its maximum at the equilibrium energy.

In our test, we investigated the self-absorption of the radiation field produced by electrons with a power law energy distribution

$$N(\gamma) \propto \gamma^{-n}, \tag{11}$$

located inside a homogeneous spherical volume that is filled by a tangled magnetic field. We compare the efficiency of the heating and the cooling processes and discuss the possible values of the equilibrium energy for different slopes of the electron energy distribution.

The emission coefficient for any electron energy distribution $N(\gamma)$ is defined by

$$j(\nu) = \frac{1}{4\pi}\int N(\gamma)P(\nu,\gamma)\mathrm{d}\gamma, \tag{12}$$

where $P(\nu,\gamma)$ is either the cyclo-synchrotron or the corrected synchrotron power, depending on the value of γ. The self-absorption coefficient for any particle distribution is given by Le Roux (1961)

$$k(\nu) = \frac{1}{8\pi m_e \nu^2}\int \frac{N(\gamma)}{\gamma p}\frac{\mathrm{d}}{\mathrm{d}\gamma}\left[\gamma p P(\nu,\gamma)\right]\mathrm{d}\gamma. \tag{13}$$

For the calculations we assume a constant intensity of the radiation field inside the source, which is approximated by

$$I(\nu) = C\frac{j(\nu)}{k(\nu)}\left\{1 - \exp[-k(\nu)R]\right\}, \tag{14}$$

where R is the source radius. The value of the parameter C depends in principle on the position within the source. In the center of our homogeneous source $C = 1$ should be used, whereas on the source surface we should apply $C = 1/2$ (Gould 1979). However, for the sake of simplicity we assume a constant value of the intensity for the whole absorbing region, and use an average value of $C = 3/4$.

Finally, the absorption or heating efficiency for any radiation field is described by (Ghisellini & Svensson 1991) as

$$\dot{\gamma}_a(\gamma) = \frac{1}{m_e c^2}\frac{1}{\gamma p}\frac{\mathrm{d}}{\mathrm{d}\gamma}\left[\gamma p\int_0^{\infty}\frac{I(\nu)}{2m_e\nu^2}P(\nu,\gamma)\mathrm{d}\nu\right]. \tag{15}$$

We can easily estimate the absorption efficiency in an asymptotic case where the particle energy is characterized by $\gamma \to 1$ ($\beta \ll 0.1$). In this case, most of the energy is emitted in the first harmonic, i.e. in a very narrow frequency range (see Fig. 1 for $\beta = 0.01$). Therefore, the cyclo-synchrotron power spectrum can be approximated by a δ-function

$$P_c(\nu,\gamma \to 1) = m_e c^2\, \dot{\gamma}_c(p)\, \delta(\nu - \nu_B) \tag{16}$$

which results in a constant value of the absorption efficiency

$$\dot{\gamma}_a(\gamma \to 1) = \frac{4}{3}c\sigma_T U_B \frac{I(\nu_B)}{2m_e\nu_B^2}\left(3\gamma + \frac{p^2}{\gamma}\right) \to \mathrm{const.} \tag{17}$$

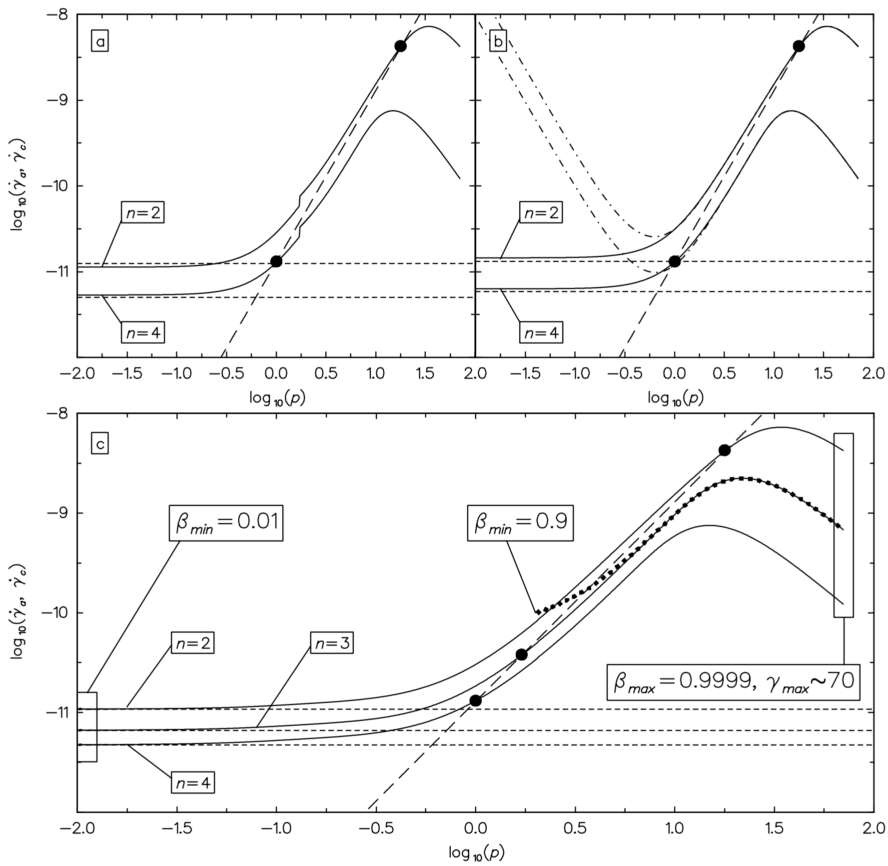

Fig. 3. The efficiency of absorption (solid lines) versus the cooling efficiency (long dashed lines) for the power law particle energy distribution ($\beta_{\rm min} = 0.1 \rightarrow \beta_{\rm max} = 0.9999$, $\gamma \simeq 70$) for $n = 2$, $n = 4$, and $n = 3$ (the last only in the main panel – **c**). The left upper panel **a)** shows the absorption efficiency calculated from the approximation of the cyclo-synchrotron emission derived by Ghisellini et al. (1998) and the standard synchrotron emission coefficient provided by Crusius & Schlickeiser (1986) and GGS98. In the upper right panel **b)**, we show the absorption efficiency calculated only from the corrected synchrotron emission. The dash-dot lines on this panel indicate the result obtained from uncorrected synchrotron emission. In the main panel **c)**, the new cyclo-synchrotron approximation and the corrected synchrotron emission were used. The short dashed lines in each panel indicate a constant value of the absorption efficiency estimated for $\gamma \rightarrow 1$ (Eq. (17)). The equilibrium energies are indicated by dots. Note that in the main panel we also show the absorption efficiency calculated for $\beta_{\rm min} = 0.9$ and $n = 3$ (bold dotted line).

Comparing this result with the cooling efficiency, which is always proportional to p^2 (Eq. (3)), we see that *for low energy particles, heating due to the self-absorption will always overcome radiative losses.* In other words, for a power law particle energy distribution, the heating term is proportional to p^2 for high values of p (in the self-absorbed regime); on the other hand, in the low energy limit $p \rightarrow 0$ (or $\gamma \rightarrow 1$), the heating term *must* always be constant. This implies, that there always is only one specific energy value for which the heating and cooling terms are equal. To illustrate this point, let us compare this result with the equilibrium solution [$N(\gamma) \propto \gamma^{-3}$] proposed in the "turbulent reactor" scenario. In this case, the (analytical) solution is obtained by considering only the relativistic regime and further assuming an infinite source (i.e. infinite self-absorption frequency). In other words, the radiation intensity is assumed to be proportional to $\nu^{5/2}$ for all frequencies, with a normalization that depends on the slope of the electron distribution (see also Rees 1967; and Mc Cray 1969). If one assumes that the particle distribution is a power law, but truncated at some low energy $\gamma_{\rm min}$ (to self consistently use the emissivity and absorption processes in the relativistic regime) then the radiation field is not $\propto\nu^{5/2}$ in the entire frequency range, but only above $\sim\gamma^2_{\rm min}\nu_B$. Below this frequency, $I(\nu) \propto \nu^2$ (e.g. Rybicki & Lightman 1979), making the electrons gain energy (through absorption) at a slightly higher rate than what is found by assuming $I(\nu) \propto \nu^{5/2}$. If instead one assumes a particle distribution extending towards mildly relativistic energies, then one obtains that $\dot\gamma_{\rm a}$, for small particle energies becomes constant and thus clearly larger than the extrapolation of the $\dot\gamma_{\rm a} \propto p^2$ law. Concerning the other energy extreme, a source of finite size becomes transparent at some finite value of the self-absorption frequency $\nu_{\rm t}$. Therefore the radiation field inside the source for frequencies close to and above $\nu_{\rm t}$ is no longer proportional to $\nu^{5/2}$, with a corresponding decrease in the energy gain rate for high-energy particles. These are the reasons a particle distribution $N(\gamma) \propto \gamma^{-3}$ is not an equilibrium solution. For any given value of the slope of the particle distribution, equilibrium is always achieved at only one specific energy.

In Fig. 3 we compare the absorption efficiency with the cooling rate of a power law particle energy distribution from $\beta_{\rm min} = 0.01$ to $\beta_{\rm max} = 0.9999$ ($\gamma_{\rm max} \simeq 70$) with three different slopes ($n = 2$, $n = 3$, and $n = 4$). We performed the computations using three different approaches for the calculation of the cyclo-synchrotron power spectrum in order to test the formulae derived in this work.

First, we used the old formula for the cyclo-synchrotron power spectrum (Eq. (2)) and the standard, uncorrected synchrotron emission coefficient (Eq. (8)). The result of these computations is presented in the upper left panel of Fig. 3. In this particular case, the transition from the old cyclo-synchrotron power spectrum to the uncorrected synchrotron emission at

$\gamma = 2$, produces a clearly visible discontinuity in the absorption efficiency. The discontinuity is related to the fact, that the uncorrected synchrotron emission overestimates the total emitted energy for $\gamma \lesssim 15$. Moreover, for low energy particles ($p < 0.1$) the level of the absorption efficiency does not agree with the constant level estimated from the δ-approximation (Eq. (17)), which is indicated by the horizontal lines.

In the second test, presented on the main panel of Fig. 3, we used our new approximation of the cyclo-synchrotron power spectrum and the corrected synchrotron emission coefficient. Since both expressions provide the correct value for the total emitted energy, the transition from one spectrum to the other (this time at $\beta = 0.9$) does not produce any discontinuity in the absorption efficiency. Moreover, with our new approximation of the cyclo-synchrotron emission, the absorption efficiency at low particle energies agrees well with the δ-approximation.

Finally, we used only the corrected synchrotron emission coefficient for the whole energy range (right panel in Fig. 3). No discontinuity is present, since we used only one formula. However, this approach, as well as the old cyclo-synchrotron power spectrum in the first test, do not agree very well with the δ-approximation. Note that for low energy particles, the uncorrected synchrotron emission gives an absorption efficiency that is a few orders of magnitude larger than the efficiency obtained from the other approximations.

These three approaches for the calculating of the absorption efficiency, qualitatively give the same results, but differ in the quantitative details, which indicate that the new approximation for the cyclo-synchrotron power spectrum together with the corrected synchrotron emission coefficient, provides the most precise description.

Our tests indeed show that the absorption efficiency for the very low-energy particles becomes independent of the particle energy and is significantly higher than the cyclo-synchrotron cooling ratio. Therefore, self-absorption will always cause a strong heating of the low-energy particles. The equilibrium energy depends strongly on the slope of the particle spectrum. For $n = 2$ the equilibrium energy is very close to the peak in the absorption efficiency (the $\dot{\gamma}_a(p)$ curve), which is related to the maximum in the self-absorbed spectrum (ν_t). If the particle spectrum is steeper ($n = 4$), the equilibrium is taken at a lower energy, but close to the energy of those particles emitting at the peak of the synchrotron spectrum. The $n = 3$ case is particularly interesting, since it corresponds to the previously claimed equilibrium solution. Contrary to this claim, heating and cooling also balance in this case only at a specific energy. This is, on one hand, due to the finite size of the source that limits the range of possible momenta of electrons that emit and absorb radiation efficiently and on the other hand, more importantly, due to the trans- and sub-relativistic regime where low energy particles always gain more energy than they loose.

The value of the equilibrium energy strongly depends on the minimum and maximum energy of the particles. In our tests the equilibrium energy for $n < 3$ depends on the self-absorption frequency (ν_t). However, for relatively low value of $\gamma_{\max}$ (e.g. 7 instead of 70 in our particular calculations), the emission should be absorbed at all frequencies of a completely optically thick source. In such a case, the equilibrium energy depends directly on $\gamma_{\max}$. For $n \geq 3$ the equilibrium energy depends on the minimal energy of the particles. In Fig. 3 we show the heating efficiency calculated for a relatively high value of the minimum particle energy $\beta_{\min} = 0.9$. The value of the equilibrium energy increases with the increasing minimum energy.

Note that for sake of simplicity, our tests assume a stationary state, where the equilibrium energy is simply given by the equilibrium between the heating and cooling rates. In reality, the system evolves and the physical conditions inside the source change. The initial power law, or any other particle distribution, will be transformed into a thermal or quasi-thermal spectrum (see GGS88). Also, the equilibrium energy in such an evolving source may be different from the energy estimated from the simple stationary analysis. This does not change that there is only one preferred equilibrium energy around which most of the particles will be accumulated, forming thermal or quasi-thermal distribution. A complete description of this time dependent evolution will be the main focus of our future study.

5. The self-absorbed spectrum

The detailed analysis of the cyclo-synchrotron emission shows that the self-absorbed part of the spectrum generated by the electrons with a power-law energy distribution, is slightly different from the well known power law relation $I(\nu) \propto \nu^{5/2}$. In this section we analyze the emission of our homogeneous source, assuming different slopes of the particle spectrum, and discuss the reasons for the deviations from the standard $\nu^{5/2}$ spectrum.

The observed intensity of the emission from the homogeneous spherical source is given by

$$I(\nu) = \frac{j(\nu)}{k(\nu)} \left(1 - \frac{2}{\tau^2}\left[1 - e^{-\tau}(\tau + 1)\right]\right), \tag{18}$$

where $\tau = 2Rk$ is the optical depth (e.g. Bloom & Marscher 1996). We calculate the intensity for a range of power law slopes, starting from $n = -1$ up to $n = 25/3$ with finite steps of $\Delta n = 2/3$. Our results are shown in Fig. 4. For $n \leq -2/3$, the spectral index is constant at $\alpha = -2$, in almost the whole self-absorbed part of the spectrum. In the range $-2/3 \leq n \leq 5/3$, the slope of the spectrum changes from $\alpha = -2$ to $\alpha = -1$ for $\nu < \nu_B$, and from $\alpha = -2$ to $\alpha = -2.5$ for $\nu > \nu_B$. Above the limiting value $n = 5/3$, the spectral index remains constant

$$\alpha = -1 \quad \text{for} \quad \nu < \nu_B,$$
$$\alpha = -5/2 \quad \text{for} \quad \nu > \nu_B.$$

Note that, for relatively steep particle distributions ($n \geq 17/3$ in this particular case) close to the low frequency cut–off, the spectrum may go over into a power law with the index $\alpha = -n$. Therefore, in such a case the self-absorbed part of the spectrum is described by three different indices $\alpha = -n \to -1 \to -5/2$. It is relatively easy to understand this specific evolution of the self-absorbed spectrum, if we analyze the limiting cases $n \leq -2/3$ and $n \geq 5/3$.

In the first case ($n \leq -2/3$), the emission in the whole frequency range is dominated by the synchrotron radiation produced by the highest energy electrons. The synchrotron power spectrum of a single particle or monoenergetic population of the particles can be approximated below the peak frequency ($\nu_p(\gamma)$) by

$$P_s(\nu, \gamma) \propto \nu^{1/3}\,\gamma^{-2/3}. \tag{19}$$

Integrating this formula over the power–law electron energy spectrum with the index $n \leq -2/3$, we obtain

$$j(\nu) \propto \nu^{1/3} \quad \text{for} \quad \nu_{\min}(\gamma_{\max}) \ll \nu \ll \nu_p(\gamma_{\max}). \tag{20}$$

Calculating the absorption in the same way, Eq. (13), we obtain

$$k(\nu) \propto \nu^{-5/3} \quad \text{for} \quad \nu_{\min}(\gamma_{\max}) \ll \nu \ll \nu_p(\gamma_{\max}), \tag{21}$$

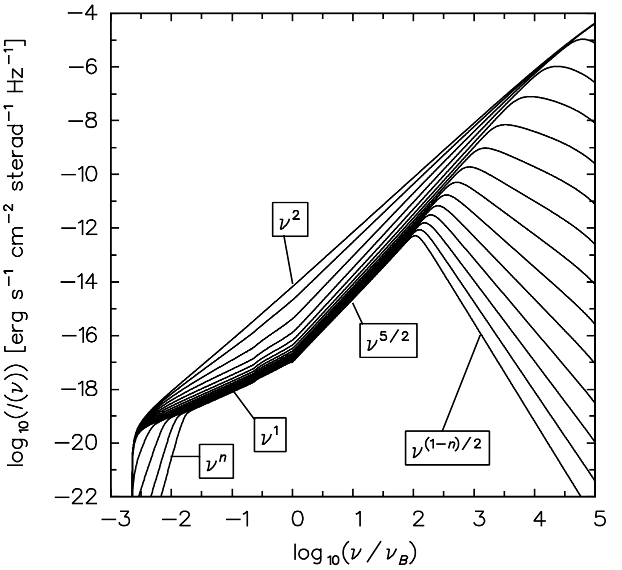

Fig. 4. The self-absorbed part of the cyclo-synchrotron and the synchrotron emission from a homogeneous spherical source. The intensities were calculated for a range of different indices of the power law particle energy spectrum, starting from γ^1 up to $\gamma^{-25/3}$ with a finite step size $\Delta n = 2/3$. The spectrum on the top [$I(\nu) \propto \nu^2$ in almost the entire frequency range shown] corresponds to $n = -1$, whereas the lowest spectrum was calculated for $n = 25/3$. The limiting values of the spectral indices presented in this figure are discussed in the Sect. 5.

and this gives $I(\nu) = j(\nu)/k(\nu) \propto \nu^2$ in the self-absorbed part of the spectrum.

In the second limiting case ($n \geq 5/3$), the spectral index above ν_B is equivalent to the well-known solution ($\alpha = -5/2$), and we only discuss the reason for the flattening of the spectrum ($\alpha = -1$) below this frequency. Around ν_B the emission is dominated by the cyclo-synchrotron radiation of the low-energy particles. However, for $\nu \ll \nu_B$ the emission becomes dominated by the tail of the synchrotron emission of the high-energy particles. Therefore, we can again use the approximation of the single-particle power spectrum (Eq. (19)). Integrating this approximation over the power law electron spectrum and neglecting the lower integration boundary, we obtain $j(\nu) \propto \nu^{1/3}\gamma_{\max}^{1/3-n}$. According to Eq. (5) the maximum energy is directly related to a given frequency $\nu \sim 1/\gamma_{\max}$. Therefore, we obtain

$$j(\nu) \propto \nu^n \quad \text{for} \quad \nu_{\min}(\gamma_{\max}) \ll \nu \ll \nu_B. \tag{22}$$

In the same way, we can integrate the absorption coefficient Eq. (13) obtaining $k \sim \nu^{-5/3}\gamma_{\max}^{-n-2/3}$, which gives

$$k(\nu) \propto \nu^{n-1} \quad \text{for} \quad \nu_{\min}(\gamma_{\max}) \ll \nu \ll \nu_B. \tag{23}$$

Finally, this gives $I(\nu) = j(\nu)/k(\nu) \propto \nu^1$ in the self-absorbed part of the spectrum below the frequency ν_B.

The low-frequency emission ($\nu \ll \nu_B$) can be absorbed only by the high-energy particles. For relatively steep particle spectra, the density of these particles can be too small to efficiently absorb the low-frequency radiation. Therefore, the source may again become optically thin and the spectral index equivalent to the index of the emission coefficient $I(\nu) \propto \nu^n$ (Eq. (22)).

When calculating the spectra presented in Fig. 4, we used our new formula for the cyclo-synchrotron power spectrum and the corrected synchrotron-emission coefficient. There is no smooth transition between these formulae at $\beta = 0.9$ for $\nu \lesssim \nu_B$ (see Fig. 2), but this is barely visible in our spectra. The effects discussed in this section depend mostly on the synchrotron emission; therefore, the corrected synchrotron emission can be used in whole energy range to get almost identical results.

The modifications of the self-absorbed spectrum that we discuss appear at relatively low frequencies. Therefore, in most astrophysical objects such effects are not observable. However, some effects might be visible in some specific physical conditions. One example might be the synchrotron radiation (in its self-absorbed portion) produced by steep power-law distributions of particles in highly magnetized sources ($B \sim 10^{7\to10}$ [G]). Note that isotropic distribution of the pitch-angles, assumed in order to derive the emissivity formulae presented in this paper, may not always be valid, especially in very highly magnetized sources.

6. Discussion

We have derived a new approximation for the cyclo-synchrotron power spectrum of a single particle and compared it with the approximation to ten first harmonics of the cyclo-synchrotron emission provided recently by Marcowith & Malzac (2003). In comparison to the other approaches, our approximation self-consistently provides the correct value of the total emitted energy over the whole range of the particle energies. Moreover, our approach describes the spectrum of the emission in the range $0.1 < \beta < 0.9$ relatively well. Finally, the approximation provides a relatively smooth connection with the corrected synchrotron emission at $\nu > \nu_B$ for $\beta = 0.9$.

All these results are useful when one needs fast computational tools to derive the cyclo-synchrotron emission and absorption, instead of using the exact expressions, which require much more computing time.

The application we will pursue is to study in detail the evolution of the emitting particle distribution subject to acceleration and/or injection of new particles, radiative and Coulomb cooling, and heating due to the synchrotron self-absorption. For the moment, we have instead analyzed a simpler process that, however, requires a careful treatment of the trans-relativistic regime. We have demonstrated that a power-law distribution of electrons emitting and absorbing cyclo-synchrotron photons can *never* be a steady solution. Energy losses and gains *always* equal each other at a particular energy γ_e, and not over a range of energies. This is contrary to previous claims that a distribution $N(\gamma) \propto \gamma^{-3}$ can be an equilibrium solution (e.g. Kaplan & Tsytovich 1973). The reason is that, at low enough, sub- or trans-relativistic energies, energy gains always exceed losses and, at the other extreme, the absorption becomes less efficient because the source becomes transparent (unless it is infinite in size). Particles therefore will tend to accumulate at γ_e, changing the shape of the energy distribution they initially belonged to.

For the steep particle spectrum ($n \geq 3$), the equilibrium energy (γ_e) is small. This motivated our detailed study of the trans-relativistic cyclo-synchrotron regime. However, we have shown that different approximations lead to very similar results for the amount of energy gains experienced by the particle. This is due to the fact that in calculating this quantity we must consider frequency-integrated expressions, with the consequent loss of details concerning the shape of the power spectrum. What matters is mainly that the frequency integrated spectrum equals the correct cooling rate (i.e. $\propto p^2$).

Exchanging photons through emission and absorption allows particles to exchange energy, independently of Coulomb collisions. This is a very important thermalization process in those magnetized, hot and rarefied plasma where Coulomb collisions are rare. Cyclo-synchrotron absorption transforms an initially non-thermal distribution into a Maxwellian in just a few cooling times (GGS88).

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