A&A 442, 587–596 (2005)
DOI: 10.1051/0004-6361:20052862

Astronomy & Astrophysics

On the metallicity dependence of Wolf-Rayet winds

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Received 10 February 2005 / Accepted 13 July 2005

ABSTRACT

We have performed a pilot study of mass loss predictions for late-type Wolf-Rayet (WR) stars as a function of metal abundance, over a range between $10^{-5} \leq (Z/Z_{\odot}) \leq 10$. We find that the winds of nitrogen-rich Wolf-Rayet stars are dominated by iron lines, with a dependence of mass loss on Z similar to that of massive OB stars. For more evolved, carbon-rich, WR stars the wind strength is also found to be dependent on the Fe abundance, so that they depend on the chemical environment of the host galaxy, but with a mass loss metallicity dependence that is less steep than for OB stars. Our finding that WR mass loss is Z-dependent is a new one, with important consequences for black hole formation and X-ray population studies in external galaxies. A further finding of our study is that the Z dependence of C-rich WR stars becomes weaker at metallicities below $Z/Z_{\odot} \lesssim 1/10$, and *mass loss no longer declines once the metal abundance drops* below $(Z/Z_{\odot}) \simeq 10^{-3}$. This is the result of an increased importance of radiative driving by intermediate mass elements, such as carbon. In combination with rapid rotation and/or proximity to the Eddington limit – likely to be relevant for massive Population III stars – this effect may indicate a role for mass loss in the appearance and evolution of these objects, as well as a potential role for stellar winds in enriching the intergalactic medium of the early Universe.

Key words. stars: Wolf-Rayet – stars: early-type – stars: mass-loss – stars: winds, outflows – stars: evolution

1. Introduction

The physics of mass loss from massive stars is fundamental for a range of astrophysical issues, comprising the formation and evolution of individual massive stars, their final fate, in particular the nature of the compact object that is left behind (neutron star or black hole), as well as the input of energy, ionizing radiation, and chemical elements into the interstellar medium (ISM) of the host galaxy. There is growing evidence that long-duration gamma-ray bursts (GRBs) are the result of the collapse of a massive star (Galama et al. 1998; Hjorth et al. 2003). As these cosmological explosions are seen up to high redshift, and consequently at very low metal abundances, one needs to establish the Z-dependent properties of their progenitors, quite likely WR stars (e.g. Woosley et al. 1999). Furthermore, the observed properties of GRB are expected to depend on the history of the wind strength of the progenitor. The question that therefore needs to be addressed is: "are the winds of WR stars metallicity dependent?"

Although there are indirect indications for such a metallicity dependence (e.g. Hadfield et al. 2005), a theoretical prediction for metallicity dependent WR winds is currently unavailable, nor is there a well-established observational relationship for it. Consequently, stellar evolution theorists need to opt for one of two rather drastic assumptions in dealing with this issue: either, *(i)* they extrapolate the mass loss - Z dependence of O stars to WR stars (e.g. Heger & Woosley 2002; Heger et al. 2003; Eldridge & Tout 2004), or *(ii)* they assume a mass loss that is independent of Z at low metallicity (e.g. Schaller et al. 1992; Maeder & Meynet 1994; Meynet & Maeder 2005).

The first assumption is based on the idea that O and WR stars share similar properties: both are hot, luminous, and generate high-speed outflows. Therefore, one might expect the winds to be driven by the same physical mechanism, i.e. that of radiation pressure on millions of spectral lines (Lucy & Solomon 1970; Castor et al. 1975, hereafter CAK). The second assumption arises from the fact that WR stars produce copious amounts of carbon and oxygen through self-enrichment, so that the radiation-driven wind may not depend on the initial metallicity of the host galaxy.

That WR winds could at all be driven by radiation pressure on spectral lines (as O star winds), was in doubt for many decades, as observations of WR stars showed particularly high wind momenta with values of the product of the mass-loss rate and the terminal flow velocity, $\dot{M}v_{\infty}$, 10–100 times larger than that of the momentum of the stellar radiation field, L_{*}/c – the so-called "wind-momentum problem" (e.g. Barlow et al. 1981), which in effect was an opacity problem (Gayley et al. 1995). Developments over the last years have shown that radiation pressure may be a viable mechanism for both initiating as well as driving the winds of WR stars after all.

One reason is the discovery that WR stars are highly clumped (e.g. Moffat & Robert 1994; Lepine & Moffat 1999), which has resulted in a downward revision of the observed WR mass-loss rates (e.g. Hamann & Koesterke 1998). A second reason is the realization that the ionization structure of WR winds is stratified: at progressively deeper layers, higher ionization stages become dominant, which increases the number of lines available for wind driving, whilst simultaneously line photons are allowed to be scattered more than once – increasing their effectiveness in transferring momentum to the gas (Lucy & Abbott 1993; Springmann 1994; Gayley 1995; de Koter et al. 1997). This combined effect of ionization stratification and multiple line scattering leads to predictions for the ratio of wind momentum over radiative momentum , $\eta = \dot{M}v_\infty/(L/c)$, of up to a factor of 10 – alleviating the wind momentum problem (but see Brown et al. 2004).

Last but not least, it appears that the "hot iron bump" (Fe X – Fe XIV) in the deep layers of WR stars has been identified as the initiation region of WR winds, conceptually proposed by Nugis & Lamers (2002), and most recently implemented numerically by Gräfener & Hamann (2005). These latter authors could self-consistently solve the dynamics for a carbon-rich WC5 star, thereby showing that radiation pressure is capable of initiating and maintaining the entire wind from below the photosphere to the region where the wind reaches its terminal velocity.

Now that it appears that radiation pressure is the principal driving mechanism for WR winds, we believe the time is ripe to explore the metallicity dependence of WR winds, as has previously been done for O stars (Vink et al. 2001; Kudritzki 2002). Empirically, the issue of an $\dot{M}$ (Z)-dependence for WR stars has been studied by Crowther et al. (2002). Comparing WR stars in the Galaxy and the Large Magellanic Cloud, they report a tentative $\dot{M}$(Z) dependence, however this result is not clear-cut, in part because the investigated baseline in Z is rather narrow (about a factor of two in Z).

As WR stars produce copious amounts of carbon and oxygen, one might argue (referring back to the second assumption mentioned earlier) that WR mass loss is likely to be *independent* of the initial chemical composition (which is set by the metallicity of the host galaxy), once intermediate mass elements become abundant in the outer layers. The relevant question is therefore whether the lines of intermediate elements do indeed dominate the Fe lines in their contribution to the total line force, and if so, whether this holds true for all metallicities, i.e. Fe mass fractions. Or alternatively, whether the Fe lines may take over the line driving starting at some minimum value of Z.

For O stars, the relative importance for the wind driving of intermediate mass elements (mainly CNO) versus that of heavy metals (notably Fe) as a function of Z has been studied in depth by Vink et al. (2001). These authors found that the intermediate elements of CNO, and to some extent even hydrogen and helium, gain importance in driving the winds of O stars at $Z \lesssim 1/30$ giving rise to a flattening of the O star mass-loss rate versus metallicity relation (see their Fig. 5c). Interestingly, the opposite behaviour has been found by Kudritzki (2002), who finds that $\dot{M}$ decreases even more rapidly at $Z/Z_\odot \lesssim 10^{-3}$. The cause of this discrepancy at very low Z in the $\dot{M}$(Z) behaviour between the two approaches has yet to be identified. It may be related to their treatments of the relevant aspects of the physics of line driving at different levels of sophistication. Kudritzki (2002) parameterizes the radiative force through so-called force multipliers (in an extension of CAK theory), enabling a relatively straightforward solution of the wind dynamics. Vink et al. (2001) calculate the radiative force exactly with a Monte Carlo approach that allows for a detailed treatment of multiple photon scatterings, albeit adopting a pre-described velocity law (but see Vink et al. 1999; Vink 2000). The global approach works very well for OB stars (Vink et al. 2000), and although its validity for WR stars is less well-established, we use it here, noting that the β-law model of Gräfener & Hamann (2005) shows a similar global energy budget as their self-consistent computation for the same parameters. This indicates that the global method may produce meaningful results (as for O stars). In any case, it is our philosophy to accept small errors in the mass-loss determination, as our goal is to explore a large parameter space, comprising a wide range of metallicities and mass-loss rates, and to establish the $\dot{M}(Z)$ scaling law that may already be retrieved from a differential study.

In the next two sections, we describe our method and assumptions. Our basic Wolf-Rayet wind models are presented in Sect. 4, followed by a study of their mass-loss properties as a function of metal abundance. We make a distinction between the nitrogen-rich Wolf-Rayet stars (WN), which may still contain some hydrogen, but with helium present in large quantities, and the carbon rich WR stars (WC) that have converted a significant fraction of helium into carbon, thereby representing a later evolutionary phase. We discuss the implications of our findings in Sect. 5, and summarize in Sect. 6.

2. Method to calculate $\dot{M}$ with Monte Carlo

The mass-loss rates are calculated with a Monte Carlo (MC) code that is used to follow the fate of a large number of photons from below the stellar photosphere through the wind, and that calculates the radiative acceleration of the wind material. The core of the approach is that the cumulative loss of radiative energy equals the gain in kinetic energy of the wind material (Abbott & Lucy 1985). We assume that each individual photon scatters coherently in the co-moving frame. In the observer's frame, this implies that each photon-ion interaction results in an exchange of momentum and energy (except for the case that the incident and emerging photon happen to have the same direction). As the absorptions and scatterings of photons in the wind depend on the wind density, this allows for a derivation of a globally consistent mass-loss rate – for a pre-described flow structure. Our approach is described in detail in Vink et al. (1999). We note that as the kinetic wind energy scales with $v_\infty{}^2$, the derived mass-loss rates depend on the choice of the terminal flow velocity.

The chemical species that are explicitly calculated in non-LTE are H, He, C, N, O and Si (see de Koter et al. 1997 for the details of the adopted atomic models). The iron-group elements, which are important for the radiative driving and consequently for $\dot{M}$, are treated in a generalized version of the "modified nebular approximation" (Lucy 1987, 2001, see also Schmutz 1991). We include Fe ionization stages up to IX, and we note that the first 30 elements of the periodic table are all included in the Monte Carlo computations for the line force. Although the cumulative effect of all these species on the radiative acceleration is properly included in our models, we will particularly highlight the roles of CNO, the dominant intermediate mass elements, and Fe, the dominant heavy metal.

As we wish to investigate the effect of a different chemical environment on the mass-loss rate of WR stars, we exclude the self-enriched element of carbon in WC stars, from what we call "Z". Therefore, where we address the $\dot{M} - Z$ dependence in WC stars, we mean a scaling with for instance the Fe abundance, realizing that the overall WR metal content is much larger (because of the large amounts of carbon produced by the WC stars themselves). Where we vary the Fe abundance, we also scale the other metal species (S, Cl, Ar, etc.) in accordance to their solar values. In the WN case, we assume that the CNO-cycle affects the relative abundances of C, N, and O in amounts that are similar to our study for Luminous Blue Variables (LBVs; Vink & de Koter 2002, and references therein), so that the total sum of C+N+O abundance is constant, and scales with the iron abundance.

We note that the stellar and ISM abundances are equal for elements such as Fe, but the stellar and ISM carbon abundance can be vastly different. It is therefore important to be very careful with our definition of Z. We also note that extragalactic metallicities are usually derived from nebular oxygen lines. The ISM oxygen and Fe abundances do not necessarily scale with the solar values, due to the differing roles of SNIa, which release Fe, and SN II, which release large amounts of α elements, but less Fe. [α/Fe] scalings can be adjusted using Table 5 by Vink et al. (2001).

The strengths of our approach are the natural way in which multiple line and continuum interactions are accounted for, and the consistent computation of an ionized stratification throughout the wind. We do however make assumptions, which are listed below.

2.1. The wind dynamics

The most important assumption in our modelling is that we calculate $\dot{M}$ with a pre-described velocity law. To be more precise, we adopt a β-type velocity law for the accelerating part of the wind:

$$v(r) = v_\infty \left(1 - \frac{R_*}{r}\right)^\beta. \quad (1)$$

Below the sonic point, a smooth transition from this velocity structure is made to a velocity that follows from the photospheric density structure. Vink et al. (2000) have shown that for O stars the predicted mass-loss rate is insensitive to the adopted value of β in the range of $\beta = 0.7-1.5$. Therefore, we routinely apply a value of $\beta = 1$ for O stars, which is consistent with the spectral line modelling of these stars (e.g. Puls et al. 1996), as well as with modified CAK theory, where $\beta \simeq 0.8$ (Pauldrach et al. 1986). There are observational indications that β may be significantly larger than 1 for denser winds, at least in the outer regions (e.g. Barlow & Cohen 1977; Antokhin & Cherepaschuk 2001), although this is still under debate (e.g. Ignace et al. 2003). On numerical grounds, Hillier & Miller (1999) proposed a "double" β law, with $\beta_1 = 1$ for the inner and $\beta_2 = 50$ for the outer wind, which appears to be a good approximation of the self-consistently derived velocity law for the WC5 star modelled by Gräfener & Hamann (2005). Here we do not apply a 2-step velocity law, but we do explore the difference between a "low" value of β ($\beta = 1$), and a larger value ($\beta = 3$) as to test the sensitivity of the velocity law assumption, and to investigate whether our choice of β influences the derived WR wind metallicity dependence significantly. We find that this is not the case (Sect. 4.5).

We stress that although our approach allows for a derivation of the mass loss that can be driven by the photons passing through the outer atmosphere, it does not treat the initiation and driving of the wind in the deepest layers at very large optical depth (above a Rosseland optical depth of 30). To do so properly would require (i) taking account of the backwarming of Fe at large depths (as this affects the temperature and ionization structure); (ii) a co-moving frame treatment of the lines (we adopt an improved Sobolev description appropriate for regions higher up); as well as (iii) accounting for ionization stages above stage IX (Gräfener & Hamann account for stages up to Fe XVII). Although one should be aware of this, we again refer to the modest differences in overall wind properties reported by Gräfener & Hamann between the case where the wind dynamics was solved for, and the case for which a velocity stratification was adopted.

2.2. Sphericity

We assume that WR winds are spherically symmetric. Given the absence of line depolarisations in 80% of the WR stars analysed with the technique of linear spectropolarimetry (Harries et al. 1998), sphericity is anticipated to be an appropriate assumption at Galactic Z (Note however that rotation may become more important at lower Z; see Sect. 5).

2.3. Line branching

In our modelling of the radiative force, all line interactions are assumed to be scatterings. Lucy (2002, 2003) has developed techniques that can accommodate for this potential deficiency, and Sim (2004) has investigated the effects of line branching on Monte Carlo mass-loss predictions, and found that neglecting line branching can overestimate $\dot{M}$ by up to a factor of three for WR stars. Sim's method however does not account for continuum absorption and subsequent thermal emission, which are included in our model. This implies that we have a channel to redistribute photons towards longer wavelength, away from the flux maximum, so that we cannot directly compare to these findings. Nonetheless, we anticipate that the neglect of line branching could cause an overprediction of the mass-loss rate by a factor of 1–3 at galactic Z. As the importance of line branching is expected to decrease at lower Z, we anticipate that by neglecting line branching we may slightly overestimate the exponent in the $\dot{M}$–Z relation.

2.4. Wind clumping

It has been established that small-scale clumping of the outflowing gas has a pronounced effect on the ionization structure of WR atmospheres, as it favours recombination over ionization processes (e.g. Hillier 1991). In addition, clumping may have a direct effect on the radiative driving, although these effects may be complex and challenging to predict (Schmutz 1997; Hillier 2003). In more quantitative terms, the observed emission line spectrum – formed through recombination – is essentially invariant for the ratio $\dot{M}/\sqrt{f}$, where f is the volume clumping factor describing the mean density in the flow $\overline{\rho} = f \times \rho$, where ρ is the density in the clumps. The interclump medium is assumed to be void. Typical values for the clumping factor in WR winds are found to be 0.1−0.25 (Hillier & Miller 1999; Hamann & Koesterke 1998). The ionization structure of a clumped wind will behave similarly to that of an unclumped wind with an $\sqrt{f}$ times lower mass-loss rate. There may be a dependence of the clumping factor on metallicity. If it holds true that weaker winds (e.g. O star winds) are less clumped than WR winds (e.g. Lamers & Leitherer 1993, but see Bouret et al. 2005), one might speculate that the winds of WR stars at lower Z may also be less clumped than those of Galactic WR stars.

Nevertheless, we emphasize that the neglect of clumping could effect the results presented in this paper.

2.5. Ion decoupling

In line-driven winds, the ions that are most effective in absorbing photons (i.e. those with many lines in the region of the flux maximum of the star; notably C, N, O, and Fe) redistribute the momentum they have gained – through friction – to ions that are only marginally effective line absorbers. These latter more "passive ions" include H and He, containing most of the mass. If the wind density becomes lower, the coupling between active and passive ions may disappear, potentially giving rise to multicomponent effects (e.g. Lucy & Solomon 1970; Castor et al. 1976). In O stars of Galactic metallicity, this ion-decoupling will only occur in the outermost regions of the wind, where the flow reaches its asymptotic velocity – and the mass-loss rate is not affected.

For the lower density winds at extremely low Z, ion-decoupling may become important for the mass-loss rate. It has been proposed that runaway instabilities may occur in a multi-component wind (Owocki & Puls 2002; Krtička & Kubát 2002), and possibly even fallback of the passive ions. Krtička et al. (2003) present a relation defining the mass loss – metallicity domain where multicomponent effects become relevant. Note that their diagram is intended for normal O stars. Ignoring this fact, and applying this diagram for the WR model parameters discussed in our study indicates that multicomponent effects start to play a role below a metal content of $Z/Z_{\odot} \sim 1/300$. However, two remarks are in order: first, at low Z the relative contribution of hydrogen and helium to the line force will increase (see Vink et al. 2001), therefore these elements become "less passive". At $Z = 1/300\,Z_{\odot}$ this contribution may amount to several tens of percents (∼10–30%). Second, and especially relevant for WC stars, the mass fraction contained in "passive ions" decreases with an increasing surface abundance of carbon and oxygen. For the WC model that we study here, already half of the mass in contained in carbon. Therefore, if ion-decoupling would occur, one may expect that the predictions presented in this paper overpredict $\dot{M}$ by at most a factor of two.

3. The WR parameters

This study is not intended to be a full parameter study of WR stars, but is deliberately limited to very specific choices of stellar properties. For an accurate modelling of higher temperatures, i.e. earlier spectral types, the line list is required to be complete for the higher ionization stages. As we are principally interested in the metallicity dependence, we choose to model "late" WR stars of both flavours: WNL, the late-type nitrogen-rich stars, and WCL, the late-type carbon-rich ones. Our strategy is as follows: we choose typical WN/WC stellar parameters such as luminosity and effective temperature, based on studies using the most sophisticated "standard" WR star models of Hillier & Miller (1998) and Gräfener et al. (2002), as well as characteristic abundances, known from spectral synthesis modelling, and evolutionary models. We then vary the metal abundance Z, such that we may study the effects of a different host galaxy environment on $\dot{M}$.

In reality, both stellar structure and evolution are expected to be Z dependent as well, however we do not take this into account at this stage (but see Sect. 5). The crucial point here is that if Fe dominates the mass loss of WR stars, mass loss is anticipated to be Z-dependent, and the precise chemical enrichment will be less relevant. However, if it turns out that lighter elements, such as the intermediate mass element carbon set the mass loss for WR stars, mass loss is not expected to be dependent on the chemical environment after all.

Although we do not attempt to reproduce a most realistic WR wind model per se, we do aim to construct a representative model. This is no trivial task for the optically thick winds of WR stars. In particular, the effective temperatures of WR stars are not well defined, since the radius is strongly wavelength dependent (e.g. Schmutz 1991). Nugis & Lamers (2000, 2002) argue that the only meaningful temperature, i.e. one that is independent on the wind strength, is the one that corresponds to the hydrostatic core, as computed in stellar evolution codes. Hence, Nugis & Lamers (2000) list effective core temperatures, termed $T_{\rm evol}$, well in excess of 100 000 K. Their luminosities are of the order of respectively $\log(L/L_{\odot}) \simeq 5.8$ for WN stars, and $\log(L/L_{\odot}) \simeq 5.3$ for WC stars.

Due to the large geometrical extent of Wolf-Rayet atmospheres the effective temperature cannot be unambiguously defined. Typically, two effective temperatures play a role, see e.g. Dessart et al. (2000) for WC stars, and Herald et al. (2001) for WN stars. The first is based on the total flux at the inner boundary radius R_* of the model, and is used to define the stellar luminosity $L_* = 4\pi R_*^2 \sigma T_*^4$. We note that recent studies incorporating a self-consistent treatment of line blanketing, such as for instance that of Herald et al. (2001) for the WN8 star WR 40, tend towards smaller core radii and larger T_*. This has led to a present-day luminosity of $\log(L/L_{\odot}) = 5.6$ for this object. The second temperature that is frequently used is the effective temperature at the radius where the Rosseland optical depth equals 2/3. In our studies, we define the stellar radius $R_{\rm eff}$ and effective temperature $T_{\rm eff}$ slightly differently, namely at the point where the thermalization optical depth measured in the center of the photometric V band (at 5555 Å) equals $1/\sqrt{3}$ (see Schmutz et al. 1990 and de Koter et al. 1996 for detailed discussions). We note that these two definitions are only marginally different.

If the mass-loss rate for Wolf-Rayet stars is a function of metal abundance, the stellar radius may be expected to depend on Z as well, because it is the wind strength that determines the location of $R_{\rm eff}$. This adds a complication to the investigation of the mass-loss dependence of Wolf-Rayet stars. Recent WC spectral analyses including line-blanketing show values for T_* in the range 57–110 kK (Dessart et al. 2000; Crowther et al. 2002; Gräfener et al. 2002) – i.e. in some case well below the expected $T_{\rm evol}$ – and large geometrical extensions of WC atmospheres. If we adopt a constant T_* and R_* for all Z models, we may find that we are comparing models with widely different $T_{\rm eff}$. If, on the other hand, we adopt a constant $T_{\rm eff}$ (by adjusting R_*) for all models, we might ignore the fact that at low metallicity such relatively cool stars may not actually exist.

In this study, we opt to keep T_* constant for both the WN and WC stars. We find that although the predicted mass-loss rates for Galactic metal content are of order 10^{-5} $M_\odot\,{\rm yr}^{-1}$, this does not lead to the formation of a truly extended optically thick wind. Therefore, in our computations the effective temperature will not increase strongly if we lower the metal abundance. We do not claim that the effective stellar radius is not affected by a change in metal content in real WR stars; it merely reflects the fact that an adjustment of the star in the sub-photospheric region due to a change in Z is not treated in our models.

3.1. The WN and WC model parameters

For our typical WN star, we choose the following stellar parameters (see Table 1): $T_* = 40\,000$ K, and $\log(L/L_\odot) = 5.62$. The stellar mass of Wolf-Rayet stars cannot be obtained from spectral synthesis modelling, and we have therefore adopted a value of $M = 20$ $M_\odot$ based on the compilation of Nugis & Lamers (2000) for late WN stars. The terminal wind velocity we adopt is $v_\infty = 840$ km s^{-1}, as found by Herald et al. (2001) for the WN8 star WR 40. The abundances were taken from Table 9 in Herald et al. (2001). Note the relatively high nitrogen abundance, as typical for WN stars of course, and the high helium abundance, $Y = 0.83$ (by mass). As helium is not *directly* relevant for the line driving at galactic metallicities (although this changes at lower Z; Vink 2000), it may well be the case that the mass-loss metallicity dependence for these stars is similar to that of O stars.

We first note that WC stars are believed to be He burning stars, theoretically expected to lie on the He-main sequence with large surface temperatures (in excess of 100 000 K). Observationally, significantly lower effective temperatures are seen, presumably due to large amounts of mass loss at Galactic metallicity. We have chosen the following stellar parameters for our WC model: $T_* = 40\,000$ K, and $\log(L/L_\odot) = 5.36$. The adopted mass is $M = 10$ $M_\odot$, again based on the compilation of Nugis & Lamers (2000) for late WC stars, and in line with the expectation that a later evolutionary phase results in a smaller mass, as a result of mass loss. The terminal wind velocity we adopt is $v_\infty = 1200$ km s^{-1}, as found by Dessart et al. (2000) for the WC8 star WR 135. Nugis & Lamers (2000) found $N_{\rm C}/N_{\rm O} \simeq 5$ for all WC subtypes, but a $N_{\rm C}/N_{\rm He}$ that is spectral-type dependent, ranging between 0.18–0.36. These abundance ratios are by number. Based on the above, it is clear that oxygen is not expected to be an important line driver, however carbon should become crucial at very low iron abundance. We adopt a ratio $N_{\rm C}/N_{\rm He}$ of 1/3, which corresponds to an equal mass contribution of these two elements. Such a high ratio maximizes the potential effect of carbon line driving, compared to that of Fe. (If we would take a lower ratio, carbon would start to dominate at a lower Z value.)

Table 1. Adopted parameters for our typical Galactic WN and WC stars. The Galactic WN model is roughly representative of the WN8 star WR 40; the typical WC model resembles the WC8 star WR 135. The relevant abundances are given by mass ratio.

Parameter	WN	WC
T_* (K)	40 000	40 000
$\log(L/L_\odot)$	5.62	5.36
M ($M_\odot$)	20	10
v_∞ (km s^{-1})	840	1200
X	0.15	–
Y	0.83	0.5
C	–	0.5
$\log \dot{M}$ ($M_\odot\,{\rm yr}^{-1}$)	−4.89	−5.34

Note that all these parameters can be adjusted to obtain a better agreement between our mass-loss predictions and the currently favoured observed rates. In particular, an increase in the carbon over helium ratio, a decrease in the stellar mass, an increase in the star's luminosity, a decrease in its terminal flow velocity, or a more rapid wind acceleration, may all cause an increase of the mass-loss rate. The purpose of this study is, however, not to establish the optimum conditions for wind driving, but to create WR models with reasonable parameters, and to investigate the Z effect on the mass-loss rate.

4. The predicted mass-loss rates

4.1. The typical WN model

We have used the method by Lucy & Abbott (1993), as shown in their Fig. 1, to search for the mass-loss rate. This mass-loss determination method yields rates that are globally consistent, and has been used for OB supergiants (Vink et al. 1999) and LBVs (Vink & de Koter 2002). Our typical WN model yields a mass-loss rate of $\log \dot{M}(M_\odot\,{\rm yr}^{-1}) = -4.89$. This number is in fair agreement with observations. Clumping-corrected mass-loss rates for WN stars give values in the range $\log \dot{M} = -4.4$ to -5.0 (e.g. Nugis & Lamers 2000; Hamann & Koesterke 2000). The predicted value is thus representative for the lower

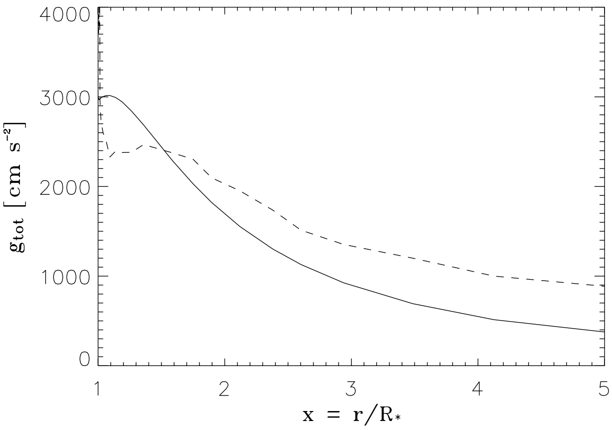

Fig. 1. The computed radiative acceleration (dashed line), and the one implied by the adopted $\beta = 1$ velocity law (solid line) versus radius – for the Galactic WN model. The cross-over point, i.e. where the model changes from an underprediction in the inner wind to an overprediction of the line acceleration in the outer wind, occurs at about 1.5 R_*.

range of observed mass loss (i.e. corresponding to weak lined WN stars).

As we have adopted a wind velocity law, and only obtained a globally consistent mass-loss rate, the system is not necessarily dynamically consistent. Although we do not aim to derive a model that is locally consistent in the transfer of bulk momentum from the photons to the gas, we may still test the quality of our velocity law by comparing the radiative acceleration that we compute to that implied by the adopted velocity law via the equation of motion. Figure 1 shows the computed radiative acceleration by the dashed line, and the one imposed by the velocity law by the solid line. Both show a similar qualitative behaviour with radius, but there are systematic differences. Our models underpredict the line force in the inner wind, and overpredict it in the outer wind. This is similar to what was found by Lucy & Abbott (1993) and Schmutz (1997). The likely reasons for these discrepancies have been listed in Sect. 2.1 (i.e. no back-warming due to Fe; no iron ionization stages above IX, no co-moving frame treatment). We restate that Gräfener & Hamann (2005) found only a modest difference between the predicted mass loss based on a self-consistent treatment and the one found using a pre-described velocity structure.

4.2. The mass-loss metallicity dependence for WN winds

Using our WN model as a starting point, we systematically vary the metallicity. The results are given in the second column of Table 2, and are plotted in Fig. 2. The figure shows an $\dot{M}$ versus Z dependence that is more or less linear between $Z/Z_\odot = 10^{-3}$–1. It flattens off for supersolar values of Z, because the iron lines – present throughout the spectral region of the bulk flux – become saturated. Phrased differently, the spectrum "runs out of continuum windows between saturated spectral lines" through which photons could escape (see Vink et al. 2001 who describe the effect for OB stars). The mass loss – metallicity dependence also flattens off for the lowest Z models at $Z/Z_\odot \lesssim 10^{-3}$. The reason is also similar to that of

Table 2. Predicted WR mass-loss rates for different metallicities for both WN and WC stars. The Galactic WN model is roughly representative of the WN8 star WR 40, the typical WC model resembles the WC8 star WR 135.

$Z/Z_\odot$	WN		WC	
	$\beta = 1$	$\beta = 3$	$\beta = 1$	$\beta = 3$
10	−4.47	−4.68	−4.93	−5.05
3	−4.64	−4.82	−5.14	−5.30
1	−4.89	−5.11	−5.34	−5.50
1/3	−5.26	−5.49	−5.62	−5.77
1/10	−5.75	−5.94	−5.99	−6.11
1/33	−6.22	−6.44	−6.22	−6.34
1/100	−6.71	−6.85	−6.39	−6.52
1/333	−7.01	−7.16	−6.57	−6.67
10^{-3}	−7.40	−7.40	−6.74	−6.77
10^{-4}	−7.91	−7.62	−6.83	−6.85
10^{-5}	−7.98	−7.67	−6.85	−6.86

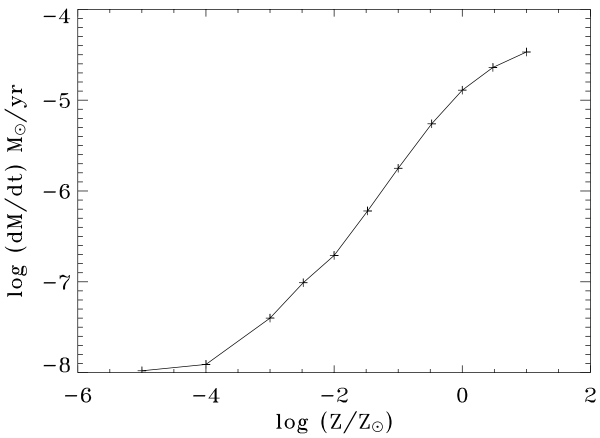

Fig. 2. The $\dot{M}(Z)$ dependence for WN stars for the case $\beta = 1$. The relation shows a power-law behaviour in the range $10^{-3} \lesssim Z/Z_\odot \lesssim 1$ with an index $m = 0.86$. At higher/lower metal abundances, the relation appears flatter (see text for explanations).

OB stars: CNO, and to some extent even hydrogen and helium (especially at extremely low Z), take over the role that Fe plays in the line driving at Galactic Z (Vink et al. 1999; Puls et al. 2000). Over the range $Z/Z_\odot = 10^{-3}$–1, the mass loss vs. metal content dependence behaves as a power-law, i.e.

$$\log \dot{M} \propto m \log(Z/Z_\odot), \quad (2)$$

with index $m = 0.86$. The power-law dependence for the WN star is similar to that of OB stars, as the line driving in both cases is primarily due to Fe lines. We note that this result is based on only one set of stellar parameters. Also, the fitting errors may be smaller than systematic errors associated with our modelling.

Our mass-loss computations have been performed with a constant wind velocity, and thus, the Z dependence actually applies to the wind *momentum*, $\dot{M}v_\infty$. In reality, v_∞ may also be Z-dependent, which would imply that the power-law index $m = 0.86$ could be an upper limit. There are no CAK-type scaling relations available for WR winds. Observationally, the dependence of v_∞ on Z is also unknown. For OB stars, index values p for a $v_\infty(Z)$ dependence given by $v_\infty = Z^p$ of the order of

0.10–0.20 have been noted (e.g. Garmany & Conti 1985; Prinja 1987; Walborn et al. 1995). Evans et al. (2004) have recently reported that there appears to be no significant difference in v_∞ between B supergiants in the Galaxy and in the Magellanic Clouds. So, even for OB stars there is no clear-cut observational evidence for a $v_\infty(Z)$ dependence. And if there is one, it is likely only a modest one.

Turning from observations to expectations, one would perhaps anticipate there to be a $v_\infty(Z)$ dependence, as the global line force is lower at lower Z, resulting in lower $\dot{M}$ and lower v_∞. Also in CAK theory the value of the force multiplier α is lower, hence v_∞ is expected to be lower (Kudritzki & Puls 2000; Puls et al. 2000). Therefore, OB wind theory predicts values of p of the order of 0.13 (e.g. Leitherer 1992). However, if the mass-loss rate is indeed lower at low metallicity there may be a *second*, less well-documented, effect that operates in the opposite direction. To some extent, the line driving is expected to *increase* in the outer wind – yielding a *larger* v_∞ – as a result of the line acceleration being inversely proportional to the density in case of optically thick lines (at very low Z optically thick lines due to CNO may gain in relative importance over the contribution of optically thin iron lines.) At Galactic Z, this second argument is also at work: it operates in B1 supergiants in relation to the bi-stability jump (Vink et al. 1999), as well as for the weak winds of B main-sequence stars, for which Babel (1996) finds particularly high terminal wind velocities.

It is clear that the $v_\infty - Z$ dependence is still relatively unexplored territory requiring future study, both observationally and theoretically. We conclude that the power-law indices m we derive for the mass-loss rate – at constant v_∞ – are likely upper limits, but that the actual values are expected to be rather close to these numbers.

4.3. The typical WC model

We now turn to carbon-rich stars. The predicted mass-loss rate for our typical WC model is found to be $\log \dot{M} = -5.34$. A recent analysis of WC stars by Dessart et al. (2000) report values in the range $-4.5 \lesssim \log \dot{M} \lesssim -5.1$. Crowther et al. (2002) find for a WC star of comparable luminosity a mass loss $\log \dot{M} = -4.7$ and a wind clumping factor $f = 0.1$. So, we probably underpredict the mass loss by about a factor two to four.

We again test the quality of our velocity law by comparing the radiative acceleration that we compute with the Monte Carlo model to that implied from the adopted velocity law via the equation of motion. Figure 3 shows the computed radiative acceleration by the dashed line, and that imposed by the velocity law by the solid line. Both show a similar qualitative behaviour with radius, but we underpredict the line force in the inner wind, and overpredict it in the outer wind.

4.4. The mass-loss metallicity dependence for WC winds

Keeping all stellar parameters fixed we now systematically vary the metal content of the WC star. The results are given in the third column of Table 2. Figure 4 shows the $\dot{M}$ versus Z dependence for $\beta = 1$. The dependence is more complex than in the case of WN stars, as several linear parts in this log–log plot can be identified. The linear part with the steepest slope occurs at $Z/Z_\odot = 1/10$–1. The power-law index of this part of the $\dot{M}(Z)$ function (see Eq. (2)) is $m = 0.66$.

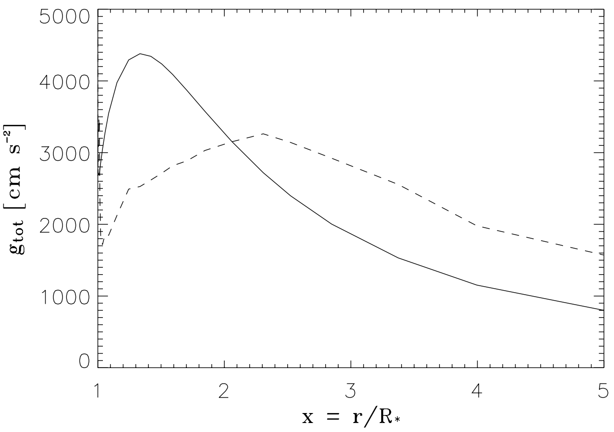

Fig. 3. The computed radiative acceleration (dashed line), and the one imposed by the adopted $\beta = 1$ velocity law (solid line) versus the wind radius – for the Galactic WC model. Note that the cross-over point, where the model changes from an underprediction in the inner wind to an overprediction of the line acceleration in the outer wind, occurs at about 2 R_*.

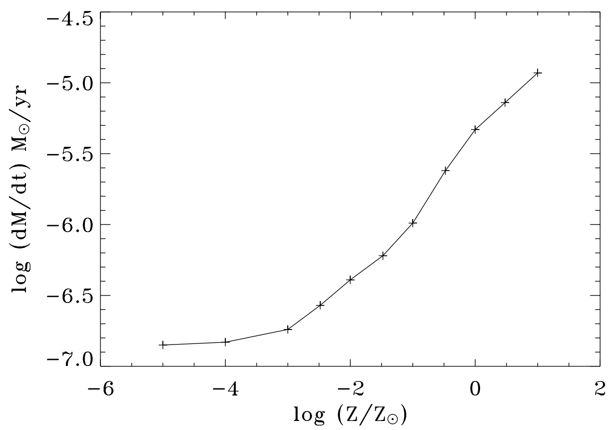

Fig. 4. The $\dot{M}(Z)$ dependence for WC stars for the case $\beta = 1$. The relationship can be devided into four linear segments. Describing the mass loss vs. metallicity relation as a power-law, i.e. $\log \dot{M} \propto m \log Z$, we find $m = 0.40$ for $(Z/Z_\odot) \gtrsim 1$; $m = 0.66$ for $10^{-1} \lesssim (Z/Z_\odot) \lesssim 1$; $m = 0.35$ for $10^{-3} \lesssim (Z/Z_\odot) \lesssim 10^{-1}$. Note that at $Z \lesssim 10^{-3}\,Z_\odot$ the mass loss becomes roughly constant. At such low metal content the wind driving is dominated by radiation pressure on carbon lines.

At higher metal content the dependence flattens ($m = 0.40$), caused by the same line saturation effect as described in Sect. 4.2 for WN stars. At metal abundances below 1/10 of the solar value, the dependence also flattens ($m = 0.35$), relative to the behaviour at Small Magellanic Cloud to Galactic abundances. A further flattening occurs at $Z \simeq 10^{-3}$ – essentially forming a plateau where mass loss no longer decreases with Z. Even though the luminosity of the WC stars is about a factor two less than that for the WN star, the predicted mass-loss rates in this very low Z regime is more than an order of magnitude

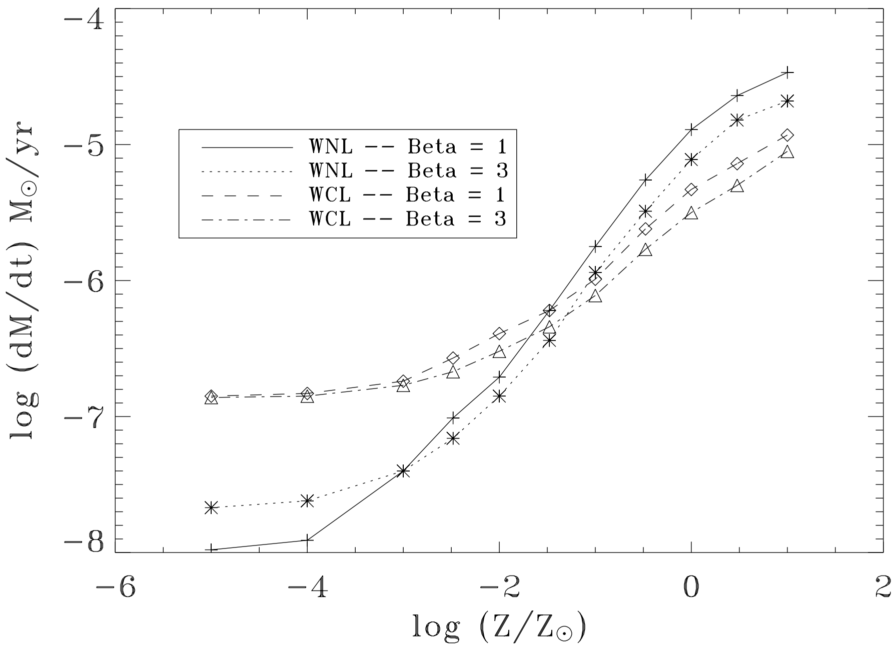

Fig. 5. The Mass-loss versus Z dependencies for both our WN and WC models, for velocity law exponent parameters β = 1 and 3. The crosses are for WN stars with β = 1, the stars for WN and β = 3. The open diamonds represent WC stars with β = 1, and the open triangles indicate WC models with β = 3. The predictions, both in terms of the absolute mass-loss as well as in the slope m of the $\dot{M}(Z)$ relation do not depend strongly on the adopted value of β.

larger. The reason for the tendency towards constant mass loss at $Z \simeq 10^{-3}$ is the ever increasing dominance of driving by spectral lines of carbon, responsible for almost half of the mass in the atmosphere.

4.5. Varying the β exponent of the velocity law

So far we have discussed WN and WC models with a β = 1 velocity law. As mentioned before, there are indications that the velocity exponent may in fact be less steep for the outer wind, whilst β = 1 may be a proper representation for the inner wind. We wish to explore whether our choice of β influences the derived WR wind metallicity dependencies. We have therefore repeated the computations of Sects. 4.2 and 4.4 for a β = 3 velocity law. The results are given in the third and fifth column of Table 2, and are plotted in Fig. 5. Although there are notable changes in the results for both the WN and WC stars between the β =1 and β = 3 velocity exponents, the general shapes are very similar, and the differences in log $\dot{M}$ of the individual models are only of the order of 0.1–0.2 dex or less. Differences in the slope m of the mass loss – metallicity relations amount to only 0.01−0.04 dex. This suggests that our results are not too sensitive to the adopted velocity law which, while reassuring, it does not remove the need for future hydrodynamic modelling.

5. Discussion and implications

Before we consider the implications of our metallicity dependent mass-loss rates for WR stars, we first relate the mass-loss rates derived for WR stars in this study to previous results for O supergiants (Vink et al. 2000). We compare the Galactic WN and WC stars to O stars having the same luminosity and effective temperature. The mass of the O stars is derived from the evolutionary tracks of Meynet et al. (1994), yielding 43.5 and 35.4 $M_\odot$ for $\log(L/L_\odot) = 5.62$ and 5.36 respectively. We find mass-loss rates $\log \dot{M}\,(M_\odot\,\mathrm{yr}^{-1}) = -5.73$ and −6.14 for the O-type counterparts of the WN and WC stars, i.e. the mass-loss rates of the WR stars are a factor of 6−7 higher. Our WR model properties differ in essentially three aspects relative to O stars: they have a much lower mass; they have a different chemical composition; and their terminal flow velocities are reduced. At a Galactic metal content, the main effect causing the difference is the reduced mass (see also Vink & de Koter 2002). The results presented in this paper show that chemical processing effects become important at very low metallicity.

At low Z, the Wolf-Rayet models show a flattening of the $\dot{M}(Z)$ relation. The limiting (minimum) mass-loss rate for the WC model is $\sim 10^{-7} M_\odot\,\mathrm{yr}^{-1}$; for the WN model it is an order of magnitude less. Given that for the WC model already half of the mass resides in carbon, we do not expect the minimum Z at which our $\dot{M}(Z)$ relation flattens to be significantly different in the case of WO stars, although these objects have significantly larger terminal velocities than we have modelled. Note that the predicted mass loss in the WC phase is not typical for only one initial mass, but for all $M_{\mathrm{init}} \gtrsim 40\,M_\odot$, as Wolf-Rayet properties in the WC phase are similar for all massive stars (see e.g. Maeder & Meynet 1994).

Given that the lifetimes of massive stars are of order 10^7 yr and that mass loss tends to increase with stellar age, we anticipate that the total mass that is lost by a massive star at $Z \lesssim 10^{-3}$ due to a continuous outflow caused by radiation pressure on spectral lines is only of the order of one $M_\odot$.

This result immediately leads to the question whether WR stars will actually form at very low Z, as the formation of a

He-rich star requires the removal of its H-rich envelope. It has sometimes been assumed that this may not occur at all within the context of single-star evolution, due to a lower mass loss at lower Z, and that the formation of WR stars at very low metal content always requires a (close) binary scenario. Perhaps surprisingly, searches for, and statistics of binary WR stars in the low Z environments of the Magellanic Clouds by Foellmi et al. (2003a,b) seem to indicate a lack of binaries. Note that so far we have discussed the formation of WR stars in an evolutionary sense. The defining observational characteristic for He-rich stars to be called WR stars is the dominance of *broad emission lines* in the spectrum. This type of spectrum is the result of line formation in a dense, trans-sonic outflow. The fact that we find the enriched stars to have a lower wind strength at lower Z implies that at metallicities lower than $Z/Z_{\odot} \lesssim 10^{-2}$ the stars would not be expected to be spectroscopically identified as WR stars (if the only mechanism contributing to the outflow of matter is a stationary line-driven wind).

Additional mechanisms may contribute to the loss of mass from massive stars at low metal content, such as stellar winds and mass ejections due to super-Eddington luminosities, LBV-type mass ejections, pulsations and/or rapid rotation, and one may speculate that this may lead to the formation of He-rich WR stars, which e.g. may well be required to form at low Z for the popular collapsor model (Woosley 1993) for long-duration GRBs to work. These additional physical effects may play a role in either *causing mass loss at low Z* or in *enhancing the mass loss by line driving*. During the LBV phase massive stars are thought to experience violent mass ejections (see e.g. Humphreys & Davidson 1994 for a review). The mechanism causing these outbursts, in which up to tens of solar masses may be ejected (Voors et al. 2000a,b; Smith et al. 2003), is not understood. Therefore, it is unknown whether this mechanism may also occur at extremely low Z, and if so, what amounts of mass can be expelled. The possible effect of pulsations on mass loss of massive stars at very low metallicity have been studied by Baraffe et al. (2001), and are found to be much weaker compared to those at solar metallicity.

In recent years, much effort has been directed towards understanding the role of rotation in stellar evolution. The role of rotation for stellar winds may be considerable, as is for instance quite dramatically illustrated by the wind shape of η Carina (van Boekel et al. 2003), which may be caused by extra radiation pressure from the stellar pole due to gravity-darkening and stellar oblateness of a rotating star (e.g. Pelupessy et al. 2000; Dwardakas & Owocki 2002). The significance for massive star evolution at very low Z is that stellar rotation is anticipated to be much faster (e.g. Meynet & Maeder 2002), which may lead to both a higher mass-loss rate (e.g. Friend & Abbott 1986; Langer 1998) as well as more efficient mixing (e.g. Maeder et al. 2004).

If our predictions of the absolute mass-loss rates in the WR phase of very low Z stars are taken at face value ($\dot{M} \sim 10^{-7}\,M_{\odot}\,\mathrm{yr}^{-1}$), one may be inclined to conclude that the effect of mass loss is too limited to significantly affect the evolution of massive stars in the early Universe, and that winds would only modestly contribute to the input of energy, momentum, and products of nucleosynthesis into the local ISM relative to supernovae. However, if mass loss by line driving occurs in conjunction with one or more of the physical phenomena discussed above (such as rotation of the star close to the Omega-Eddington limit), *it may be critical in causing a much larger mass loss*. Recent numerical simulations suggest that stars in the early Universe at low Z were more massive than in the present-day Universe (e.g. Bromm et al. 1999; Abel et al. 2002), so that these stars were more luminous (e.g. Schaerer 2002; Marigo et al. 2003), and closer to the Eddington limit. Therefore, our key finding that the mass loss flattens off at low Z may indicate a role for mass loss in determining the properties and evolution of massive stars in the early Universe, contrary to assumptions made in many cosmological studies where the neglect of mass loss is primarily based on CAK-type wind models for O stars (e.g. Salvaterra & Ferrara 2003; Madau et al. 2004).

6. Summary and conclusions

We have performed a pilot study of Monte Carlo mass loss computations for late-type Wolf-Rayet (WR) stars as a function of metal abundance, over the range between $10^{-5} \leq (Z/Z_{\odot}) \leq 10$. Although our predictions take the effects of multiple scatterings and ionization stratifications into account, we have made assumptions that might affect the quantitative outcomes of our study, such as the exponents of the $\dot{M} - Z$ dependence, as well as the ranges over which these exponents are valid. We have found that the winds of nitrogen-rich Wolf-Rayet stars are dominated by a myriad of Fe lines, so that the dependence of mass loss on Z is similar to that of OB stars. We find that the wind momentum scales as $Z^{0.86}$ in the range $10^{-2} \leq Z/Z_{\odot} \leq 1$.

For more evolved WR stars, where carbon is an abundant species, the winds are still found to be dependent on the Fe abundance, and hence they depend on the chemical environment of the host galaxy, but with a mass loss versus metallicity dependence that is less steep than that of their nitrogen-rich counterparts. At lower metallicities, this dependence however becomes much weaker, and it no longer declines anymore once the metal abundance drops below $(Z/Z_{\odot}) \simeq 10^{-3}$. To be more precise, we find that the wind momentum scales as $Z^{0.66}$ for the range between $1/10 \lesssim Z/Z_{\odot} \lesssim 1$. At lower Z, it becomes approximately $Z^{0.35}$, and mass loss no longer declines below $(Z/Z_{\odot}) \simeq 10^{-3}$.

Our finding that WR mass loss is Z-dependent is a new one, with important consequences for black hole formation and X-ray population studies in external galaxies (e.g. Soria et al. 2005), as well as the prediction of ionizing radiation from young starburst clusters and galaxies. Finally, we note our finding that mass loss no longer declines once the metal abundance drops below $(Z/Z_{\odot}) \simeq 10^{-3}$, owing to an increased importance of radiative driving by intermediate mass elements, such as carbon. In combination with rapid rotation and/or proximity to the Eddington limit – likely to be relevant for massive Population III stars – this effect may indicate a role for mass loss in the appearance and evolution of these objects, as well as a potential role for stellar winds in enriching the intergalactic medium of the early Universe.

Acknowledgements. We thank the referee, Goetz Gräfener, for constructive comments that have helped to improve this paper. J.S.V. is supported by PPARC.

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