

Source blending effects on microlensing time-histograms and optical depth determination

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ABSTRACT

Source blending in microlensing experiments is known to modify the Einstein time of the observed events. In this paper, we have conducted Monte-Carlo calculations, using analytical relationships to quantify the effect of blending on the observed event time distribution and optical depth. We show that short events are affected significantly by source blending and that, for moderately blended sources, the optical depth τ is globally overestimated, because of an underestimation of the exposure. For high blending situations, on the other hand, blending leads to an underestimation of the optical depth. Our results are in agreement with the most recent optical depth determinations toward the Galactic Center of the MACHO and OGLE-II collaborations that use clump giants (less affected by the blending effect) as sources. The blending-corrected, lower optical depth toward the Galactic Bulge is now in good agreement with the value inferred from galactic models, reconciling theoretical and observational determinations.

Key words. Galaxy: stellar content – galaxies: structure – gravitational lensing

1. Introduction

Microlensing is one of the most efficient ways to measure the amount of (sub)stellar matter, dark or luminous, in our Galaxy. By measuring the amplification of the flux of a source star, one can statistically derive the mass, distance and transverse velocity of the deflector. When a deflector passes near the line of sight between the observer and the source, the flux of the source is amplified by a factor (Paczynski 1986)

$$A(u) = \frac{u^2 + 2}{u\sqrt{u^2 + 4}} \quad \text{where} \quad u(t) = \sqrt{u_{\min}^2 + \left(\frac{t}{t_E}\right)^2}$$

if the origin of time is taken at the moment of maximum amplification. The light curve depends on two parameters only, u_{\min} and t_E , where u_{\min} is the minimum impact parameter (the minimum distance to the line of sight, normalized to the Einstein radius of the lens R_E), and t_E is the Einstein time, i.e. the time required to cross the Einstein radius.

The Einstein radius is given by the lens mass M , the observer-source distance L and the ratio x of the lens-observer distance to L :

$$R_E^2 = \frac{4GM}{c^2} Lx(1-x).$$

When a microlensing event occurs, a theoretical light curve is adjusted to give u_{\min} and t_E : u_{\min} is related to the maximum amplification by $u_{\min} = f(A_{\max})$, where the function f is defined by $f(x) \equiv \sqrt{\frac{2x}{\sqrt{x^2-1}} - 2}$. The Einstein time is obtained from the

event duration t_d during which $A > A_T$ (where A_T is the threshold amplification, usually taken as $3/\sqrt{5}$, which corresponds to $u_T = 1$) by:

$$t_E = t_d / \sqrt{u_T^2 - u_{\min}^2}.$$

The observed optical depth is estimated from the Einstein times of the events as (Alcock et al. 1997):

$$\tau_{\text{obs}} = \frac{\pi}{4E} \sum_i t_{Ei} / \varepsilon(t_{Ei}),$$

where E is the exposure, i.e. the number of observed sources times the duration of the experiment (in stars \times years), and ε is the detection efficiency. This optical depth can be compared with the one obtained from galactic models using

$$\tau = \int \frac{4\pi G}{c^2} x(1-x)L^2 \rho(xL) dx \quad (1)$$

for a constant observer-source distance, where ρ is the galactic mass density distribution.

A long-standing unsolved problem concerning the microlensing experiments towards the Galactic bulge is the significant discrepancy between the optical depths derived from microlensing experiments and the one calculated from theoretical models, as identified originally by Kiraga & Paczynski (1994).

Using Difference Image Analysis, and using main sequence stars as microlensing sources, the MACHO project

(Alcock et al. 2000) deduced an optical depth $\tau_{-6} = 2.91^{+0.47}_{-0.45}$ at 2σ with 99 observed events, where $\tau_{-6} \equiv \tau/10^{-6}$. The OGLE collaboration derived $\tau_{-6} = 3.3 \pm 2.4$ at 2σ with 9 observed events (Udalski et al. 1994). More recently, the MOA collaboration has determined an optical depth $\tau_{-6} = 2.59^{+0.84}_{-0.64}$ (Sumi et al. 2003).

On the other hand, all Galactic models predict a typical optical depth $\tau \simeq 1-1.2 \times 10^{-6}$ (see for example Peale 1998; Bissantz et al. 1997). As shown by Peale (1998), one can reproduce the observed optical depth only if the bulge mass is equal to $M_b = 3.3 \times 10^{10} M_\odot$, whereas gas dynamics calculations based on DIRBE observations to derive the gravitational potential of our Galaxy yield a maximum mass for the bulge $M_{b,\max} \simeq 2 \times 10^{10} M_\odot$ (Bissantz et al. 1997; Englmaier et al. 1999; Sevenster et al. 1999). More recently, Han & Gould (2003) derived an optical depth $\tau_{-6} = 0.98$, for events due to bulge stars, and $\tau_{-6} = 1.63$ when including the events due to the disk.

Using brighter stars (in particular red clump giants), the MACHO and the OGLE-II collaborations have derived new optical depth determinations, namely $\tau_{-6} = 2.17^{+0.47}_{-0.38}$ for 62 events for the MACHO group (Popowski et al. 2004), and $\tau_{-6} = 1.96^{+0.41}_{-0.34}$ for 33 events for the OGLE-II group (Sumi et al. 2005). These values, closer to the theoretical expectations, are significantly lower than the aforementioned ones based on main sequence star sources. This raises two important questions: 1) what is the origin of the systematic discrepancy between optical depth measured using faint (main sequence) and bright (clump giant) stars; and 2) which one should be compared with theoretical calculations. We demonstrate here that this difference is largely due to blending effects, and that the optical depth measured using main sequence stars is systematically biased.

Micro-lensing experiments are subject to different biases such as source blending (when the observed lensed source is blended with other unresolved stars), lens blending (when the lens is a luminous star, its own luminosity may affect the observed light curve), and amplification bias (the amplified source is a star below the detection limit close to another star above the detection limit). Lens blending has been addressed by Han (1998); the effect is found to result in an underestimation of the real optical depth by $\simeq 10\%$. Amplification bias also has been studied by Han (1997): its effect is an overestimation of the optical depth by a factor $\simeq 1.7^{+0.7}_{-0.4}$, depending on the size of the seeing disk. Alard (1997) studied source blending by taking it into account directly in Eq. (1). The effect on the optical depth is an overestimation of $\sim 15\%$, but the effect on t_E -histograms was not quantified. In this paper, we study in detail the aforementioned first kind of blending, i.e. when the amplified source is a star above the detection limit, blended with a star above the detection limit.

In the case of source blending, an observed source is the superposition of different source stars, all above the detection limit. When an event occurs, only one of the stars composing the observed source will be amplified, in general¹. The

observed flux is no longer the baseline flux multiplied by the amplification, but instead, if one notes F_s as the flux of the amplified source and F_o the flux of the other unresolved sources,

$$A_{\text{obs}} = \frac{A(u(t))F_s + F_o}{F_s + F_o}.$$

It is very difficult to account correctly for this effect for, as shown by Han (1999), there is an almost perfect degeneracy between unblended and blended light curves. Moreover, it is very difficult to determine the amount of blending by photometric means (see Woźniak & Paczyński 1997). Nevertheless, using the astrometric shift during a microlensing event, Goldberg & Woźniak (1998) have shown that about half of the sources observed by the OGLE collaboration suffer some amount of blending.

To quantify the effect of source blending on the duration of a microlensing event and on the inferred optical depth, we first outline in Sect. 2 the relations between t_E and u_{\min} with and without blending. In Sect. 3, we derive the distribution function to be used for blending. The consequences on the Einstein time histograms and on the optical depth are examined in Sects. 4 and 5, respectively. Section 6 is devoted to the conclusions.

2. Analytical relationships

We will follow the previous calculations by Han (1999) with slightly different notations.

When a star suffers blending, the total observed flux is the sum of the flux from the source F_s and the flux from the other unresolved stars F_o . We define the dimensionless parameters B as $B \equiv F_s/(F_s + F_o)$ and $\bar{B} \equiv 1 - B$.

The observed amplification of a microlensing event thus reads:

$$A(t) = B \frac{u^2 + 2}{u \sqrt{u^2 + 4}} + \bar{B}.$$

Using the observed light curve, one can derive some quantities, such as the impact parameter or the duration of the event. We will label “obs” the quantities derived when one supposes that $B = 1$ (i.e. when one ignores that the source is blended). The quantities corrected for blending will be called “true”, and will have no index.

The observed impact parameter $u_{\min,\text{obs}}$ is

$$u_{\min,\text{obs}} = f(A_{\max,\text{obs}}) \\ = f\left(B \frac{u_{\min}^2 + 2}{u_{\min} \sqrt{u_{\min}^2 + 4}} + \bar{B}\right)$$

so that the duration of the event

$$t_{d,\text{obs}}^2 = t_E^2 \left(f^2 \left(\frac{A_T - \bar{B}}{B} \right) - u_{\min}^2 \right)$$

is underestimated.

We can then relate the observed Einstein time to the true Einstein time, the true impact parameter and the amount of blending by:

$$t_{E,\text{obs}} = t_E \sqrt{f^2 \left(\frac{A_T - \bar{B}}{B} \right) - u_{\min}^2} \left| \sqrt{u_T^2 - u_{\min,\text{obs}}^2} \right. \quad (2)$$

¹ The probability that more than one source is amplified is negligible.

These relations allow us to relate the “observed” and “real” quantities, for a known amount of blending. As shown by Han (1999), the curves characterized by (t_E, u_{\min}, B) and the corresponding $(t_{E,\text{obs}}, u_{\min,\text{obs}}, B = 1)$ lie very close to each other, and the difference is smaller than the observational errors.

3. Blending distribution function

In order to estimate the effect of blending, one needs to know the distribution function (DF) of the blending factor B . We first need to estimate the number of stars whose projection lies inside a small surface ΔS . For that purpose, we suppose that the projected density of stars is constant and that the stars are distributed randomly. By doing so, we neglect all systematic effects in the spatial distribution of stars. Note that this approximation is used by the MACHO group to compute their detection efficiency toward the LMC (Alcock et al. 2001). Then, the probability that there are n stars inside ΔS is given by the Poisson law: $\varphi(n) = \bar{n}^n e^{-\bar{n}} / n!$, where \bar{n} is the mean number of stars lying inside ΔS .

The second step is to calculate the probability that there are n stars inside the seeing disk, of surface ΔS , of an observed source. This probability $P(n)$ is the probability that there are n stars inside ΔS , knowing that there is at least one star inside ΔS . We thus have

$$P(n) \equiv \varphi(n | n \geq 1) = \frac{\varphi(n)}{\sum_{i=1}^{\infty} \varphi(i)} = \frac{\bar{n}^n e^{-\bar{n}}}{n! (1 - e^{-\bar{n}})}.$$

The only free parameter in this law² is \bar{n} , the mean number of stars inside a seeing disk. This parameter depends on the size of the seeing disk, and we will usually take $\bar{n} \approx 1.257$, which ensures that half of the observed sources are unblended (i.e. $P(1) = 0.5$), as inferred by Goldberg & Woźniak (1998) from the OGLE data. Once the number \bar{n} is given, F_o and F_s are randomly determined for a given bulge luminosity function (LF).

For this latter, we use the LF obtained towards Baade’s Window with the Hubble Space Telescope (HST) (Holtzman et al. 1998). Since there are very few observed bright stars, we have proceeded as follows to obtain a combined LF: for bright stars ($V < 18.5$) the HST LF is matched to a power law function, with a -2 exponent. We restrict our calculations to sources fainter than $V_{\text{sup}} = 16$, which corresponds to the majority of observed sources (Alcock et al. 1997), and we performed calculations with different values of this parameter (V_{sup} ranging from 15.5 to 16.5).

The faint cutoff V_{inf} is more difficult to determine. We tried different limiting magnitudes between $V = 21.5$ and $V = 22.5$, which correspond to the lower cutoff of the MACHO LF (Alcock et al. 1997). Since we are only concerned by source blending and not by amplification bias, there is no effect arising from the part of the LF below the detection limit.

² In the following, we will refer to P as the Poisson law, although this is not exactly the standard Poisson law because of the different normalization factor.

4. Effect on t_E -histograms

Blending affects the Einstein time histogram in two different ways. The first one is to decrease t_E to $t_{E,\text{obs}}$ (Eq. (2)). The second one is to reduce the number of observed events: if the maximum amplification is lower than the threshold amplification, the event is missed. This occurs if B is lower than B_{\min} , where B_{\min} is a function of u_{\min} :

$$B_{\min} \equiv (A_T - 1) \left(\frac{u_{\min}^2 + 2}{u_{\min} \sqrt{u_{\min}^2 + 4}} - 1 \right)^{-1}.$$

The Einstein time histograms are calculated using a Monte-Carlo code originally developed by Chabrier & Méra (Méra et al. 1998). We have simulated 10^7 events which result, once including detection efficiency, in about $\sim 10^6$ observed events. The Galactic model (model 1) used in the calculations is the following: a bar-shaped bulge with a density given by the model of Zhao (1996). The bar is in the plane of the disk, with an angle $\phi = 13^\circ$ with respect to the Sun-Galactic center line, the near end of the bar lying in the first quadrant. The bulge mass is taken to be $2.2 \times 10^{10} M_\odot$.

The bar rotates rigidly with an angular speed $\Omega_{\text{bar}} = 63.6 \text{ km s}^{-1} \text{ kpc}^{-1}$, the velocity dispersion is $\sigma_{\text{bulge}} = 110 \text{ km s}^{-1}$ in all directions. The model of the disk is a classical double exponential disk (Bahcall & Soneira 1980), with a scale length $R_d = 2700 \text{ pc}$ and a scale height $h = 300 \text{ pc}$. The local normalization of the disk is $\rho_\odot = 0.05 M_\odot / \text{pc}^3$. The rotation velocity is $v_{\text{rot,disk}} = 210 \text{ km s}^{-1}$ and the isotropic velocity dispersion is $\sigma_{\text{disk}} = 20 \text{ km s}^{-1}$. The velocity distribution of each star is assumed to be Gaussian, with the mean velocity equal to the rotation velocity and the afore mentioned dispersions. The Sun is in the galactic plane, at a distance of $R_\odot = 8 \text{ kpc}$ from the Galactic center. Its velocity is equal to that of the local standard of rest, $v_\odot = v_{\text{LSR}} = 210 \text{ km s}^{-1}$. The visibility function of the sources is the one used by Kiraga & Paczyński (1994), with $\beta = -1$. The mass function is the one derived by Méra et al. (1998).

We stress that the aim of the present paper is not to examine the validity of different galactic models, but to study the effect of source blending on the microlensing event time distribution and optical depth.

To compute the time-histograms, we also need to take into account the detection efficiency. We used the clump giant efficiency toward the bulge: this efficiency is equal to the sampling efficiency and is not corrected for blending (Alcock et al. 1997).

Figure 1 shows the effect of blending on the theoretical histogram (fraction of events as a function of the Einstein time) for different values of \bar{n} in the Poisson distribution function of B (Sect. 3). As expected, the effect of blending is to decrease the mean Einstein time when the fraction of unblended stars f decreases. At the maximum of the histogram (between 4 and 6 days), there are 25% more events in the case of the Poisson DF ($\bar{n} \approx 1.25$) compared to the unblended histogram. This difference increases in the short-time region, and reaches 50% for $t_E = 2.5$ days, but due to the decrease of the efficiency, there are very few observed events in this region. The mean Einstein time is 10% to 15% lower for the three Poisson DF. If we convert this

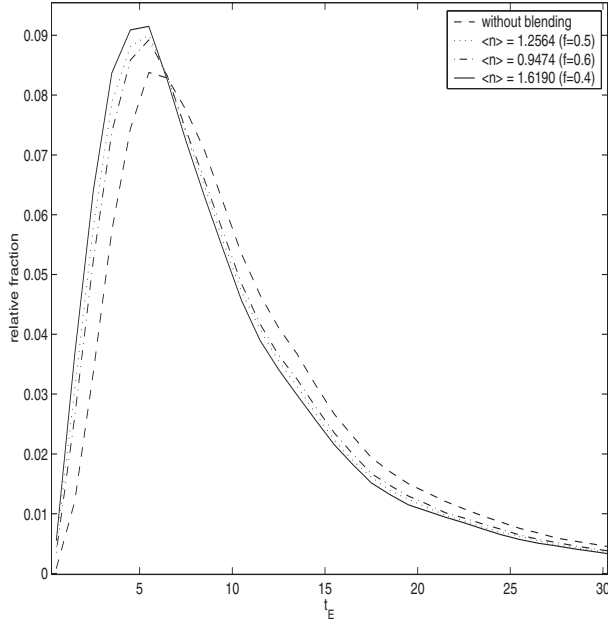


Fig. 1. Einstein time histogram for different amounts of blending. The curves give the relative number of short time events (below 66 days), binned in one-day time intervals. The curves are computed assuming a Poisson law with different values of \bar{n} corresponding to different fractions of unblended stars (f).

difference into a mean lens mass ($\langle t_E \rangle \propto \sqrt{\langle m \rangle}$), we obtain a difference larger than 20% between the unblended case and the Poisson law, i.e. the observed mean mass (derived from observations assuming that there is no blending) and the real mean mass (that could be derived from observations if one knows the amount of blending) i.e. $\langle m \rangle_{\text{obs}} / \langle m \rangle_{\text{real}} < 0.8$.

5. The observed optical depth

Source blending has three effects on the estimation of the optical depth τ . The first one is to lower t_E , which tends to underestimate the optical depth. The second one, as we just showed, is to underestimate the exposure (when a single source is thought to be observed, there are in fact many sources that may be lensed), which tends to overestimate the optical depth. The last one is due to the efficiency ε which, depending on the variations of the efficiency with t_E , can either underestimate or overestimate τ . In order to quantify the resulting global effect, we performed the same Monte-Carlo calculations as described previously.

We first need to relate the true exposure to the observed one. If $n(i)$ is the real number of sources inside the observed source i , the ratio of the true to the observed exposure is given by:

$$\frac{E}{E_{\text{obs}}} = \frac{\sum_{\text{sources}} n(i)}{N_{\text{sources}}}$$

where N_{sources} is the number of monitored sources. The true exposure is given by

$$E = \sum_{n=1}^{\infty} n \times N(n)$$

where $N(n) = N_{\text{sources}} \times P(n)$ is the number of observed sources composed of n blended sources. Therefore

$$\frac{E}{E_{\text{obs}}} = \sum_{n \geq 1} n P(n) = \frac{\sum_{n \geq 1} n \varphi(n)}{1 - \varphi(0)} = \frac{\bar{n}}{1 - e^{-\bar{n}}}$$

It is then straightforward to compute the ratio of the observed optical depth to the real optical depth, for a given DF of the blending B . The luminosity of the source is calculated for each event with the bulge LF. Then, knowing the real number $n(i)$ of sources, we can calculate the amount of blending. The ratio of the true to observed optical depth thus reads:

$$\frac{\tau}{\tau_{\text{obs}}} = \frac{E_{\text{obs}}}{E} \sum \frac{t_E}{\varepsilon(t_E)} \bigg/ \sum \frac{t_{E,\text{obs}}}{\varepsilon(t_{E,\text{obs}})}$$

Figure 2 shows the ratio τ/τ_{obs} for different values of the fraction of unblended stars f . f is related to the parameter of the Poisson law by

$$f = \frac{\bar{n} e^{-\bar{n}}}{1 - e^{-\bar{n}}} = P(1).$$

In order to test the sensitivity of this ratio to the Galactic model, we have performed the same calculations for two other models. Model 2 is obtained by changing only the bar angle (set to $\phi = 20^\circ$), the mass of the bulge ($1.2 \times 10^{10} M_\odot$) and the disk scale length ($R_d = 3500$ pc), while the local normalization of the disk ρ_\odot is unchanged. Model 3 is obtained by changing the velocity parameters: $(\sigma_{\text{bulge}}, \sigma_{\text{disk}}) = (100, 30) \text{ km s}^{-1}$, $\Omega_{\text{bar}} = 50.0 \text{ km s}^{-1} \text{ kpc}^{-1}$ and $v_{\text{rot,disk}} = 180 \text{ km s}^{-1}$. As seen in Fig. 2, the ratio $\frac{\tau}{\tau_{\text{obs}}}$ is not very sensitive to the Galactic model. This was expected since, forgetting the ε term, $t_{E,\text{obs}} = t_E \times g(u_{\text{min}}, B)$ with

$$g(u_{\text{min}}, B) \equiv \frac{\sqrt{f^2 \left(\frac{A_\tau - B}{B} \right) - u_{\text{min}}^2}}{\sqrt{u_1^2 - u_{\text{min,obs}}^2}} \Theta(B - B_{\text{min}})$$

where Θ is the Heaviside function (equal to 0 for a negative argument). Then the ratio τ_{obs}/τ can be written:

$$\tau_{\text{obs}}/\tau = \frac{\bar{n}}{1 - e^{-\bar{n}}} \langle g(u_{\text{min}}, B) \rangle,$$

which is independent of the galactic model. The ratio is also quite insensitive to the cutoff values of the LF since all the curves in the case of model 1 are very close to each other and the differences are of the order of the errors in the Monte-Carlo calculations.

The effect of blending is to *overestimate the optical depth*, for each of the three models considered. The ratio of the true optical depth to the observed one can reach 75%. This result is consistent with the one derived by Alard (1997) using a different method. By taking into account the blending directly in Eq. (1), he found an overestimation of $\sim 15\%$, whereas we obtain $\sim 20\%$ for $f = 0.5$.

We can relate our results to the last determinations of the optical depth using clump giant stars as sources. Popowski et al. (2004) argue that, using clump giant and Differential Image Analysis, they can derive an optical depth with no blending effect. This comes from the fact that, due to their luminosity, clump giant stars are unlikely to suffer large amounts of

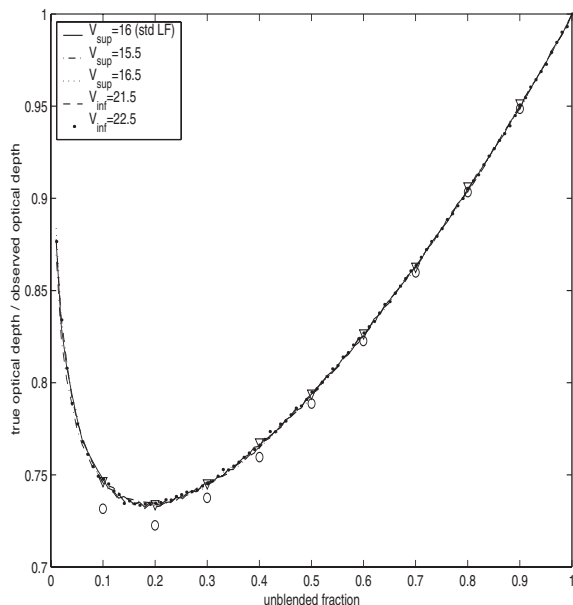


Fig. 2. Ratio τ/τ_{obs} as a function of the unblended source fraction for different values of the LF cutoff. Standard values are $V_{\text{inf}} = 22$ and $V_{\text{sup}} = 16$ and these values are assumed except when indicated by the legend. Triangles and circles are computed using the same LF as the continuous line, but with model 2 and model 3 (see text).

blending. Note, however, that Sumi et al. (2005) suggest that even clump giant stars can suffer some amount of blending, at least in the OGLE-II sample. Using the last MACHO value as the true optical depth, the corresponding observed optical depth for *blended* sources, assuming a Poisson law with $f \approx 0.5$, would be $\tau_{-6} = 2.71^{+0.59}_{-0.47}$, compatible with the value obtained from main sequence sources, $\tau_{-6} = 2.91^{+0.47}_{-0.45}$ (Alcock et al. 2000).

Note that for very crowded fields, $\bar{n} > 10$, the ratio τ_{obs}/τ is greater than one, so that in that case blending effects yield an *underestimation* of the optical depth.

6. Conclusion

We have shown in this paper that blending can have an important effect both on the Einstein time histograms and the inferred optical depth.

For t_E histograms, the effect is to lower the observed t_E and then to decrease the derived mean lens mass. The change in the histogram is about 15% in the low- t_E region corresponding to the maximum of the expected histogram, and can decrease substantially the inferred mean lens mass (the difference can reach 20%).

For the optical depth, we have shown that, for all values considered for \bar{n} , the effect is to overestimate the optical depth as a result of an underestimation of the exposure. In the case of the Poisson law we used, with an unblended fraction around 50%, this effect can reach 20%. These results are quite insensitive to the cutoff of the LF and to the galactic model. We note, however, that in the case of very high blending, for $\bar{n} > 10$, the effect of blending is to underestimate the optical depth. This can occur for microlensing experiments towards

very crowded fields, like M 31 (AGAPE project, Ansari et al. 1997, and references therein).

Finally, to relate the observed optical depth to the one derived from different models, it is necessary to combine these results with the effect of other biases, like lens blending (Han 1998) and amplification bias (Han 1997; Alard 1997). The effect of lens blending is to underestimate the optical depth by about 10%, which can cancel, at least in part, the effect of source blending derived presently in the case of moderate blending (for the Poisson law used in these calculations, this corresponds to an unblended fraction larger than 80%). For the amplification bias, Han (1997) has shown that the effect would be to overestimate the optical depth. Therefore, the net effect of the various blending effects is very likely to overestimate the correct optical depth by a substantial fraction. Taking blending effects into account brings into agreement the optical depth derived with main sequence sources (Alcock et al. 2000; Udalski et al. 1994), the one derived using brighter stars (Popowski et al. 2004; Sumi et al. 2005) and the one derived from usual Galactic models (Bissantz et al. 1997; Englmaier et al. 1999; Sevenster et al. 1999) at the 2σ level, and thus reconciles experimental and theoretical determinations without drastic changes in galactic modelling. Further observational determinations of the exact amount of blending are needed to determine precisely the net effect of blending.

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