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# The CdC2000 Bordeaux *Carte du Ciel* catalogue ($+11° \leq \delta \leq +18°$)*

M. Rapaport[1], C. Ducourant[1,5], J. F. Le Campion[1], A. Fresneau[2], R. W. Argyle[3], C. Soubiran[1], R. Teixeira[5,1], J. I. B. Camargo[1], J. Colin[4], G. Daigne[1], J. P. Périé[1], and Y. Réquième[1]

[1] Observatoire Aquitain des Sciences de l'Univers, CNRS-UMR 5804, BP 89, 33270 Floirac, France
e-mail: ducourant@obs.u-bordeaux1.fr
[2] Observatoire Astronomique de Strasbourg, 11 rue de l'Université, 67000 Strasbourg, France
[3] Astronomy Survey Unit, Institute of Astronomy, Madingley Road, Cambridge, CB3 0HA, UK
[4] Observatoire de la Côte d'Azur, Boulevard de l'Observatoire, BP 4229, 06304 Nice Cedex 4, France
[5] Instituto de Astronomia, Geofísica e Ciências Atmosféricas, Universidade de São Paulo, Rua do Matão, 1226, Cidade Universitária, 05508-900 São Paulo, SP, Brazil

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**ABSTRACT**

*Context.* This work is part of a program of proper motion measurements in the Bordeaux *Carte du Ciel* zone.
*Aims.* We present the CdC2000 catalogue issued from the reduction of the complete Bordeaux *Carte du Ciel* zone observed at the begining of last century with the Bordeaux astrograph.
*Methods.* Plates have been individually reduced using the TYCHO-2 Catalogue as the astrometric reference.
*Results.* This catalogue is a positional catalogue of 344 781 stars covering the declination zone $+11° \leq \delta \leq +18°$. The average epoch of positions is 1914.7. The data are from the 512 *Carte du Ciel* plates archived at the Bordeaux Observatory and scanned with the APM Cambridge automatic measuring machine. Astrometric standard errors are about 0.10″ to 0.12″ on positions and 0.6 mag on photographic magnitudes.
*Conclusions.* A detailed study of errors and a comparison of various reduction methods are presented in order to take into account the significant propagated errors from the reference catalogue. The CdC2000 provides positions at the epoch of the plates for 50 682 TYCHO-2 stars. The associated positions are proved to be $\sqrt{2}$ more precise than the positions given by the TYCHO-2 catalogue at the epoch of the plates (0.11″ instead of 0.15″). This work is part of a program of proper motion measurements in the Bordeaux *Carte du Ciel* zone.

**Key words.** astrometry – catalogs – reference systems – surveys

## 1. Introduction

Accurate proper motions for a large set of stars are of great importance to many areas of astronomy, in particular for galactic studies. The understanding of the structure and of the dynamic of our galaxy is improved with the release of each high-quality proper motion catalogue.

At the present time the HIPPARCOS (ESA 1997) and TYCHO-2 (Høg et al. 2000) catalogs are the largest high-quality proper motion surveys. The HIPPARCOS catalog provides the most accurate proper motions (precision of 1 mas/yr) for 120 000 bright stars. The TYCHO-2 catalogue is complete down to $V = 11$ with a mean standard error on proper motions of 2.5 mas/yr.

The UCAC2 catalogue (Zacharias et al. 2004) is the most recently published astrometric catalogue. It provides proper motions down to $R = 16$ with an accuracy varying from 1 to over 7 mas/yr depending on first epoch material; most accurate ones rely on AC2000.2 positions (Urban et al. 1998) and concern objects mostly brighter than $V = 13$.

For fainter objects there is a lack of high accuracy proper motions. Indeed the precise determination of proper motions relies heavily on the time base between the various catalogues compared. In this magnitude domain there is no all-sky astrometric catalogue with ancient positions such as AC2000.2.

The *Carte du Ciel* program was a project that started in 1887 in 18 observatories spread around the world. It aimed at the production of a chartof the entire sky up to the 16th magnitude (*Carte du Ciel* project) and of a catalogue of positions and magnitudes complete up to the 14th magnitude (Astrographic Catalogue project). The *Carte du Ciel* plates would contain triple exposures, each of 20 min, slightly shifted, usually with a triangular pattern. The Astrographic Catalogue plates would

* Table 2 is an extract of CdC2000 catalogue which is only available in electronic form at the CDS via anonymous ftp to cdsarc.u-strasbg.fr (130.79.128.5) or via http://cdsweb.u-strasbg.fr/cgi-bin/qcat?J/A+A/449/435

contain triple exposures (10 min, 5 min, 40 s) usually aligned. While the Astrographic program was completed and published (at the epoch in each observatory and later in AC2000.2 (Urban et al. 2001)) the *Carte du Ciel* project never led to the publication of a catalogue. Very often the *Carte du Ciel* plates remained unexploited in the Observatories.

As one participant of the *Carte du Ciel* project, the Bordeaux observatory owns 512 *Carte du Ciel* plates covering the declination range $+11° \leq \delta \leq +18°$. It was recently decided to digitize them in order to salvage this astronomical treasure before its deterioration and to provide a valuable first epoch material in proper motion programs. The plate processing started in December 2000 at the APM automatic measuring machine in Cambridge (UK) and finished in November 2001.

In 1998 the PM2000 project of proper motions measurement in the Bordeaux *Carte du Ciel* zone ($11° \leq \delta \leq 18°$) started at Bordeaux. The idea was to re-observe this zone with the Bordeaux automatic CCD meridian circle (Rapaport et al. 2001) and to simultaneously measure the Bordeaux *Carte du Ciel* plates at the APM (Cambridge). Proper motion would result from the comparison of the CCD meridian observations with the ancient plate measurements (Ducourant et al. 2006). We present here the work that concerns the measurement and reduction of the photographic plates and the resulting positional catalogue. The CdC2000 catalogue is the first complete reduction of a *Carte du Ciel* zone and provides a precious ancient positional data set for proper motion measurements.

In Sect. 2 we describe the plate material. In Sect. 3 we briefly describe the methods used to remove reseau and to cross-identify objects in the triple exposures. The analysis of systematic errors in the measurements on the APM machine is developed in Sect. 4, while the problem of identification of triplets is briefly mentioned in Sect. 5. The classical reduction method with an analysis of the errors is presented in Sect. 6. The astrometric reduction used in this paper, inspired by Jefferys (1980, 1981), is developed in Sect. 7. The content of the catalogue is given in Sect. 8.

## 2. Plate material

The Bordeaux *Carte du Ciel* plate archive consists of 552 photographic plates (2.4° × 2.4°) taken with the Bordeaux *Carte du Ciel* astrograph ($\phi = 44°50', \lambda = 00°32'$) between 1893 and 1937. Forty plates of this archive were lost over time and the work presented here deals with the measurements of the 512 remaining plates.

No filter was used for the exposures and the resulting mean central wavelength is about 430 nm. The plates contain triple exposures (20 min each) and a reseau grid.

We give in Fig. 1 the sky coverage of our archive and in Fig. 2 the distribution of epochs of the plates. The characteristics of the astrograph and of the plates are given in Table 1.

## 3. Plate measurements

The plates were scanned with the APM at Cambridge. A 10 micron flying spot was used to raster scan each plate producing square pixels of side 7.5 micron which were then binned by a factor of 2 to match the grain size of the emulsion. The raw data file thus formed was 143 Mbyte in size but this was reduced to approximately 0.5 Mbyte by keeping only those features that appear as contiguous pixels 3$\sigma$ above the plate background.

All the plates have a reseau grid which, when scanned, produces a characteristic series of images all of which are highly elliptical and which can therefore be removed through post processing. This procedure used the crowded-field software described by Irwin (1997).

Photographic magnitudes were derived from the measured fluxes as $B = K - 2.5 \log_{10}(\mathrm{Flux})$, where $K$ is the unknown instrumental constant defined using TYCHO-2 $B$ magnitudes.

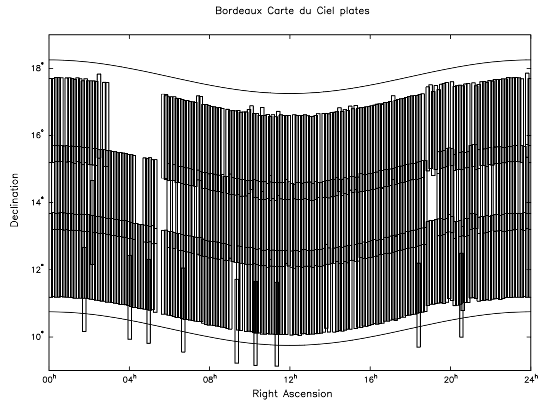


**Fig. 1.** Sky coverage with the Bordeaux *Carte du Ciel* plate material in equatorial coordinates (J2000). Each vertical strip corresponds to a single plate.

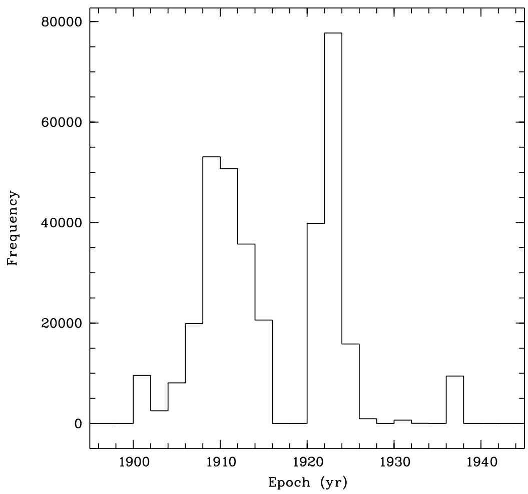


**Fig. 2.** Histogram of the epochs of the Bordeaux *Carte du Ciel* plate archive.

**Table 1.** *Carte du Ciel* astrograph and plate characteristics.

| | |
|---|---|
| Focal length | 3,43 m |
| Aperture | 33 cm |
| Nb exposures | 3 |
| Exposure time | 20 min |
| Triplets separation (epoch ≤ 1910) | 0.08 mm |
| Triplets separation (epoch ≥ 1910) | 0.16 mm |

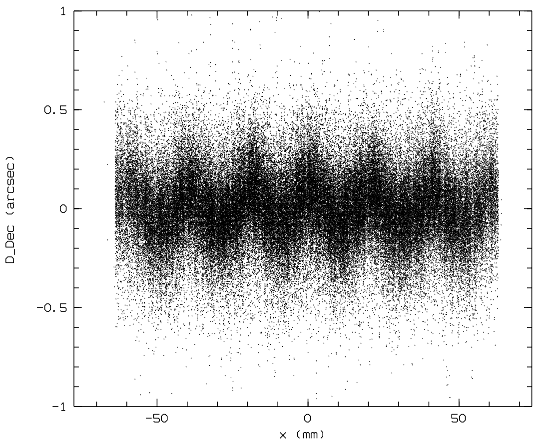


**Fig. 3.** Residuals in declination of TYCHO-2 stars covering the whole *Carte du Ciel* zone as a function of $x$.

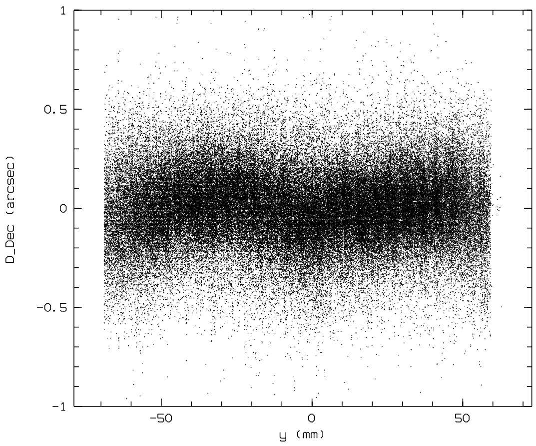


**Fig. 4.** Residuals in declination of TYCHO-2 stars covering the whole *Carte du Ciel* zone as a function of $y$.

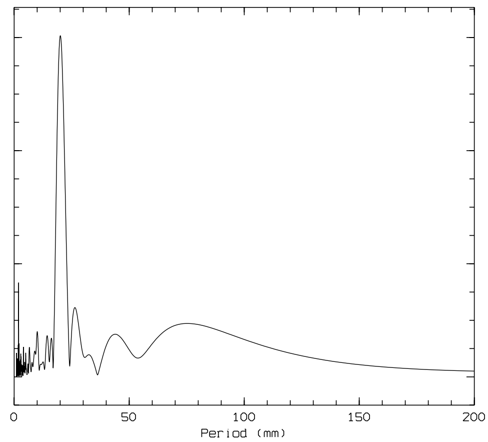


**Fig. 5.** Square root of the power spectrum of the residuals in declination (arbitrary units) as a function of $x$ position on the plate. The amplitude of the maximal peak is 30 mas.

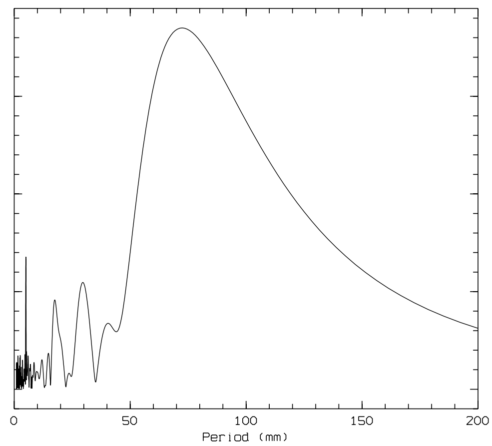


**Fig. 6.** Square root of the power spectrum of the residuals in declination (arbitrary units) as a function of $y$ position on the plate. The amplitude of the maximal peak is 18 mas.

The photographic magnitudes were derived with a mean rms of 0.6 mag.

## 4. Systematic error on measured positions

Ewans & Irwin (1995) found that the APM measured positions were contaminated by periodic systematic errors in the $y$ coordinate. To investigate this effect, we reduced all the plates with the TYCHO-2 reference catalogue using a 12 constants model and we compared the calculated position of the TYCHO-2 stars with their catalogue positions. No systematic effect could be detected in the residuals in right ascension. We give in Figs. 3 and 4 the residuals in declination as a function of the measured $x$ and $y$ coordinates.

In both diagrams a signal is clearly visible. An analysis of the power spectrum of the residuals in declination as function of the $x$ coordinate led to the detection of a signal with an amplitude of 30 mas and a period of 20 mm as shown in Fig. 5.

This result is in good agreement with Evans & Irwin (1995). A second signal could be detected in the residuals indeclination expressed as a function of the $y$ coordinate. The analysis of this signal led to the measurement of a signal with an amplitude of 18 mas and a period of 72 mm as shown in Fig. 6.

This last signal was not detected before. The $y$ measurements (expressed in mm) provided by the APM have been corrected for the two signals as follows:

$$y = y\prime - 0.018 \cdot 2\cos\left(\frac{y}{72.464} - 21.816\right)$$

with (1)

$$y\prime = y_{\rm mes} - 0.030 \cdot 2\cos\left(\frac{x}{20.080} + 162.988\right)$$

where $y_{\rm mes}$ are the uncorrected measurements from APM expressed in millimeters.

## 5. Triplets identification

A classic problem of *Carte du Ciel* plates is the automatic identification of triplets of measurements in the scanned data. The strategy used here was to compute distances between all measured objects. The histogram of these distances exhibits

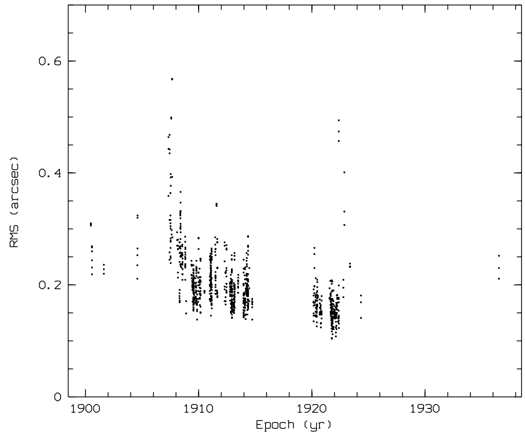


**Fig. 7.** Mean rms of plate reductions as a function of epoch for the 512 plates.

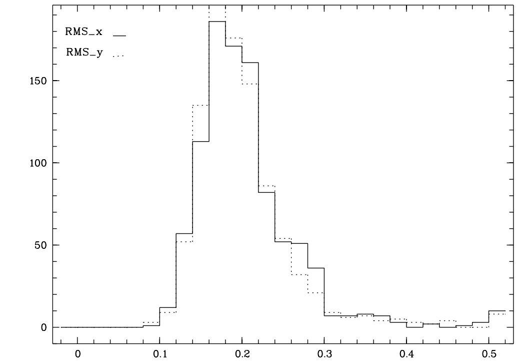


**Fig. 8.** Histogram of rms of plates reductions in right ascension and declination expressed in ″. The associated mean values are: $\varepsilon_{\mathrm{O-C},x} = 0\farcs21 \pm 0.07$, $\varepsilon_{\mathrm{O-C},y} = 0\farcs19 \pm 0.07$ and $\sqrt{\varepsilon^2_{\mathrm{O-C},x} + \varepsilon^2_{\mathrm{O-C},y}} = 0\farcs28 \pm 0.08$.

three preferential distances corresponding to the three exposures. All triplets were identified and separated in three independent exposures.

As shown in Table 1 the distance between exposures is about 0.08 mm before 1910 and about 0.16 mm after 1910. Figure 7 gives the residuals of the reduction of each plate (12 constants model) as a function of the epoch of the plate.

After 1910 the resulting astrometry of the plates is better than before 1910. The small distance between the three exposures on the plates before 1910 explains this poor astrometric quality, the separation of the three exposures being more difficult and contamination much more frequent.

## 6. Astrometric reduction

### 6.1. The "classical" astrometric reduction

In the classical astrometric reduction of a plate, the conversion of the measured coordinates $(x, y)$ to standard coordinates $(X, Y)$ is performed by a mathematical model whose coefficients are known as plate constants. We tested two different plate models to reduce the *Carte du Ciel* plates: the 8 constants model used for the reduction of the Astrographic Catalogue plates (Urban et al. 2001), and the 12 constants model (complete polynomial of second order) traditionally used at Bordeaux (Ducourant 1989). These two models gave very similar results and we decided to use the complete polynomial of second order model:

$$\begin{aligned} X - x &= a + bx + cy + dx^2 + ey^2 + fxy \\ Y - y &= g + hx + iy + jx^2 + ky^2 + lxy \end{aligned} \tag{2}$$

where $(X, Y)$ are the standard coordinates, $(x, y)$ are the measured coordinates and $(a, b, c, d, e, f, g, h, i, j, k, l)$ are the unknown plate constants determined from the resolution of Eq. (2) by the least squares method.

### 6.2. The "classical" estimation of error measurements

In the classical procedure of reduction considered in this section, the positions of the reference stars are considered to be known perfectly, while the measurements $(x, y)$ are subject to random errors, characterized by their rms $(\varepsilon_{\mathrm{mes},x}, \varepsilon_{\mathrm{mes},y})$ on the coordinates $x$ and $y$. An estimation of $(\varepsilon_{\mathrm{mes},x}, \varepsilon_{\mathrm{mes},y})$ is given by the quantity $\varepsilon_{\mathrm{O-C}} = \sqrt{\frac{\Sigma r_i^2}{(N-6)}}$ where $N$ is the number of equations associated with each coordinate and $r_i$ is the residual associated with the $i$th equation.

We calculated for each exposure of all plates, an estimation of the measurement errors: $\varepsilon_{\mathrm{O-C},x}, \varepsilon_{\mathrm{O-C},y}$.

Figure 8 presents the histograms of these quantities.

The mean values of these quantities for the complete set of exposures are:

$$\begin{aligned} \varepsilon_{\mathrm{O-C},x} &= 0\farcs21 \pm 0.07 \\ \varepsilon_{\mathrm{O-C},y} &= 0\farcs19 \pm 0.07 \\ \sqrt{\varepsilon^2_{\mathrm{O-C},x} + \varepsilon^2_{\mathrm{O-C},y}} &= 0\farcs28 \pm 0.08. \end{aligned} \tag{3}$$

### 6.3. Another determination of the precision of measurements

Most of the *Carte du Ciel* plates are characterized by the presence of three separate images of each star. If $(x_1, y_1)$, $(x_2, y_2)$, $(x_3, y_3)$ are the actual coordinates of the three images of a given star, the quantities $(x_2 - x_1)$, $(x_3 - x_2)$,$(x_1 - x_3)$ are constant for all stars present on a given plate. Let us consider now the set of measured quantities $(x_2 - x_1)$,$(x_3 - x_2)$, $(x_1 - x_3)$ for each star on the plate (same formulation in $y$). Their variances $V_1, V_2, V_3$ are related to the standard errors $\sigma_1, \sigma_2, \sigma_3$ of the measurements on each image by the relation

$$\begin{aligned} \sigma_1^2 + \sigma_2^2 &= V_1 \\ \sigma_2^2 + \sigma_3^2 &= V_2 \\ \sigma_3^2 + \sigma_1^2 &= V_3 \end{aligned} \tag{4}$$

from which we deduce for each plate the standard errors $\sigma_1, \sigma_2, \sigma_3$ of the measurements in the two coordinates for each exposure of the triplets. The mean value and rms of these quantities for the set of 512 plates are:

$$\begin{aligned} \sigma_{1,x} &= 0\farcs120 \pm 0.003 & \sigma_{1,y} &= 0\farcs100 \pm 0.003 \\ \sigma_{2,x} &= 0\farcs100 \pm 0.003 & \sigma_{2,y} &= 0\farcs110 \pm 0.002 \\ \sigma_{3,x} &= 0\farcs120 \pm 0.003 & \sigma_{3,y} &= 0\farcs100 \pm 0.002. \end{aligned} \tag{5}$$

The values obtained for each exposure are nearly equal, and we can consider that the mean precision of the measurements on the plates is well estimated by

$$\begin{aligned} \varepsilon_{\mathrm{mes},x} &= 0\farcs11 \\ \varepsilon_{\mathrm{mes},y} &= 0\farcs10. \end{aligned} \tag{6}$$

The technical characteristics of the measuring machine allow us to estimate independently that the accuracy of measurements on the Bordeaux *Carte du Ciel* plates is about 2.5 $\mu$ on each coordinate, corresponding to 0.15″ on the sky. These estimations are consistent with the precision $\varepsilon_{\mathrm{mes},x}$ and $\varepsilon_{\mathrm{mes},y}$ obtained in Eq. (6).

From these considerations, we evaluate the precision of measurements on the *Carte du Ciel* plates to be between $0\farcs10$–$0\farcs15$.

These values are clearly smaller than the values ($\varepsilon_{\mathrm{O-C},x}$ and $\varepsilon_{\mathrm{O-C},y}$) obtained in Eq. (3) by the "classical" error estimation. This discrepancy comes from the fact that in our classical reduction error calculation we considered the reference catalogue free of error. In this special case where the mean epoch of TYCHO-2 (1991.25) is far from the epoch of the plates, the propagation of the catalogue errors can no longer be neglected. This is demonstrated in the next section.

### 6.4. The propagated errors of the TYCHO-2 reference catalogue

Let us now consider the TYCHO-2 reference stars present on the *Carte du Ciel* plates. The TYCHO-2 catalogue gives the accuracy of its proper motions and positions at the epoch of the catalogue. Thus we can determine for each plate the mean value of the precision in both coordinates $\alpha, \delta$ of all TYCHO-2 stars present on the plate. For the set of all plates, the mean values of these quantities are

$$\begin{aligned} \varepsilon_{\mathrm{cat},x} &= 0\farcs15 \pm 0.02 \\ \varepsilon_{\mathrm{cat},y} &= 0\farcs15 \pm 0.02. \end{aligned} \tag{7}$$

Comparing Eqs. (6) and (7), we notice that the catalogue errors are slightly larger than the measurements errors; thus it is not completely correct, at least from a theoretical point of view, to consider that the reference star positions are perfectly known, as is explicitly supposed in classical astrometric reduction. We are in the situation described in several books or papers (Bevington 1969; Eichhorn 1985) where the problem of fitting a straight line through a set of points $(x_i, y_i)$ with uncertainties on both $(x_i, y_i)$ is considered. Bevington suggests that one should combine the uncertainties on both quantities $(x_i, y_i)$ and assign them to the dependent variable alone. We see that in the case of the *Carte du Ciel* plates considered here, the combination of measurements and catalogue errors obtained in Eqs. (6) and (7) give

$$\begin{aligned} \varepsilon_{x,t} &= 0\farcs19 \\ \varepsilon_{y,t} &= 0\farcs18 \end{aligned} \tag{8}$$

which are very close to the values obtained in Eq. (3) for the classical reduction. Nevertheless several authors (Eichhorn et al. 1971; Eichhorn 1988; Jefferys 1980, 1981) developed new reduction methods for the situation described in this section and we decided to use the algorithm developed by Jefferys for the reduction of our plates, as presented in next section.

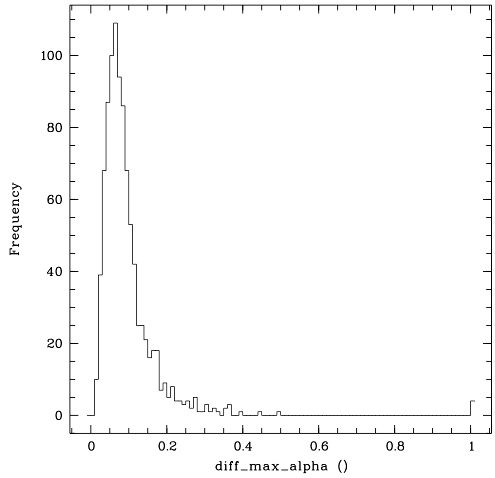

**Fig. 9.** Histogram of the maximal difference in right ascension (arcseconds) between the final right ascensions using the classical reduction procedure and Jefferys's algorithm.

## 7. The reduction of the *Carte du Ciel* plates using Jefferys's method

### 7.1. The method

The condition equations corresponding to a complete 2nd degree model are:

$$\begin{aligned} X_i - x_i &= a + bx_i + cy_i + dx_i^2 + ey_i^2 + fx_iy_i \\ Y_i - y_i &= g + hx_i + iy_i + jx_i^2 + ky_i^2 + lx_iy_i. \end{aligned} \tag{9}$$

The standard coordinates $(X_i, Y_i)$ of the reference stars are affected by the errors of the TYCHO-2 catalogue. The measured coordinates $(x_i, y_i)$ of the reference stars on the plates are affected by the measurement errors. The two types of errors have been shown previously to be of the same order ($0\farcs10$–$0\farcs15$) in each coordinate. So, in the method presented in this section, $(X_i, Y_i)$ and $(x_i, y_i)$ are considered as observational data with their associated precision, $\varepsilon_{\mathrm{mes},x}, \varepsilon_{\mathrm{mes},y}$ for the measurements and $\varepsilon_{\mathrm{cat},x}, \varepsilon_{\mathrm{cat},y}$ for the catalogue errors while $a, b, c, \cdots, j, k, l$ are the unknown plate constants. Using the iterative procedure described in Jefferys (1980, 1981), we determined the plate constants with their covariance matrix. We then determined the standard coordinates of each target star with their formal precision.

### 7.2. Comparison with the classical reduction procedure

For all stars of each plate, we determined the maximal difference between the final positions obtained using both the classical reduction procedure and Jefferys's algorithm. The histogram of the maximal differences for the two coordinates is given in Figs. 9 and 10. Their mean values for the set of plates are $0\farcs09$ in right ascension and $0\farcs08$ in declination.

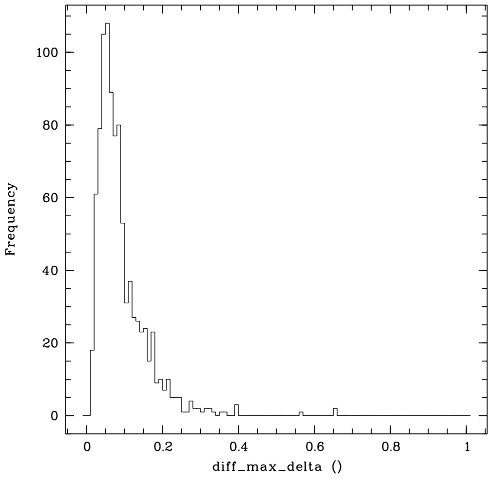


**Fig. 10.** Histogram of the maximal difference in declination (arcseconds) between the final declinations using the classical reduction procedure and Jefferys's algorithm.

The differences between the results of to the two methods are not completely negligible.

### *7.3. Position accuracy of the target stars*

After the *Carte du Ciel* plates have been reduced with Jefferys's method, it is easy to calculate the precision for the positions of the target stars on each plate. This precision depends on:

- the positions $(x, y)$ of the stars on the plates;
- the number $N$ and the repartition of the reference stars through the matrix $(M^t M)$ of the normal equations associated with the system;
- the precision $\varepsilon_{\rm mes}$ of the measurements and $\varepsilon_{\rm cat}$ of the catalogue positions.

After some development, we obtain the precision on target stars positions $\sigma_\alpha$ and $\sigma_\delta$

$$\sigma_\alpha^2 \cos^2\delta = \varepsilon_{\rm mes}^2 + \left(\varepsilon_{\rm mes}^2 + \cos^2\delta\ \varepsilon_{\rm cat}^2\right)\left(1, x, \ldots, y^2\right)\left(M^t M\right)^{-1}\begin{pmatrix}1\\ x\\ \vdots\\ y^2\end{pmatrix} \tag{10}$$

and a similar equation in declination.

If no catalogue error was present, we would obtain from (10) the well-known expression:

$$\sigma_\alpha^2 \cos^2\delta = \varepsilon_{\rm mes}^2 \left[1 + \left(1, x, \ldots, y^2\right)\left(M^t M\right)^{-1}\begin{pmatrix}1\\ x\\ \vdots\\ y^2\end{pmatrix}\right]. \tag{11}$$

The first term in the bracket of Eq. (11) corresponds to the part of the variance for $\alpha$ that originates in the measurements of $x$ (same in $y$), while the second term corresponds to the plate constant variance (Eichhorn et al. 1963).

If we now suppose that the $N$ reference stars have an uniform density on the plate, we can give a simple analytical expression for the plate constants variance and then for the precision of a star on the plate, using calculations similar to those of Eichhorn et al. (1963) or Veiga et al. (1994). The result of the calculation can be written as:

$$\sigma_\alpha^2 \cos^2\delta = \varepsilon_{\rm mes}^2 + \frac{\varepsilon_{\rm mes}^2 + \cos^2\delta\ \varepsilon_{\rm cat}^2}{N} \times\left(\frac{7}{2} - 18\left(x^2+y^2\right) + 180\left(x^4+y^4\right) + 144x^4y^4\right). \tag{12}$$

Near the center of the plate, $x \simeq y \simeq 0$ and Eq. (12) becomes:

$$\sigma_\alpha^2 \cos^2\delta = \varepsilon_{\rm mes}^2 + \frac{7}{2}\left(\frac{\varepsilon_{\rm mes}^2 + \cos^2\delta\ \varepsilon_{\rm cat}^2}{N}\right). \tag{13}$$

For the Bordeaux *Carte du Ciel* plates, we gave in Eqs. (6) and (7) a determination of the errors $\varepsilon_{\rm mes}$ and $\varepsilon_{\rm cat}$. If, in order to simplify the calculations, we consider that both quantities are about 0″15, Eq. (13) becomes:

$$\sigma_\alpha \cos\delta \simeq 0''15\left(1 + \frac{7}{2N}\right) \tag{14}$$

with a similar result for $\delta$.

A typical value of $N$ for the Bordeaux plates of *Carte du Ciel* is 70 and only 16 plates contain less than 20 reference stars.

We can conclude that a characteristic value of the formal precision on both coordinates for one image of a target star near the center of the plate is equal to

$$\sigma_\alpha \cos\delta \simeq \sigma_\delta \simeq 0''16. \tag{15}$$

On the other hand, Eq. (12) allows us to analyze the variation of the precision of the position for a target star according to its position $x, y$ on the plate. The value of the bracket begins to decrease when the stars are away from the center of the plate, and increases again when $x$ and $y \sim 0.3$ (we consider that the coordinates of the edges of the plate are ±0.5, ±0.5). Then the formal accuracy near the center of the plate given in Eq. (15) is significant for most of the *Carte du Ciel* plates.

If we continue to move away from the center of the plate, the precision decreases due to the propagation of errors on the plates constants, and becomes

$$\begin{aligned} \sigma_\alpha \cos\delta &\simeq 0''15\left(1 + \tfrac{10}{N}\right) \text{ for } x = y = 0.4\\ \sigma_\alpha \cos\delta &\simeq 0''15\left(1 + \tfrac{26}{N}\right) \text{ for } x = y = 0.5. \end{aligned} \tag{16}$$

If we again consider that 70 reference stars are present on the plate, we obtain

$$\begin{aligned} \sigma_\alpha \cos\delta &\simeq 0''17 \text{ for } x = y = 0.4\\ \sigma_\alpha \cos\delta &\simeq 0''20 \text{ for } x = y = 0.5. \end{aligned} \tag{17}$$

A confirmation of these results is obtained by the analysis of the dispersion of the 4915 target star positions detected on several contiguous plates and therefore near the edges of the plates. The mean dispersion of their positions is:

$$\begin{aligned} \sigma_\alpha \cos\delta &= 0''17\\ \sigma_\delta &= 0''20. \end{aligned} \tag{18}$$

We can then say that the formal accuracy for the two coordinates of the image of a target star decreases from 0″16 near the center to 0″20 for a star situated at the extreme edge of the plate, the reduction being made using Jefferys' algorithm which takes into account the errors on the positions of the reference stars. The positions for a star given in our catalogue will be the weighted mean of the positions of the three images whose measurements are independent and the formal accuracy of the mean position of a star will decrease from 0″10 near the center to 0″12 at the edge of the plate.

### *7.4. Comparison with other expressions of the position accuracy of target stars*

Equations (12) and (13) give the formal accuracy of the position of a target star when errors occur in reference stars as well as in the measurements of images on the plates. Pascu et al. (1990) proposed a slightly different formula giving this accuracy near the center of the plate. Keeping our notations, the formula of Pascu can be written:

$$\sigma^2_{\mathrm{target}} = \varepsilon^2_{\mathrm{mes}} + \left(\frac{\varepsilon^2_{\mathrm{mes}} + \varepsilon^2_{\mathrm{cat}}}{N - m}\right) \tag{19}$$

where $m$ is the number of degrees of freedom, i.e. the number of plate constants determined in the plate solution. The formula of Pascu (19) presents two noticeable differences with our result Eq. (13):

- Eq. (13) does not contain the parameter $m$;
- a coefficient $\frac{7}{2}$ appears in Eq. (13).

It is easy to prove that the origin of the coefficient $\frac{7}{2}$ comes from the plate model adopted. So we would obtain the value 1 for this coefficient if we used the three constant linear model

$$\begin{aligned} X_i - x_i &= a + bx_i + cy_i \\ Y_i - x_i &= d + ex_i + fy_i. \end{aligned} \tag{20}$$

The disappearance of this parameter in Eq. (13) is a consequence of the theoretical developments leading to Eqs. (12) and (13). However, from a practical point of view, the numerical results given by the formulae of Pascu et al. and of this work are not very different.

## 8. The CdC2000 Bordeaux *Carte du Ciel* catalogue

The CdC2000 catalogue is a positional catalogue containing 344 781 objects. The position given in our catalogue for a star on a plate is the mean of the three positions derived from the independent reduction and in the case of overlapping plates, the mean of all positions of the star given for the mean epoch associated with these positions.

In Table 2 we give an extract of the CdC2000 Bordeaux *Carte du Ciel* catalogue. Columns (1) and (2) give the observed position of the stars (equator and mean equinox of J2000) at the epoch given in Col. (3), Col. (4) gives the photographic $B$ magnitude of the star and the precision for both coordinates is given in Col. (5). Column (6) gives the TYCHO-2 identificator for TYCHO-2 stars.

**Table 2.** Extract of the Bordeaux CdC2000 *Carte du Ciel* plates catalogue.

| $\alpha$ h m s | $\delta$ ° ′ ″ | Epoch yr | $B$ photo | $\sigma_{\alpha*,\delta}$ ″ | Tycho-2 id. |
|---|---|---|---|---|---|
| 0 8 44.357 | 16 29 26.87 | 1923.683 | 12.6 | 0.04 | 1178 1113 |
| 0 8 44.706 | 17 19 09.38 | 1923.683 | 14.4 | 0.10 | |
| 0 8 48.242 | 12 15 54.82 | 1921.737 | 15.8 | 0.10 | |
| 0 8 48.949 | 13 15 20.97 | 1921.737 | 12.8 | 0.13 | |
| 0 8 48.992 | 16 57 37.98 | 1923.683 | 14.0 | 0.10 | |
| 0 8 50.601 | 15 42 41.62 | 1923.683 | 13.7 | 0.10 | |
| 0 8 50.614 | 16 47 59.92 | 1923.683 | 13.8 | 0.10 | |
| 0 8 50.754 | 12 13 47.98 | 1921.737 | 13.4 | 0.10 | |
| 0 8 52.466 | 14 30 50.53 | 1921.687 | 13.2 | 0.06 | 600 589 |
| 0 8 52.498 | 12 22 32.81 | 1921.737 | 15.7 | 0.10 | |
| 0 8 52.596 | 12 00 18.31 | 1921.737 | 15.6 | 0.12 | |
| 0 8 53.630 | 16 38 49.36 | 1923.683 | 15.5 | 0.12 | |
| 0 8 53.746 | 12 26 46.18 | 1921.737 | 14.8 | 0.10 | |
| 0 8 53.765 | 17 14 54.64 | 1923.683 | 13.4 | 0.10 | |
| 0 8 55.073 | 11 50 09.98 | 1921.737 | 13.6 | 0.08 | 597 134 |
| 0 8 55.609 | 16 00 03.52 | 1923.683 | 14.7 | 0.10 | |
| | ... | | | | |

Figure 11 gives the histogram of the photographic magnitudes of the stars.

Figure 12 gives the histogram of the precisions of positions as given in the CdC2000 catalogue.

## 9. Error budget

### *9.1. The precision on the position of target stars*

The formal precision on the stellar positions of the stars, obtained from the reduction algorithm used (Jefferys 1981), is given by the Eq. (12). In the analysis developed in Sect. 7, we concluded that:

- propagation of errors due to the plate constants was seen only near the edges of the plates;
- each star has about three measurement per plate, thus the values of the precision decrease to about 0″10–0″12.

### *9.2. Improvement on the precision of TYCHO-2 stars positions*

As noticed by Jefferys (1981), the algorithm used in this work is able to give an estimation of the covariance matrix of the "adjusted observations". In the case of the reduction of the *Carte du Ciel* plates, the positions of the reference TYCHO-2 stars are considered as observations; thus we are able to estimate the improved precision of their positions at the epoch of the plates. The formulae (15) and (16) of Jefferys (1981) allow us to give an estimation of this improvement; after some algebraic manipulations, we obtain the formula giving the expression of the improved precision $\varepsilon_{\mathrm{imp}}$ of the positions of TYCHO-2 stars.

$$\varepsilon^2_{\mathrm{imp}} = \varepsilon^2_{\mathrm{cat}} \left(1 - \frac{\varepsilon^2_{\mathrm{cat}}}{\varepsilon^2_{\mathrm{cat}} + \varepsilon^2_{\mathrm{mes}}}\right) \tag{21}$$

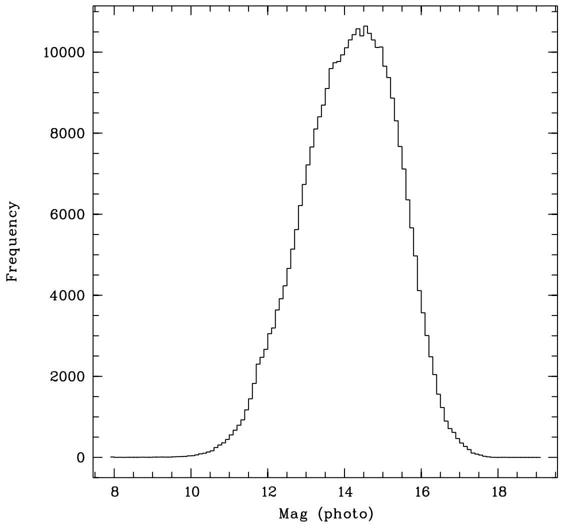


**Fig. 11.** Histogram of photographic magnitudes of stars of the CdC2000 catalogue.

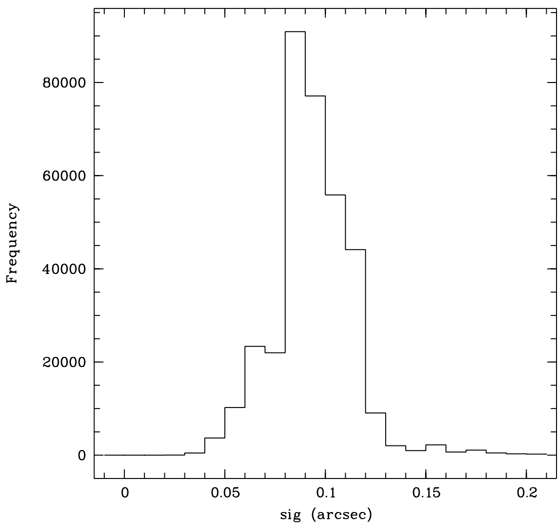


**Fig. 12.** Histogram of precisions of positions $\sigma_\alpha \cos(\delta) = \sigma_\delta$ of the CdC2000 catalogue.

where $\varepsilon_{\rm mes}$ represents the measurement errors, $\varepsilon_{\rm cat}$ represents errors on TYCHO-2 star positions at the epoch of the plates. We showed in previous sections that these two errors are nearly equal so that we obtain:

$$\varepsilon_{\rm imp}^2 = \frac{1}{2}\varepsilon_{\rm cat}^2 \tag{22}$$

$$\varepsilon_{\rm imp} = 0.11''. \tag{23}$$

This result shows the interest of such reduction of ancient plates allowing an improvement of the positions of a reference catalogue such as TYCHO-2 at the epoch of the plates.

## 10. Conclusion

We have presented the CdC2000 catalogue which provides astrometric positions of the stars present on 512 Carte du Ciel plates archived at Bordeaux Observatory. This catalogue contains the positions of 344 781 stars down to magnitude 17. The mean positional precision is 0.10–0.12″with mean epoch 1914.7.

Plates were scanned with the APM Cambridge automatic measuring machine and reduced in the frame of the TYCHO-2 catalogue. Due to the long interval of time between the epochs of TYCHO-2 positions and CdC plates (about 80 years), the propagated TYCHO-2 errors are non-negligible, becoming equal to or even greater than the measurement errors. It is then necessary to take into account these catalogue errors in the reduction procedure. The very general method developed by Jefferys (1980, 1981) is shown to be perfectly adapted to our problem and it was used in this paper to reduce the Bordeaux Carte du Ciel plates. The method of reduction used allows us to determine rigorously the formal error on the positions obtained. The typical value of this error is $0\overset{''}{.}10$–$0\overset{''}{.}12$ on each coordinate, for the major part of the surface of the plates. Jefferys' algorithm used in this work is able to provide an estimation of the improvement obtained on the 50 682 TYCHO-2 reference star positions at the epoch of the plates. The result (improvement of 0.11″ on precision) is clearly non-negligible.

These two results on targets and reference stars show that the reduction of the carte du Ciel plates gives an interesting contribution to the ground-based astrometry of stars and allow us to reach an accuracy of proper motions necessary for refined stellar kinematics investigations.

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