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The gravitational wave radiation of pulsating white dwarfs revisited: the case of BPM 37093 and PG 1159-035

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ABSTRACT

We compute the emission of gravitational radiation from pulsating white dwarfs. This is done by using an up-to-date stellar evolutionary code coupled with a state-of-the-art pulsational code. The emission of gravitational waves is computed for a standard $0.6\,M_\odot$ white dwarf with a liquid carbon-oxygen core and a hydrogen-rich envelope, for a massive DA white dwarf with a partially crystallized core for which various $\ell = 2$ modes have been observed (BPM 37093) and for PG 1159-035, the prototype of the GW Vir class of variable stars, for which several quadrupole modes have been observed as well. We find that these stars do not radiate sizeable amounts of gravitational waves through their observed pulsation g-modes, in line with previous studies. We also explore the possibility of detecting gravitational waves radiated by the f-mode and the p-modes. We find that in this case the gravitational wave signal is very large and, hence, the modes decay very rapidly. We also discuss the possible implications of our calculations for the detection of gravitational waves from pulsating white dwarfs within the framework of future space-borne interferometers like LISA.

Key words. stars: evolution – stars: white dwarfs – stars: oscillations – gravitational waves

1. Introduction

Gravitational waves are a direct consequence of General Relativity. Much effort has been made to detect them, but due to the intrinsic experimental difficulties involved in detection and data analysis no definite result has yet been obtained. Supernova core collapse, binary systems involving compact objects and pulsating neutron stars are, amongst others, promising sources of gravitational waves – see Schutz (1999) for a comprehensive review of the subject. Moreover, with the advent of the current generation of terrestrial gravitational wave detectors, like LIGO (Abramovici et al. 1992), VIRGO (Acernese et al. 2004), GEO600 (Willke et al. 2004), or TAMA (Takahashi et al. 2004), and of space-borne interferometers like LISA (Bender et al. 1998, 2000), gravitational wave astronomy will probably soon be a possible reality.

Despite its potential interest, the emission of gravitational waves by pulsating white dwarfs has been little explored up to now. Apart from the pionering work of Osaki & Hansen (1973), only the gravitational wave radiation of rotating white dwarfs undergoing quasi-radial oscillations has been studied so far – see Benacquista et al. (2003) and references therein. White dwarfs are the most common end-point of the evolution of low- and intermediate-mass stars. Hence, white dwarfs constitute, by far, the most numerous stellar remnants in our Galaxy, outnumbering neutron stars. Moreover, the relative simplicity of their physics allows us to obtain very detailed models which can be ultimately compared with their observed properties. Among white dwarfs there are three specific families of variable stars, known as ZZ Ceti (or DAV, with hydrogen-rich envelopes and $T_{\rm eff} \sim 12\,000$ K), V777 Her (or DBV, with helium-rich envelopes and $T_{\rm eff} \sim 25\,000$ K) and GW Vir stars (or variable PG 1159 objects, with envelopes which are rich in carbon, oxygen and helium, and $T_{\rm eff}$ ranging from ~80 000 to 150 000 K), which show periodic variations in their light curves – see Gautschy & Saio (1995; 1996) for reviews. The typical periods are within ~100 s and ~2000 s and, consequently, lay in the region of frequencies to which LISA will be sensitive. The luminosity changes of these variable stars have been successfully explained as due to nonradial g-mode pulsations.

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At present, there is a general consensus that variable white dwarfs are very interesting targets for pulsational studies. Their very simple internal structures allow us to predict theoretically the pulsational frequencies with a very high degree of detail and sophistication. Also, they have a very rich spectrum of frequencies which may give us information about the stellar mass, the core composition, the mass of the surface helium and hydrogen layers (if present), the angular speed of rotation and the strength of the magnetic field – see, for instance, Pfeiffer et al. (1996) and Bradley (1998, 2001), amongst others. Consequently, it is not surprising that in recent years ZZ Ceti and V777 Her white dwarfs, as well as GW Vir stars, have been the preferred targets for the network called the "Whole Earth Telescope" (WET). WET observations have been of an unprecedent quality, and in some cases have allowed us to disentangle the internal structure and evolutionary status of several white dwarf stars by applying the powerful tools of asteroseismology (Nather 1995; Kawaler 1998).

BPM 37093 is the most massive pulsating white dwarf ever found (Kanaan et al. 1992). It is a massive ZZ Ceti star – that is, with a hydrogen-rich atmosphere – with a stellar mass of $\sim 1.05\,M_{\odot}$, and an effective temperature $T_{\rm eff} \simeq 11\,800$ K. BPM 37093 has been thoroughly studied (both theoretically and observationally) because presumably it should have a sizeable crystallized core (Winget et al. 1997). Hence, for BPM 37093 we have detailed models (Montgomery & Winget 1999; Córsico et al. 2005) and extensive observational data (Kanaan et al. 2005). One of the most apparent modes of BPM 37093 has a period $P = 531.1$ s, very close to the frequency of maximum sensitivity of LISA and pulsates with $\ell = 2$. $\ell = 1$ modes do not radiate gravitational waves and $\ell = 2$ modes are relevant for the emission of gravitational waves, thus making BPM 37093 an especially suitable target for LISA. Moreover, the distance to BPM 37093 is known ($d = 16.8$ pc). Consequently, a detailed study of the possibilty of detecting the gravitational waves emitted by this star is of the maximum interest, but still remains to be done. On the other hand, PG 1159-035, the prototype of the GW Vir class of objects, has a complex spectrum with several $\ell = 2$ modes (Winget et al. 1991). Unfortunately there is no reliable parallax determination for PG 1159-035. Werner et al. (1991) provide $d \sim 800^{+600}_{-400}$ pc, whereas Kawaler & Bradley (1994) obtained $d \simeq 400 \pm 40$ pc. However, a spectroscopic determination of its mass ($M_\star \simeq 0.54\,M_{\odot}$) is available. These are, to the best of our knowledge, the only two known white dwarf pulsators with confirmed quadrupole g-modes.

In this paper we compute the emission of gravitational waves from pulsating white dwarfs. We first compute the gravitational waves radiated by a typical $0.6\,M_{\odot}$ white dwarf with a carbon-oxygen core and a $10^{-4}\,M_\star$ hydrogen envelope, which we regard as our fiducial model. For this model white dwarf we first compute the gravitational waves emitted by g-modes. Then we compute the gravitational waves emitted by BPM 37093 and PG 1159-035, the only two known white dwarfs with quadrupole g-modes. As it will be shown below, we find that the fluxes radiated away by these two stars in the form of gravitational waves are very small. This is why we also explore other possibilities. In particular we also compute the fluxes radiated by the f- and p-modes, independently of the lack of observational evidence for these modes in pulsating white dwarfs. The paper is organized as follows. In Sect. 2 we briefly review the basic characteristics of nonradial pulsation modes. In Sect. 3 we discuss the numerical codes used to compute the nonradial pulsation modes of the white dwarf models presented here. Section 4 gives the expressions for the emission of gravitational waves from pulsating white dwarfs. Finally in Sect. 5 we present our results and in Sect. 6 we summarize our findings and we draw our conclusions.

2. Nonradial pulsation modes

Unno et al. (1989) and Cox (1980) give details of nonradial stellar pulsations. Here we give a brief overview of the basic properties of nonradial modes. Briefly, nonradial modes are the most general kind of stellar oscillations. There exist two subclasses of nonradial pulsations, namely *spheroidal* and *toroidal* modes. Of interest in this work are the spheroidal modes, which are further classified into g-, f- and p-modes according to the main restoring force acting on the oscillations, gravity for the g- and f-modes and pressure for the p-modes.

For a spherically symmetric star, a linear nonradial pulsation mode can be represented as a standing wave of the form $\Psi'_{k,\ell,m}(r,\theta,\phi,t) = \Psi'_{k,\ell,m}(r)\ Y^m_\ell(\theta,\phi)\ {\rm e}^{{\rm i}\sigma_{k,\ell,m}t}$, where the symbol "$'$" indicates a small Eulerian perturbation of a given quantity Ψ (like the pressure, gravitational potential, etc.) and $Y^m_\ell(\theta,\phi)$ are the corresponding spherical harmonics. Geometrically, ℓ is the number of nodal lines on the stellar surface and m is the number of such nodal lines in longitude. In the absence of any physical agent able to remove spherical symmetry (like magnetic fields or rotation), the eigenfrequencies $\sigma_{k,\ell,m}$ are dependent on ℓ but are $2\ell + 1$ times degenerate in m. $\Psi'_{k,\ell,m}(r)$ is the radial part of the eigenfunctions, which for realistic models necessarily must be computed numerically together with $\sigma_{k,\ell,m}$. The index k (known as the radial order of the mode) represents, in the frame of simple stellar models (like those of white dwarf stars which we shall study below), the number of nodes in the radial component of the eigenfunction. Generally speaking, g-modes are characterized by low oscillation frequencies (long periods) and by displacements of the stellar fluid essentially in the horizontal direction. At variance, p-modes have high frequencies (short periods) and are characterized by essentially radial displacements of the stellar fluid. Finally, there is a single f-mode for a given ℓ (≥ 2) value. This mode does not have any node in the radial direction ($k = 0$) and possesses an intermediate character between g- and p-modes. Its eigenfrequency lies between that of the low order g- and p-modes, and generally slowly increases when ℓ increases. For g-modes (p-modes), the larger the value of k the lower (higher) the oscillation frequency.

3. Numerical codes

We compute the nonradial pulsation modes of the white dwarf models considered in this work with the help of the same pulsational code described in detail in Córsico at al. (2001a, 2002). The code, which is based on a standard finite differences scheme, provides very accurate oscillation frequencies and nonradial eigenfunctions, and has been employed in numerous works on white dwarf pulsations – see, for instance, Córsico et al. (2004) and references therein. The code solves the fourth-order set of equations governing Newtonian, linear, nonradial stellar pulsations in the adiabatic approximation following the dimensionless formulation given in Unno et al. (1989). To build up the white dwarf models needed for our pulsational code we employed the LPCODE evolutionary code described in detail in Althaus et al. (2003, 2005). Our evolutionary code contains very detailed physical ingredients. A full description of these physical ingredients is out of the scope of this paper and, consequenlty, the reader is referred to Althaus et al. (2003, 2005) for an extensive description of them. Instead, we will only summarize here the most important inputs. For instance, the equation of state includes partial ionization, radiation pressure, ionic contributions, partially degenerate electrons and Coulomb interactions. For the white dwarf regime, we include an updated version of the equation of state of Magni & Mazzitelli (1979). The code uses OPAL radiative opacities – including carbon- and oxygen-rich compositions – for arbitrary metallicity from Iglesias & Rogers (1996) and molecular opacities from Alexander & Ferguson (1994). High-density conductive opacities are taken from Itoh et al. (1994) and the references cited there, whereas neutrino emission rates are those of Itoh et al. (1996), and references therein. The stellar models for BPM 37093 and PG 1159-035 discussed below have been derived from full evolutionary calculations that take into account the history of the progenitor stars – see Althaus et al. (2003, 2005) for details. During the white dwarf cooling phase, the effects of time-dependent element diffusion have been considered in the calculations.

4. Gravitational waves from a pulsating white dwarf

The basic formalism for deriving the gravitational wave radiation of pulsating objects (either white dwarfs or neutron stars) is well known – see, for instance, Osaki & Hansen (1973). We will extend it to the case in which a white dwarf has a partially crystallized core. Generally speaking, the amplitude of a gravitational wave emitted from any slow-moving source in the quadrupole approximation is given by (Misner et al. 1973)

$$h_{ij}^{\rm TT} = \frac{2G}{c^4 d}\ddot{Q}_{ij}^{\rm TT}, \tag{1}$$

where "TT" stands for the traceless-transverse gauge, d for the distance, and Q is the quadrupole moment of the mass distribution, which is defined as

$$Q_{ij} = \int_{R^3} \rho(\boldsymbol{r})(3x_i x_j - \delta_{ij}r^2)\mathrm{d}^3 r. \tag{2}$$

As previously stated, we assume that the spatial and temporal behavior of the the perturbed density profile is provided by the following expression:

$$\rho(\boldsymbol{r}, t) = \rho_0(r) + \rho'(r)\mathrm{Re}\left(Y_\ell^m(\theta,\phi)\mathrm{e}^{\mathrm{i}\sigma t}\right) \tag{3}$$

where ρ_0 is the unperturbed density profile, $\rho'(r)$ stands for the radial perturbation of the density profile, $Y_\ell^m(\theta,\phi)$ are the spherical harmonics, and σ is the pulsational frequency. As we are dealing with $\ell = 2$ modes, and since the emission of gravitational waves in this case will be the same for all the values of m, we shall choose the simplest case. That is, we adopt $\ell = 2$ and $m = 0$. Additionally, it must be taken into account that BPM 37093 has a sizeable crystallized core. Therefore, the appropriate boundary conditions differ from those of an ordinary star. Particularly, the boundary condition at the stellar center (when crystallization has not yet set in) is that given by Osaki & Hansen (1973). However, when the core of the white dwarf undergoes crystallization we switch the fluid internal boundary conditions to the so-called "hard sphere" boundary conditions (Montgomery & Winget 1999). Within this approximation the nonradial eigenfunctions are inhibited from propagating in the crystallized region of the core. Consequently, and keeping in mind that for the axisymmetric case $Q_{11} = Q_{22} = -\frac{1}{2}Q_{33}$ and $Q_{ij} = 0$ if $i \neq j$ (Osaki & Hansen 1973), the following expression can be easily obtained

$$\begin{aligned} Q_{33} &= \int_0^{2\pi}\mathrm{d}\phi\int_0^{\pi}\sqrt{\frac{5}{16\pi}}\sin\theta\left(\cos^2\theta - 1\right)\left(3\cos^2\theta - 1\right)\mathrm{d}\theta \\ &\quad\times\int_{R_0}^{R_\star} r^4\rho'(r)\cos(\sigma t)\,\mathrm{d}r \end{aligned} \tag{4}$$

where $R_\star$ and R_0 are the stellar radius and the radial coordinate of the crystallization front, respectively. By using Poisson's equation

$$4\pi G\rho'(r) = \frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\frac{\mathrm{d}\Phi'}{\mathrm{d}r}\right) - \frac{6}{r^2}\Phi' \tag{5}$$

and Eq. (4) it can be shown after a straightforward calculation that the dimensionless strain, $h_{33}^{\rm TT}$, is given by

$$\begin{aligned} h_{33}^{\rm TT} &\approx 5\times10^{-18}\left(\frac{M_\star}{M_\odot}\right)\left(\frac{R_\star}{R_\odot}\right)^2\left(\frac{\nu}{1\,\mathrm{mHz}}\right)^2\left(\frac{1\,\mathrm{pc}}{d}\right) \\ &\quad\times\left[A(R_\star) - F_\mu F_R^2 A(R_0)\right]\cos(\sigma t) \end{aligned} \tag{6}$$

being $\nu = 2\pi\sigma$ the frequency of the signal and

$$\begin{aligned} A(r) &\equiv y_4 - 2y_3 \\ y_3 &\equiv \frac{\Phi'}{gr} \\ y_4 &\equiv \frac{1}{g}\frac{\mathrm{d}\Phi'}{\mathrm{d}r} \\ F_\mu &\equiv \frac{M_0}{M_\star} \\ F_R &\equiv \frac{R_0}{R_\star}. \end{aligned} \tag{7}$$

In these expressions Φ' is the perturbed gravitational potential, g is the gravitational acceleration at a given radius, $M_\star$ and $R_\star$ are the mass and radius of the star, and M_0 and R_0 are the mass and radius of the crystallized core of the white dwarf. Obviously, in the non-crystallized case, M_0 and R_0 are identically zero (corresponding to the stellar center). The advantage of the previously described formalism is that the quantities y_3 and y_4 can be easily tabulated for a typical stellar model, whereas the rest can be observationally obtained. Finally,

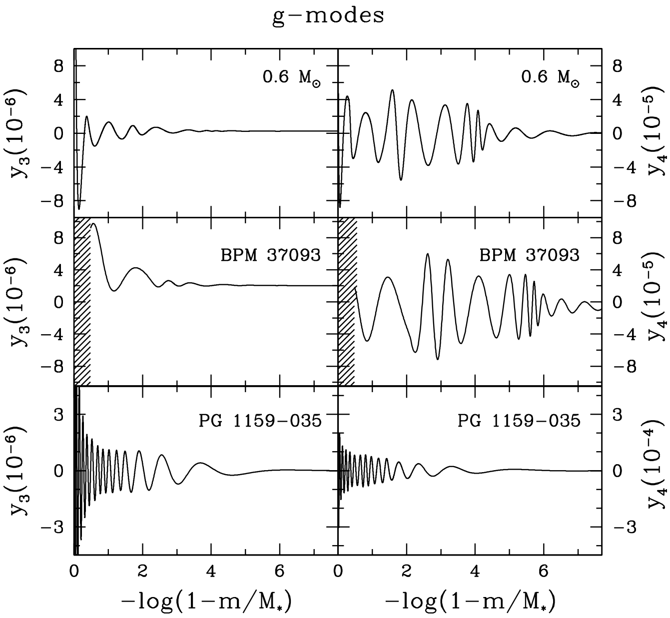

Fig. 1. Run of the y_3 – *left panels* – and y_4 – *right panels* – eigenfunctions for the g-modes of a typical 0.6 $M_\odot$ white dwarf – *top panels* – for BPM 37093 – *central panels* – and for PG 1159-035 – *bottom panels* – as a function of the mass coordinate $\log(1-m/M_\star)$. For the case of BPM 37093 the crystallized core is shown as a hatched area. See text for details.

the luminosity radiated in the form of gravitational waves is (Osaki & Hansen 1973):

$$L_{\rm GW} \approx 10^{36}\left(\frac{M_\star}{M_\odot}\right)^2\left(\frac{R_\star}{R_\odot}\right)^4\left(\frac{\nu}{1\,{\rm mHz}}\right)^6 \times\left[A(R_\star)-F_\mu F_R^2 A(R_0)\right]^2. \quad (8)$$

5. Results

Figure 1 shows the run, as a function of the mass coordinate, of the functions y_3 – left panels – and y_4 – right panels – discussed in Sect. 4 for the g-modes of our fiducial model (a typical 0.6 $M_\odot$ white dwarf made of carbon and oxygen with a liquid core and hydrogen envelope of 10^{-4} M_*), for BPM 37093 – a massive ($M_\star \simeq 1.05$ $M_\odot$) white dwarf, with a sizeable crystallized core – and for PG 1159-035, the other known white dwarf with unambiguously identified quadrupole g-modes. The fractional change in radius due to pulsations, $\delta R_\star/R_\star$, must not necessarily be the same for each pulsation mode. However, the linear theory of nonradial pulsations does not provide any indication of the value of the fractional change in radius, since the governing equations are homogeneous and the normalization of the eigenfunctions is arbitrary (Cox 1980). In addition, $\delta R_\star/R_\star$ is poorly constrained by the observations, since the luminosity variations of pulsating white dwarfs are almost exclusively caused by changes in temperature, not by their radius variations (Robinson et al. 1982). Thus, we have adopted, somewhat arbitrarily, that the fractional change in radius due to pulsations is $\delta R_\star/R_\star = 10^{-4}$ for all the considered modes, which is a typical value for pulsating white dwarfs, and reasonably reproduces the amplitude of the observed light curve (Robinson et al. 1982).

We discuss the results obtained for our fiducial model (top panels of Fig. 1). We have chosen to display the quadrupole ($\ell = 2$) g-mode with radial order $k = 25$, which has a period $P = 678.22$ s. As can be seen the functions y_3 and y_4 are small everywhere in the star, their amplitudes being of the order of $\sim 10^{-6}$ and $\sim 10^{-4}$, respectively. Moreover, the amplitudes are only significant for the central regions of the white dwarf. For the case of BPM 37093 – central panels – we show the quadrupole g-mode with $k = 27$. This g-mode has a period $P = 536.4$ s, which is very close to one of the observed periods, $P = 531.1$ s. Since BPM 37093 is a massive white dwarf, a sizeable region of its core is crystallized. This region is clearly marked in the central panels of Fig. 1 as a shaded area. We note that in this region the amplitudes of both y_3 and y_4 are null. For the PG 1159-035 model – bottom panels – we display the quadrupole mode with $k = 30$, which best fits the observed period of $P = 423.2$ s. This mode has a period $P = 423.8$ s, thus providing an excellent fit to the observational data.

In Table 1 we summarize the most important results for several g-modes of the models computed so far. We have assumed that all of the observed periods in BPM 37093 and in PG 1159-035 in Table 1 are $\ell = 2$, following the works by Kannan et al. (2005) and Winget et al. (1991), respectively. The maximum dimensionless strain, $h_{\rm max}$ for BPM 37093 has been computed adopting the measured distance to the source ($d = 16.8$ pc), whereas for our fiducial model we have adopted a distance $d = 50$ pc, which we consider to be representative of a typical white dwarf. For the case of PG 1159-035 we have adopted a distance of 400 pc, in line with the determinations of Werner et al. (1991) and Kawaler & Bradley (1994). Column 7 provides the luminosity radiated away in the form of gravitational waves, $L_{\rm GW}$, computed with Eq. (8). In the last column of Table 1 we list the kinetic energy of each of the modes. In general, the agreement between the computed and the observed periods is rather good for all the modes, both for the case of BPM 37093 and for PG 1159-035. However, the amplitudes of the dimensionless strains are small in all cases. This is also the case for the luminosities radiated away in the form of gravitational waves. In the best of the cases BPM 37093 radiates away $\sim 10^{19}$ erg/s in the form of gravitational waves, whereas PG 1159-035 radiates away a much more modest amount, only $\sim 10^{17}$ erg/s.

For our fiducial model – third section of Table 1 – the larger the radial order k, the smaller the dimensionless strain and the smaller the luminosities radiated away in the form of gravitational waves. In particular, an increase from $k = 1$ to $k = 10$ produces a reduction of a factor of almost 4×10^2 in the dimensionless strain and of 3×10^6 in the flux of gravitational waves. The reductions when considering the $k = 20$ mode are much more modest. This is because low-k modes sample the core more than high-k modes, and since the core has a higher density, larger mass motions are produced, and hence more gravitational wave losses are produced. Nevertheless, both the dimensionless strains and the fluxes of gravitational waves are in this case much larger than those found for BPM 37093 and PG 1159-035. For the case of BPM 37093 pulsations occur

Table 1. Summary of the gravitational wave emission of the g-modes of BPM 37093 and PG 1159-035. Our fiducial model is also shown for comparison. In all cases we have adopted $\delta R_\star/R_\star = 10^{-4}$. The first column lists the model. The second column corresponds to its respective mass. In the third column we show the radial order, k, of the computed g-mode. The observed and the computed periods (in seconds) are given in Cols. 4 and 5, respectively. In Col. 6 we give the maximum dimensionless strain, $h_{\rm max}$. Column 7 lists the luminosity in the form of gravitational waves and Col. 8 lists the kinetic energy of the modes.

Model	$M/M_\odot$	k	$P_{\rm o}$ (s)	$P_{\rm c}$ (s)	$h_{\rm max}$	$L_{\rm GW}$ (erg/s)	$\log(E_K)$ (erg)
BPM 37093	1.10	26	511.7	516.7	4.8×10^{-28}	9.2×10^{18}	46.4
		27	531.1	536.4	5.4×10^{-28}	1.1×10^{19}	46.5
		28	548.4	555.8	5.3×10^{-28}	9.9×10^{18}	46.6
		29	582.0	574.9	6.4×10^{-28}	1.3×10^{19}	46.7
		30	600.7	593.0	6.4×10^{-28}	1.2×10^{19}	46.8
		32	633.5	630.4	5.5×10^{-28}	8.3×10^{18}	46.9
PG 1159-035	0.54	25	352.7	358.9	1.0×10^{-30}	5.0×10^{16}	44.0
		30	423.8	423.2	2.3×10^{-30}	1.7×10^{17}	43.8
		50	694.9	684.5	3.5×10^{-31}	1.6×10^{15}	43.4
		55	734.2	752.9	2.0×10^{-31}	4.1×10^{14}	43.3
		60	812.6	818.1	1.8×10^{-31}	2.8×10^{14}	43.2
		70	968.7	950.1	6.0×10^{-32}	2.5×10^{13}	42.8
0.6 $M_\odot$	0.6	1	–	66.6	6.9×10^{-25}	1.0×10^{28}	47.0
		10	–	310.3	1.8×10^{-27}	3.1×10^{21}	44.8
		20	–	555.2	1.5×10^{-27}	6.6×10^{20}	45.1

only in a small region of the star as a result of its crystallized core. Thus, despite its mass being much larger than that of our fiducial model, the emission of gravitational waves is strongly inhibited. For the case of PG 1159-035, the most important reason why so few gravitational waves are radiated away is its small mass (and average density).

Given the results obtained for the quadrupole g-modes studied up to now we ask whether other modes, namely the f- and p-modes, of pulsating white dwarfs can radiate away a measurable amount of gravitational waves. Thus, we have extended our calculations to incorporate such modes, despite the lack of observational evidence for them. Obviously, for these modes we do not know the appropriate value of $\delta R_\star/R_\star$, since the estimate $\delta R_\star/R_\star = 10^{-4}$ is based on observed g-modes in white dwarfs. However, for the calculations reported here we adopted the same value. Figure 2 shows the run of the functions y_3 and y_4 for f-modes of our fiducial model, for BPM 37093 and for PG 1159-035. As is the case for all f-modes the radial order is zero, and their respective periods are (from top to bottom): 11.35 s, 3.7 s and 20.9 s. The functions y_3 and y_4 are much larger than in the case previously studied. In this case the functions y_3 and y_4 have large amplitudes everywhere and do not vanish at the surface. Consequently, we expect that a large number of gravitational waves can be radiated away. This is indeed the case, as can be observed in Table 2. For all the f-modes of the three models presented here the dimensionless strains are several orders of magnitude larger and, moreover, the luminosities radiated away are much larger than the optical luminosities, even of the order of 10^{41} erg/s in the case of BPM 37093. This, in turn, could have important consequences since it could provide one of the possible reasons why these modes have not been observed thus far: if they are excited they are quickly damped by emission of gravitational waves.

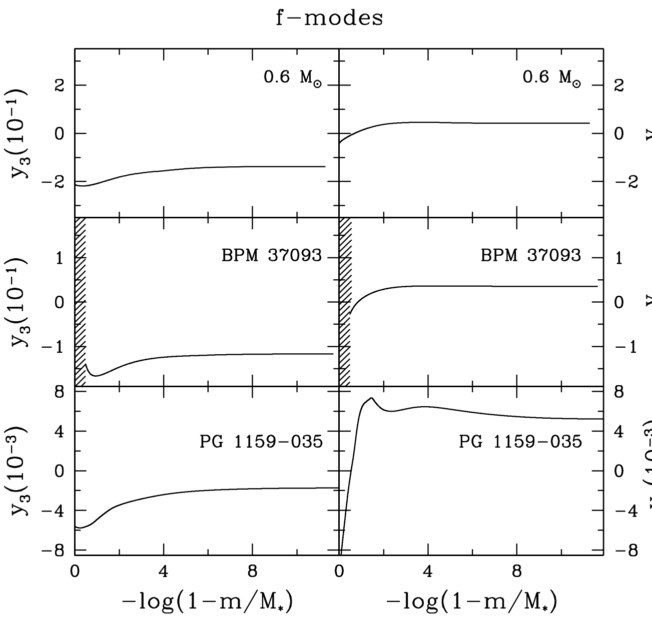

Fig. 2. Same as Fig. 1 for the f-mode of our 0.6 $M_\odot$ fiducial model, for BPM 37093 and for PG 1159-035.

Now we turn our attention to the p-modes. Again, we show the run of the y_3 and y_4 functions of the p-modes in terms of the mass coordinate for our three models in Fig. 3. We have chosen to show the $k = 25$ mode for all three cases. Their respective periods are $P = 0.56$ s for our fiducial 0.6 $M_\odot$ model, $P = 0.11$ s for BPM 37093 and $P = 2.26$ s for PG 1159-035. The amplitudes of the y_3 and y_4 functions for p-modes are much smaller than those of the corresponding f-modes, and comparable to the corresponding g-modes studied before. In addition, in contrast to the situation for the f-modes, the amplitudes of y_3 and y_4 are almost negligible in regions

Table 2. Summary of the gravitational wave emission of the f- and p-modes of BPM 37093 and PG 1159-035. Our fiducial model is also shown for the sake of comparison. In all cases we have adopted $\delta R_\star/R_\star = 10^{-4}$. We show the model (first column), its respective mass (second column), the considered mode (third column), the radial order (fourth column), the respective frequency of the mode (fifth column), the dimensionless strain (sixth column), the luminosity radiated away in the form of gravitational waves (seventh column) and the kinetic energy (last column).

Model	$M/M_\odot$	Mode	k	ν (Hz)	$h_{\rm max}$	$L_{\rm GW}$ (erg/s)	$\log(E_K)$ (erg)
BPM 37093	1.10	f	0	2.7×10^{-1}	6.6×10^{-19}	3.4×10^{41}	49.2
		p	1	9.3×10^{-1}	9.1×10^{-20}	7.7×10^{40}	47.8
PG 1159-035	0.54	f	0	4.8×10^{-2}	1.2×10^{-22}	2.1×10^{35}	45.5
		p	1	5.9×10^{-2}	7.1×10^{-23}	5.1×10^{34}	44.6
$0.6\,M_\odot$	0.60	f	0	8.8×10^{-2}	5.6×10^{-20}	2.3×10^{39}	47.0
		p	1	1.8×10^{-1}	6.4×10^{-21}	1.3×10^{38}	48.6
		p	5	4.6×10^{-1}	6.1×10^{-21}	7.5×10^{38}	46.0

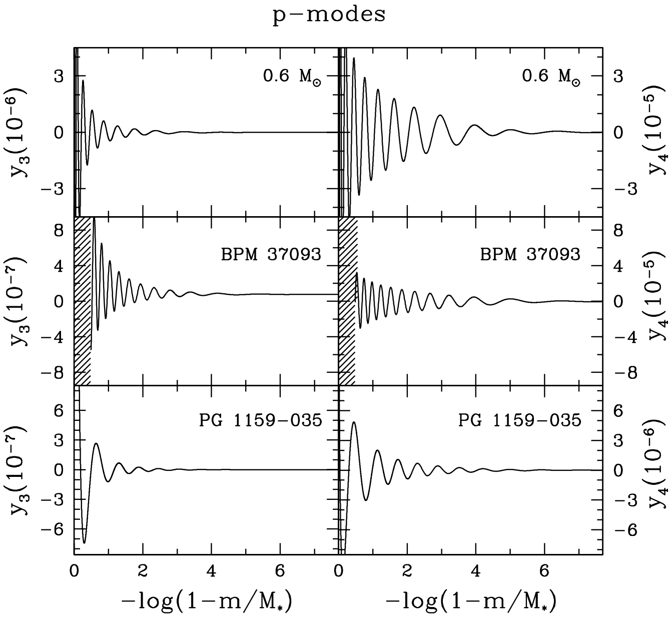

Fig. 3. Same as Fig. 1 for the p-modes of our $0.6\,M_\odot$ fiducial model, for BPM 37093 and for PG 1159-035.

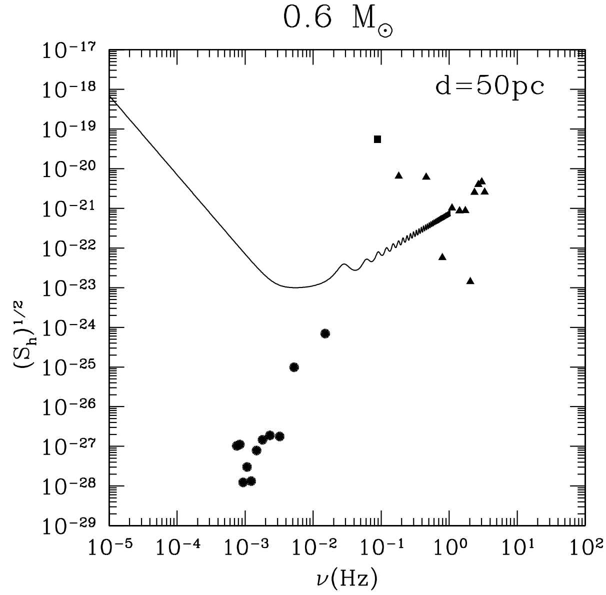

Fig. 4. A comparison of the signal produced by the quadrupole g-modes of our fiducial model – circles – by the f-mode – square – and by the p-modes – triangles – with the spectral distribution of noise of LISA for a one-year integration period, and assuming that the source is located at 50 pc.

close to the surface of the white dwarf. However, because the pulsation frequencies of p-modes are considerably higher than those of the f- and g-modes, the dimensionless strains – see Table 2 – are consequently large and the corresponding gravitational wave luminosities are very large as well, although roughly one order of magnitude smaller than those obtained for the f-modes.

In order to check whether LISA would be able to detect the pulsating white dwarfs studied here we have assumed that the integration time of LISA will be one year. The signal-to-noise ratio, η, is given by:

$$\eta^2 = \int_{-\infty}^{+\infty} \frac{\tilde{h}^2(\sigma)}{S(\sigma)} \frac{\mathrm{d}\sigma}{2\pi} \tag{9}$$

where $S(\sigma) = S_{\rm h}(\sigma)\tau$ is the sensitivity of LISA, τ is the integration period, $\tilde{h}(\sigma)$ is the Fourier Transform of the dimensionless strain, and σ has been previously defined. It can easily be shown that for a monochromatic gravitational wave $\eta = h(\sigma)/S_{\rm h}^{1/2}(\sigma)$. We have adopted a signal-to-noise ratio $\eta = 5$. We have furthermore used the integrated sensitivity of LISA as obtained from `http://www.srl.caltech.edu/~shane/sensitivity`. The results of this procedure are shown in Figs. 4–6 for our fiducial model ($0.6\,M_\odot$ carbon-oxygen white dwarf), for BPM 37093 and for PG 1159-035, respectively. In all three figures the g-modes are shown as circles, the f-mode is shown as a square and the p-modes are displayed as triangles. As done previously, for our fiducial model we have adopted a distance of 50 pc, for PG 1159-035 we have assumed a distance of 400 pc, and for BPM 37093 we have used its measured distance (16.8 pc). As can be seen, for none of the three cases studied here will LISA be able to measure the dimensionless strains of the g-modes, even at a reduced signal-to-noise ratio. Most of the p-modes will not be observed as well, either because their frequencies are too high to be observed by LISA or because they are too weak. Moreover, given that these modes radiate huge amounts

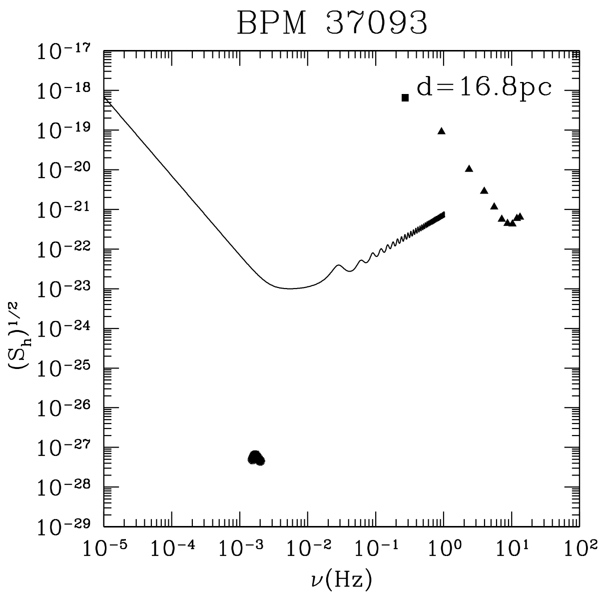

Fig. 5. Same as Fig. 4 for the case of BPM 37093. The distance in this case is known, $d = 16.8$ pc.

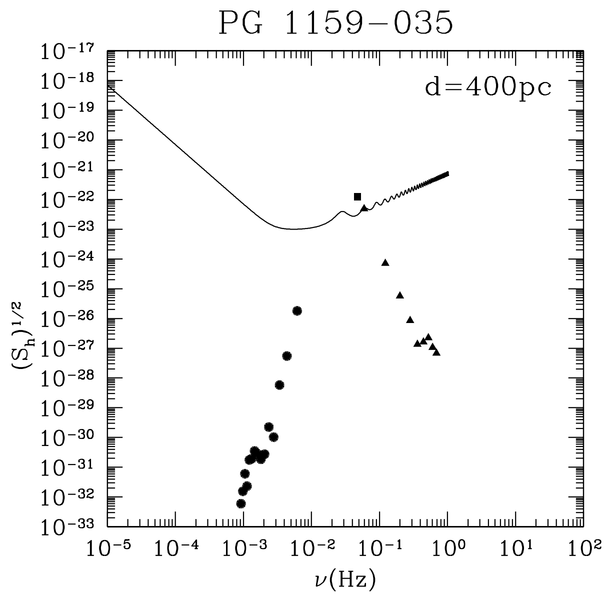

Fig. 6. Same as Fig. 4 for the case of PG 1159-035. Since the distance to PG 1159-035 is not accurately known we have adopted $d = 400$ pc, which is a reasonable estimate.

of energy in the form of gravitational waves they become quickly damped and, consequently, it will be very difficult to detect them. Particularly, and given that for the p-modes studied here the time-averaged dissipation rates of pulsations due to radiative (photons) heat leakage and neutrino losses are much smaller than the luminosity radiated as gravitational waves, an estimate of the damping timescale, $\tau_{\rm d}$, can be easily obtained by considering the kinetic energy of the mode, E_K, which can be easily computed from our numerical models:

$$\tau_{\rm d} \simeq \frac{2E_K}{L_{\rm GW}}. \tag{10}$$

For instance, for the p-mode with $k = 1$ of BPM 37093 one obtains $\tau_{\rm d} \simeq 0.5$ yr, which clearly is too short to allow a detection.

Finally, the three f-modes of the models presented here lay in the appropriate range of frequencies and, additionally, they are well over the sensitivity curve of LISA. However, as was the case for the p-modes, they also radiate very large numbers of gravitational waves and, hence, they will be quickly damped, hampering the possibility of detection. In this case we obtain a damping timescale $\tau_{\rm d} \simeq 2.9$ yr for the f-mode of BPM 37093. Although the luminosity radiated away in the form of gravitational waves is larger for the f-mode than for the p-mode considered previously, the damping timescale is larger. This is so because the kinetic energies involved are quite different: $E_K = 1.54 \times 10^{49}$ erg for the f-mode and $E_K = 6.42 \times 10^{47}$ erg for the p-mode with $k = 1$. For the sake of completeness we present in Table 2 all the relevant information for the p- and f-modes that may be detected.

6. Discussion and conclusions

In this paper we have computed the gravitational wave emission of pulsating white dwarfs. We have started by computing the gravitational wave radiation of white dwarfs undergoing nonradial g-mode pulsations, which are currently observed in a handful of pulsating white dwarfs. We have focused on three model stars. Our fiducial model corresponds to an otherwise typical 0.6 $M_\odot$ model white dwarf with a carbon-oxygen fluid core and a hydrogen envelope. We have also paid attention to two additional white dwarf models, corresponding to two stars for which quadrupole g-modes have been observed so far, namely, BPM 37093 and PG 1159-035. We have shown that in these cases the gravitational wave signal is too weak to be observed by future space-borne interferometers, like LISA. We have found that the luminosities in the form of gravitational waves radiated away by these stars and the corresponding dimensionless strains are very small in all the cases, in agreement with the pioneering work of Osaki & Hansen (1973). Hence, all these sources contribute to the Galactic noise and no individual detections are expected, despite the proximity of the sources. For completeness we have computed the gravitational wave emission of white dwarfs undergoing nonradial f- and p-mode oscillations, even if these modes have not been observationally detected. We have found that for white dwarfs undergoing this kind of pulsation the luminosities in the form of gravitational waves radiated away are very large in all the cases, in line with the earlier results of Osaki & Hansen (1973). Consequently, these modes, if excited, should be very short-lived, thus hampering their eventual detection.

It may seem that for the case of pulsating white dwarfs undergoing g-mode oscillations there could still be a possibility of *indirect detection* by measuring the secular rate of change of the period of the observed modes. However, this is not the case. The secular rate of change of the period of a pulsating white dwarf is given by

$$\frac{\dot{P}}{P} = -a\frac{\dot{T}}{T} + b\frac{\dot{R}_\star}{R_\star} \tag{11}$$

where T is the temperature of the isothermal core and a and b are constants of order unity which depend on the chemical composition, thicknesses of the H atmosphere and He buffer, equation of state, and other ingredients involved in the modeling of white dwarfs. For DA white dwarfs in the ZZ Ceti instability strip, the second term of the right hand side of Eq. (11) is usually negligible and, thus, the secular rate of change of the period only reflects the speed of cooling – see, for instance, Isern et al. (1992) and references therein – whereas for GW Vir stars this term is relevant, but can be accounted for by the theoretical models and, hence, the speed of cooling can also be derived. However, any additional source of cooling – like gravitational waves – would eventually translate into an anomalous rate of period change (Isern et al. 1992):

$$\frac{\dot{P}_{\rm o}}{\dot{P}_{\rm c}} - 1 = \frac{L_{\rm GW}}{L + L_\nu} \tag{12}$$

where $P_{\rm o}$ is the observed period, $P_{\rm c}$ is the computed period without taking into account the emission of gravitational waves, L_ν is the neutrino luminosity – which is important for hot white dwarfs – and the rest of the symbols are as previously defined. The rate of secular period change has been measured for some pulsating white dwarfs. Particularly, for G117-B15A (Kepler et al. 2000) it has been possible to measure the secular variation of the main observed period of 215.2 s, $\dot{P} = (2.3 \pm 1.4) \times 10^{-15}$ s s^{-1}, with unprecedented accuracy. This white dwarf is the most stable optical clock known, and has been used to pose tight constraints on the mass of the axion (Córsico et al. 2001b) and the rate of variation of gravitational constant (Benvenuto et al. 2004). Other pulsating white dwarfs – like L 19-2 and R 548 – also have determinations of the secular rate of period change but are not as accurate as that of G117-B15A. Note, however, that for a 0.6 $M_\odot$ white dwarf undergoing quadrupole g-mode oscillations with a period of $P \sim 200$ s the right hand side of Eq. (12) is 2×10^{-10} and $P_{\rm c} \sim 10^{-15}$ s s^{-1}. In other words the gravitational radiation will produce a change in $\dot{P} \sim 10^{-25}$ s s^{-1}, making impossible such an indirect detection even if accurate observational data and reliable theoretical models eventually become available. Again, the only case of interest here would be the case in which $L_{\rm GW} \sim L + L_\nu$, which may be true for the f- and p-mode pulsators.

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References

Abramovici, A., Althouse, W. E., Drever, R. W. P., et al. 1992, Science, 256, 325
Acernese, F., Amico, P., Arnaud, N., et al. 2004, Class. & Quantum Grav., 21, S385
Alexander, D. R., & Ferguson, J. W. 1994, ApJ, 437, 879
Althaus, L. G., Serenelli, A. M., Córsico, A. H., & Montgomery, M. H. 2003, A&A, 404, 593
Althaus, L. G., Serenelli, Panei, J. A., Córsico, A. H., García-Berro, E., & Scóccola, C. G. 2005, A&A, 435, 631
Bender, P. L., et al. 1998, LISA: Laser Interferometer Space Antenna for the detection and observation of GW, Pre-fase A Report (Garching: Max Planck-Institut für Quantenoptik)
Bender, P. L., et al. 2000, LISA: a conerstone mission for the observation of gravitational waves, ESA-SCI(2000)11, System and Technology Study Report
Benvenuto, O. G., García-Berro E., & Isern, J. 2004, Phys. Rev. D, 69, 082002
Benacquista, M., Sedrakian, D. M., Hairapetyan, M. V., Shahabasyan, K. M., & Sadoyan, A. A. 2003, ApJ, 596, L223
Bradley, P. A. 1998, Baltic Astron., 7, 111
Bradley, P. A. 2001, ApJ, 552, 326
Córsico, A. H., Althaus, L. G., Benvenuto, O. G., & Serenelli, A. M. 2001a, A&A, 380, L17
Córsico, A. H., Benvenuto, O. G., Althaus, L. G., Isern, J., & García-Berro, E. 2001b, New Astron., 6, 197
Córsico, A. H., Althaus, L. G., Benvenuto, O. G., & Serenelli, A. M. 2002, A&A, 387, 531
Córsico, A. H., García-Berro, E., Althaus, L. G., & Isern, J. 2004, A&A, 427, 923
Córsico, A. H., Althaus, L. G., Montgomery, M. H., García-Berro, E., & Isern, J. 2005, A&A, 429, 277
Costa, J. E. S., Kepler, S. O., & Winget, D. E. 1999, ApJ, 522, 973
Cox, J. P. 1980, Theory of stellar pulsation (Princeton University Press)
Gautschy, A., & Saio, H. 1995, ARA&A, 33, 75
Gautschy, A., & Saio, H. 1996, ARA&A, 34, 551
Iglesias, C. A., & Rogers, F. J. 1996, ApJ, 464, 943
Isern, J., Hernanz, M., & García-Berro, E. 1992, ApJ, 392, L23
Itoh, N., Hayashi, H., &, Kohyama, Y. 1994, ApJ, 436, 418
Itoh, N., Hayashi, H., Nishikawa, A., & Kohyama, Y. 1996, ApJS, 102, 411
Kanaan, A., Nitta, A., Winget, D. E., et al. 2005, A&A, 432, 219
Kanaan, A., Kepler, S. O., Giovannini, O., & Diaz, M. 1992, ApJ, 390, L89
Kawaler, S. D. 1998, Baltic Astron., 7, 11
Kawaler, S. D., & Bradley, P. A. 1994, ApJ, 427, 415
Kepler, S. O., Mukadam, A. S., Winget, D. E., et al. 2000, ApJ, 534, L185
Magni, G., & Mazzitelli, I. 1979, A&A, 72, 134
Misner, C. W., Thorne, K., & Wheeler, J. A. 1973, Gravitation (San Francisco: Freeman)
Montgomery, & M. H., Winget, D. E. 1999, ApJ, 526, 976
Nather R. E. 1995, Baltic Astron., 4, 117
Osaki, Y., & Hansen, C. J. 1973, 185, 277
Pfeiffer, B., Vauclair, G., Dolez, N., et al. 1996, A&A, 314, 182
Robinson, E. L., Kepler, S. O., & Nather, R. E. 1982, ApJ, 259, 219
Schutz, B. F. 1999, Class. & Quantum Grav., 16, A131
Takahashi, R., et al. 2004, Class. & Quantum Grav., 21, S403
Unno, W., Osaki, Y., Ando, H., Saio, H., & Shibahashi, H. 1989, Nonradial Oscillations of Stars (University of Tokyo Press), 2nd edition
Werner, K., Heber, U., & Hunger, K. 1991, A&A, 244, 437
Willke, B., Aufmuth, P., Aulbert, C., et al. 2004, Class. & Quantum Grav., 21, S417
Winget, D. E., Nather, R. E., Clemens, J. C., et al. 1991, ApJ, 378, 326
Winget, D. E., Kepler, S. O., Kanaan, A., Montgomery, M. H., & Giovannini, O. 1997, ApJ, 487, L191