

## Can pulsating PG 1159 stars place constraints on the occurrence of core overshooting?

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**Abstract.** The present letter is aimed at exploring the influence of overshooting during the central helium burning in pre-white dwarf progenitors on the pulsational properties of PG 1159 stars. To this end we follow the complete evolution an intermediate-mass white dwarf progenitor from the zero age main sequence through the thermally pulsing and born-again phases to the domain of the PG 1159 stars. Our results suggest that the presence of mode-trapping features in the period spacings of these hot pulsating stars could result from structure in the carbon-oxygen core. We find in particular that in order to get enough core structure consistent with observational demands, the occurrence of overshoot episodes during the central helium burning is needed. This conclusion is valid for thick helium envelopes like those predicted by our detailed evolutionary calculations. If the envelope thickness were substantially smaller, then the occurrence of core overshooting would be more difficult to disentangle from the effects related to the envelope transition zones.

**Key words.** stars: evolution – stars: interiors – stars: white dwarfs – stars: oscillations – stars: convection

### 1. Introduction

Convective overshooting is a longstanding problem in the theory of stellar structure and evolution. It is well known on theoretical grounds that during many stages in their lives stars experience overshoot episodes, that is partial mixing beyond the formally convective boundaries as predicted by the Schwarzschild criterium of convective stability (Zahn 1991; Canuto 1992; Freytag et al. 1996; see also Renzini 1987). In particular, core overshooting taking place during central burning is an important issue because it has significant effects on the stellar structure and evolution. Over the years, considerable observational effort has been devoted to demonstrating the occurrence of core overshooting. Indeed, confrontation of stellar models with a wide variety of observational data suggests that convective overshoot takes place in real stars (see Stothers & Chin 1992; Alongi et al. 1993; Kozhurina-Platais et al. 1997; Herwig et al. 1997; von Hippel & Gilmore 2000, among others).

However, most of the evidence about the occurrence of core overshooting relies primarily on observational data from the very outer layers of stars from where radiation emerges. A more promising and direct way of placing constraints on the physical processes occurring in the very deep interior of stars is by means of the study of their pulsational properties. Pulsating

white dwarf stars are particularly important in this regard. In fact, white dwarfs constitute the end product of stellar evolution for the vast majority of stars, and the study of their oscillation spectrum through asteroseismological techniques has become a powerful tool for probing the otherwise inaccessible inner regions of these stars (Bradley 1998; Metcalfe et al. 2002; Metcalfe 2003). White dwarf asteroseismology has also opened the door to peer into the physical processes that lead to the formation of these stars. In particular, Straniero et al. (2003) have raised the issue of using pulsating white dwarfs to constrain the efficiency of extra mixing episodes in the core of the white dwarf progenitors.

This letter is aimed at specifically assessing the feasibility of employing pulsating PG 1159 stars to demonstrate the occurrence of core overshoot during the core helium burning phase. Pulsating PG 1159 stars (or variable GW Virginis) are very hot hydrogen-deficient post-AGB stars with surface layers rich in helium, carbon and oxygen that exhibit *g*-mode luminosity variations. These stars are thought to have experienced a very late helium-shell flash during their early cooling phase after hydrogen burning has almost ceased – a born-again episode; see Fujimoto (1977), Schönberner (1979). As shown by Kawaler & Bradley (1994), variable PG 1159 stars are particularly important to infer fundamental properties about pre-white dwarfs in general, such as the location of chemical interfaces and envelope masses. Specifically, we present an adiabatic pulsation study based on detailed stellar models,

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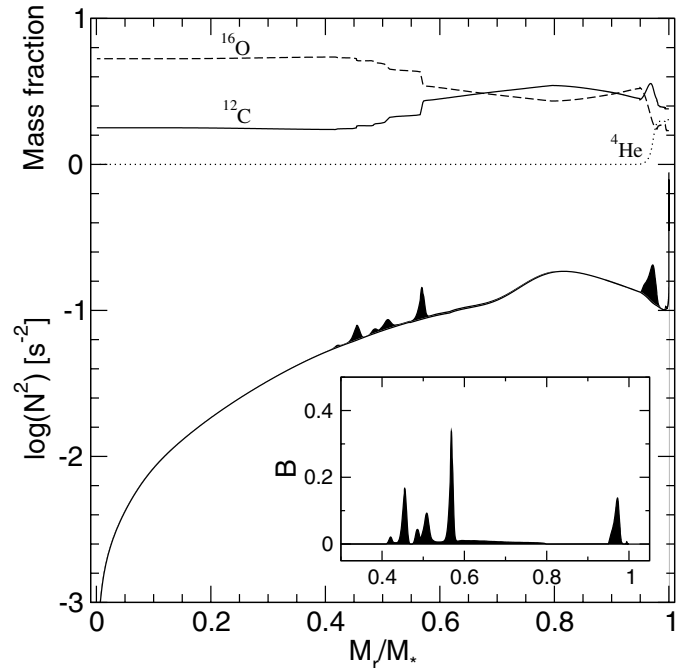
the evolution of which has been followed from the zero-age main sequence through the thermally pulsing phase and born-again episode to the PG 1159 state. This fact allows us to obtain PG 1159 models with a physically sound internal structure consistent with the predictions of the theory of stellar evolution.

## 2. Details of evolutionary and pulsational computations

The adiabatic pulsational analysis presented in this work are based on evolutionary stellar models that take into account the complete evolution of the progenitor star (see Althaus et al. 2005, for details)<sup>1</sup>. Specifically, the evolution of an initially  $2.7 M_{\odot}$  stellar model from the zero-age main sequence has been followed all the way from the stages of hydrogen and helium burning in the core up to the tip of the AGB where helium thermal pulses occur. After experiencing 10 thermal pulses, the progenitor departs from the AGB and evolves toward high effective temperatures, where a final thermal pulse takes place soon after the early white dwarf cooling phase is reached – a very late thermal pulse and the ensuing born-again episode, see Blöcker (2001) for a review. During this episode, most of the residual hydrogen envelope is engulfed by the helium-flash convection zone and completely burnt. After the occurrence of a double-loop in the Hertzsprung-Russell diagram, the now hydrogen-deficient, quiescent helium-burning  $0.5895 M_{\odot}$  remnant evolves at constant luminosity to the domain of PG 1159 stars with a surface chemical composition rich in helium, carbon and oxygen: ( ${}^4\text{He}$ ,  ${}^{12}\text{C}$ ,  ${}^{16}\text{O}$ ) = (0.306, 0.376, 0.228). This is in good agreement with surface abundance patterns observed in pulsating PG 1159 stars (Dreizler & Heber 1998; Werner 2001). Also, the surface nitrogen abundance (about 0.01 by mass) predicted by our models is in line with that detected in pulsating PG 1159 stars (see Dreizler & Heber 1998). We mention that abundances changes are described by means of a time-dependent scheme that simultaneously treats nuclear evolution and mixing processes due to convection and overshooting. A treatment of this kind is particularly necessary during the extremely short-lived phase of the born-again episode, for which the assumption of instantaneous mixing is inadequate. Overshooting is treated as an exponentially decaying diffusion process and has been considered during all evolutionary phases. In particular, overshooting occurring toward the end of core helium burning phases yields a sharp variation of the chemical composition in the carbon/oxygen core. Radiative opacities are those of OPAL (including carbon- and oxygen-rich compositions, Iglesias & Rogers 1996), complemented, at low temperatures, with the molecular opacities from Alexander & Ferguson (1994).

As for pulsational calculations, we have computed adiabatic  $g$ -mode periods by employing an updated version of the pulsation code described in Córscico et al. (2001). We limited our calculations to the degrees  $\ell = 1, 2$  because the periods observed in pulsating PG 1159 stars have been identified

<sup>1</sup> The stellar models have also recently been employed in Gautschy et al. (2005) for nonadiabatic pulsation studies of PG 1159 stars.

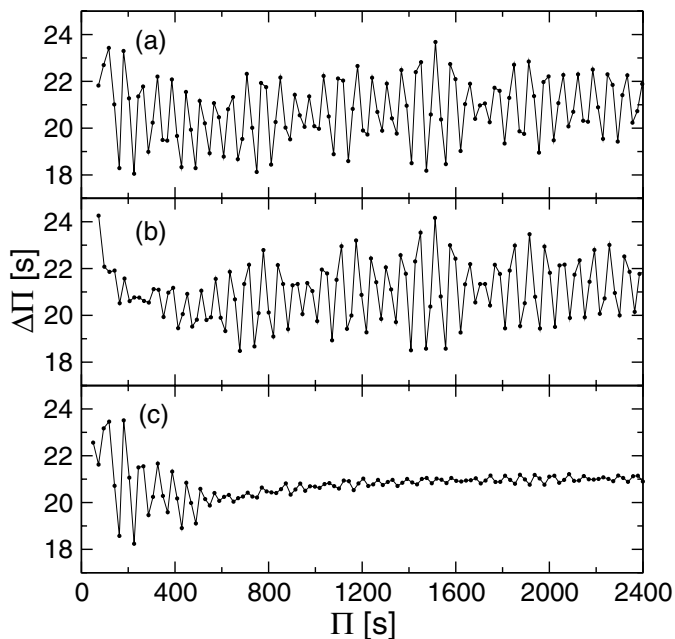


**Fig. 1.** The run of the squared Brunt-Väisälä frequency in terms of the mass coordinate, corresponding to a  $0.5895 M_{\odot}$  PG 1159 model at  $T_{\text{eff}} = 139\,000$  K and  $\log(L/L_{\odot}) = 2.31$ . The mass of the helium content is of  $0.0052 M_{\odot}$ . Dark regions denote the contributions of the Ledoux term  $B$  (shown in the inset) to the Brunt-Väisälä frequency. The chemical profile of the main nuclear species is displayed in the upper zone of the plot.

with  $\ell = 1, 2$ . To get values of periods as precise as possible, we have employed about 2700–3000 mesh-points to describe our background stellar models. The prescription we follow to assess the run of the Brunt-Väisälä frequency ( $N$ ) is the so-called “Ledoux Modified” treatment (see Tassoul et al. 1990), appropriately generalized to include the effects of having several nuclear species with varying abundance in a given region. In this numerical treatment the contribution to  $N$  from any change in composition is almost completely contained in the Ledoux term  $B$ ; this fact renders the method particularly useful to infer the relative weight that each chemical transition region have on the mode-trapping properties of the model.

## 3. Results and discussion

We begin by examining Fig. 1 in which a representative spatial run of the Brunt-Väisälä frequency of a PG 1159 model is displayed. The model is characterized by a stellar mass of  $0.5895 M_{\odot}$ , an effective temperature of  $\approx 139\,000$  K and a luminosity of  $\log(L/L_{\odot}) = 2.31$ . In addition, the plot shows the internal chemical stratification of the model for the main nuclear species (upper region of the plot), and for illustrative purposes the profile of the Ledoux term  $B$  (inset). The figure emphasizes the role of the chemical interfaces on the shape of the Brunt-Väisälä frequency. At the core region there are several peaks at  $M_r/M_* \approx 0.4$ – $0.6$  resulting from steep variations in the inner oxygen/carbon profile. The stepped shape of the carbon and oxygen abundance distribution within the core is



**Fig. 2.** The forward period spacing  $\Delta\Pi$  vs. period  $\Pi$  for dipole ( $\ell = 1$ ) modes for the same PG 1159 model analyzed in Fig. 1. Panel **a**) corresponds to pulsational calculations in which the Ledoux term  $B$  is computed self-consistently, whereas panel **b**) corresponds to the situation in which the Ledoux term has been artificially suppressed in the region at  $M_r/M_* \approx 0.96$ , and panel **c**) correspond to the case in which  $B = 0$  at  $M_r/M_* \approx 0.4-0.6$ . See text for details.

typical for situations in which extra mixing episodes beyond the fully convective core during central helium burning are allowed (Straniero et al. 2003). In particular, the sharp variation around  $M_r \approx 0.56 M_*$  induced by mechanical overshoot could be a potential source of mode trapping in the core region. The bump in  $N$  at  $M_r \approx 0.96 M_*$  is other possible source of mode trapping, in this case associated with modes trapped in the outer layers. This feature is caused by the chemical transition of helium, carbon and oxygen resulting from nuclear processing in prior evolutionary stages.

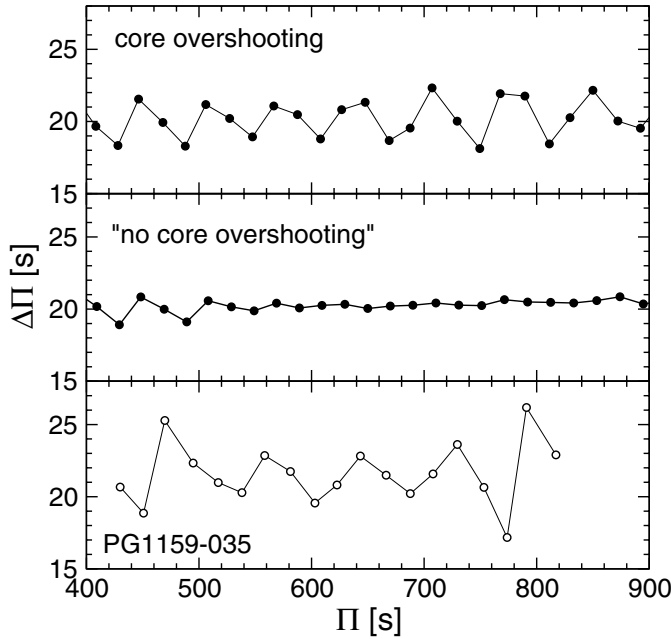
The influence of the chemical composition gradients on the pulsation pattern is clearly shown in panel (a) of Fig. 2, in which the  $\ell = 1$  forward period spacing ( $\Delta\Pi$ ) is plotted in terms of the periods ( $\Pi$ ) for the same model analyzed in Fig. 1. The plot shows very rapid variations in  $\Delta\Pi$  everywhere in the period spectrum, with trapping amplitude (measured as the interval in  $\Delta\Pi$  between a period spacing maximum and the adjacent minimum) up to about 6 s and a trapping cycle (measured as the interval in  $\Pi$  between the period spacing minima) of  $\approx 70$  s. The rather complex period-spacing diagram shown by Fig. 2 is typical of models characterized by several chemical interfaces. In order to disentangle the effect of each chemical composition gradient on mode trapping, we follow the procedure of Charpinet et al. (2000). Specifically, we minimize – although no completely eliminate – the effects of a given chemical interface simply by forcing the Ledoux term  $B$  to be zero in the specific region of the star in which such interface is located. In this way, the resulting mode trapping will be only due to the remainder chemical interfaces. Specifically,

we have recomputed the entire  $g$ -mode period spectrum assuming (1)  $B = 0$  at the region of the O/C/He chemical interface ( $M_r/M_* \approx 0.96$ ), and (2)  $B = 0$  at the region of O/C chemical interface ( $M_r/M_* \approx 0.4-0.6$ ) (see inset of Fig. 1). The results are shown in panels (b) and (c) of Fig. 2, respectively. By comparing the different cases illustrated, an important conclusion emerges from this figure: the chemical transition region at  $M_r \approx 0.96 M_*$  is responsible for the non-uniformities in  $\Delta\Pi$  only for  $\Pi \lesssim 500$  s (panel c), whereas the chemical composition gradients in the core region ( $M_r \approx 0.4-0.6 M_*$ ) cause the mode-trapping structure in the rest of the period spectrum (panel b).

From the above discussion, it is clear that *the mode-trapping features predicted by full evolutionary PG 1159 models are dominated by the core chemical structure left by prior overshoot episodes. This is particularly true for the range of periods observed in GW Vir stars.* On its hand, the more external chemical transition has a minor influence, except in the regime of short periods. This finding is clearly at odds with previous results reported by Kawaler & Bradley (1994). Indeed, these authors have found that the mode-trapping properties of their PG 1159 models are fixed mainly by the outer O/C/He transition region, to such a degree that they have been able to employ mode-trapping signatures as a sensitive locator of this transition region. We note that our PG 1159 stellar models differ considerably from those employed by Kawaler & Bradley (1994), particularly concerning the details of the treatment of the evolutionary stages that lead to the formation of PG 1159 stars. Of particular interest is the presence of a much less pronounced chemical transition in the C/O core of the Kawaler & Bradley (1994) models, as compared with the rather abrupt overshoot-induced chemical gradients at  $M_r \approx 0.4-0.6 M_*$  in our full PG 1159 evolutionary models. In addition, we note that our evolutionary treatment predict thick helium envelopes. Thus, trapping structure predicted by our models is due mostly to the core chemical gradients. If we artificially minimize the effect of these gradients we immediately recover the results of Kawaler & Bradley (1994) for the case of thick helium envelopes.

In addition to the issue of core overshooting, other relevant point regarding the core chemical stratification of our models are the adopted reaction rates during the central helium burning phase. Following a suggestion of an anonymous referee, we have explored the possibility of a smaller rate of the  $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$  nuclear reaction (see Kunz et al. 2002). We found that with a lower rate of that reaction, the resulting central abundances of oxygen and carbon are quite similar, partially smoothing the chemical steps in the core. However, appreciable structure remains that is still able to give clear pulsational signals associated to the occurrence of prior core overshooting.

In Fig. 3 we compare the predictions of our calculations with the pulsational spectrum of PG 1159-035, the prototype of GW Vir stars. We do not intend to perform here a detailed asteroseismological fit to this star. Instead, we want to show that the structure associated with the observed period spacing of PG 1159-035 could be reflecting the presence of chemical gradients in its inner core, a fact that is borne out by Fig. 3. In fact, we find that according to the predictions of the theory of



**Fig. 3.** The forward period spacing  $\Delta\Pi$  vs. period  $\Pi$  for dipole ( $\ell = 1$ ) modes for the same model analyzed in Fig. 2. Upper panel corresponds to pulsational computations in which the Ledoux term  $B$  is computed self-consistently (core overshooting), whereas middle panel correspond to the case in which  $B = 0$  at  $M_c/M_* \approx 0.4-0.6$  ("no core overshooting"). The observed period spacings for PG 1159-035 are plotted in lower panel. See text for details.

post-AGB evolution for the chemical stratification for the pulsating PG 1159 stars, the occurrence of core overshoot episodes during central helium burning is required to be consistent with seismological demands of these stars.

We judge that the period-spacing distribution exhibited by PG 1159 stars bears the signature of core overshoot episodes and that improved observations would eventually turn these pulsating pre-white dwarfs into a powerful tool for shedding new lights on the mixing and central burning processes occurred in the white dwarf progenitors of intermediate masses. However, it is not unconceivable that stellar winds during the PG 1159 stage could reduce the helium content considerably,

with the consequent result that the trapping features induced by core overshooting could not be disentangled from that caused by the O/C/He transition zone.

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