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# Magnetic flux in the inter-network quiet Sun from comparison with numerical simulations *(Research Note)*

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**ABSTRACT**

Khomenko et al. estimate the mean magnetic field strength of the quiet Sun to be 20 G. The figure is smaller than several existing estimates, and it comes from the comparison between observed Zeeman polarization signals and synthetic signals from numerical simulations of magneto-convection. The numerical simulations require an artificially large magnetic diffusivity, which smears out magnetic structures smaller than the grid scale. Assuming a turbulent cascade for the unresolved artificially smeared magnetic fields, we find that their unsigned magnetic flux is at least as important as that explicitly shown in the simulation. The unresolved fields do not produce Zeeman polarization but contribute to the unsigned flux. Since they are not considered by Khomenko et al., their mean magnetic field strength has to be regarded as a lower limit. This kind of bias is not specific of a particular numerical simulation or a spectral line. It is to be expected when observed quiet Sun Zeeman signals are compared with synthetic signals from simulations.

**Key words.** Sun: fundamental parameters – Sun: magnetic fields – Sun: photosphere

## 1. Rationale

The observational properties of the quiet Sun magnetic fields are not well established (see, e.g., Sánchez Almeida 2004, and references therein). We are in the way of characterizing them, a process whose end is difficult to predict partly due to contradictions among existing measurements. In this sense, Khomenko et al. (2005) estimate a mean field strength or unsigned flux density $\langle B\rangle$ of only 20 G, a value smaller than the unsigned flux densities inferred by other means (e.g. Faurobert et al. 2001; Domínguez Cerdeña et al. 2003; Sánchez Almeida et al. 2003; Trujillo Bueno et al. 2004; Bommier et al. 2005; Domínguez Cerdeña et al. 2006). This *Note* argues that the unsigned flux estimated by Khomenko et al. should be regarded as a lower limit rather than an unbiased estimate. If the conclusion is correct, it may reconcile the result of Khomenko et al. with other existing estimates claiming $\langle B\rangle$ to be of the order of 100 G. Moreover, the argumentation and the caveat it entails are of general nature. They must be considered whenever the quiet Sun unsigned flux is derived by comparison of observed Zeeman signals with numerical simulations. Such approach to study the quiet Sun magnetism is bound to play a major role in the future.

The paper by Khomenko et al. (2005) compares the polarization observed in a quiet Sun region with the synthetic polarization produced by realistic numerical simulations of solar magneto-convection. The numerical simulations have a mean magnetic field strength $\langle B\rangle$ that decreases exponentially with time $t$,

$$\langle B(t)\rangle \simeq \langle B(0)\rangle \exp(-t/\tau_{\rm D}), \qquad (1)$$

with an e-folding time scale $\tau_{\rm D} \simeq 50$ min (Khomenko & Vögler 2005, private communication). As usual, the angle brackets denote volume average. The snapshot of the simulation with $\langle B(t)\rangle \simeq 20$ G is the one that best reproduces the histograms of observed polarization signals. This agreement is used to indicate that the observed inter-network region has $\langle B\rangle$ close to 20 G. There is a natural way to reconcile Khomenko et al. (2005) result with the much larger fluxes from other measurements. The numerical simulations do not provide information on structures smaller than a few grid points. They miss small scale magnetic structures that would show up if the simulations had finner more realistic resolution. The simulations only include a part of the magnetic fields, and so, it is to be expected that the estimate by Khomenko et al. (2005) only provides a lower limit to the true solar $\langle B\rangle$. This consideration can be a serious caveat or an academic remark depending on the importance of the neglected magnetic fields. There is no definite way of evaluating the importance of the bias except for improving the spatio-temporal resolution of the simulations to reach realistic values. This approach, however, demands a computer power exceeding by orders of magnitude the current resources.

Our work provides a preliminary estimate using the tools available at present.

## 2. Energy and unsigned flux density in the unresolved magnetic fields

The simulations assume a magnetic diffusivity $\eta_{\rm t} = 1.1 \times 10^{11}$ cm$^2$ s$^{-1}$, which is some three orders of magnitude larger than the Ohmic diffusivity to be expected at the base of the photosphere ($\eta \simeq 10^8$ cm$^2$ s$^{-1}$; Kovitya & Cram 1983). This unrealistically high diffusivity is needed for technical reasons to stabilize the numerical MHD code, and to prevent the artificial built-up of energy at the grid scale (Vögler 2003; Vögler et al. 2005; Nordlund & Stein 1990). How can the numerical simulations be realistic if they employ such artificially large magnetic diffusivity? They can if there is a physical mechanism that diffuses away the magnetic energy at the rate imposed by the artificial diffusivity. Somehow the existence of such mechanism is implicit when the numerical simulations are used to represent and characterize the real Sun. Lacking of obvious alternatives, we will describe such mechanism as turbulent diffusion, similar to that required for the astrophysical dynamos to operate within reasonable time scales (e.g., Parker 1979, Chap. 17). This association allows us to carry out simple estimates. The turbulent diffusion is produced by a complex unresolved magnetic field with spatial scales $l$ so small that the diffusion time scale for the true small Ohmic diffusivity is similar to the diffusion time scale for the structures in the simulation (Parker 1979; Biskamp 2003). Assuming that the high magnetic diffusivity used by the numerical code is of turbulent nature, one can evaluate the magnetic energy in the turbulent magnetic field implicit in the simulation. This complex magnetic field does not produce polarization signals, so that the observables remain as those synthesized by Khomenko et al. (2005). Nevertheless, it contains energy and unsigned flux that must be included when estimating the true unsigned flux density corresponding to the observed polarization signals. The section aims at showing how such turbulent magnetic field can indeed contain a significant amount of energy and unsigned flux.

In order to estimate the magnetic energy in the turbulent cascade from the minimum resolvable scale $L$, to the diffusive scale $l$, we need a model for the MHD turbulence. The MHD turbulence is a topic of active research and there is no unique and final way to approach the problem (see, e.g., Boldyrev 2005; Brandenburg et al. 2005, and references therein). However, one can estimate the magnetic energy in the turbulent cascade using a few approximate prescriptions which are now in use. They all lead to the conclusion that this energy may be significant.

We start off by assuming a Kolmogorov spectrum for the turbulent cascade (e.g., Biskamp 2003). Then the total magnetic energy per unit mass corresponding to a (spatial) wavenumber $k$ is,

$$E_k = C_{\rm K}\, \varepsilon^{2/3}\, k^{-5/3}, \tag{2}$$

with

$$2\pi/L < k < 2\pi/l. \tag{3}$$

The symbol $\varepsilon$ stands for the energy dissipated per unit time and unit mass by the turbulent cascade (see, e.g., Biskamp 2003, Sect. 5.3.2). The Kolmogorov constant $C_{\rm K}$ is a numerical factor of the order of 1.7. Then the magnetic energy per unit mass in unresolved fields $E_{\rm t}$ is,

$$E_{\rm t} = \int_{2\pi/L}^{2\pi/l} E_k\, {\rm d}k \simeq \frac{3C_{\rm K}}{2(2\pi)^{2/3}} (\varepsilon L)^{2/3}, \tag{4}$$

where we have taken into account that $l \ll L$. Now, the mean magnetic field of the numerical simulations have an exponential decrease with an e-folding time scale $\tau_{\rm D}$ (Eq. (1)). Assuming that the standard deviation among the field strengths scales with the mean field[1],

$$\sigma_B = \langle (B - \langle B\rangle)^2\rangle^{1/2} \propto \langle B\rangle, \tag{5}$$

then,

$$\left\langle B^2 \right\rangle = \sigma_B^2 + \langle B\rangle^2 \propto \langle B\rangle^2, \tag{6}$$

implying a magnetic energy $\left\langle B^2\right\rangle/(8\pi)$ decreasing exponentially with a time scale half $\tau_{\rm D}$ (compare Eqs. (1) and (6)). The energy dissipated per unit time by the turbulent cascade turns out to be

$$\varepsilon \simeq -\frac{1}{\rho}\frac{\rm d}{{\rm d}t}\left[\frac{\left\langle B^2\right\rangle}{8\pi}\right] = \frac{\left\langle B^2\right\rangle}{8\pi} 2\,\tau_{\rm D}^{-1}\,\rho^{-1}, \tag{7}$$

with $\rho$ representing the mass density required to transform energy per unit mass ($E_{\rm t}$ and $E_k$) into energy per unit volume $\left\langle B^2\right\rangle/(8\pi)$. The dissipated energy has a compact expression in terms of the Alfven speed $v_{\rm A}$,

$$\varepsilon \simeq v_{\rm A}^2\, \tau_{\rm D}^{-1}, \tag{8}$$

with,

$$v_{\rm A}^2 = \frac{\left\langle B^2\right\rangle}{4\pi\rho}. \tag{9}$$

If the symbol $B_{\rm t}$ stands for the turbulent magnetic field strength,

$$E_{\rm t} = \frac{\left\langle B_{\rm t}^2\right\rangle}{8\pi\rho} = \frac{\left\langle B_{\rm t}^2\right\rangle}{\left\langle B^2\right\rangle}\frac{v_{\rm A}^2}{2}. \tag{10}$$

Using Eqs. (4), (8), and (10),

$$\frac{\left\langle B_{\rm t}^2\right\rangle}{\left\langle B^2\right\rangle} = \frac{3C_{\rm K}}{(2\pi)^{2/3}}\left(\frac{L}{\tau_{\rm D}\, v_{\rm A}}\right)^{2/3}. \tag{11}$$

The variables defining our problem are, $\rho \simeq 3\times 10^{-7}$ g cm$^{-3}$ (the mass density at the base of the photosphere), $\left\langle B^2\right\rangle^{1/2} = 28$ G (assuming $\sigma_B \simeq \langle B\rangle \simeq 20$ G), $L \simeq 100$ km (Appendix A), and $\tau_{\rm D} = 50$ min (Eq. (1)). They render,

$$\frac{\left\langle B_{\rm t}^2\right\rangle}{\left\langle B^2\right\rangle} \simeq 0.56. \tag{12}$$

[1] A condition typical of many probability density functions suitable for describing the variable $B$, e.g., a Maxwellian distribution, and exponential distribution, a uniform distribution, etc.

We are interested in the mean magnetic field strength (or unsigned flux density), since this parameter is used to characterize the observations. Assuming the relationship (5) with the same ratio $\sigma_B/\langle B\rangle$ for both $\langle B\rangle$ and $\langle B_{\rm t}\rangle$,

$$\frac{\langle B_{\rm t}\rangle}{\langle B\rangle} = \sqrt{\frac{\left\langle B_{\rm t}^2\right\rangle}{\langle B^2\rangle}} \simeq 0.75. \qquad (13)$$

The analysis above has been repeated using the so-called Iroshnikov-Krishnan spectrum of MHD turbulence (Biskamp 2003),

$$E_k = C_{\rm IK}\,(v_{\rm A}\,\varepsilon)^{1/2}\,k^{-3/2}. \qquad (14)$$

(MHD turbulence simulations seem to yield spectra in between Kolmogorov and Iroshnikov-Krishnan; see Boldyrev 2005). Following the procedure explained above, one finds,

$$\frac{\left\langle B_{\rm t}^2\right\rangle}{\langle B^2\rangle} = \frac{2C_{\rm IK}}{\sqrt{\pi/2}}\Big(\frac{L}{\tau_{\rm D}\,v_{\rm A}}\Big)^{1/2}. \qquad (15)$$

With $C_{\rm IK} \simeq C_{\rm K}$, and for the numerical values used in Eq. (12),

$$\frac{\left\langle B_{\rm t}^2\right\rangle}{\langle B^2\rangle} \simeq 1.30, \qquad (16)$$

yielding,

$$\frac{\langle B_{\rm t}\rangle}{\langle B\rangle} \simeq 1.14. \qquad (17)$$

The estimates above assume the inertial range of the turbulence to begin at a scale larger than $L$. For this reason we can describe the magnetic energy for $k > 2\pi/L$ using laws characteristic of the inertial range (Eqs. (2) and (14)). However, this assumption is not decisive. Consider a turbulent cascade whose inertial range begins at an unresolved scale $\Lambda < L$. Let us represent the unknown spectrum in the range between $L$ and $\Lambda$ as a power law of index $\alpha$,

$$E_k \simeq \frac{\left\langle B^2\right\rangle}{8\pi\rho}\left(\frac{L}{2\pi}\right)^{1-\alpha} k^{-\alpha}, \qquad (18)$$

with $2\pi/L < k \le 2\pi/\Lambda$. The factor multiplying $k^{-\alpha}$ guarantees the continuity of $E_k$ at $k = 2\pi/L$; it is equal to the mean energy per unit mass and spatial frequency existing in the resolved scales, so that for $k = 2\pi/L$,

$$E_k \simeq \frac{\left\langle B^2\right\rangle}{8\pi\rho}\,\frac{1}{2\pi/L}. \qquad (19)$$

The exponent $\alpha$ is also imposed by continuity arguments. When the inertial range begins, i.e. at $k = 2\pi/\Lambda$, then $E_k$ is given by Eq. (2). This constraint leads to

$$(\alpha-1)\,\ln(L/\Lambda) = \ln\left(\frac{\left\langle B^2\right\rangle}{8\pi\rho}\right) - \ln C_k - \frac{2}{3}\ln\left(\frac{\epsilon\Lambda}{2\pi}\right). \qquad (20)$$

The definition of $E_{\rm t}$ in Eq. (4) implies,

$$E_{\rm t} = \int_{2\pi/L}^{2\pi/\Lambda} E_k\,{\rm d}k + \int_{2\pi/\Lambda}^{2\pi/l} E_k\,{\rm d}k \ge \int_{2\pi/L}^{2\pi/\Lambda} E_k\,{\rm d}k$$
$$\simeq \frac{\left\langle B^2\right\rangle}{8\pi\rho}\,\frac{1}{\alpha-1}, \qquad (21)$$

where we have assumed $\alpha > 1$ and $L \gg \Lambda$. (Intermediate values of $\Lambda$ provide magnetic energies in between Eq. (11) and the limit considered here.) The expression (21), together with Eq. (10), render

$$\sqrt{\frac{\left\langle B_{\rm t}^2\right\rangle}{\langle B^2\rangle}} \gtrsim \frac{1}{\sqrt{\alpha-1}} \simeq 1. \qquad (22)$$

Once more the unsigned flux in unresolved fields turns out to be significant. The ratio in Eq. (22) has been evaluated using Eq. (20) with the same parameters leading to Eq. (13), which provide an exponent $\alpha$ varying from 2.1 to 1.8 when $\Lambda$ goes from 10 km to 0.1 km.

Equations (13), (17) and (22) suggest that the implicit turbulent field accounting for the used magnetic diffusivity has a magnetic flux similar to that explicitly shown in the simulations.

Some numerical simulations of turbulent magneto-convection show a range of wavenumbers where the magnetic energy exceeds the kinetic energy (super equipartition range; see Biskamp 2003; Maron et al. 2004; Brandenburg et al. 2005). The magnetic energy densities used above are much smaller than the kinetic energy density of the solar granulation. If this super equipartition range would have to be included in our estimate, one needs to increase $\varepsilon$ with respect to the values employed above. The turbulent magnetic energy would increase accordingly, leading to $\langle B_{\rm t}\rangle$ larger than the values in Eqs. (13) and (17). Consequently, the estimates above are probably conservative, a conclusion reinforcing the importance of the unresolved magnetic fields.

## 3. Conclusions and discussion

The numerical simulations of solar magneto-convection require artificially large magnetic diffusion, which smears out all the spectrum of magnetic structures smaller than the grid scale. The use of such high magnetic diffusivity can be understood as the effect of a complex unresolved turbulent magnetic field with spatial scales so small that the diffusion time scale for the true small Ohmic diffusivity is similar to the diffusion time scale for the structures in the simulation. Such an implicit magnetic field does not contribute to the polarimetric signals synthesized by Khomenko et al. (2005). Assuming a turbulent cascade for the unresolved artificially smeared magnetic fields, we find that their unsigned magnetic flux is at least as important as that explicitly shown in the simulation. Should this magnetic flux is considered, the Zeeman polarization signals measured by Khomenko et al. are consistent with an unsigned flux of $2 \times 20$ G (i.e., $\langle B_{\rm t}\rangle \simeq \langle B\rangle$; Eqs. (13), (17) and (22)). In other words the unsigned flux assigned by Khomenko et al. (2005) must be regarded as a conservative lower limit. This conclusion is not specific of the simulations analyzed here. A bias is to be expected whenever the quiet Sun unsigned magnetic flux is inferred as the unsigned flux of numerical simulations reproducing observed Zeeman polarization signals.

The calculations that we describe represent only a first approximation to estimating the bias. They are based on the theory of MHD turbulence, which remains to be completed.

The conclusions have to be backed up or rejected by numerical simulations with realistic Ohmic diffusivities.

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## Appendix A: $L \simeq 100$ km

The spatial discretization of the MHD equations solved by Vögler (2003) is based on a four-order finite difference scheme. Then the components of the magnetic field are approximated by a four-order polynomial, so that the first and second partial derivatives appearing in the MHD equations are represented by a third-order polynomial and a second-order polynomial, respectively. We will estimate $L$ as the smallest wavelength of a sinusoidal magnetic field for which the error involved in these polynomial approximations is insignificant.

Let us denote by $f(x)$ one of the components of the magnetic field vector, with $k$ the wavenumber along the spatial coordinate $x$,

$$f(x) = \sin(kx). \tag{A.1}$$

The numerical code represents it in the surroundings of $x_0$ as the polynomial $f_a(x)$,

$$f_a(x) = \sum_{n=0}^{4} f^{(n)}(x_0)\frac{(x-x_0)^n}{n!}. \tag{A.2}$$

The fifth order term in the Taylor expansion of $f(x)$ provides the error of the approximation, $\Delta f(x)$,

$$\Delta f(x) = f(x) - f_a(x) \simeq f^{(5)}(x_0)\frac{(x-x_0)^5}{5!}. \tag{A.3}$$

As usual, the symbol $f^{(n)}$ denotes the $n$th derivative of $f(x)$. The approximation (A.2) is carried out at each pixel, so that

$$|x - x_0| \leq \Delta x, \tag{A.4}$$

with $\Delta x$ the pixel size. Among the various polynomials used to represent the MHD equations, the computation of the second derivatives,

$$f^{(2)}(x) = -k^2 f(x), \tag{A.5}$$

corresponds to the lowest order and, therefore, it is the worst approximation. If the error associated with the computation of the second derivatives is tolerable, then the polynomial approximation is tolerable. According to Eq. (A.3), the error $\Delta f^{(2)}(x)$ is

$$\Delta f^{(2)}(x) = f^{(2)}(x) - f_a^{(2)}(x) \simeq k^5 \cos(kx_0)\,\frac{(x-x_0)^3}{3!}, \tag{A.6}$$

where we have taken into account that $f^{(5)}(x_0) = k^5\cos(kx_0)$. The average (unsigned) error within the range of the approximation (Eq. (A.4)) is,

$$\begin{aligned}\overline{|\Delta f^{(2)}(x)|} &= \frac{1}{2\Delta x}\int_{x_0-\Delta x}^{x_0+\Delta x} |\Delta f^{(2)}(x)|\,\mathrm{d}x \\ &\simeq k^5\,|\cos(kx_0)|\frac{\Delta x^3}{24}.\end{aligned} \tag{A.7}$$

The factor $|\cos(kx_0)|$ varies with $x_0$ but its average is the same as the average of $|f(x)|$. Then the mean relative error when computing the second derivatives, $\delta$, is given by,

$$\overline{|\Delta f^{(2)}(x)|}\Big/\overline{|f^{(2)}(x)|} = \delta \simeq \frac{k^3\Delta x^3}{24}. \tag{A.8}$$

This expression provides a relationship between the relative error of the approximation, the wavelength of the sinusoidal $\lambda$, and the pixel size,

$$\lambda = \frac{2\pi}{k} \simeq 4.7\,\Delta x\,(\delta/0.1)^{-1/3}. \tag{A.9}$$

We define $L$ as the smallest wavelength for which the error is insignificant, namely, $\lambda$ assuring $\delta \leq 0.1$. According to Eq. (A.9), and keeping in mind that $\Delta x = 21$ km,

$$L \simeq 4.7\,\Delta x \simeq 100\text{ km}. \tag{A.10}$$

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