

Accretion torque on magnetized neutron stars

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ABSTRACT

The conventional picture of disk accretion onto magnetized neutron stars has been challenged by the spin changes observed in a few X-ray pulsars, and by theoretical results from numerical simulations of disk-magnetized star interactions. These indicate possible accretion during the propeller regime and the spin-down torque increasing with the accretion rate. Here we present a model for the accretion torque exerted by the disk on a magnetized neutron star, assuming accretion continues even for rapid rotators. The accretion torque is shown to have some different characteristics from that in the conventional model, but in accord with observations and numerical calculations of accretion-powered magnetized neutron stars. We also discuss its possible applications to the spin evolution in X-ray pulsars.

Key words. accretion, accretion disks – pulsars: general – stars: magnetic field – X-ray: stars

1. Introduction

The interaction between a magnetized, rotating star and a surrounding accretion disk takes place in a variety of astrophysical systems, including T Tauri stars, cataclysmic variables, and neutron star X-ray binaries (Frank et al. 2002). Magnetic fields play an important role in transferring angular momentum between the accreting star and the disk. A detailed model for the disk-star interaction was developed by Ghosh & Lamb (1979a,b). In this model the stellar magnetic field lines are assumed to penetrate the accretion disk, and become twisted because of the differential rotation between the star and the disk. The resulting torque can spin up or down the central star, depending on the star's rotation, magnetic field strength, and mass accretion rate. The total torque N exerted on a star with mass M contains two components,

$$N = N_0 + N_{\text{mag}}. \quad (1)$$

The torque N_0 carried by the material falling onto the star is generally taken to be

$$N_0 = \dot{M} R_0^2 \Omega_0 \quad (2)$$

where $\Omega_0 \approx (GM/R_0^3)^{1/2}$ is the (Keplerian) angular velocity at the inner radius R_0 of the disk, G the gravitational constant, and \dot{M} the accretion rate. The magnetic torque N_{mag} results from the azimuthal field component B_ϕ generated by shear motion between the disk and the vertical field component B_z ¹,

$$N_{\text{mag}} = - \int_{R_0}^{\infty} R^2 B_\phi B_z dR. \quad (3)$$

¹ Here we have adopted cylindrical coordinate (R, ϕ, z) centered on the star, and the disk is assumed to be located on the $z = 0$ plane, perpendicular to the star's spin and magnetic axes.

The torque N_{mag} can be positive and negative, depending on the value of the “fastness parameter” $\omega = \Omega_*/\Omega_0 = (R_0/R_c)^{3/2}$, where Ω_* is the angular velocity of the star and $R_c \equiv (GM/\Omega_*^2)^{1/3}$ the corotation radius. A rapidly rotating star ($\omega \lesssim 1$) can be spun down with steady accretion. The Ghosh & Lamb picture has been modified and extended in several directions by, for example, Königl (1991), Campbell (1992), Lovelace et al. (1995), Wang (1995), Yi (1995), Li & Wang (1996), Li et al. (1996).

However, the above standard model has been challenged by recent observations on the spin changes in several X-ray pulsars (Bildsten et al. 1997) and accreting millisecond pulsars (Galloway et al. 2002), and by theoretical results from 2- and 3-dimensional numerical simulations of disk accretion to magnetized, rotating stars (Romanova et al. 2002, 2003, 2004). These observational and theoretical facts have raised a number of puzzling issues, including (1) abrupt spin reversals in X-ray pulsars, (2) spin-down in millisecond X-ray pulsars throughout the outburst, (3) spin-down rates increasing with accretion rate, and (4) accretion in the “propeller” regime. These have motivated more recent investigations on the torques exerted by an accretion disk on a magnetized star (Nelson et al. 1997; Yi et al. 1997; Torkelsson 1998; Lovelace et al. 1999; Locsei & Melatos 2004; Rappaport et al. 2004).

In this paper, attempting to solve some of the puzzles related to disk-accreting neutron stars, we have constructed a new model for the accretion torque based on some modifications of the Ghosh & Lamb picture. We introduce the model in Sect. 2, and apply it to spin evolution in X-ray pulsars in Sect. 3. In Sect. 4 we discuss the similarities and differences between

this work and the competent model proposed by Lovelace et al. (1999).

2. Model

Here we adopt the typical assumptions. The central star contains a rotation-axis aligned dipolar magnetic field. At a cylindrical radius R from the star, the vertical component of the field $B_z(R) = B_*(R_*/R)^3$, where B_* is the stellar surface field strength. This field is anchored in the stellar surface and also threads a thin-Keplerian accretion disk. The region above and below the disk contains low-density gas and the system exists in a steady-state. Assume that the magnetosphere is nearly force-free and reconnection takes place outside the disk (Wang 1995), the azimuthal component B_ϕ of the field on the surface of the disk generated by rotation shear is given by

$$\frac{B_\phi}{B_z} = \begin{cases} \gamma \frac{(\Omega_* - \Omega_K)}{\Omega_K}, & \Omega_* \leq \Omega_K \\ \gamma \frac{(\Omega_* - \Omega_K)}{\Omega_*}, & \Omega_* > \Omega_K, \end{cases} \quad (4)$$

where the parameter γ is of order unity, and $\Omega_K = \Omega_K(R)$ is the Keplerian angular velocity in the disk. This form guarantees that the azimuthal pitch $|B_\phi/B_z|$ cannot be larger than γ over significant distances, for reasons of equilibrium and stability of the field above and below the disk plane (Aly 1984, 1985; Uzdensky et al. 2002).

We assume that Eq. (4) applies in both accretion ($\omega < 1$) and “soft” propeller ($\omega \gtrsim 1$) cases. It is conventionally thought that, when \dot{M} drops sufficiently, an accreting neutron star will transit from an accretor to a propeller, in which mass ejection occurs. However, this point of view is not fully consistent with recent observations of transient, accretion-driven X-ray pulsars and with the energetics considerations (see Rappaport et al. 2004 for a discussion). In stead, Spruit & Taam (1993) argued that even when $\omega \gtrsim 1$ accretion cannot stop, and the disk may find a new accretion state with the inner disk radius close to the corotation radius R_c . Rappaport et al. (2004) further developed this idea and applied the results to compute the torques expected during the outbursts of the transient millisecond pulsars.

The inner edge R_0 of an magnetically truncated accretion disk is usually assumed to be determined by the condition that the magnetic stresses dominate over viscous stresses in the disk and begin to disrupt the Keplerian motion of the disk material (Campbell 1992; Wang 1995), i.e.,

$$-R_0^2 B_{\phi 0} B_{z 0} = \dot{M} \frac{d(R^2 \Omega)}{dR} \Big|_0, \quad (5)$$

where the subscript 0 denotes quantities evaluated at R_0 . The limitation of this definition is obvious if one combines Eqs. (4) and (5), and takes the limit as Ω approaches Ω_0 : the left hand side of Eq. (5) becomes zero while the right hand side keeps to be a certain value, suggesting that it is not possible to calculate R_0 when $\omega \rightarrow 1$ (Li & Wickramasinghe 1997). However, the critical fastness parameter ω_c beyond which spin-down occurs is always close to 1 (Wang 1995, 1997; Li & Wang 1996). Since in our model accretion occurs with ω extending from 0 to $\gtrsim 1$, we in stead take R_0 to be at the magnetospheric radius R_m

determined by equating the ram pressure of the accreting matter with the magnetic pressure due to the dipole field of the neutron star (Davidson & Ostriker 1973; Lamb et al. 1973),

$$R_0 \simeq R_m = \left(\frac{\mu^4}{2GM\dot{M}^2} \right)^{1/7}, \quad (6)$$

where $\mu = B_* R_*^3$ is the magnetic moment of the neutron star. Observational evidence supporting this expression has been found in the transient X-ray pulsar A0535+26 (Li 1997). We have not taken $R_0 = R_c$ for $\omega > 1$ as in Rappaport et al. (2004). The reason is that, if the disk has already been disrupted at $R_m (> R_c)$, the accretion flow may pile up around the magnetosphere (Romanova et al. 2004), it is unknown whether the accretion flow can still keep the “disk” structure between R_c and R_m .

We now estimate the torque exerted on the neutron star by the surrounding accretion disk. When the disk rotation is Keplerian, we integrate the magnetic torque in Eq. (3) over the disk from R_0 to infinity combining with Eq. (4), and get the following expression,

$$N_{\text{mag}} = \begin{cases} \left(\frac{\gamma \mu^2}{3R_0^3} \right) \left(1 - 2\omega + \frac{2\omega^2}{3} \right), & \omega \leq 1 \\ \left(\frac{\gamma \mu^2}{3R_0^3} \right) \left(\frac{2}{3\omega} - 1 \right), & \omega > 1. \end{cases} \quad (7)$$

For the material torque N_0 , Eq. (2) implies 100% efficient angular momentum transfer. It actually results from the magnetospheric torque arising from the shearing motion between the corotating magnetosphere and the non-Keplerian boundary layer in the disk, which connects the magnetosphere and the outer Keplerian disk. The multi-dimensional calculations by Romanova et al. (2002, 2004) indicate that most of the angular momentum carried by the accreting material in the magnetosphere has been transferred to the magnetic field. From the investigations of magnetized accretion disks (e.g., Ghosh & Lamb 1979a; Campbell 1987; Erkut & Alpar 2004) we find that the angular velocity in the boundary layer can be roughly represented with the following form

$$\Omega(R) = \Omega_0 (R/R_0)^n, \quad (8)$$

and the magnetospheric torque can be evaluated according to Eq. (4),

$$\begin{aligned} N_0 &= - \int_{R_0-\Delta}^{R_0} R^2 B_\phi B_z dR \\ &= \gamma \mu^2 \int_{R_0-\Delta}^{R_0} R^{-4} \left[1 - \left(\frac{\Omega_*}{\Omega_0} \right) \left(\frac{R}{R_0} \right)^{-n} \right] dR \\ &\simeq \gamma \delta \frac{\mu^2}{R_0^3} (1 - \omega) \\ &= \left(\sqrt{2} \gamma \delta \right) \dot{M} (GM R_0)^{1/2} (1 - \omega) \end{aligned} \quad (9)$$

where Δ is the width of the boundary layer and $\delta = \Delta/R_0 \ll 1$ (Ghosh & Lamb 1979a).

Obviously the real form of $\Omega(R)$ must be more complicated than Eq. (8). However, Eq. (9) presents a reasonable order-of-magnitude estimate of the magnetospheric torque. Thus we let

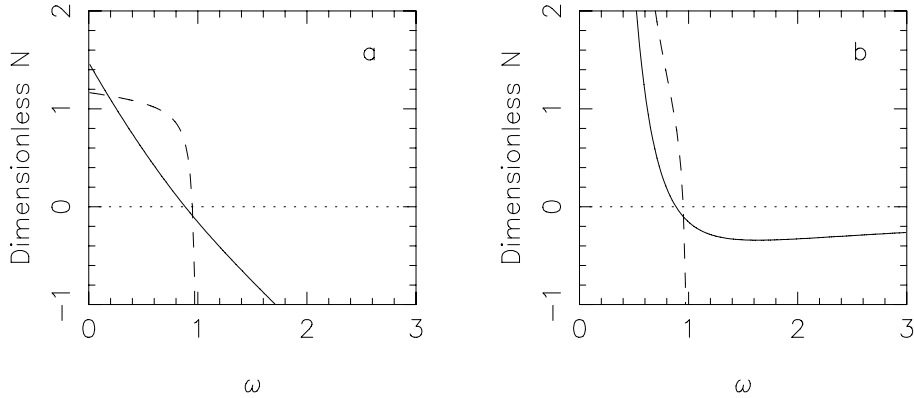


Fig. 1. The solid lines show the dimensionless torque **a)** $N/M(GMR_0)^{1/2}$ and **b)** $N/(\mu^2/3R_c^3)$ as a function of the fastness parameter ω . The dashed lines represent the standard model.

the torque N_0 take the following form in both accretion and propeller cases (see also Menou et al. 1999),

$$N_0 = \xi \dot{M}(GMR_0)^{1/2}(1 - \omega), \quad (10)$$

where the parameter $\xi = \sqrt{2}\gamma\delta$ depends on the structure of the magnetosphere and $0 < \xi \leq 1$. This implies that *the accreting material in the boundary layer may lose some fraction of the angular momentum, probably, through the viscous torque and/or the magnetic centrifugal winds/outflows* (e.g. Erkut & Alpar 2004). In the present analysis we always take $\xi = 1$, and Eq. (10) recovers to Eq. (2) when $\omega \ll 1$. The total torque exerted on the star by the disk is then

$$N = N_0 + N_{\text{mag}} = \begin{cases} \dot{M}(GMR_0)^{1/2} \left[\xi(1 - \omega) + \frac{\sqrt{2}\gamma}{3} \left(1 - 2\omega + \frac{2\omega^2}{3} \right) \right], & \omega \leq 1 \\ \dot{M}(GMR_0)^{1/2} \left[\xi(1 - \omega) + \frac{\sqrt{2}\gamma}{3} \left(\frac{2}{3\omega} - 1 \right) \right], & \omega > 1. \end{cases} \quad (11)$$

Figures 1a and 1b show the dependence of the dimensionless torque $N/M(GMR_0)^{1/2}$ and $N/(\mu^2/\sqrt{2}R_c^3)$ on ω in solid curves with $\xi = \gamma = 1$, respectively. The dashed curves in the figures represent the results in the disk model of Wang (1995) for comparison. In Fig. 1a there is no singularity problem in this work when $\omega \rightarrow 1$, the parameter space range for ω for spin-down with accretion is also much wider. The critical fastness parameter ω_c , at which $N = 0$, is $\omega_c \approx 0.884$. Furthermore, it indicates that, for constant \dot{M} , the spin-down torque increases with Ω_* ($\propto \omega$), in line with the numerical simulation results by Romanova et al. (2004).

Figure 1b shows how the torque varies with $\omega \propto \dot{M}^{-3/7}$ when μ and Ω_* are invariant. One can see that the spin-down torque is not a monotonous function of ω (or \dot{M}) when $\omega > \omega_c$. The spin-down torque takes the maximum value when $\omega \approx 1.634$. Thus a given spin-down torque can correspond to two values of ω . When $\omega > 1.634$, the spin-down torque increases with \dot{M} , which is opposite to the dependence in the standard model, but consistent both with the observational fact in the X-ray pulsar GX 1+4 that the X-ray flux appears to be increasing with the spin-down torque (Chakrabarty 1995), and with the numerical simulation results by Romanova et al. (2004).

Finally we point out that the derived results in this simplified model should be regarded as qualitative, and one should not take the values above very seriously.

3. Applications to X-ray pulsars

The long-term, continuous monitoring of accreting X-ray pulsars by *BATSE* on board *Compton Gamma-Ray Observatory* has provided new insight into the spin evolution of these systems (Bildsten et al. 1997). For the X-ray pulsar Cen X-3, the *BATSE* data show that it exhibits 10–100 day intervals of steady spin-up and spin-down, with a transition timescale $\lesssim 10$ days. Similar bimodal torque (or torque reversal) behavior has been observed in other pulsars, including 4U 1626–67, GX 1+4, and OAO 1657–415. These pulsars seem to be subject to instantaneous torques of roughly comparable magnitude $\lesssim \dot{M}(GMR_c)^{1/2}$, and differentiate themselves only by the timescale for reversals of sign. It is hard for the standard Ghosh & Lamb type model to explain a sudden torque reversal with nearly constant $|N|$ unless the \dot{M} variation is discontinuous and fine tuned.

Many mechanisms have been proposed to explain the torque reversals in accretion-powered X-ray pulsars. They can be roughly divided into two categories, one for the change of the direction of disk rotation, the other for bimodal magnetic torque. In the former type of models, van Kerkwijk et al. (1998) suggested that the accretion disk may be subject to a warping instability because of the irradiation from the neutron star. The inner part of the accretion disk will flip over and rotate in the opposite direction, which would lead to a torque reversal. The spin-down can also be explained if the disk rotation is retrograde (Makishima et al. 1988; Nelson et al. 1997). In the latter, Torkelsson (1998) suggested that the magnetic torque reversals may be explained by the presence of the intrinsic magnetic field in the accretion disk. The orientation of the disk field is not determined by the difference between the angular velocities of the star and of the disk but is rather a free parameter. Thus the direction of the magnetic torque between the two is arbitrary. A bimodal magnetic torque was also proposed by Lovelace et al. (1999, see below). Locsei & Melatos (2004) presented a model of the disk-magnetosphere interaction, which adds diffusion of the stellar magnetic field to the disk. In

certain conditions, the system possesses two stable equilibria, corresponding to spin-up and spin-down.

An alternative model for the transitions between spin-up and spin-down was suggested by Yi et al. (1997), in which the torque reversals are caused by alternation between a Keplerian thin disk and a sub-Keplerian, advection-dominated accretion flow (ADAF) with small changes in the accretion rate. When \dot{M} becomes smaller than a critical value \dot{M}_{cr} , the inner part of the accretion disk may make a transition from a primarily Keplerian flow to a substantially sub-Keplerian ADAF, in which the angular velocity $\Omega'(R) = A\Omega_{\text{K}}(R)$ with $A < 1$ (Narayan & Yi 1995). In this case the corotation radius becomes $R'_c = A^{2/3}R_c$ (Here we use the prime to denote quantities in ADAF). The dynamical changes in the disk structure lead to the slow ($\omega < \omega_c$, spin-up) and rapid ($\omega' > \omega'_c$, spin-down) rotator stage alternatively with $\dot{M} \sim \dot{M}_{\text{cr}}$. This scenario is attractive because it is consistent with the transition time scales – the standard disk-ADAF transition will occur on a thermal time $t_{\text{th}} \sim (\alpha\Omega_0)^{-1} \sim 10^3$ s, while the interval between torque transition is set by the changes in \dot{M} on a global viscous time scale. However, some issues in this model suggest that it needs to be improved. First, at equilibrium spin the critical fastness parameter $\omega_c = \omega'_c \simeq 0.875$ (see also Wang 1995). The narrow range of ω' ($\sim 0.875-1$) for spin-down requires that the physical parameters (e.g., \dot{M} , B_*) should still be fine-tuned for various X-ray pulsars². Second, as the Ghosh & Lamb model, this model also predicts decreased spin-down rate when \dot{M} increases.

In the present work, we have assumed that accretion can continue even when $\omega \gtrsim 1$. During the disk transition the inner radius of the disk may change little, i.e., $R'_0 \simeq R_0$, since the total (thermal and kinetic) energy density in the disk remains to be unchanged (Yi et al. 1997). The torque in the ADAF case can be derived in the similar way as in the Keplerian disk case,

$$N' = \begin{cases} A\dot{M}(GMR'_0)^{1/2} \left[\xi(1 - \omega') + \frac{\sqrt{2}\gamma}{3A} \left(1 - 2\omega' + \frac{2\omega'^2}{3} \right) \right], & \omega' \leq 1 \\ A\dot{M}(GMR'_0)^{1/2} \left[\xi(1 - \omega') + \frac{\sqrt{2}\gamma}{3A} \left(\frac{2}{3\omega'} - 1 \right) \right], & \omega' > 1 \end{cases} \quad (12)$$

where $\omega' = (R'_0/R_c)^{3/2} = \omega/A$. Note that the critical fastness parameter ω'_c (for $N' = 0$) equals ω_c , but the equilibrium period would become longer by a factor of $1/A$, and the system begins to evolve towards the newly determined equilibrium after the transition. Figure 2 shows the torque N and N' in units of $\dot{M}(GMR_c)^{1/2}$ as a function of ω with $\xi = \gamma = 1$. For ω between ~ 0.2 and ~ 0.8 , if \dot{M} is around \dot{M}_{cr} , small changes in \dot{M} can cause the torque reversal behavior with comparable spin-up/down torques $\lesssim \dot{M}(GMR_c)^{1/2}$.

We have applied the model to explain the spin change in several X-ray pulsars. As an illustration, Fig. 3 shows the fit to the observed spin reversal in GX 1+4. Similar as in Yi et al. (1997), we assume that \dot{M} keeps a linear increase or decrease as an approximation to more complex \dot{M} variations on longer

² For oblique rotators, there could be no spin-down allowed with ω or $\omega' < 1$ (Wang 1997).

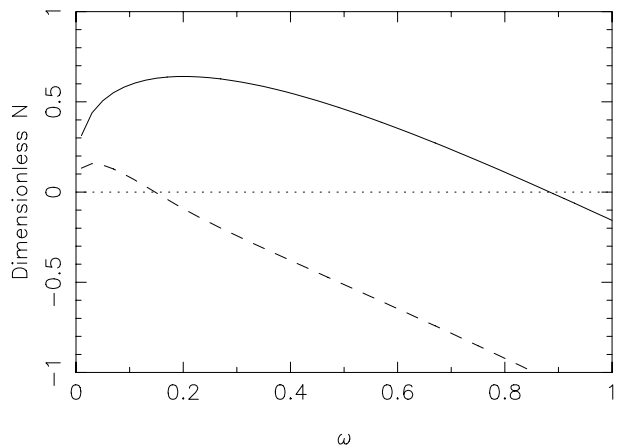


Fig. 2. The dimensionless torque $N/\dot{M}(GMR_c)^{1/2}$ (solid line) and $N'/\dot{M}(GMR_c)^{1/2}$ (dashed line) as a function of the fastness parameter ω .

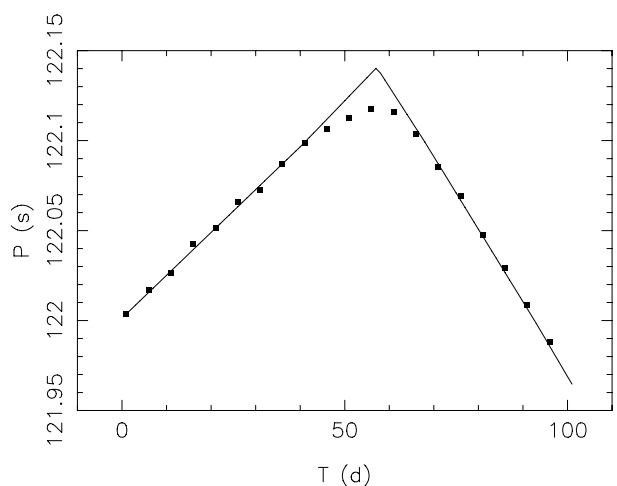


Fig. 3. Torque reversal event in GX 1+4. The points are the observed data and the solid line corresponds to the fitting result.

timescales. The transition occurs on a timescale $\ll |\Omega_*/\dot{\Omega}_*|$ before and after the transition. In Fig. 3, the observed torque reversal event is reproduced by taking $B_* = 1.0 \times 10^{13}$ G, $\dot{M} = 6.0 \times 10^{15}$ gs^{-1} , $d\dot{M}/dt = 3.25 \times 10^{15}$ $\text{gs}^{-1} \text{yr}^{-1}$, and $A = 0.3$ (we take the neutron star mass $M = 1.4 M_\odot$, and radius $R_* = 10^6$ cm). We note, however, that our estimated values (of B_* and \dot{M}) can always be rescaled by changing A and γ . The critical accretion rate for the transition is assumed to be $\dot{M}_{\text{cr}} = 6.5 \times 10^{15}$ gs^{-1} .

4. Discussion

Assuming accretion continues during the (soft) propeller stage, we have constructed a model for the magnetized torque on accreting neutron stars, which changes continuously when ω varies from 0 to $\gtrsim 1$ (without the singularity problem), and the spin-down torque increases with \dot{M} . Following Yi et al. (1997), we have applied the model to explain the torque reversal phenomena observed in a few X-ray pulsars.

The same problem has been investigated by Lovelace et al. (1999). These authors have developed a model for magnetic “propeller”-driven outflows for rapidly rotating neutron stars accreting from a disk. They find that the inner radius R_0 of the disk depends not only on μ and \dot{M} , but also on the star’s rotation rate Ω_* . The most interesting result is that for a given value of Ω_* , there may exist two solutions of R_0 , with one $>R_c$ and the other $<R_c$, corresponding to two possible equilibrium configurations. In a transition of R_0 , the propeller varies between being “off” and being “on”, leading to jumps between spin-up and spin-down. The transitions are assumed to be stochastic or chaotic in nature, and could be triggered by small variations in the accretion flow and magnetic field configuration. The ratio of spin-down to spin-up torque is also shown to be of order unity.

This model shows some similar features as our work. For example, in both models (1) discontinuous transitions of the magnetic torque are invoked to account for the observed abrupt spin changes in X-ray pulsars, (2) the torques are of comparable magnitude, and (3) the spin-down torques increases with \dot{M} . Especially, Eq. (10) suggests loss of angular momentum at the inner edge of the disk, probably by the magnetic-driven outflows proposed by Lovelace et al. (1999).

However, there exist significant differences between the two works. First, in the propeller regime, Lovelace et al. (1999) let most of the accreting material be ejected from the star, while we still allow mass accretion to guarantees no significant variations in the X-ray luminosities during the transition of spin evolution, implying that $\omega \gg 1$ in Lovelace et al. (1999), and $\omega \gtrsim 1$ in this work. Second, the torque reversals in our model are caused by the change of the dynamical structure of the disk, as suggested by Yi et al. (1997), at a certain, critical mass accretion rate. In Lovelace et al. (1999) the transition is stochastic, and could take place at any luminosities. Third, we have adopted the traditional Alfvén radius as the inner radius R_0 of the disk in both accretion and propeller phases. In Lovelace et al. (1999) R_0 depends on Ω_* also, and its value increases with Ω_* in the propeller case. Obviously the hypothesis of Ω_* -depending R_0 is more realistic, but how R_0 changes with Ω_* has not been well understood³. Future observations are expected to provide more stringent constraints on the mechanisms suggested in both models.

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³ Romanova et al. (2004) indeed observed increasing R_0 with Ω_* in their calculations. But this change is caused by the fact that the initial dipolar field becomes non-dipolar when the star rotates very rapidly.