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The role of rotation on Petersen diagrams. The $\Pi_{1/0}$ (Ω) period ratios

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ABSTRACT

The present work explores the theoretical effects of rotation in calculating the period ratios of double-mode radial pulsating stars with special emphasis on high-amplitude δ Scuti stars (HADS). Diagrams showing these period ratios vs. periods of the fundamental radial mode can be employed as a good tracer of non-solar metallicities and are known as Petersen diagrams (PD). In this paper we consider the effect of moderate rotation on both evolutionary models and oscillation frequencies and show that such effects cannot be completely neglected as has been done until now. In particular we find that differences in period ratios of some hundredths can be obtained even for low-to-moderate rotational velocities (15−50 km s^{-1}). The main consequence is, therefore, the "confusing scenario" generated when trying to fit the metallicity of a given star using this diagram without previous knowledge of its rotational velocity.

Key words. δ Sct – stars: rotation – stars: variables: RR Lyr – stars: oscillations – stars: fundamental parameters – Cepheids

1. Introduction

Radial pulsators, and more particularly double-mode pulsators, have been extensively studied using the well-known Petersen diagrams (PD), which show the ratio between the fundamental radial mode and the first overtone as a function of the fundamental mode. Typically, the stars concerned are double-mode Cepheids, RR Lyrae and high-amplitude δ Scuti stars (HADS). Recently, analysis of the data from large-scale projects like OGLE (Optical Gravitational Lensing Experiment Szymanski 2005; Udalski et al. 1997), NSVS (Northern Sky Variability Survey Woźniak et al. 2004), ASAS (All Sky Automated Survey Pojmanski 2002, 2003), or MACHO (Alcock et al. 2000) has permitted deeper studies of the observational properties of such double-mode pulsators.

These period ratios were first studied by Petersen (1973, 1978), and they have been used for decades as metallicity indicators, as a function of stellar mass and age, to test mass-luminosity and/or radius-luminosity relations. In addition, they also have been used, for instance, to determine the distance modulus to the SMC (Kovács 2000).

While non-radial pulsators like β Cephei or low-amplitude δ Scuti stars (LADS) generally show moderate-to-fast rotational velocities ($v \sin i$), the double-mode radial pulsators can be considered as slow-to-moderate rotating stars. This fact has lead to systematic neglect the rotation effect on theoretical period-ratios, in particular F-1O ratios in PD. However, we remind the reader that such objects are generally faint enough to make the observation of their rotational velocities ($v \sin i$) difficult. Figure 1 shows the location of all known double-mode HADS (from Poretti et al. 2005) in the HR diagram, compared with the location of all LADS with measured $v \sin i$. As can be seen, the narrow band (in effective temperature) occupied by double-mode HADS overlaps the region where LADS are located. In luminosity, HADS occupy a broader range. Unfortunately, the number of known HADS constitutes a very poor sample, specially when compared with LADS. This means that we cannot discard, a priori, the possibility of HADS in a wider range of effective temperatures and larger $v \sin i$ measurements. In addition, this represents a lower limit to their rotational velocities. Although most of HADS present $v \sin i \leq 20$ km s^{-1}, when varying the angle of inclination i of the star, velocities up to 50 km s^{-1} could be reached.

From the theoretical side, only a few works examine the effect of rotation on period ratios and mainly focused on very rapidly rotators, but only partially. In Perez Hernandez et al. (1995), second order effects of rotation on oscillation periods are taken into account in order to discriminate between radial and non-radial modes. The authors express the relative period change as $\delta\Pi_n^{(\mathrm{rot})}/\Pi_n = Z_n(\Pi_{\mathrm{rot}})/\Pi_n)^{-2}$, where Π_n is the unperturbed period of the mode with radial order n, and $\Pi_{(\mathrm{rot})}$ is the

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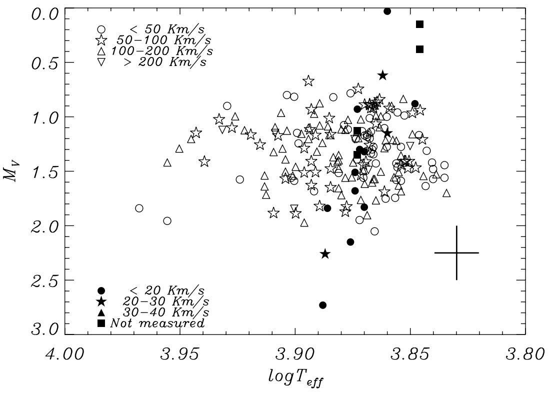

Fig. 1. Absolute magnitudes M_V as a function of the effective temperature of all known δ Scuti stars (empty symbols), compared with those of all double-mode HADS known to date (filled symbols). Different symbol types represent the observed $v \sin i$ ranges (taken from Rodríguez et al. 2000). The cross represents typical errors on absolute magnitudes and effective temperature for δ Scuti stars.

rotation period. The coefficient Z_n depends on both the radial order of the mode and the structure of the star. Such values were estimated by interpolating from computations derived for a polytropic model with index 3. Recently, Pamyatnykh (2003) studied the behaviour of the period ratios of radial modes when near-degeneracy effects due to rotation are included for a typical 1.8 $M_\odot$ δ Scuti stellar model. He showed that very large and non-regular perturbations to such ratios are expected to occur. More recently, Suárez et al. (2005) proposed a limit of validity for the perturbation theory (up to second order) in terms of rotational velocity for rotating models. Such a limit is given by the behaviour of period ratios when near degeneracy is considered, which clearly complicates a naive interpretation of the PD.

In the present work, we aim to analyse the consequences of neglecting the effect of rotation on radial period ratios, even for low rotational velocities. To do so, up-to-date techniques taking properly the rotation into account in the modelling (in equilibrium models and in oscillation frequencies) are used. Similarly to Pamyatnykh (2003) and Suárez et al. (2005), near degeneracy is also considered.

The paper is structured as follows. A general description of the modelling, focusing on how rotation is taken into account, is given in Sect. 2. Fundamental-to-first harmonic ratios in the presence of rotation, $\Pi_{1/0}(\Omega)$, are introduced in Sect. 3, and a discussion of their impact on PD analysis is proposed in Sect. 4. Finally, the conclusions are given in Sect. 5.

2. The modelling

The evolutionary code CESAM (Morel 1997) was used, because is particularly adapted for our purposes. The numerical precision and the mesh grid, around 2000 mesh points on the basis of B-splines, of equilibrium models were adapted according to the oscillation computation requirements.

Following Kippenhahn & Weigert (1990), a first order effect of rotation was taken into account in equilibrium equations. In particular, the spherical symmetric contribution of the centrifugal acceleration was included by means of an effective gravity $g_{\rm eff} = g - \mathcal{A}_{\rm c}(r)$, where g represents the local gravity component, r the radial distance and $\mathcal{A}_{\rm c}(r) = \frac{2}{3} r^2 \Omega$, the centrifugal acceleration of matter elements at a distance r from the centre of the star. This spherically symmetric contribution of the rotation does not change the shape of the hydrostatic equilibrium equation. Although the non-spheric components of the centrifugal acceleration were not considered, they were included as a perturbation in the oscillation computation. The total angular momentum of models was assumed to be globally conserved along the evolution of the star. Input physics was then adapted for intermediate mass stars.

Theoretical oscillation spectra were computed from the equilibrium models described in the previous section. For this purpose the oscillation code *Filou* (Tran Minh & Léon 1995; Suárez 2002) was used. This code, based on a perturbative analysis, provides adiabatic oscillations corrected for the effects of rotation up to second order (centrifugal and Coriolis forces).

Furthermore, for moderate-high rotational velocities, the effects of near degeneracy were expected to be significant (Soufi et al. 1998). Two or more modes, close in frequency, are rendered *degenerate* by rotation under certain conditions, corresponding to selection rules. In particular, these rules select modes with the same azimuthal order m and degrees ℓ that differ by 2 (Soufi et al. 1998). If we consider two generic modes $\alpha_1 \equiv (n, \ell, m)$ and $\alpha_2 \equiv (n', \ell', m')$ under the afore-mentioned conditions, near degeneracy occurs for $|\sigma_{\alpha_1} - \sigma_{\alpha_2}| \leq \sigma_\Omega$, where σ_{α_1} and σ_{α_2}, represent the eigenfrequency associated to modes α_1 and α_2 respectively, and σ_Ω represents the stellar rotational frequency. In certain cases, such an effect may be dominant in the behaviour of the radial period ratios studied here (Pamyatnykh 2003; Suárez et al. 2005). However, due to its complexity, we believe such effects should be analysed separately (Suárez et al., in prep.) so, for the sake of clarity, they have not been included in the present work.

3. The $\Pi_{1/0}(\Omega)$ ratios

In general, low-order radial period ratios can be considered as dependent on the distribution of mass (or density) throughout the star and on the thermodynamical properties of the stellar matter. In Fig. 2, a classic PD for 1.8 $M_\odot$ evolutionary tracks is depicted. It illustrates the well-known dependence on the metallicity of $\Pi_{1/0} = \Pi_1/\Pi_0$ ratios similar to those shown, for instance, in Petersen (1973), Petersen & Christensen-Dalsgaard (1996), and Petersen & Christensen-Dalsgaard (1999). As can be seen, the period ratios increase when decreasing the stellar initial metal content. This property is commonly used to discriminate Pop. I from Pop. II stars in the context of radial pulsators. The shaded region indicates the typical $\Pi_{1/0}$ values found for Pop. I stars in the range of $\Pi_{1/0} = [0.772, 0.776]$. As a reference, the observed period ratios of two stars – RV Ari (Pop. I) and SX Phe (Pop. II) – are also depicted (values obtained from Poretti et al. 2005, and references therein). For a given chemical composition, the period ratio $\Pi_{1/0}$ is determined by mass and the radius (or Π_0 which is scaled by the mean density $\bar{\rho}$, and then

$$\Pi_{1/0} = \Pi_{1/0}(M, R, Z).$$

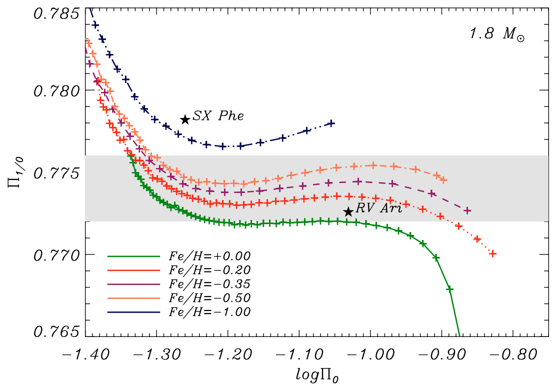

Fig. 2. Typical PD (Π_0 in d) containing different tracks of 1.8 $M_\odot$ evolutionary models computed with different initial metal content [Fe/H], from solar composition (*bottom*, solid line) to −1.00 (*top*, triple dot-dashed line). Crosses represent non-rotating models. The shaded area corresponds to typical values found for Pop. I stars. The two filled-star symbols represent the observed $\Pi_{1/0}$ of the double-mode high-amplitude δ Scuti stars RV Ari and SX Phe, as an illustration of (Pop. I) and (Pop. II) stars, respectively. (For clarity, colours are used in the *on-line* version of the paper).

In addition, period ratios can be easily written in terms of observed quantities through the so-called pulsation constant $Q_{n,\ell} = \Pi_{n,\ell}\sqrt{\bar\rho/\bar\rho_\odot}$. In the context of radial modes, this constant can be expressed as

$$Q_{1,0} = \Pi_{1,0}\left(\frac{g}{g_\odot}\right)^{1/2}\left(\frac{L}{L_\odot}\right)^{-1/4}\left(\frac{T_{\rm eff}}{T_{\rm eff,\odot}}\right),$$

allowing us to assign to each observed radial period $\Pi_{n,0}$ a value $Q_{n,0}$, which can be compared with theoretical predictions.

However, it is well known that rotation modifies the structure of stars and thereby the cavity where modes propagate. It is thus plausible to consider the period ratios to be Ω-dependent. Moreover, the angle of inclination of the star is generally unknown (only rotational projected velocities (v sin i) are provided by observations), and thereby it follows that

$$\Pi_{1/0}\,(\Omega) = \Pi_{1/0}\,(M, R, Z, \Omega(i)).$$

From a theoretical point of view, following Soufi et al. (1998) and Suárez et al. (2006), the adiabatic oscillation eigenfrequencies $\omega_{n,\ell,m}$ can be expressed in terms of a perturbative theory as:

$$\omega_{n,0,0} = \omega^{(0)}_{n,0,0} + \omega^{(2)}_{n,0,0}$$

for radial modes ($\ell = 0, m = 0$), where $\omega^{(0)}_{n,0,0}$ represents the unperturbed frequency and $\omega^{(2)}_{n,0,0}$ the second order correction term. Both terms implicitly include the symmetric component of the centrifugal force (mainly the departure from sphericity of the star) given by the equilibrium model. First-order correcting terms $\omega^{(1)}_{1,0,0}$ (Coriolis force effect) are proportional to the azimuthal order m, and therefore they are zero for radial modes. From now on, we keep only the subscript n for the sake of brevity. Figure 3 illustrates the effect of rotation on normalised[1]

[1] The normalisation constant is $(G\,M/R^3)^{1/2}$ which is proportional to the model mean density $\bar\rho$.

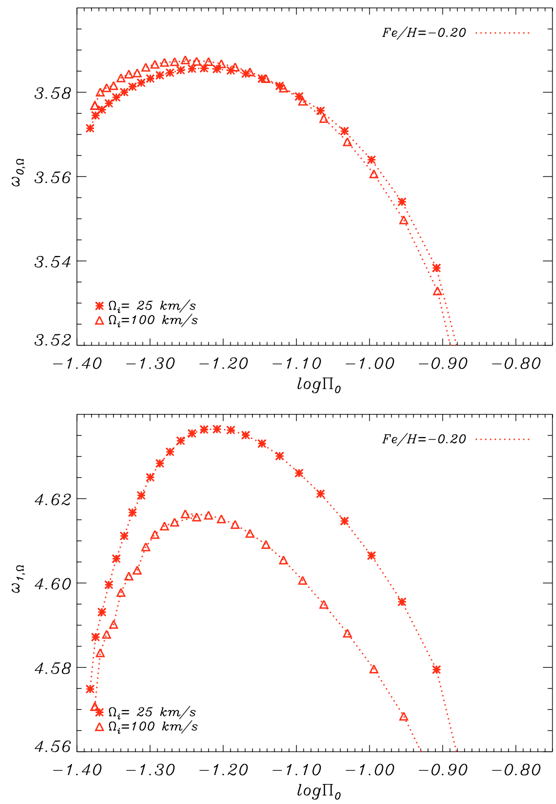

Fig. 3. Fundamental radial order mode $\omega_{0,\Omega}$ (*top panel*) and the first overtone $\omega_{1,\Omega}$ (*bottom panel*) normalised frequencies as a function of the logarithm of the fundamental radial order period (in d). This figure is available in colour in electronic form.

frequencies corresponding to the fundamental mode $\omega_{0,\Omega}$ (top panel) and the first overtone $\omega_{1,\Omega}$ (bottom panel) for a given metallicity. As can be seen, the first overtone frequencies are visibly more affected by rotation than the fundamental ones. Without entering into detail (a theoretical work analysing the behaviour of $\Pi_{1/0}\,(\Omega)$ is currently in preparation), this suggests the possibility that $\omega_{1,\Omega}$ is proportional to the distribution of the density inside the star (affected by rotation) rather than simply to the mean density.

4. The $\Pi_{1/0}\,(\Omega)$ period ratios and the metallicity determinations

In order to analyse the impact of considering $\Pi_{1/0}\,(\Omega)$ rather than $\Pi_{1/0}$ period ratios on metallicity determinations, these quantities, obtained from models taking rotation $\Pi_{1/0}\,(\Omega)$ into account, are compared with those $\Pi_{1/0}$. To do so, several evolutionary tracks were computed as described in Sect. 2 for six different metallicities: [Fe/H] = 0, −0.1, −0.2, −0.35, −0.50, and −1.00 dex; and three different initial rotational velocities $\Omega_{\rm i}$ = 25, 50, and 100 km s^{-1}. The case of rotational velocities between 50 and 100 km s^{-1} is purely illustrative, since there are no HADS known in that range. The discussion is thus mainly focused on models with rotational velocities up to 50 km s^{-1}.

During the evolution, the rotational velocity of models decreases up to 0.75 $\Omega_{\rm i}$ at TAMS, due to global conservation of the total angular momentum (see Sect. 2 for more details).

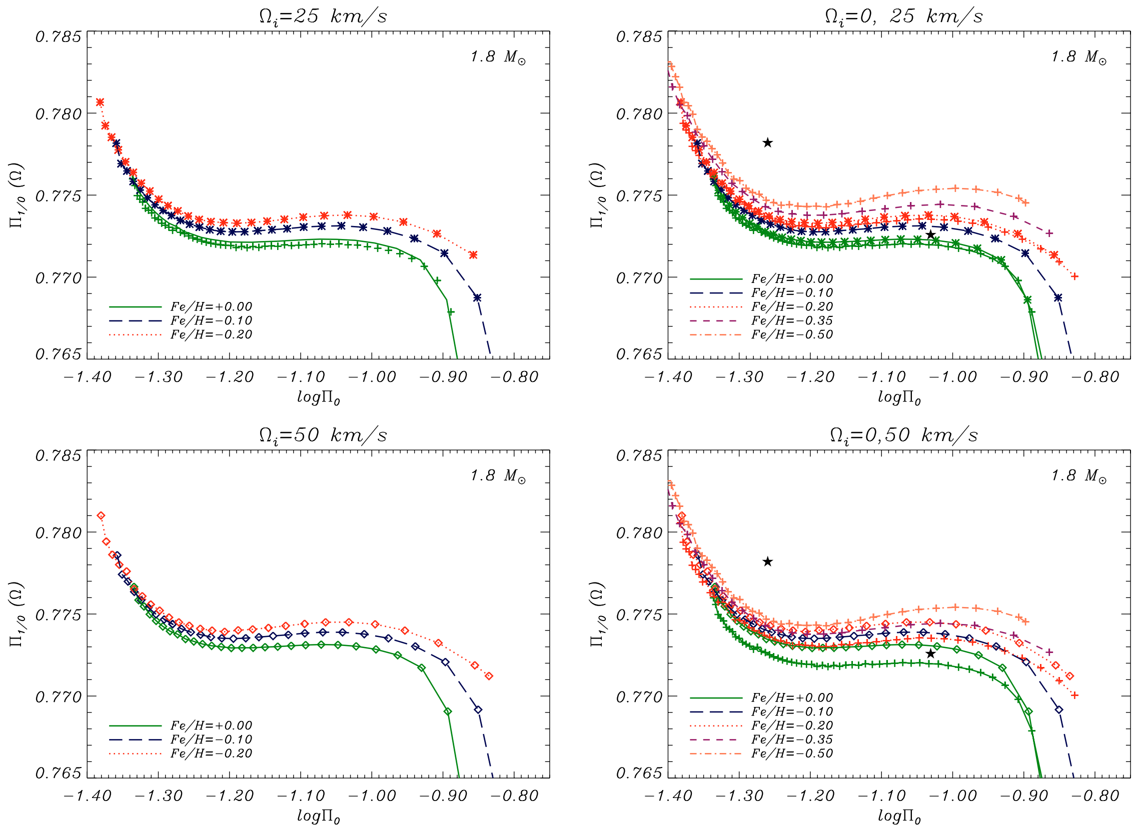

Fig. 4. Theoretical PD including rotation effects (RPD). The $\Pi_{1/0}\,(\Omega)$ period ratios were computed in the manner described in Sect. 2 for a set of evolutionary 1.8 $M_{\odot}$ tracks obtained for different metallicities. Tracks for three initial rotational velocities are considered: 25 and 50 $\mathrm{km\,s^{-1}}$ (*from top to bottom*). *Left panels* show only rotating models. *Right panels* show the comparison between rotating and non-rotating tracks (classic PD). For convenience, the following symbols are used: crosses, representing non-rotating models; asterisks, representing models evolved with $\Omega_{\mathrm{i}} = 25\,\mathrm{km\,s^{-1}}$; diamonds and those evolved with $\Omega_{\mathrm{i}} = 50\,\mathrm{km\,s^{-1}}$. As in Fig. 2, filled star symbols represent the observed values for SX Phe and RV Ari. (For clarity, colours are used in the *on-line* version of the paper.)

The mass of models is fixed to 1.8 $M_{\odot}$ which typically corresponds to a δ Scuti star.

Adiabatic oscillations are then computed from these rotating models, thereby obtaining the corresponding $\Pi_{1/0}\,(\Omega)$ period ratios. In Fig. 4 such *rotational* PD (hereafter RPD) are displayed from top to bottom for tracks computed from $\Omega_{\mathrm{i}} = 25$ to $50\,\mathrm{km\,s^{-1}}$, respectively. It can be noticed that $\Pi_{1/0}\,(\Omega)$ period ratios increase for increasing rotational velocities (left panels). Such an effect is similar to decreasing the metallicity in classic PD. As explained in the previous section, this can be understood in terms of variations of the density distribution in stellar model interiors due to rotation effects (Suárez et al., in preparation). In addition, the shift to larger period ratios is dependent on the metallicity. In particular, the higher the rotational velocity the closer the models of different metallicity in RPD (left panels). This means that the effect of rotation on period ratios is systematically larger for increasing metallicity values (up to the solar value, in the present study).

In Fig. 4, $\Pi_{1/0}\,(\Omega)$ (right panels) are displayed together with the classic ones, $\Pi_{1/0}$. A first quantitative comparison is performed in term of period ratios differences

$$\delta\Pi_{1/0}(\Omega,[\mathrm{Fe/H}]) = \Big[\Pi_{1/0}\,(\Omega) - \Pi_{1/0}\Big]_{[\mathrm{Fe/H}]}$$

for a given metallicity. For the lowest initial rotational velocity considered, $\Omega_{\mathrm{i}} = 25\,\mathrm{km\,s^{-1}}$, $\delta\Pi_{1/0}$ reach up $2\text{–}3\times10^{-3}$, and for $\Omega_{\mathrm{i}} = 50\,\mathrm{km\,s^{-1}}$, differences increase up to $6\text{–}8\times10^{-3}$. The main effect on PD (right panels) is to shift and compress tracks of the same metallicity (and mass) toward higher period ratios with respect to classic tracks.

The previous differences in period ratios can also be analysed in terms of metallicity. For brevity, tracks will be specified from now on with the subscript corresponding to the rotational velocity being considered. For instance, the track computed with $\Omega_{\mathrm{i}} = 25\,\mathrm{km\,s^{-1}}$ and $[\mathrm{Fe/H}] = -0.1$ will be called $[-0.1]_{25}$. As will be shown, such analysis leads to a *confusing* scenario: for $\Omega_{\mathrm{i}} = 25\,\mathrm{km\,s^{-1}}$ (top, right) the tracks of solar metallicity are similar to classic ones. However, when decreasing [Fe/H], rotating and non-rotating tracks are located quite close. In particular, rotating $[-0.10]_{25}$ tracks may be confused with $[-0.20]_{0}$ ones, and $[-0.20]_{25}$ may be confused with non-rotating $[-0.35]_{0}$ ones. Analysis of the middle right panel reveals that $[0.00]_{50}$ tracks are located close to $[-0.20]_{0}$ ones. Similarly, $[-0.10]_{50}$ tracks may be confused with $[-0.20,-0.35]_{0}$ ones, and finally, $[-0.20]_{50}$ tracks are close to $[-0.50]_{0}$ ones. As can be seen, the mix-up is critical for pop. I stars when considering 1.8 $M_{\odot}$ rotating models evolved with $\Omega_{\mathrm{i}} = 25, 50\,\mathrm{km\,s^{-1}}$.

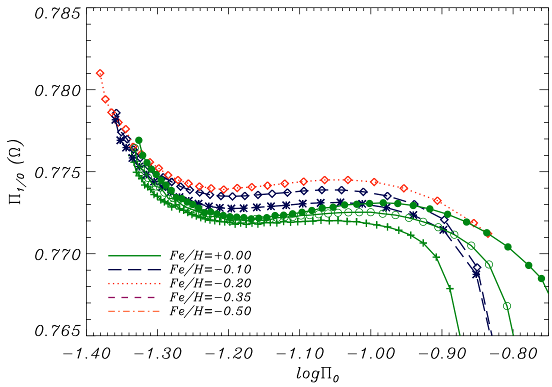

Fig. 5. RPD (Π_0 in d) illustrating the effect of considering different masses. Symbols have the same meaning as those in Fig. 4, except for empty and filled circles, which correspond to 1.9 and 2 $M_\odot$ for non-rotating models and solar metallicity, respectively. (For clarity, colours are used in the *on-line* version of the paper.)

Up to this point, all the discussion has been based on the results obtained only for 1.8 $M_\odot$ models. When other masses and metallicites are taken into account, the *confusion* in metallicities and rotational velocities significantly increases. In order to illustrate this, two solar metallicity tracks for higher mass (1.9 and 2.00 $M_\odot$) non-rotating models are displayed in the RPD of Fig. 5 where they are compared with other rotating non-solar tracks. As can be seen, the effect of increasing the mass of the models goes in the same direction as increasing the rotational velocity, that is, it increases the period ratios. It is worth noting that such a shift toward larger period ratios systematically occurs for any *rotating track*. Therefore the previous discussion based on 1.8 $M_\odot$ models is equivalent, whatever the mass and/or rotational velocity considered. Such behaviour may extend the *confusing scenario* to Pop. II stars. Considering 1.8 $M_\odot$ models, it would be necessary to consider initial rotational velocities up to 100 km s^{-1}. However, this lower limit rapidly decreases when increasing the mass of the models. For instance, 2 $M_\odot$ and Ω_i = 50 km s^{-1} tracks may be misinterpreted with $[-1.00]_0$ ones.

The construction of complete RPD for a wide range of metallicities, initial rotational velocities, and masses becomes necessary for the exhaustive analysis of double-mode pulsators (work currently in progress); however, this exceeds the scope of this paper.

5. Conclusions

The impact of taking the effect of rotation on Petersen Diagrams into account has been examined here, focusing on main sequence double-mode pulsators. Detailed seismic models were computed considering the effects of rotation on both equilibrium models and on adiabatic oscillation frequencies. For 1.8 $M_\odot$ stellar models, period ratios were calculated for different rotational velocities (Rotational Petersen Diagrams) and metallicities, and then compared with classic non-rotating ones (PD).

Analysis of these RPD reveals that the difference in period ratios increases with the rotational velocity for a given metallicity. It remains around 10^{-3} for rotational velocities up to 50 km s^{-1} and can reach up 10^{-2} for rotational velocities close to 100 km s^{-1}. This difference has been found as strong enough to produce a significantly confusing scenario when analysing RPD in terms of metallicity variations. In particular, for 1.8 $M_\odot$ stellar models, differences in metallicity up to δ[Fe/H] ~ 0.30 dex can be found when considering models evolved with initial rotational velocities of 50 km s^{-1}. Furthermore, such confusion may still increase when including other stellar masses, rotational velocities, and metallicities, as well as other physical parametrisation (work in progress).

The results presented here should thus be taken into account when analysing double-mode pulsators with Petersen Diagrams, in particular when accurate metallicity and/or mass determinations are required.

A work on the detailed analysis of period ratios in the presence of near degeneracy is currently in preparation. It will provide new constraints on the modelling of HADS.

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