On the “injection problem” at the solar wind termination shock

R. Kallenbach¹, M. Hilchenbach², S. V. Chalov³, J. A. le Roux⁴, and K. Bamert⁵

¹ International Space Science Institute, Hallerstrasse 6, 3012 Bern, Switzerland
e-mail: kallenbach@soho.unibe.ch

² Max Planck Institute for Solar System Research, Postfach 20, 37191 Katlenburg-Lindau, Germany
e-mail: hilchenbach@mpae.mpg.de

³ Institute for Problems in Mechanics of the Russian Academy of Sciences, Prospect Vernadskogo 101-1,
117526 Moscow, Russia
e-mail: chalov@ipmnet.ru

⁴ Institute of Geophysics and Planetary Physics, University of California, Riverside, CA 92521, USA
e-mail: Jakobus@citrus.ucr.edu

⁵ Institut für Experimentelle und Angewandte Physik, University of Kiel, Leibnizstrasse 19, Kiel 24098, Germany
e-mail: bamert@physik.uni-kiel.de

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Abstract. This article presents an integrated analytical model on the injection efficiencies of the different ion species of the Anomalous component of the Cosmic Rays (ACRs) at the solar wind termination shock. We find that the injection into diffusive (first-order Fermi) acceleration is dominated by parallel ion diffusion and not by perpendicular diffusion unless the angle Ψ between the shock normal and the heliospheric magnetic field is almost exactly 90° ($89.3^\circ < \Psi \approx 90^\circ$). In steady state the threshold speed for injection into first-order Fermi acceleration at a not exactly perpendicular solar wind termination shock – with the Parker shock angle $\Psi \approx 89.3^\circ$ – adjusts itself self-consistently. Increased anisotropic ACR flux amplifies Alfvénic turbulence which in turn suppresses parallel diffusion. It therefore increases the injection threshold and decreases the ACR flux until equilibrium is reached. For this equilibrium situation, we estimate the injection efficiencies of different species of suprathermal ions at the termination shock. We consider the following pre-acceleration processes: 1) momentum diffusion in compressional (ion-acoustic and magnetosonic) turbulence in the upstream supersonic solar wind and adiabatic cooling during convection to the termination shock; 2) reflection, transmission, and acceleration in the electric potential of the termination shock; and 3) momentum diffusion (stochastic or second-order Fermi acceleration) in the subsonic solar wind downstream of the termination shock in the inner heliosheath region. Our model results are compared to data from instruments on board the SOHO, ACE, Ulysses, and Voyager spacecraft.

Key words. solar wind – plasmas – acceleration of particles – turbulence – shock waves – solar system: general

1. Introduction

Anomalous Cosmic Rays (ACRs) are thought to originate from suprathermal pick-up ions derived from the interstellar gas (Fisk et al. 1974), ionized in the heliosphere and convected to the solar wind termination shock where they are accelerated (Pesses et al. 1981). The energy threshold for the injection of suprathermal ions into diffusive acceleration at the solar wind termination shock has been estimated to be of order 100 (Zank et al. 2001) to 1000 keV/amu (Jokipii 1992). Ions in the energy range above this threshold can be observed in-situ near the termination shock using the LECP instrument on board the Voyager 1 spacecraft (Krimigis et al. 2003; McDonald et al. 2003; Fisk 2003). Suprathermal ions in the energy range from 1 to 100 or 1000 keV/amu i.e. in the energy range below the commonly presumed injection threshold may be studied by “remote” imaging of the energetic neutral atoms (ENAs) that originate from the heliospheric interface region and reach Earth’s orbit on undistorted trajectories. In the energy range 58–88 keV/amu ENAs have been observed with the SOHO/CELIAS/HSTOF sensor near Earth’s orbit (Hilchenbach et al. 1998). Upper limits on the ENA flux in the energy range 10–50 keV/amu have been found with the IMAGE/HENA instrument near Earth’s orbit (E. Roelof, private communication). However, ENAs may only provide observational data to verify models on the injection of pick-up ions at the termination shock if the ENA flux from the outer heliosphere is not obscured by ENAs created in the inner and middle heliosphere.

In this article, we give theoretical estimates for the suprathermal particle flux in the heliosphere inside and outside the termination shock. We describe the processes of momentum diffusion (stochastic or second-order Fermi acceleration) in compressional

Fig. 1. *Left:* quiet-time proton spectra in the CME-dominated slow solar wind and in polar coronal holes during solar activity maximum (Gloeckler 2003). The average suprathermal proton flux in the slow solar wind is about a factor 5 higher than during the quiet times and even higher in the CIR-dominated slow solar wind during solar activity minimum. Therefore, the shown suprathermal proton flux is a lower limit for the flux in the inner and middle heliosphere. Flux of suprathermal H^+ and He^+ upstream and downstream of Jupiter’s quasi-perpendicular bow shock. *Right:* comparison of suprathermal tails of H^+ at 1 AU and at about 5 AU. Adapted from Gloeckler (2003).

and Alfvénic turbulence in the supersonic solar wind, the interaction of suprathermal ions with the electric field of the termination shock, and momentum diffusion in Alfvénic turbulence of the heliosheath plasma downstream of the termination shock. The power spectral density of Alfvénic fluctuations in the heliosheath is yet unknown, but we try to estimate it from theory on turbulence transmission through shocks (McKenzie & Westphal 1969; Vainio & Schlickeiser 1998) and from the recently observationally verified efficiency of Alfvén wave amplification by suprathermal and energetic particles (Bamert et al. 2004). In a self-consistent manner, these Alfvénic fluctuations determine the injection threshold at the termination shock. We do not model the stochastic acceleration of suprathermal ions in compressional turbulence downstream of the termination shock because of the high uncertainty of its fluctuation levels.

The study of this article extends the work by Chalov et al. (2003) in so far as it includes 1) a more detailed assessment of momentum diffusion in ion-acoustic solar wind turbulence; 2) fully analytical approximations which are easy to handle; 3) an estimate of the self-consistent injection threshold at the solar wind termination shock; and 4) in particular a more detailed comparison to spacecraft data. We compare our model results with suprathermal pick-up ion flux data from ACE/SWICS at 1 AU and Ulysses/SWICS at heliocentric distances out to 5 AU (Gloeckler 2003), with ENA flux data in the energy range 58–88 keV/amu from SOHO/CELIAS/HSTOF (Hilchenbach et al. 1998), and with ion flux data at energies larger than 100 keV/amu from Voyager 1. In particular, we refer to a recent evaluation of the spectra of suprathermal pick-up ions transmitted through the quasi-perpendicular bow shock of Jupiter (Gloeckler et al. 2004). The study presented in this article should give guidelines to estimate the contributions from the regions inside and outside the termination shock to the ENA flux observed at 1 AU. This is of importance for future space missions such as the Interstellar Boundary Explorer (IBEX).

2. Pre-acceleration of suprathermal ions in the supersonic solar wind

The majority of suprathermal ions in the middle heliosphere is thought to be derived from pick-up ions of interstellar origin. Pick-up ions in the supersonic solar wind are selectively accelerated in co-rotating interaction regions (Balogh et al. 1999) and at interplanetary coronal mass ejections (Bamert et al. 2002). Even in the quiet solar wind, undisturbed by interplanetary shocks, ubiquitous suprathermal tails in the pick-up ion distributions are observed (Gloeckler 2003), (Fig. 1). These pick-up ions, pre-accelerated in the supersonic solar wind and convected to the outer heliosphere, may provide a seed population for injection into diffusive acceleration at the termination shock.

The acceleration process that creates the suprathermal tails of pick-up ions remains uncertain, however. They may be created through acceleration of pick-up ions which are multiply reflected at quasi-perpendicular shocks of co-rotating interaction regions (le Roux et al. 2000) and their successors in the outer heliosphere, or through selective acceleration at shocks driven by coronal mass ejections (Kallenbach 2002). If these shocks are responsible for the pre-acceleration of pick-up ions the process may cease during solar activity minimum at a heliocentric distance of about 15 AU, where the shocks of merged interaction regions cease to exist (Gazis et al. 1999). The process may cease even closer to the Sun during solar activity maximum because the shocks of interplanetary coronal mass ejections – which supply the majority of energetic particles during activity maximum – weaken rapidly with heliocentric distance.

The suprathermal tails may also arise from stochastic acceleration in turbulent waves. Dwyer et al. (2004) suggest that suprathermal ions predominantly originate in the compressed region between the forward and reverse shock of co-rotating interaction regions. The momentum diffusion parameter of ions trapped in compressional fluctuations of the solar wind plasma has been derived by Bykov & Toptygin (1993). Compressional fluctuations may be classified as ion-acoustic turbulence (Gurnett et al. 1979a; Mangeney et al. 1999) and magnetosonic turbulence. Schwadron et al. (1996) have studied the correlation between suprathermal ion flux and the strength of magnetic field fluctuations near co-rotating interaction regions in order to verify the model on statistical acceleration in magnetosonic wave turbulence (Fisk 1976). Stochastic acceleration also occurs in non-compressional Alfvénic solar wind turbulence. The strength of these types of turbulence relative to the energy density of the bulk plasma is predicted to persist or even to increase in the solar wind out to the region of the termination shock (Zank et al. 1996).

Any of the acceleration processes of suprathermal ions has to compete with the adiabatic cooling in the radially expanding solar wind. The dynamics of suprathermal ions convected with the solar wind plasma is described by the pitch-angle averaged Fokker-Planck equation (Chalov et al. 2003):

$$\frac{\partial f}{\partial t} + \mathbf{V}_{\text{SW}}(\mathbf{r}, t) \cdot \frac{\partial f}{\partial \mathbf{r}} = \frac{1}{v^2} \frac{\partial f}{\partial v} \left[v^2 D_{vv}(\mathbf{r}, v, t) \frac{\partial f}{\partial v} \right] + \frac{v}{3} \frac{\partial f}{\partial v} \nabla \cdot \mathbf{V}_{\text{SW}}(\mathbf{r}, t) + Q(\mathbf{r}, v, t) + S(\mathbf{r}, v, t). \quad (1)$$

The momentum diffusion parameter D_{vv} is the sum of the parameters $D_{vv;\text{A}}$, $D_{vv;\text{m}}$, and $D_{vv;\text{ia}}$ related to Alfvénic, magnetosonic and ion-acoustic turbulence. The scaling of these parameters with heliocentric distance r and particle speed v is conveniently expressed in powers of the dimensionless variables $\rho = r/r_E$ and $u = v/V_{\text{SW}}$ with $r_E = 1 \text{ AU} = 1.5 \times 10^{11} \text{ m}$. The solar wind bulk speed is assumed to be constant over heliocentric distance with a typical value of $V_{\text{SW}} \approx 440 \text{ km s}^{-1}$ corresponding to 1 keV/amu. This is an idealization which does not apply very well in the outer heliosphere, where the solar wind is slowed down by mass-loading from the interstellar gas, but the problem remains analytically treatable and gives an approximate assessment. The process of acceleration at quasi-perpendicular shocks during multiple reflections (le Roux et al. 2000) will not be described here in detail again. The resulting suprathermal population will simply be introduced into the Fokker-Planck Eq. (1) as a mean source term in the form of some power law $Q_{\text{MRI}}(\rho, u) = Q_{\text{MRI};0} \rho^{-1} u^{-\alpha}$. However, we discuss the different processes of stochastic acceleration in the solar wind in more detail.

2.1. Momentum diffusion in Alfvénic solar wind turbulence

We assume small-scale Alfvénic turbulence scaling as a power law, $\delta \tilde{B}_A^2(k) = \langle \delta B_A^2 \rangle (s-1)^{-1} (kL_A/2\pi)^{-s} L_A/2\pi$, with spectral index $s \approx 3/2 \dots 5/3$ above a minimum wavenumber $k_{\text{min}} = 2\pi/L_A$. The correlation length L_A is about 0.03 AU at a heliocentric distance $r_E = 1 \text{ AU}$ (Goldstein et al. 1995). To estimate the approximate efficiency for momentum diffusion in this Alfvénic turbulence we apply standard quasi-linear theory (Hasselmann & Wibberenz 1968). For further use in this article we introduce the parameter describing the total relative Alfvénic magnetic field fluctuations

$$\zeta_A = \frac{\langle \delta B_A^2 \rangle}{B_0^2} = \frac{2\pi (s-1)^{-1}}{L_A} \left(\frac{kL_A}{2\pi} \right)^s \frac{\delta \tilde{B}_A^2(k)}{B_0^2} \Leftrightarrow \frac{\delta \tilde{B}_A^2(k)}{B_0^2} = \frac{L_A (s-1)}{2\pi} \left(\frac{kL_A}{2\pi} \right)^{-s} \zeta_A. \quad (2)$$

The parallel scattering mean free path $\Lambda_{\parallel}$ follows from cyclotron-resonant interaction $k = \Omega/v$ with the Alfvénic fluctuations, where $\Omega = \mathcal{R}^{-1} \Omega_p$ is the angular speed of the gyro-motion of an ion with atomic mass-to-charge ratio $\mathcal{R}$. We also give the parallel spatial diffusion parameter $\kappa_{\parallel}$:

$$\Lambda_{\parallel} = \frac{3v^2}{8\pi\Omega^2} \frac{B_0^2}{\delta \tilde{B}_A^2(\Omega/v)} = \frac{3(s-1)^{-1} v^2}{4L_A \Omega^2 \zeta_A} \left(\frac{kL_A}{2\pi} \right)^s, \quad \kappa_{\parallel} = \frac{v\Lambda_{\parallel}}{3} = \frac{(s-1)^{-1} v^3}{4L_A \Omega^2 \zeta_A} \left(\frac{kL_A}{2\pi} \right)^s. \quad (3)$$

The momentum diffusion parameter $D_{vv;\text{A}}$ is, with $k = \Omega/v$,

$$D_{vv;\text{A}} = \frac{v^2 V_A^2}{9\kappa_{\parallel}} = \frac{4L_A \Omega^2 \zeta_A V_A^2 (s-1)}{9v} \left(\frac{2\pi}{kL_A} \right)^s = \frac{8\pi\Omega(s-1)}{9} \zeta_A u^{s-1} V_A^2 \left(\frac{V_{\text{SW}}}{\Omega L_A} \right)^{s-1} \quad \text{with} \quad u := \frac{v}{V_{\text{SW}}}. \quad (4)$$

This momentum diffusion parameter applies for super-Alfvénic ions, i.e. for ions with speed v' in the solar wind frame which is large compared to the Alfvén speed V_A , $v' \gg V_A$. The Alfvén speed is $V_A = [B_0^2 / (\mu_0 n_p m_p)]^{1/2}$ with n_p the proton density, m_p the proton mass, and B_0 the ambient magnetic field.

In the heliocentric frame, where ions have speed v , this momentum diffusion parameter is valid for $v^2 \gg V_{\text{SW}}^2$ or $u^2 \gg 1$. For suprathermal ions with $u \geq 2$, such as the observed ions in the suprathermal tails of the solar wind (Gloeckler 1999), the above momentum diffusion parameter can be applied. For more refined models on the diffusion parameters in Alfvénic fluctuations, we refer to Schlickeiser (2002).

The rms-amplitude of the average Alfvénic fluctuations in the solar wind $\langle \delta B_A^2 \rangle^{1/2}$ scales as $\rho^{-3/2}$ (Zank et al. 1996). The angular speed of the gyro-motion Ω scales with B_0 , which in turn scales with ρ^{-1} in the outer heliosphere. At 1 AU the proton

angular speed Ω_p is about 0.5 s^{-1} . The Alfvén speed V_A is assumed to be about 45 km s^{-1} at 1 AU and does not vary much with heliocentric distance. The correlation length L_A increases less than or about proportional to ρ (Zank et al. 1996), and most power spectra are Kolmogorov-like with spectral index $s = 5/3$.

Therefore, the parameter $D_{vv:A}$ scales approximately with ρ^{-1} . The relative magnetic turbulence power in the undisturbed slow solar wind ranges from 0.01 (Bamert et al. 2004) to 0.03 (Forsyth et al. 1996). However, only a fraction of 0.15 of the turbulence represents parallel Alfvén waves (Bieber et al. 1996) so that we take $\zeta_{A:E} \approx 3 \times 10^{-3}$ as typical value.

The momentum diffusion parameter $D_{vv:A}$ becomes dimensionless in units of the convection time scale $\tau_{\text{conv};E} = r_E V_{\text{SW}}^{-1}$ at 1 AU by multiplying it by $r_E V_{\text{SW}}^{-1}$ and dividing it by V_{SW}^2 . It then reads

$$D_A = D_{A:E} \rho^{\gamma_A} u^{\delta_A}, \quad \delta_A \approx 2/3, \quad \gamma_A \approx -1, \quad D_{A:E} = 25 \mathcal{R}^{-1/3} \zeta_{A:E}, \quad \zeta_{A:E} \approx 0.003, \quad (5)$$

where the subscript E denotes conditions at Earth’s orbit. The ratio $\mathcal{R}$ is the mass-to-charge ratio of the accelerated ion in atomic units.

2.2. Statistical acceleration in magnetosonic solar wind turbulence

The interaction of suprathermal ions with magnetosonic fluctuations differs from the interaction with Alfvénic fluctuations because magnetosonic waves are compressional and have an electric field amplitude along the ambient magnetic field. The dimensionless diffusion parameter arising from large-scale magnetosonic fluctuations, which have a correlation length of $L_{m:E} \approx 3 \text{ AU}$, is (Chalov et al. 2003):

$$D_m = D_{m:E} \rho^{-0.7} u^2, \quad D_{m:E} = \zeta_{m:E} \frac{r_E}{9 L_{m:E}} \approx 0.04 \zeta_{m:E}, \quad \zeta_{m:E} = \frac{\langle \delta V_{\text{SW};m:E}^2 \rangle^{1/2}}{V_{\text{SW}}}. \quad (6)$$

The $\rho^{-0.7}$ -dependence of D_m means that the rate of statistical acceleration in magnetosonic waves increases relatively to the rate of adiabatic cooling with heliocentric distance. This relative increase is proportional to $\rho^{0.3}$ as the adiabatic cooling scales as ρ^{-1} . For the region in the vicinity of the termination shock, the parameter D_m can be written in the form

$$D_m \approx D_{m:TS} \rho^{-1} u^2 \approx D_{m:E} \rho_{\text{TS}}^{0.3} \rho^{-1} u^2 \approx 0.08 \rho^{-1} u^2 \quad \text{for } \rho \approx \rho_{\text{TS}} \approx 85 \text{ and } \zeta_{m:E} \approx 0.5. \quad (7)$$

2.3. Stochastic acceleration in ion-acoustic solar wind turbulence

Another type of compressional fluctuations in the solar wind are ion-acoustic fluctuations. Ion-acoustic waves mainly propagate along the ambient magnetic field and have an electric field amplitude along the ambient magnetic field.

We need to distinguish two types of ion-acoustic turbulence regions: 1) regions, where the size of the turbulence region L_{turb} along the magnetic field is large compared to the parallel scattering mean free path $\lambda_{\parallel}$ and 2) regions, where $L_{\text{turb}} \ll \lambda_{\parallel}$. There is a basic difference in the momentum diffusion in these two types of regions. If $L_{\text{turb}} \gg \lambda_{\parallel}$ the bulk plasma fluid transfers momentum contained in the velocity fluctuations to the suprathermal ions. According to Bykov & Toptygin (1993) the detailed scattering mechanism that determines $\lambda_{\parallel}$ is not important, and the momentum diffusion parameter only depends on the total amplitude of the velocity fluctuations, but not on its spectral distribution. In the solar wind and near collisionless shocks, we can use the mean free path for pitch-angle scattering in Alfvénic turbulence as upper limit for the parallel mean free path to verify the condition $L_{\text{turb}} \gg \lambda_{\parallel}$.

In the case $L_{\text{turb}} \ll \lambda_{\parallel}$ the ion only gains energy in the fluctuating electric field of the compressional turbulence. We will treat the acceleration due to the Landau resonance between ions and ion-acoustic waves. The power spectral density of the electric field fluctuations then enters the momentum diffusion parameter.

2.3.1. Large regions of ion-acoustic turbulence

For the situation that suprathermal ions are trapped in the region of ion-acoustic turbulence, i.e., $\lambda_{\parallel} \ll L_{\text{turb}}$, the momentum diffusion in ion-acoustic fluctuations also scales with u^2 as in the case of magnetosonic fluctuations (le Roux 2004):

$$D_{ia} = D_{ia:E} \rho^{-1} u^2, \quad D_{ia:E} = \zeta_{ia:E} \frac{2r_E}{5L_{ia:E}}, \quad \zeta_{ia:E} = \frac{\langle \delta V_{\text{SW};ia:E}^2 \rangle^{1/2}}{V_{\text{SW}}}. \quad (8)$$

Fairly large regions of ion-acoustic turbulence have been observed in the downstream regions of interplanetary shocks at frequencies of a few kHz i.e. in the range of the proton plasma frequency (Gurnett et al. 1979b). The e-folding lengths for the electric field turbulence has been found to range between $1.6 \times 10^7 \text{ m}$ and $2 \times 10^9 \text{ m}$. For such large turbulence regions the condition $\lambda_{\parallel} \ll L_{\text{turb}}$ may be valid. Mean free paths of protons at 60 keV upstream of interplanetary shocks range at 10^9 m (Bamert et al. 2004). In the downstream region, the mean free paths could be smaller by more than a factor 10 and even smaller for suprathermal protons at energies below 60 keV.

The dispersion relation of ion-acoustic waves is

$$\omega \approx c_{se} k \left(\frac{1 + 3T_p/T_e}{1 + k^2 \lambda_D^2} \right)^{1/2} = \xi_T c_{se} k (1 + k^2 \lambda_D^2)^{-1/2}, \quad \xi_T = \sqrt{1 + 3T_p/T_e}, \quad c_{se}^2 = \frac{\gamma_e k_B T_e}{m_p}, \quad \lambda_D^2 = \frac{\epsilon_0 k_B T_e}{n_e e^2}, \quad (9)$$

where the electron temperature is $T_e \approx 10^5$ K at 1 AU, the Debye length is $\lambda_D \approx 15$ m, the polytropic index of the solar wind electrons is $\gamma_e \approx 1.5$, and the electron density is of order $n_e \approx 3 \times 10^6 \text{ m}^{-3}$. The condition $3T_p \approx T_e$ has often been observed in connection with ion-acoustic waves in the solar wind (Gurnett et al. 1979a). The speed of ion-acoustic waves is about $c_{se} \approx 3 \times 10^4 \text{ m/s}$.

The power spectral densities in these ion-acoustic turbulence regions have been found to be as large as $\delta \tilde{E}^2 \approx 10^{-13} \text{ V}^2 \text{ m}^{-2} \text{ Hz}^{-1}$ at 1 AU and at a frequency of about 3 kHz; at smaller heliocentric distance the wave intensity has been found to be larger, further out it is smaller. We estimate the rms-amplitude of the speed fluctuations $\langle \delta V_{SW;ia;E}^2 \rangle^{1/2}$ from the power spectral density $\delta \tilde{E}^2(f)$:

$$\langle \delta V_{SW;ia;E}^2 \rangle = \int_{f_{\min}}^{f_{\max}} \delta \tilde{V}_{SW;ia}^2(f) df = \int_{f_{\min}}^{f_{\max}} \frac{e^2 \delta \tilde{E}^2(f)}{m_p^2 4\pi^2 f^2} df \approx \frac{e^2 \delta \tilde{E}^2(\bar{f}) \Delta f}{4\pi^2 (s+1) m_p^2 \bar{f}^2}$$

with $\Delta f = \frac{f_{\max} - f_{\min}}{2}$ and $\bar{f} = \frac{f_{\max} + f_{\min}}{2}$. (10)

The above expression is valid for a rather peak-shaped power spectral density as observed. The peak-shaped structure of the power spectral density comes from the fact that the ion-acoustic waves are limited in frequency to about the proton plasma frequency at the upper frequencies and by strong damping at the low frequencies. From standard plasma physics we can estimate the damping length as a function of frequency. The wavelength $\lambda_{eq} = 2\pi/k_{eq}$, for which the damping length L_{eq} equals the wavelength, follows from the condition that the damping rate γ_{ia} equals the frequency ω_{ia} of the ion-acoustic wave. Using the standard expression for the damping of ion-acoustic waves at $T_e \gg T_p$, the condition $\gamma_{eq;ia} \approx -\omega_{eq;ia}$ yields

$$\frac{\gamma_{eq;ia}}{\omega_{eq;ia}} \approx -\sqrt{\frac{\pi}{2}} \sqrt{\frac{m_e}{m_p}} \frac{1}{\lambda_D^3 k_{eq}^3} \approx -1 \Rightarrow \lambda_{eq} \approx 2\sqrt{2\pi} \sqrt{\frac{m_p}{m_e}} \lambda_D \Rightarrow \lambda_{eq} \approx 200 \lambda_D \approx 3000 \text{ m}. \quad (11)$$

The corresponding frequency of the ion-acoustic wave is about 10 Hz. The damping length $L_{10 \text{ Hz}} \approx \lambda_{eq} \approx 3000 \text{ m}$ is rather short. As the damping length scales approximately with the square of the wave frequency, it is of order $L_{3 \text{ kHz}} \approx 3 \times 10^8 \text{ m}$ at 3 kHz. This is a value that approximately corresponds to the observed size of the turbulence region downstream of the shocks (Gurnett et al. 1979b). We do not know, whether there is a non-linear process such as cascading that redistributes the wave power among the frequencies between 10 Hz and a few kHz, and how strong this process is. Observations suggest that we can take $\Delta f \approx 1 \text{ kHz}$ and $\bar{f} \approx 3 \text{ kHz}$. The correlation length may approximately correspond to the damping length at 3 kHz, i.e. $L_{ia;E} \approx 3 \times 10^8 \text{ m}$.

From Eq. (8) the momentum diffusion parameter is

$$D_{ia;E} \approx \frac{er_E}{10\pi^2 m_p V_{SW} L_{ia;E}} \sqrt{\frac{\delta \tilde{E}^2(\bar{f}) \Delta f}{\bar{f}^2}} \approx 12 \sqrt{\delta E_{13}^2}, \quad (12)$$

where the dimensionless parameter δE_{13}^2 denotes the power spectral density at 3 kHz divided by $10^{-13} \text{ V}^2 \text{ m}^{-2} \text{ Hz}^{-1}$.

The above values only apply for the strongest ion-acoustic waves near interplanetary shocks. At solar activity maximum, these shocks are mainly CME-driven shocks. If a few CMEs per day with a typical size of the turbulence region $L_{turb} \approx 3 \times 10^{-3} \text{ AU}$ pass the heliosphere, the time-averaged power spectral density of the ion-acoustic turbulence may be about 3×10^{-2} of that observed downstream of CME-driven shocks. This still would yield $D_{ia;E} \approx 0.3$.

Ion-acoustic activity may also be present in the ambient solar wind. The over-all power spectral density of ion-acoustic fluctuations in the heliosphere may drop inversely with heliocentric distance, however, this is not well known, in particular it is unknown for the outer heliosphere. As baseline we take for the average momentum diffusion coefficient in ion-acoustic turbulence with $\lambda_D \ll L_{turb}$ the value

$$D_{uu;ia} = D_{ia;E} \rho^{-1} u^2 \quad \text{with} \quad D_{ia;E} = 0.3. \quad (13)$$

This estimate is very uncertain and needs observational verification.

2.3.2. Small ion-acoustic wave structures

In the quiet slow solar wind, ion-acoustic turbulence may also be present in the form of weak double layers (Salem et al. 2003). Although these structures are subject to further investigation, we model the potential efficiency of stochastic ion acceleration in their electric field fluctuations. Their typical net potential $\delta\phi$ is reported to be about 1 mV, and they are observed as spikes that

last about 1 ms and typically occur at a rate of 1 s^{-1} at 1 AU. This gives a rough estimate for their spatial extension $L_{\text{turb}} \approx V_{\text{SW}} \times 1 \text{ ms} \approx 500 \text{ m} \approx 30\lambda_D$ (Mangeney et al. 1999), which in fact is smaller than the value for the maximum wavelength $\lambda_{\text{max}} \approx 200\lambda_D$ mentioned above. For such small values of L_{turb} the condition $\lambda_{\parallel} \ll L_{\text{turb}}$ is unlikely to apply. Therefore, another approach than that of le Roux (2004) needs to be taken. The ion is not energized by transfer of momentum from the bulk fluid i.e. by the velocity fluctuations but by the electrostatic field fluctuations in a narrow bandwidth given by the Landau resonance.

In quasi-linear theory the momentum diffusion parameter in electrostatic field fluctuations is calculated as (Treumann & Baumjohann 1997)

$$D_{v'v'} = \frac{\pi e^2}{\mathcal{R}^2 m_p^2} \int \delta \tilde{E}^2(\omega = kv'\mu) d\mu, \quad (14)$$

with v' the ion speed in the solar wind frame. The normalized integral over the cosine of the pitch angle μ takes into account that the Landau resonance condition can be fulfilled for many different speeds v' . The resonance condition $\omega = kv'\mu$ and the dispersion relation (9) constrains k as a function of v' and μ to

$$k = \lambda_D^{-1} \sqrt{\frac{\xi^2 c_{se}^2}{\mu^2 v'^2} - 1} = \lambda_D^{-1} \sqrt{x^{-1} - 1} \quad \text{with} \quad x = \frac{(\mu v - \mu_{\text{SW}} V_{\text{SW}})^2}{\xi_T^2 c_{se}^2}; \quad x_{\text{min/max}} = (\lambda_D^2 k_{\text{max/min}}^2 + 1)^{-2}. \quad (15)$$

We already have made the transformation to the heliocentric reference system. The integral over the pitch-angles will then be performed with the variable x . The minimum wave number follows from $L_{\text{turb}} \approx 30\lambda_D$ to $k_{\text{min}} \approx 0.06\lambda_D^{-1}$ and thus $x_{\text{max}} \approx 0.993$. The maximum wave number is not much above λ_D^{-1} according to the full dispersion relations of the ion-acoustic waves in kinetic theory, but even with $k_{\text{max}} = 3\lambda_D^{-1}$ we obtain $x_{\text{min}} \approx 0.01$ which will turn out to be less important in the evaluation of the integral over x .

The power spectral density $\delta \tilde{E}^2(\omega)$ of the electrostatic fluctuations of the ion-acoustic waves is up to $\delta \tilde{E}_0^2 f_{\text{kHz}}^{-3}$ with $\delta \tilde{E}_0^2 \approx 10^{-12} \text{ V}^2 \text{ m}^{-2} \text{ Hz}^{-1}$ and $f_{\text{kHz}} = \omega/(2\pi \text{ kHz})$ in average over the spikes (Lacombe et al. 2002). Typical mean power spectral densities in the slow solar wind during solar activity minimum are $\delta \tilde{E}^2(f = 1 \text{ kHz}) \approx 10^{-18} \text{ V}^2 \text{ m}^{-2} \text{ Hz}^{-1}$ (Issautier et al. 1999). In our evaluation we allow for a general power law $\delta \tilde{E}^2(f_{\text{kHz}}) = \delta \tilde{E}_0^2 f_{\text{kHz}}^{-s}$ with spectral index s . Then, the diffusion parameter is

$$D_{v'v'} = \frac{\pi e^2}{\mathcal{R}^2 m_p^2} \int_{\mu_{\text{min}}}^{\mu_{\text{max}}} \delta \tilde{E}_0^2 (2\pi \text{ kHz})^s (kv'\mu)^s d\mu = \frac{\pi e^2 \delta \tilde{E}_0^2}{\mathcal{R}^2 m_p^2} \left(\frac{\lambda_D 2\pi \text{ kHz}}{\xi_T c_{se}} \right)^s \frac{\xi_T c_{se}}{2v} \int_{x_{\text{min}}}^{x_{\text{max}}} \frac{dx}{\sqrt{x}} (1-x)^{-s/2} = \frac{\pi e^2 \delta \tilde{E}_0^2}{\mathcal{R}^2 m_p^2} \mathfrak{N}^s \left(\frac{\xi_T c_{se}}{V_{\text{SW}}} \right) u^{-1} I_x, \quad (16)$$

$$\mathfrak{N} = \frac{2\pi \text{ kHz} \lambda_D}{\xi_T c_{se}}, \quad I_x > \frac{1}{2} \int_0^{x_{\text{max}}} dx (1-x)^{-s/2} \approx \frac{(1-x_{\text{max}})^{(2-s)/2}}{s-2}.$$

With $s \approx 3$ and $V_{\text{SW}} = 440 \text{ km s}^{-1}$ we obtain $I_x \approx 10$, $\mathfrak{N}^3 \approx 10$, and $\xi_T c_{se}/V_{\text{SW}} \approx 0.1$. Multiplying by $r_E V_{\text{SW}}^{-2} = 0.77 \text{ s}^2 \text{ m}^{-1}$ and taking the average $\delta \tilde{E}^2(f = 1 \text{ kHz}) \approx 10^{-18} \text{ V}^2 \text{ m}^{-2} \text{ Hz}^{-1}$ (Issautier et al. 1999) gives

$$D_{uu;W} \approx D_{W;E} \rho^{-1} u^{-1}, \quad D_{W;E} \approx 0.2 \mathcal{R}^{-2}, \quad (17)$$

where it has been assumed that the power spectral density of the electrostatic fluctuations scales as ρ^{-1} . The latter assumption is based on the suggestion that the weak double layers represent the interplanetary potential of the solar wind in a spiky form (Lacombe et al. 2002).

2.4. Solutions of the Fokker-Planck equation

Using the above diffusion coefficients, we first solve the homogeneous part of the above equation i.e. with $Q = 0$ and $S = 0$ and then add a source $Q_{\text{PUI}}(\mathbf{r}, v)$ of freshly ionized pick-up ions or a source $Q_{\text{MRI}}(\mathbf{r}, v)$ of pick-up ions pre-accelerated at quasi-perpendicular shocks during multiple reflections. We first assume $D_{\text{ia}} \propto u^2$ due to turbulence regions which are larger than the mean free path for pitch-angle scattering. Then, we add $D_W \propto u^{-2}$ to the problem with D_W resulting from e.g. weak double layers which are smaller than the mean free path for pitch-angle scattering.

1. First, the homogeneous part of the transport equation is solved. We begin with the case that the momentum diffusion parameter D_{uu} scales with u^2 and inverse heliocentric distance ρ^{-1} , $D = D_{2;E} \rho^{-1} u^2$, $D_{2;E} = D_{\text{ia};E} + D_{\text{m};\text{TS}}$. Any scaling law of D_2 close to ρ^{-1} may be approximated over some range of heliocentric distance, e.g. close to the termination shock, as it was done for the scaling of the large-scale magnetosonic fluctuations. We then obtain a homogeneous solution f_{hom} :

$$-\frac{\partial f}{\partial \rho} + \frac{1}{\rho} \frac{2u}{3} \frac{\partial f}{\partial u} + \frac{D_{2;E}}{\rho} \frac{1}{u^2} \frac{\partial}{\partial u} \left[u^4 \frac{\partial f}{\partial u} \right] = 0 \quad \Rightarrow \quad f_{\text{hom}}(u, \rho) = f_0 \rho^{-\beta} u^{-\alpha} \quad \text{with} \quad \beta = \frac{2}{3} \alpha - \alpha(\alpha - 3) D_{2;E}$$

$$\text{or} \quad \alpha = \left(\frac{3}{2} + \frac{1}{3D_{2;E}} \right) \times \left(1 + \sqrt{1 - \frac{4\beta}{4 + 9D_{2;E} + 4/(9D_{2;E})}} \right). \quad (18)$$

In the range $D_{2,E} \approx 0.01 \dots 3$ the spectral index α can be approximated by

$$\alpha \approx 1.8 \left(\frac{3}{2} + \frac{1}{3D_{2,E}} \right) \quad (19)$$

within an error of about 5%. For fairly small values of $D_{2,E}$ the spectral index of the suprathermal tails is in the range $\alpha \approx 5 \dots 6$ that agrees with observations (Gloeckler 2003). For a momentum diffusion parameter $D_{2,E} = D_{ia,E} + D_{m,E} \approx 0.2$ and $\beta = 1$ the spectral index is $\alpha \approx 5.4$. A momentum diffusion parameter $D_{2,E} \approx 0.2$ would approximately correspond to the values expected from observations on magnetosonic and ion-acoustic turbulence, although more detailed observations are required.

2. Second, we solve the transport equation including the sources. The local source of freshly ionized interstellar hydrogen atoms scales as ρ^{-2} outside the ionization cavity around the Sun extending out to about 7.5 AU. For interstellar helium atoms, the ρ^{-2} scaling is valid further inward, in particular in the upwind direction of the interstellar medium, i.e. $Q(u, \rho) = \rho^{-2} q(u)$ for $\rho > 1$. The inhomogeneous solution f_{inhom} then scales as ρ^{-1} i.e. $\beta = 1$.

If the distribution function is factorized as $f_{\text{inhom}} = f_0 \rho^{-1} u^{-\alpha} g_{\text{inj}}(u)$, then we obtain a differential equation for $g_{\text{inj}}(u)$:

$$\begin{aligned} \left[(2\alpha - 4) D_{2,E} - \frac{2}{3} \right] \frac{dg_{\text{inj}}}{du} - D_{2,E} u \frac{d^2 g_{\text{inj}}}{du^2} &= q(u) u^{\alpha-1} \Rightarrow \\ \left[(2\alpha - 3) D_{2,E} - \frac{2}{3} \right] g_{\text{inj}}(u) - D_{2,E} u \frac{dg_{\text{inj}}(u)}{du} &= \int_0^u q(u') u'^{\alpha-1} du' \quad \text{if } g_{\text{inj}}(0) = 0. \end{aligned} \quad (20)$$

The source function is assumed to represent a sphere with radius $u_{\text{sp}} = 1$ around the solar wind speed $U_{\text{sw}} = 1$, $q(u') = q_{\text{shell}}(u') = u' \sqrt{1 - u'^2/4}$ for $u' \leq 2$ and $q(u') = 0$ for $u' > 2$. We use this shell distribution as the source function rather than the standard distribution of pick-up ions convected in the solar wind by Vasyliunas & Siscoe (1976). The shell distribution of freshly ionized pick-up ions extends to $u = 2$ with higher phase space density than the adiabatically cooled convected pick-up ion distribution and, therefore, is better suited for injection into stochastic acceleration.

The above integral over the source function is limited to the range $0 < u' < 2$. This means that for $u > 2$ the injection function is a constant $g_{\text{inj};2} = g_{\text{inj}}(u > 2)$. It follows then from the above differential equation that

$$g_{\text{inj};2} \approx \left[(2\alpha - 3) D_{2,E} - \frac{2}{3} \right]^{-1} \int_0^2 q(u') u'^{\alpha-1} du'. \quad (21)$$

An alternative explanation for the observed spectral index $\alpha \approx 5 \dots 6$ of suprathermal tails is the acceleration of ions in the motional electric field of quasi-perpendicular shocks during multiple reflections (le Roux et al. 2000). If this source is in average

$$Q(\rho, u) = Q_{\text{MRI}} \rho^{-2} u^{-\alpha}, \quad (22)$$

then the suprathermal tails adopt the same spectral index α and their phase space density $f = f_0 \rho^{-1} u^{-\alpha}$ is related to the source strength by

$$f_0 = Q_0 [2\alpha/3 - \alpha(\alpha - 3) D_{2,E} - 1]^{-1}. \quad (23)$$

However, if the quasi-perpendicular shocks cease outside a heliocentric distance of about 15 AU (Gazis et al. 1999), and if there is no strong wave activity outside 15 AU, the transport equation is dominated by convection and adiabatic cooling. Therefore, the heliocentric scaling of the distribution function $f \propto \rho^{-\beta} u^{-\alpha'}$ has a power-law index of $\beta = 2\alpha'/3 \approx 3$. Between 15 AU and 85 AU, the phase space density would decrease by an additional factor 30 compared to the ρ^{-1} scaling. Voyager 1 observations (Krimigis et al. 2003) indicate a decrease by only a factor 3 compared to the ρ^{-1} scaling of the phase space density of the suprathermal pick-up ions in the middle heliosphere (Gloeckler 2003).

3. Third, we treat the case including a momentum diffusion parameter $D_W \propto u^{-1}$, which results from weak double layers or other electric field fluctuations with scale sizes smaller than the scattering mean free path of the ions. First, we try to solve the homogeneous transport equation for this situation with a product of the power law function that solves the equation for $D_W = 0$ and another function $g_{\text{hom}}(u)$:

$$\begin{aligned} -\frac{\partial f}{\partial \rho} + \frac{1}{\rho} \frac{2u}{3} \frac{\partial f}{\partial u} + \frac{D_{2,E}}{\rho} \frac{1}{u^2} \frac{\partial}{\partial u} \left[u^4 \frac{\partial f}{\partial u} \right] + \frac{D_{W,E}}{\rho} \left[\frac{1}{u^2} \frac{\partial f}{\partial u} + \frac{1}{u} \frac{\partial^2 f}{\partial u^2} \right] &= 0 \quad \text{with } f_{\text{hom}}(u, \rho) = \rho^{-\beta} u^{-\alpha} g_{\text{hom}}(u) \Rightarrow \\ \left[\beta - \frac{2\alpha}{3} + \alpha(\alpha - 3) D_{2,E} \right] u^{-\alpha} g(u) + D_{W,E} u^{-\alpha-3} \alpha^2 g_{\text{hom}}(u) & \\ - \left[2D_{2,E}(\alpha - 2) - \frac{2}{3} \right] u^{-\alpha+1} \frac{dg_{\text{hom}}}{du} - 2\alpha D_{W,E} u^{-\alpha-2} \frac{dg_{\text{hom}}}{du} + \left[D_{2,E} u^{-\alpha+2} + D_{W,E} u^{-\alpha-1} \right] \frac{d^2 g_{\text{hom}}}{du^2} &= 0. \end{aligned} \quad (24)$$

The first term relates the spectral index of the power-law distribution in speed with that in heliocentric distance by $\beta = 2\alpha/3 - D_{2,E}\alpha(\alpha - 3)$ as above. The differential equation for $g_{\text{hom}}(u)$ is solved to first-order derivatives:

$$\frac{dg_{\text{hom}}}{du} = \frac{\alpha}{2u} \frac{g_{\text{hom}}(u)}{1 + (u/u_W)^3} \quad \text{with} \quad u_W = \left[\frac{\alpha D_{W,E}}{(\alpha - 2) D_{2,E} - 1/3} \right]^{1/3} \Rightarrow g_{\text{hom}}(u) = \left[\frac{(u/u_W)^3}{1 + (u/u_W)^3} \right]^{\alpha/6}. \quad (25)$$

For this solution it can be shown that the ratio of the term with the second-order derivative d^2g_{hom}/du^2 to the term linear in g_{hom} varies between $(\alpha - 2)/(4\alpha)$ at $u = 0$ to $-2D_{2,E}/[(\alpha - 2) D_{2,E} - 1/3]$ at $u \rightarrow \infty$, i.e., the term with the second-order derivative remains below the level of about 0.2 times the term linear in g_{hom} for all u .

The speed u_W characterizes the importance of stochastic acceleration by small-scale electrostatic fluctuations such as the weak double layers. Large $D_{W,E}$ results in large u_W , while $D_{W,E} \rightarrow 0$ results in $u_W \rightarrow 0$ and $g_{\text{hom}}(u) \rightarrow 1$. The complete solution $f_{\text{hom}} = \rho^{-1} u^{-\alpha} g_{\text{hom}}$ is a power law $u^{-\alpha}$ for $u \gg u_W$, where typically $u_W < 2$. Therefore, in the range $u > 2$ i.e. outside the pick-up ion shell, where the suprathermal tails can be observed, the theory predicts power laws, as $g_{\text{hom}}(u) \rightarrow 1$ for $u \rightarrow \infty$.

4. Fourth, we include the source term in addition to the momentum diffusion terms proportional to u^2 and u^{-1} into the transport equation. It turns out that the momentum diffusion proportional to u^{-1} increases the overall strength of the suprathermal tails, i.e., the efficiency for injection of freshly ionized pick-up ions into the process of stochastic acceleration is somewhat increased.

If the distribution function is factorized as $f(u, \rho) = \rho^{-1} u^{-\alpha} g_{\text{hom}}(u) g_{\text{inj}}(u)$, then we need to solve for g_{inj} :

$$\left[(2\alpha - 4) D_{2,E} - \frac{2}{3} \right] u^3 \frac{dg_{\text{inj}}}{du} - D_{2,E} u^4 \frac{d^2g_{\text{inj}}}{du^2} + 2\alpha D_{W,E} \frac{dg_{\text{inj}}}{du} - D_{W,E} u \frac{d^2g_{\text{inj}}}{du^2} = \frac{u^{\alpha+2}}{g_{\text{hom}}(u)} q(u). \quad (26)$$

The source function $q(u)$ is constraint to the range $0 < u < 2$ and is peaked at $u \approx 1.4$. Therefore, it can be shown that the first two terms on the left-hand side of the equation dominate if $u_W \ll 1$, while the second two terms dominate for $u_W \gg 1$. In the first case, the injection efficiency is the same as in Eq. (21), while in the second case, the injection efficiency is calculated as

$$g_{\text{inj}}(u > 2) \approx \int_0^2 du' \frac{u'^{\alpha+2}}{(2\alpha + 1) D_{W,E}} q(u') g_{\text{hom}}^{-1}(u') = \frac{(2\alpha - 3) D_{2,E} - 2/3}{(2\alpha + 1) D_{W,E}} \times \frac{1}{(2\alpha - 3) D_{2,E} - 2/3} \times \int_0^2 du' u'^{\alpha-1} q(u') \times u'^3 \left[1 + \left(\frac{u_W}{u'} \right)^3 \right]^{\alpha/6}. \quad (27)$$

This equation has been written in a form which eases comparison to Eq. (21) in order to verify how much small-scale electrostatic fluctuations can boost the injection efficiency. At $\alpha = 6$, the last term in the integral increases linearly with $D_{W,E}$ while the pre-factor scales inversely with $D_{W,E}$. Some increase in injection efficiency is due to the additional u'^3 factor in the above integral. However, only for $\alpha > 6$ a larger increase in the injection efficiency is expected due to the dominance of the last term in the above integral.

We conclude that stochastic acceleration in small-scale electrostatic fluctuations (ion-acoustic waves), with $L_{\text{turb}} \ll \lambda_{\parallel}$ described by a momentum diffusion parameter $D_W \propto u^{-1}$, is mainly effective at low speeds. The spectral index α at larger speeds is determined by the diffusion parameter $D_2 \propto u^2$. However, the momentum diffusion described by $D_W \propto u^{-1}$ can boost the injection efficiency to some extent. We note, that for the injection of solar wind ions, the ion-acoustic waves have the “advantage” that their phase speed can be quite low for frequencies around the proton plasma frequency f_{pp} . Consequently, the resonance between ion-acoustic waves and the tails of the bulk solar wind is more efficient than the resonance with Alfvén waves.

5. We leave the discussion of stochastic acceleration in Alfvénic turbulence to Sect. 4. It will be shown there that in a steady state, the stochastic acceleration in Alfvénic turbulence is only efficient for speeds up to $u_A \approx (2D_A)^{3/4}$. From the values in Eq. (5) we can conclude that stochastic acceleration in Alfvénic turbulence is rather inefficient in the supersonic solar wind, even for ten times higher mean turbulence levels in the solar wind than assumed in Eq. (5). However, in the heliosheath plasma the situation may be different because the much longer convection time scale in the subsonic flow simply leaves more time for stochastic acceleration.

The above solutions give rough estimates for stationary average phase space densities in the heliosphere. We give some baseline phase space densities for the slow and fast solar wind over the heliosphere. We do not know whether they apply for the outer heliosphere because of the lack of observational data. However, we note that a phase space density $f \propto \rho^{-1} u^{-5}$ is, within a factor 3, an approximate solution that fits both the suprathermal tails in the slow solar wind observed with SWICS/ULYSSES near Jupiter’s orbit (Fig. 1 of Gloeckler 2003) as well as the energetic ion distributions below 1 MeV/amu observed with Voyager 1 at 85 AU (Fig. 4 of Krimigis et al. 2003, and Fig. 2, *left*). Possibly, the low-energy part of the spectra at 85 AU represent suprathermal tails of the slow solar wind convected towards the termination shock. However, the approximate match of phase space densities at 85 AU with the model values may as well be incidental. We discuss the interpretations of the Voyager 1 data in Sect. 5.

As baseline for our work we take the phase space densities

$$f_{\text{UP;p}}(\rho, u) \approx f_{\text{UP;p;0}} \rho^{-1} u^{-5}, \text{ with } f_{\text{UP;p;0}} \approx 50 \text{ s}^3 \text{ km}^{-6} \text{ for H}^+, \quad \text{and} \\ f_{\text{UP;He}}(\rho, u) \approx f_{\text{UP;He;0}} \rho^{-1} u^{-5} \text{ with } f_{\text{UP;He;0}} \approx 4 \text{ s}^3 \text{ km}^{-6} \text{ for He}^+ \quad (28)$$

in the slow solar wind. These spectra are somewhat harder than those observed at 5 AU and at 1 AU (Gloeckler 2003), but are as hard as those observed at 85 AU (Krimigis et al. 2003). For the fast solar wind the pre-factors $f_{\text{UP;p;0}}$ and $f_{\text{UP;He;0}}$ are typically ten times larger and the spectral index is typically $\alpha \approx 8$.

We also estimate the typical spatial scale with which the phase space densities vary in the heliosphere. For suprathermal tails with spectral index α , the ratio of the adiabatic cooling rate to the inverse convection timescale is $2\alpha/3$. Therefore, spatial variations occur on a scale $3\rho/(2\alpha)$. Variations of at least a factor 5 are observed in the suprathermal ion flux (Gloeckler 2003), for instance when comparing quiet-time flux with the average flux. This implies that the “acceleration site” of the quiet-time suprathermal tails may be as distant as about $3 \ln(5)\rho/(2\alpha)$, which is about 0.5 AU near Earth for spectral indices $\alpha \approx 5 \dots 6$. This needs to be kept in mind when correlating measured turbulence levels and suprathermal ion flux. The tails during quiet times may be remnants from acceleration in regions with higher turbulence such as the downstream regions of interplanetary shocks or regions of stream-stream interactions.

All the processes of stochastic acceleration in plasma waves described above prefer pick-up ions over bulk solar wind ions. They require that the ions are faster than the wave phase speed in the solar wind frame. While this is usually the case for pick-up ions, the vast majority of solar wind ions is slower than the wave phase speeds, even in the case of ion-acoustic waves. Therefore, an additional process such as direct acceleration at the shock may be required for the bulk solar wind ions. On the other hand, direct acceleration of ions at interplanetary shocks is again more efficient for pick-up ions because part of the pick-up ions has the same velocity as the shock, which, for instance, is necessary for shock surfing to operate (Lee et al. 1996).

3. Acceleration of suprathermal ions at the termination shock

Two processes are commonly discussed in the context of ion acceleration near the termination shock: (i) diffusive (first-order) Fermi acceleration and (ii) shock drift acceleration in the motional electric field of the shock. In fact, during diffusive acceleration at the termination shock ions seem to gain most energy in the motional electric field as well (Jokipii 1992). In the following we derive an expression for the injection threshold speed into first-order Fermi acceleration from quasi-linear theory. Then we apply quasi-linear theory to estimate the approximate strength of Alfvén wave amplification by ACRs and the resulting increase of the injection threshold speed. This leads to a self-consistent adjustment of the ACR intensity. Additionally, the effect of the electric shock potential on low-energy suprathermal ions is quantified.

3.1. The injection threshold into diffusive acceleration

The standard theory of diffusive shock acceleration (first-order Fermi process) predicts a power law for the energetic ion distribution,

$$F_{\text{ACR}}(u) = \gamma_{\text{sh}} u^{-\gamma_{\text{sh}}} \int_{u_{\text{inj}}}^u du' u'^{\gamma_{\text{sh}}-1} f_{\text{UP}}(u', \rho_{\text{TS}}) = \frac{\gamma_{\text{sh}} f_{\text{UP;0}}}{\alpha - \gamma_{\text{sh}}} \frac{u^{-\gamma_{\text{sh}}}}{\rho_{\text{TS}}} \left[u_{\text{inj}}^{\gamma_{\text{sh}}-\alpha} - u^{\gamma_{\text{sh}}-\alpha} \right] \quad \text{for } u \geq u_{\text{inj}}, \quad (29)$$

where the spectral index is $\gamma_{\text{sh}} = 3\xi_{\text{sh}}/(\xi_{\text{sh}} - 1)$ with $\xi_{\text{sh}} = V_{\text{SW;up;sh}}/V_{\text{SW;ds;sh}} = n_{\text{ds;sh}}/n_{\text{up;sh}} \approx 3$ the compression ratio of the subshock. The index α ($\alpha \approx 5 \dots 6$) characterizes the distribution f_{UP} that is injected from the supersonic solar wind. This distribution undergoes some modification which we will discuss in Sect. 3.2. Here, we concentrate on providing a number for the injection threshold w_{inj} , where w is the ion speed in units of the downstream solar wind speed $V_{\text{SW;ds}}$, $w = v/V_{\text{ds}}$ with the shorter notation V_{ds} for $V_{\text{SW;ds}}$.

Diffusive acceleration at a shock operates if the ion is diffusively returned to the shock from the downstream plasma. The mean speed of this diffusive motion must be larger than the convection speed with which the ion is transported in downstream direction. In the steady state of first-order Fermi acceleration this is the upstream convection speed $V_{\text{SW;up}}$ in both the upstream and the downstream plasma. The diffusion speed is the spatial diffusion parameter $\kappa_{\text{eff}} = v\Lambda_{\text{eff;ds}}/3$ divided by the gradient scale r_{G} of the distribution function of super-Alfvénic ions. The gradient scale cannot be smaller than the gyro-radius r_{g} (Jokipii 1992; Zank et al. 2001). Taking the gyro-radius r_{g} as gradient scale gives an upper limit for the “diffusion speed” and thus a lower limit for the injection threshold speed w_{inj} . According to standard kinetic theory we get the injection condition

$$V_{\text{SW;up}} < \frac{v\Lambda_{\text{eff;ds}}}{3r_{\text{g}}} \Rightarrow 3\xi_{\text{sh}} < w \frac{\Lambda_{\text{eff;ds}}}{r_{\text{g}}} \\ \text{with } \xi_{\text{sh}} = \frac{V_{\text{SW;up}}}{V_{\text{ds}}}, \quad \Lambda_{\text{eff;ds}} = \Lambda_{\perp;\text{ds}} \sin^2 \Psi + \Lambda_{\parallel;\text{ds}} \cos^2 \Psi, \quad \text{and} \quad \frac{\Lambda_{\perp}}{\Lambda_{\parallel}} = \left[1 + \Lambda_{\parallel}^2/r_{\text{g}}^2 \right]^{-1} \approx \frac{r_{\text{g}}^2}{\Lambda_{\parallel}}, \quad (30)$$

where Ψ is the angle of the ambient magnetic field to the shock normal. In case of a spherical termination shock and the heliospheric magnetic field in the ecliptic plane in the form of a Parker spiral, we can use $\Psi_{\text{Parker}} = \arctan(\Omega_S r / V_{\text{SW}}) \approx \arctan \rho$ with the Carrington angular speed $\Omega_S \approx 2\pi/27 \text{ days} \approx 2.9 \times 10^{-6} \text{ s}^{-1}$. At the termination shock the Parker angle is $\Psi_{\text{Parker}}(\rho_{\text{TS}}) \approx 89.3^\circ$ and thus $\cos[\Psi_{\text{Parker}}(\rho_{\text{TS}})] \approx 0.012$. The ratio ξ_{sh} has been chosen to be the ratio between the far upstream and the downstream bulk speed. This is an upper limit for use in the injection condition because the ACRs do not necessarily need to cross the full upstream pre-cursor of the termination shock for acceleration.

In the following we express lengths in units of r_E (1 AU) and speeds in units of the downstream plasma speed V_{ds} , $w = v/V_{\text{ds}}$. The gyro-radius in units of r_E is

$$\rho_{g;\text{ds}} = w V_{\text{SW};\text{ds}} / r_E \Omega_{\text{ds}} \approx 0.7 w \mathcal{R} 10^{-4}, \quad (31)$$

where the magnetic field $B_{\text{ds}} = 0.044 \text{ nT}$ and the solar wind speed $V_{\text{SW};\text{ds}} \approx 45 \text{ km s}^{-1}$ has been taken from Voyager 1 measurements (Krimigis et al. 2003), although these results are considered controversial by McDonald et al. (2003) and Fisk (2003) and may have to be revised.

The parallel mean free path is

$$\begin{aligned} \lambda_{\parallel;\text{ds}} &= \frac{\Lambda_{\parallel;\text{ds}}}{r_E} = \frac{3v^2}{8\pi r_E \Omega^2} \frac{B_{\text{ds}}^2}{\delta \tilde{B}_{\text{ds}}^2 (\Omega/v)} \\ &= \frac{3L_{\text{A};\text{E}}}{16\pi^2 r_E (s-1)} \left(\frac{2\pi w V_{\text{SW};\text{ds}}}{\Omega_E L_{\text{A};\text{E}}} \right)^{2-s} \frac{\rho_{\text{TS}}^2}{\zeta_{\text{A};\text{E}} a_{\text{ds}}} \approx \frac{50 w^{1/3} \mathcal{R}^{1/3}}{a_{\text{ds}}}, \end{aligned} \quad (32)$$

where $\Omega_E \approx 0.5 \mathcal{R}^{-1} \text{ s}^{-1}$, $\zeta_{\text{A};\text{E}} \approx 3 \times 10^{-3}$ in the quiet slow solar wind, $L_{\text{A}}/\zeta_{\text{A}} \propto \rho^2$, and $\Omega L_{\text{A}} = \text{const.}$ has been assumed. The factor $a_{\text{ds}} = a_{\text{TS}} a_{\text{up}}$ characterizes the increase of ζ_{A} at the termination shock (a_{TS}) or anywhere in the outer heliosphere inside the termination shock (a_{up}) above the level given by the relation $L_{\text{A}}/\zeta_{\text{A}} \propto \rho^2$. In particular, this includes the amplification of Alfvénic turbulence by pick-up ions outside 10 AU which can be up to $a_{\text{up}} \approx 10$ (e.g. Heber & Burger 1999). The Rankine-Hugoniot conditions would give $a_{\text{TS}} \approx 4$ (McKenzie & Westphal 1969; Chalov & Fahr 2000) right at the termination shock.

The effective mean free path perpendicular to the termination shock is approximately

$$\begin{aligned} \lambda_{\text{eff};\text{ds}} &= \lambda_{\perp;\text{ds}} \sin^2 \Psi + \lambda_{\parallel;\text{ds}} \cos^2 \Psi \approx \lambda_{\parallel;\text{ds}} \cos^2 \Psi \left[1 + \tan^2 \Psi \left(1 + \frac{\lambda_{\parallel;\text{ds}}^2}{\rho_g^2} \right)^{-1} \right] \\ &\approx 50 w^{1/3} \mathcal{R}^{1/3} \frac{\cos^2 \Psi}{a_{\text{ds}}} \left[1 + \tan^2 \Psi \left(1 + \frac{10^{12}}{2 w^{4/3} \mathcal{R}^{4/3} a_{\text{ds}}^2} \right)^{-1} \right] \approx 50 w^{1/3} \mathcal{R}^{1/3} \frac{\cos^2 \Psi}{a_{\text{ds}}}. \end{aligned} \quad (33)$$

The last approximation means that perpendicular diffusion is neglected. It is valid if the amplification factor a_{ds} is not too large and $\cos \Psi$ not too small. It applies for the Parker angle $\cos[\Psi_{\text{Parker}}(\rho_{\text{TS}})] \approx 0.01$ as long as $a_{\text{ds}} w^{4/3} \mathcal{R}^{4/3} \ll 10^8$. We note that the above expressions give radial mean free paths $\lambda_{rr} \approx \lambda_{\parallel} \cos^2 \Psi_{\text{Parker}} + \lambda_{\perp} \sin^2 \Psi_{\text{Parker}}$ for the upstream solar wind region that can reconcile observed radial mean free paths of ACRs at high rigidities in the outer heliosphere (Hill et al. 2002). With Eq. (33) expressed for the upstream region with $a_{\text{ds}} = 1$ the radial mean free path is about 0.05 AU for protons with an energy of 1 MeV and for a strict Parker spiral. The somewhat smaller values 0.1 AU (GV/R_p), with R_p the rigidity, to 0.67 AU (GV/R_p) by Hill et al. (2002) could represent the influence of pick-up ion driven Alfvénic turbulence and the influence of the perpendicular mean free path in two-dimensional magnetohydrodynamic turbulence in the outer heliosphere (Parhi et al. 2003) which is not evaluated in detail here.

From Eq. (33) we estimate the injection threshold. For $\Psi_{\text{Parker}} \approx 89^\circ$ the expression for the injection threshold yields the value $w_{\text{inj}} = (0.0016 a_{\text{ds}})^3$. This very small value suggests that the “injection problem” in this form may only be a problem that applies for a strictly perpendicular shock. For a not exactly perpendicular termination shock, the injection threshold is simply given by the condition that the ions must be super-Alfvénic in the upstream and downstream plasma. The simple condition that the ions should be super-Alfvénic apparently advantages the pick-up ions for injection into diffusive acceleration.

However, the “injection problem” may arise for the following reasons: 1) the quasi-linear theory does not apply for only slightly super-Alfvénic particles. In this article, we do not address this problem which has to be tackled by numerical simulations. 2) Once ACRs are accelerated, they generate Alfvén waves themselves and increase the factor a_{ds} . In that sense the ACR production is “self-limiting”. High ACR flux means high Alfvén wave amplification and thus a large factor a_{ds} and an increasing injection threshold w_{inj} , which causes the ACR production to be less efficient. In fact, the injection threshold speed w_{inj} may easily be in the highly super-Alfvénic range, where quasi-linear theory is usually a good approach. We pursue these thoughts in the following. First, we show that the acceleration times of ACRs are sufficiently short to allow for acceleration within the convection time scale of the heliosheath plasma and within the ionization time of the singly charged pick-up ions to become doubly charged.

3.1.1. The acceleration time scale of ACRs

The typical time for an ion to be accelerated by the first-order Fermi process from speed v_0 to v_1 is (e.g. Zank et al. 2001, and references therein)

$$t_{\text{acc}} = \frac{3}{V_{\text{SW};\text{up}} - V_{\text{SW};\text{ds}}} \int_{v_0}^{v_1} \left(\frac{v \Lambda_{\text{eff};\text{up}}}{3 V_{\text{SW};\text{up}}} + \frac{v \Lambda_{\text{eff};\text{ds}}}{3 V_{\text{SW};\text{ds}}} \right) \frac{dv}{v} \Rightarrow \tau_{\text{acc}} = \frac{t_{\text{acc}} V_{\text{SW};\text{ds}}}{r_E} = \frac{1}{\xi_{\text{sh}} - 1} \left(1 + \frac{a_{\text{TS}}}{\xi_{\text{sh}}} \right) \int_{w_0}^{w_1} dw \lambda_{\text{eff};\text{ds}} \Rightarrow$$

$$d\tau_{\text{acc}} = \frac{\xi_{\text{sh}} + a_{\text{TS}}}{\xi_{\text{sh}} (\xi_{\text{sh}} - 1)} dw \lambda_{\text{eff};\text{ds}} \Rightarrow$$

$$\frac{d\tau_{\text{acc}}}{dw} w \approx 50 \frac{\xi_{\text{sh}} + a_{\text{TS}}}{\xi_{\text{sh}} (\xi_{\text{sh}} - 1)} \left[\mathcal{R}^{1/3} \frac{\cos^2 \Psi}{a_{\text{ds}}} w^{4/3} + 2 \times 10^{-12} a_{\text{ds}} \mathcal{R}^{5/3} w^{8/3} \right] \quad \text{for } w < \mathcal{R}^{-1} \left(\frac{10^{12}}{2a_{\text{ds}}^2} \right)^{3/4}, \quad (34)$$

where $\xi_{\text{sh}} \approx 3$. The parameter τ gives the acceleration time in units of the downstream convection time of about 0.1 yr. We have approximated $\cos^2 \Psi \approx \cot^2 \Psi$. It follows from the above expression that for $w > \mathcal{R}^{-1} (5 \times 10^{11} \cos^2 \Psi / a_{\text{ds}}^2)^{3/4}$ the acceleration is dominated by perpendicular diffusion.

The maximum time for ACRs to remain singly charged (Ellison et al. 1999) is $\tau_{\text{acc};\text{max}} \approx 50$. The convection time scale τ_{conv} in the heliosheath is approximately the same. This means that the upper speed limit in ACR acceleration is quite high. We need, however, to explore the factor $a_{\text{ds}} = a_{\text{TS}} a_{\text{up}} \approx 4a_{\text{up}}$ in more detail. It turns out that the factor a_{up} can be quite large.

The above acceleration times are upper limits because the ions also gain energy in the motional electric field during each shock transition (Jokipii 1992).

3.1.2. Alfvén wave generation upstream of the termination shock

The factor a_{up} of Alfvén wave generation upstream of the termination shock is derived from the efficiency of Alfvén wave generation through pick-up ions and through anisotropic energetic particles. Here, we report on recent observations near interplanetary travelling shocks (Bamert et al. 2004) and try to extrapolate these results to the situation of the termination shock. The increase of wave power is given by a factor

$$G_A(k) = \frac{\Gamma_p F_p(k)}{P(k, \infty)}, \quad \Gamma_p = \frac{3\pi^2 v^6}{\gamma_{\text{sh}} (\gamma_{\text{sh}} - 2) \Omega_p n_p V_{\text{SW}} V_A} \propto \rho^3, \quad P(k, \infty) = \frac{L_A \zeta_A (s-1)}{2\pi} \left(\frac{2\pi v}{\Omega_p L_A} \right)^s \propto \rho^0, \quad (35)$$

where $P(k, \infty)$ indicates the unamplified turbulence level far upstream of the shock. The shock parameter γ_{sh} in Bamert et al. (2004) is $\gamma_{\text{sh}} \approx 4.9$, close to what one may expect for the termination shock, $\gamma_{\text{sh}} \approx 4.5$. The amplification $G_A(k)$ of Alfvén waves was found as $G_A(k) = 15 (k/k_1 \text{ MeV})^{\gamma_{\text{sh}}-13/3}$ and is thought to be caused by an anisotropic flux of energetic particles (Lee 1983). Anisotropies are in fact observed at 85 AU (Krimigis et al. 2003), however the phase space density of energetic ions is a factor of 3000 lower than F_p during the Bastille Day event (Bamert et al. 2004) (Fig. 2). The phase space density of the ACRs displayed in Fig. 2 (*solid line*) is $F_{\text{ACR}}(v_1 \text{ MeV}) = j_{\text{ACR}}(1 \text{ MeV}) \times 1 \text{ MeV} / v_1^4$ with $v_1 \text{ MeV} = 1.38 \times 10^4 \text{ km s}^{-1}$ and hence $F_{\text{ACR}}(v_1 \text{ MeV}) = 10^{-7} \text{ s}^3 \text{ km}^{-6}$, while Fig. 2b gives $F_p(v_1 \text{ MeV}) = 3 \times 10^{-4} \text{ s}^3 \text{ km}^{-6}$ for the Bastille Day event. A higher value, by a factor 10, is given for F_p during the Bastille Day event in Smith et al. (2001). This may be due to different view directions of the ACE and HSTOF sensors. We take a factor $F_{\text{ACR}}(v_1 \text{ MeV}) = 10^{-4 \pm 0.5} \times F_p(v_1 \text{ MeV})$ as baseline to estimate the wave power generated upstream of the termination shock. With a spectral index γ_{sh} this is $F_{\text{ACR}}(u < u_1 \text{ MeV}) \approx 0.5 u^{-4.5} \text{ s}^3 \text{ km}^{-6}$ for the lower energy range in Fig. 2a expressed in terms of $u = v/V_{\text{SW}}$, where $u_1 \text{ MeV} \approx 30$. It is close – perhaps incidentally – to the phase space density of the suprathermal tails of protons extrapolated from observations in the slow solar wind in the inner and middle heliosphere (Sect. 2.4), $f_{\text{UP};\text{p}}(\rho, u) \approx 50 \rho^{-1} u^{-5} \text{ s}^3 \text{ km}^{-6}$.

The unmodulated ACR phase space density at 1 MeV (see Fig. 2) may still be a factor 100 higher right at the termination shock. This can be expected from the high-energy part of the spectra in Fig. 2a. As Γ_p scales with ρ^3 and $P(k, \infty)$ does not vary with ρ , one may expect a wave amplification near the termination shock of perhaps $15 \times 85^3 \times 10^{-2} \approx 10^5$ for a quasi-parallel shock. The Alfvén wave generation scales as $\cos \Psi$ (Lee 1983), where Ψ is the angle of the ambient magnetic field to the shock normal. This yields typically $G_A(k) \approx 10^5 \cos \Psi (k/k_1 \text{ MeV})^{\gamma_{\text{sh}}-13/3}$ at the termination shock. Introducing the resonance condition $\Omega = kv\mathcal{R}$ to the heliosheath speed scale $w = 300v/v_1 \text{ MeV}$ the amplification factor becomes

$$G_A(w) \approx 10^5 \times \cos \Psi \times (w\mathcal{R}/300)^{13/3-\gamma_{\text{sh}}}. \quad (36)$$

With $a_{\text{TS}} \approx 4$ and Eqs. (33) and (31) we can derive the parameter

$$\eta_c = \lambda_{\parallel} / \rho_g \approx \frac{50 w^{1/3} \mathcal{R}^{1/3}}{0.7 \times 10^{-4} a_{\text{TS}} G_A(w) \cos \Psi w \mathcal{R}} \approx \frac{1.8}{\cos \Psi} \times 300^{-2/3} \left(\frac{w\mathcal{R}}{300} \right)^{\gamma_{\text{sh}}-15/3} \approx \frac{0.04}{\cos \Psi} \left(\frac{\mathcal{R}E/m}{1 \text{ MeV/amu}} \right)^{\gamma_{\text{sh}}-5}, \quad (37)$$

Fig. 2. a) Voyager 1 and Voyager 2 ACR proton energy spectra in the outer heliosphere at about 85 AU during the intensity enhancement in 2002 (2002/2092002/364), along with the solar minimum spectra observed by Voyager 1 in 1998/1999/182 (*dashed lines*) and the predicted shock source spectra for a weak shock (*dotted lines*). From McDonald et al. (2003). **b)** Phase space density of protons upstream of the main interplanetary shock driven by the Bastille Day CME at 1 AU. The dash-dotted line denotes the phase space density that the protons would assume if they were propagated from the shock to the upstream region without amplifying Alfvén waves of the ambient solar wind. The dashed line denotes the model prediction by Lee (1983). **c)** Power spectral density of magnetic field fluctuations measured at the same time and location as the proton spectrum in **b)**. The increase of the wave power – with respect to the ambient solar wind level (*dash-dotted line*) – due to amplification by the energetic protons shown in panel **b)** is by about a factor 30 at $k = 4k_1$ MeV. Panels **b)** and **c)** from Bamert et al. (2004).

which describes the strength of scattering. If η_c is close to unity the scattering is strong i.e. the parallel mean free path comparable to the gyro-radius. With $\gamma_{sh} = 4.5$ and $\cos \Psi = 0.01$ and with the wave amplification $G_A \approx 10^5 \cos \Psi$, it turns out that scattering can be rather strong, but still $\lambda_{||} \ll \rho_g$. We obtain values for η_c which are close to the value $\eta_c = 14$ assumed by Ellison et al. (1999) for the direct acceleration from pick-up ion energies of about 4 keV/amu to MeV-energies. The above expression for η_c means that at about 16 MeV/amu and for $\cos \Psi = 0.01$ the ACR protons will drive the Alfvén fluctuations to the limit $\eta_c \approx 1$.

3.1.3. Self-consistent adjustment of the injection threshold

With the expected rather high amplification of Alfvén waves – even at a quasi-perpendicular shock – the parallel mean free path is strongly reduced and the perpendicular mean free path is increased accordingly. In Eq. (33) we need to include the full expression for the effective mean free path. We calculate the injection threshold w_{inj} for first-order Fermi acceleration to operate. We use $\cos^2 \Psi \approx \cot^2 \Psi$ and F_{rel} as the ratio between the proton ACR phase space density with spectral index γ_{sh} and the ACR phase space density $F_{ACR;p;5}(v_1 \text{ MeV}) \approx 10^{-5} \text{ s}^3 \text{ km}^{-6}$ which causes an amplification of about $a_{up} \approx 10^5 \cos \Psi$ for Alfvén waves resonating with 1 MeV protons. The ratio F_{rel} also expresses so-to-say the ratio between the modelled ACR phase space density and the phase space density $F_{ACR;p;5}$ expected from the observation of the unmodulated high-energy tail of the ACRs. We perform some algebra:

$$3\xi_{sh} < w \frac{\lambda_{eff;ds}}{\rho_g} \quad \text{with} \quad \lambda_{eff;ds} = 50w^{1/3}\mathcal{R}^{1/3} \frac{\cos^2 \Psi}{a_{ds}} \left[1 + \left(\cos^2 \Psi + \frac{10^{12}}{2a_{ds}^2 w^{4/3} \mathcal{R}^{4/3}} \right)^{-1} \right],$$

$$\rho_g = 0.7w\mathcal{R}10^{-4}, \quad a_{ds} = a_{TS} \left[1 + F_{rel} 10^5 \cos \Psi \left(\frac{w\mathcal{R}}{300} \right)^{13/3 - \gamma_{sh}} \right], \quad \xi_{sh} \approx 3, \quad \text{and} \quad F_{rel} = \frac{F_{ACR;p}(w_1 \text{ MeV} = 300)}{F_{ACR;p;5}(w_1 \text{ MeV} = 300)};$$

$$\begin{aligned}
F_{\text{ACR;p};5}(w_1 \text{ MeV} = 300) &= \frac{50 \text{ s}^3 \text{ km}^{-6}}{30^{4.5}} \approx 10^{-5} \text{ s}^3 \text{ km}^{-6} \Rightarrow \\
1 &< 1.3 \left(\frac{w\mathcal{R}}{300} \right)^{\gamma_{\text{sh}}-4} \frac{\cos \Psi}{F_{\text{rel}}} \left[1 + \left(\cos^2 \Psi + 0.0015 F_{\text{rel}}^{-2} \left(\frac{w\mathcal{R}}{300} \right)^{2\gamma_{\text{sh}}-10} \right)^{-1} \right] \Rightarrow \\
0 &< 0.002 x^{6-\gamma_{\text{sh}}} \cos \Psi F_{\text{rel}}^2 - F_{\text{rel}} + 1.3 x^{\gamma_{\text{sh}}-4} \cos \Psi \quad \text{with } x := w\mathcal{R}/300 \Rightarrow \\
F_{\text{rel}} &= \frac{1 \pm \sqrt{1 - 0.01 x_{\text{inj}}^2 \cos^2 \Psi}}{0.004 x^{6-\gamma_{\text{sh}}} \cos \Psi} \approx 1.3 \cos \Psi x_{\text{inj}}^{\gamma_{\text{sh}}-4} \Rightarrow x_{\text{inj}} \approx \left(\frac{F_{\text{rel}}}{1.3 \cos \Psi} \right)^{1/(\gamma_{\text{sh}}-4)} \quad \text{or} \quad w_{\text{inj}} \approx \frac{300}{\mathcal{R}} \left(\frac{F_{\text{rel}}}{1.3 \cos \Psi} \right)^{1/(\gamma_{\text{sh}}-4)} \\
\text{or } u_{\text{inj}} &\approx \frac{30}{\mathcal{R}} \left(\frac{F_{\text{rel}}}{1.3 \cos \Psi} \right)^{1/(\gamma_{\text{sh}}-4)}. \tag{38}
\end{aligned}$$

In the last line we have expressed the injection threshold in both the speed units of the far upstream (u_{inj}) and the downstream plasma (w_{inj}).

Equation (38) shows that the injection threshold is lower for ions with high mass-to-charge ratio. This is in general agreement with the observed composition of the ACRs (Cummings et al. 2002). Equation (38) qualitatively predicts that the injection threshold rises when the ACR intensity rises. This is due to the fact that increasing ACR intensity causes increasing Alfvénic turbulence and, thus, a reduction of the diffusion speed and consequently a higher injection threshold. Therefore, some level F_{rel} will result in a self-consistent manner.

To estimate the level on which the ACR intensity adjusts itself self-consistently we additionally need to know from which source the ACRs are “fed”. We treat the most simple case that power-law distributions are injected into first-order Fermi acceleration. The level F_{rel} is derived from the phase space density of Eq. (29) at high speeds $u \gg u_{\text{inj}}$ and from the expression in Eq. (38) for the injection threshold as a function of F_{rel} :

$$\begin{aligned}
F_{\text{ACR;p}}(u) &\approx \frac{\gamma_{\text{sh}} f_{\text{UP;0;p}}}{\alpha - \gamma_{\text{sh}}} \rho_{\text{TS}}^{-1} u^{-\gamma_{\text{sh}}} u_{\text{inj;p}}^{\gamma_{\text{sh}}-\alpha} \quad \text{with } u_{\text{inj;p}} = 30 \left(\frac{F_{\text{rel}}}{1.3 \cos \Psi} \right)^{1/(\gamma_{\text{sh}}-4)}; \quad F_{\text{rel}} = \frac{F_{\text{ACR;p}}(u = 30) 30^{4.5}}{50 \text{ s}^3 \text{ km}^{-6}}; \\
f_{\text{TS;p}} := f_{\text{UP;0;p}} \rho_{\text{TS}}^{-1} &\Rightarrow F_{\text{rel}} \approx \left[\frac{\gamma_{\text{sh}}}{\alpha - \gamma_{\text{sh}}} \frac{f_{\text{TS;p}} 30^{4.5-\alpha}}{50 \text{ s}^3 \text{ km}^{-6}} \right]^{(\gamma_{\text{sh}}-4)/(\alpha-4)} \times (1.3 \cos \Psi)^{(\alpha-\gamma_{\text{sh}})/(\alpha-4)}. \tag{39}
\end{aligned}$$

The ACR levels F_{rel} are plotted for protons in Fig. 3 (*left*) as a function of the injected level $f_{\text{UP;0}}$ and for spectral indices $\alpha = 5$ and $\alpha = 8$ of the injected distribution. In Fig. 3 (*right*) the corresponding injection thresholds are shown. As their dependence on the injected phase space density seems to be very strong, we have evaluated them for a weaker termination shock with $\gamma_{\text{sh}} = 6$ as well. In that case the dependence of the injection threshold on the injected phase space density is weaker. A larger spectral index $\gamma_{\text{sh}} = 6$ in Fig. 3 (*right*) would also correspond to the situation where $\gamma_{\text{sh}} = 4.5$ but the Alfvén wave amplification parameter Γ_p in Eq. (35) is proportional to $v^{9/2}$ instead of v^6 . That means, in case there is some modification to the strong dependence of Γ_p on v , i.e. $\propto v^6$, or on the resonant k , i.e. k^{-6} , respectively, then the injection threshold speed varies much less with the injected phase space density.

3.1.4. Self-consistent injection efficiencies

The injection efficiency is commonly defined as the ratio of the pressure of the ACRs of species S at the termination shock with respect to their dynamical pressure in the upstream solar wind:

$$\begin{aligned}
\eta_{\text{inj;S}} &\approx \frac{p_{\text{ACR;S}}}{p_{\text{SW}}} = \frac{4\pi}{3n_{\text{SW;S}}(\rho_{\text{TS}})m_S V_{\text{SW}}^2} \int_{u_{\text{inj;S}}}^{v_{\text{max;S}}} m v^2 F_{\text{ACR;S}}(v) v^2 dv = \frac{4\pi V_{\text{SW}}^3}{3n_{\text{SW;S}}(\rho_{\text{TS}})} \int_{u_{\text{inj;S}}}^{u_{\text{max;S}}} F_{\text{ACR;S}}(u) u^4 du \\
&\approx \frac{n_{\text{SW;p}}}{n_{\text{SW;S}}} \frac{0.0007 \text{ s}^3}{\text{km}^6} \int_{u_{\text{inj;S}}}^{u_{\text{max;S}}} F_{\text{ACR;S}}(u) u^4 du; \quad u = v/V_{\text{SW}}, \tag{40}
\end{aligned}$$

where we have used $n_{\text{SW}}(\rho) \approx 3\rho^{-2} \text{ cm}^{-3}$ and $\rho_{\text{TS}} = 85$. The parameter $v_{\text{max;S}}$ is the highest speed up to which the power law of first-order Fermi acceleration applies. It can be derived from Eq. (34). As mentioned above, the maximum time for ACRs to remain singly charged (Ellison et al. 1999) is $\tau_{\text{acc;max}} \approx 50$, and the convection time scale τ_{conv} in the heliosheath

Fig. 3. *Left:* flux of ACRs relative to the expected maximum unmodulated ACR flux at the termination shock, corresponding to $F_{ACR,p}(v_{1 \text{ MeV}}) \approx 10^{-5} \text{ s}^3 \text{ km}^{-6}$, as a function of the injected phase space density $f_{TS,p} \approx f_{TS,0,p} u^{-\alpha}$, where $f_{TS,0,p}$ is the parameter of the abscissa. The range of extrapolated phase space densities from observations at 1 AU and 5 AU is shown in grey shade. *Right:* injection threshold energy of protons corresponding to the relative ACR proton flux levels shown in the left panel for spectral indices $\gamma_{sh} = 4.5$ and $\gamma_{sh} = 6$.

is approximately the same. As the wave amplification factor a_{up} is quite large the acceleration at high speeds is dominated by perpendicular diffusion. We only need to evaluate the term containing $w^{8/3}$ in Eq. (34). After some algebra we obtain

$$w_{\max,S} \approx 2000 \mathcal{R}^{-5/8} F_{\text{rel}}^{-3/8} \left(\frac{0.01}{\cos \Psi} \right)^{3/8} \Rightarrow u_{\max} \approx 200 \mathcal{R}^{-5/8} F_{\text{rel}}^{-3/8} \left(\frac{0.01}{\cos \Psi} \right)^{3/8}$$

$$\text{or } \left(\frac{E}{m_S} \right)_{\max} \approx 40 \frac{\text{MeV}}{\text{amu}} \mathcal{R}^{-5/4} F_{\text{rel}}^{-3/4} \left(\frac{0.01}{\cos \Psi} \right)^{3/4}. \quad (41)$$

The injection speed is given by Eq. (38).

We need to insert the full function F_{ACR} of Eq. (29) which is based on an injected power law $f_{UP,S} = f_{UP,0,S} \rho^{-1} u^{-\alpha}$ convected to the termination shock, where, for instance, $f_{UP,p;0} = 50 \text{ s}^3 \text{ km}^{-6}$ and $\alpha \approx 5$. We abbreviate $f_{UP,0,S} \rho_{TS}^{-1}$ as $f_{TS,S}$ to allow for a more general expression for the injected population at the termination shock. We get

$$\eta_{inj,S} = \frac{n_{SW,p}}{n_{SW,S}} \frac{0.0007 \text{ s}^3}{\text{km}^6} f_{TS,S} \frac{\gamma_{sh}}{\alpha - \gamma_{sh}} \left[\frac{u_{\max,S}^{5-\gamma_{sh}} - u_{inj,S}^{5-\gamma_{sh}}}{5 - \gamma_{sh}} u_{inj,S}^{\gamma_{sh}-\alpha} - \frac{u_{\max,S}^{5-\alpha} - u_{inj,S}^{5-\alpha}}{5 - \alpha} \right]$$

$$\text{with } \frac{u_{\max,S}^{5-\alpha} - u_{inj,S}^{5-\alpha}}{5 - \alpha} \rightarrow \ln \left(\frac{u_{\max,S}}{u_{inj,S}} \right) \text{ for } \alpha \rightarrow 5, \quad u_{inj,S} \approx \frac{30}{\mathcal{R}} \left(\frac{F_{\text{rel}}}{1.3 \cos \Psi} \right)^{1/(\gamma_{sh}-4)}, \quad u_{\max,S} \approx 200 \mathcal{R}^{-5/8} F_{\text{rel}}^{-3/8} \left(\frac{0.01}{\cos \Psi} \right)^{3/8},$$

$$F_{\text{rel}} \approx \left[\frac{\gamma_{sh}}{\alpha - \gamma_{sh}} \frac{f_{TS,p} 30^{4.5-\alpha}}{50 \text{ s}^3 \text{ km}^{-6}} \right]^{(\gamma_{sh}-4)/(\alpha-4)} \times (1.3 \cos \Psi)^{(\alpha-\gamma_{sh})/(\alpha-4)}. \quad (42)$$

These injection efficiencies are evaluated numerically and plotted in Fig. 4. They will be re-evaluated after having included the effect of the shock potential (see Fig. 7).

3.2. The effect of the shock potential

Any shock has a cross-shock electric potential and an associated electric field. Non-parallel shocks have additionally a motional electric field $\mathbf{V}_{up} \times \mathbf{B}_{up}$ in the upstream plasma viewed from the shock frame. The motional electric field is parallel to the shock front. The electric fields will accelerate, decelerate and/or reflect suprathermal ions. A recent analysis of data from the SWICS instrument on board Ulysses (Gloeckler et al. 2004) indicates that the distribution functions of suprathermal ions are considerably modified when they pass Jupiter’s bow shock (Fig. 5). A possible explanation of this phenomenon is the interaction of the ions with the shock potential and the convective (motional) electric field in the foreshock region.

We do not present a detailed discussion of the direct pre-acceleration of pick-up ions at the quasi-perpendicular termination shock during multiple reflections of pick-up ions (le Roux et al. 2000). Rather, we give a rough estimate of the modification of a nearly isotropic power-law distribution function of suprathermal ions already present in the upstream supersonic solar wind and convected through the termination shock. In order to do so we calculate a transmission coefficient for a given ion speed u .

Fig. 4. Injection efficiencies of H⁺, He⁺, and O⁺ ACR ions as a function of the injected phase space density of the proton suprathermal tails characterized as in Fig. 3.

Fig. 5. Flux of suprathermal H⁺ and He⁺ upstream and downstream of Jupiter’s quasi-perpendicular bow shock. The suprathermal tails of H⁺ are enhanced by a factor 10, while the He⁺ tails are enhanced by a factor 4–5. Data from Gloeckler et al. (2004).

This transmission coefficient is based on the distinction of ion pitch-angles that allow for transmission and those pitch-angles which lead to a reflection of the ion. Ions which are reflected will encounter the termination shock again, but eventually with a different pitch-angle and somewhat higher speed due to the acceleration in the motional electric field near the shock. As the total number of pick-up ions is conserved during the passage through the shock, the downstream spectrum is to first order given by the normalized product of the transmission function times the upstream spectrum. We neglect the modification of the spectral index of the suprathermal ions by the passage through the termination shock. The data of Fig. 5 indicate that the spectral index of protons does not change very much.

We begin with the motion of test particles in the shock potential described in the shock frame:

$$\ddot{x} = \omega_1 \mathcal{R}^{-1} \dot{y} + \mathcal{R}^{-1} \frac{V_S^2}{2L_S}, \quad \ddot{y} = -\omega_1 \mathcal{R}^{-1} \dot{x} - \omega_1 \mathcal{R}^{-1} V_{SW}, \quad 0 \leq x \leq L_S, \quad (43)$$

where V_S describes the strength of a linear electrical potential across the shock with a width of L_S . The width of the cross-shock potential L_S may be of the order of the electron inertial length, $L_E \approx c/\omega_{pe}$, or the proton inertial length $L_p \approx c/\omega_{pp}$, or at most the proton gyro-radius $r_g \approx V_{SW}/\Omega_{ds}$, where Ω_{ds} is the downstream angular proton gyro-frequency. The parameter V_S must be of the order of the solar wind speed because the cross-shock potential slows down the supersonic solar wind to its downstream speed. The coordinates are chosen such that x denotes the motion perpendicular to the shock front, while y describes the motion parallel to it. The downstream region is in negative x -direction. The term $-\omega_1 \mathcal{R}^{-1} V_{SW}$ describes the acceleration in the convective (motional) electric field, $\mathbf{E}_m = \mathbf{V}_{SW} \times \mathbf{B}_{up}$, measured in the frame of the stationary termination shock, where ω_1 is the upstream proton gyro-frequency at the termination shock. Note, that without the shock potential, the motion with drift speed $\dot{x} = -V_{SW}$

superposed with a gyration with angular frequency $\omega_q = \omega_1 \mathcal{R}^{-1}$ solves Eq. (43). In the region of the shock potential, the first-order solution for $\dot{y}$ is

$$\dot{y} = -\omega_q (v_{xL} + V_{SW}) t + v_{yL} \Rightarrow \ddot{x} = -\omega_q^2 (v_{xL} + V_{SW}) t + \omega_q v_{yL} + \frac{V_S^2}{2L_S \mathcal{R}}. \quad (44)$$

From that we can derive the condition for transmission through the shock, $x = 0$:

$$\begin{aligned} 0 &= L_S + v_{xL} t + \frac{1}{2} \left(\omega_q v_{yL} + \frac{V_S^2}{2L_S \mathcal{R}} \right) t^2 - \frac{\omega_q^2}{6} (v_{xL} + V_{SW}) t^3 \Rightarrow \\ 0 &= 1 + \frac{v_{xL}}{V_q} (\omega_q t) + \frac{1}{2V_q} \left(v_{yL} + \frac{V_S^2}{2V_q \mathcal{R}} \right) (\omega_q t)^2 - \frac{v_{xL} + V_{SW}}{6V_q} (\omega_q t)^3 \quad \text{with } V_q = L_S \omega_q. \end{aligned} \quad (45)$$

For typical values of the solar wind density and the heliospheric magnetic field near the termination shock, $n_{SW;TS} \approx 10^6 \text{ m}^{-3}/85^2$ and $B_{TS} \approx 0.05 \text{ nT}$, the proton angular gyro-frequency is $\omega_1 \approx 0.005 \text{ Hz}$, and the electron inertial length is $L_e \approx 4 \times 10^5 \text{ m}$, the proton inertial length is $L_p \approx 2 \times 10^7 \text{ m}$, and the gyro-radius of a solar wind proton is about $r_g \approx 10^8 \text{ m}$. The parameter V_q ranges from $0.004 V_{SW}$ if the shock width $L_S \approx L_e$ to V_{SW} in case of $L_S \approx r_g$. Most theoretical models indicate that the shock width L_S has a value somewhere between L_e and L_p . Therefore, for times $t < \omega_q^{-1}$, the quadratic and linear term in the above equation are larger than the cubic term for most velocities (v_{xL}, v_{yL}) in the above equation. In fact, the ions with $v_{xL} \approx -V_q$ and $v_{yL} \approx -V_S^2 / (2V_q \mathcal{R})$ belong to the minority of ions which undergo the process of shock surfing or multiple reflections and gain substantial energy in the motional electric field of the shock. We refer to (Lee et al. 1996) for a full treatment of the dynamics of this minority population.

We continue with the solution to the quadratic part of Eq. (45), which is valid for the majority of ions:

$$\begin{aligned} t_{\text{trans}} &= -\omega_q^{-1} \frac{v_{xL}}{v_{yL} + V_S^2 / (2V_q \mathcal{R})} \left[1 - \sqrt{1 - \frac{2V_q}{v_{xL}^2} \left(v_{yL} + \frac{V_S^2}{2V_q \mathcal{R}} \right)} \right] \\ t_{\text{trans}} &> 0 \text{ for } v_{xL} < 0, |v_{yL}| < \frac{V_S^2}{2V_q \mathcal{R}}, \text{ and } v_{xL}^2 - 2V_q v_{yL} - V_S^2 \mathcal{R}^{-1} > 0. \end{aligned} \quad (46)$$

The time t_{trans} after which an ion with velocity (v_{xL}, v_{yL}) is transmitted through the shock is usually smaller than $t_q = \omega_q^{-1} \approx 200 \text{ s } \mathcal{R}^{-1}$. This time must be a real value which requires the discriminant of the quadratic equation to be larger than zero. This leads to the above shown relation between v_{xL} and v_{yL} . Defining μ' by $v_{xL} = v\mu'$ and $v_{yL} = v\sqrt{1 - \mu'^2} \approx v(1 - \mu'^2)$, the condition for transmission for $V_q \ll V_S$ and $v \gg 2V_q$ translates to

$$\mu'^2 \geq \mu'_{\text{min}}^2 = \frac{2V_q v + V_S^2 \mathcal{R}^{-1}}{v(v + 2V_q)} \Rightarrow 1 - \mu'_{\text{min}}^2 \approx 1 - \frac{u_S^2}{u^2 \mathcal{R}} \text{ with } u_S^2 := \frac{V_S^2}{V_{SW}^2} \text{ and } u := \frac{v}{V_{SW}}. \quad (47)$$

The value $1 - \mu'_{\text{min}}^2$ is the transmission coefficient for ions with speed u and mass-to-charge ratio $\mathcal{R}$, if the velocity distribution is isotropic. The transmission is larger for higher speeds and for larger mass-to-charge ratio. The ions that are reflected by the shock potential are accelerated in the motional electric field near the shock until they fulfill the condition for transmission.

Three populations approach the termination shock from the upstream solar wind: 1) the bulk solar wind ions idealized as a pencil beam $f_{\text{bulk}} \propto \delta(u - 1, \mu - 1)$; 2) the freshly ionized pick-up ions in a shell distribution $q(u)$; and 3) the suprathermal tails $f_{ST} \propto u^{-\alpha}$ for $u > u_{\text{min}}$. The suprathermal tails at the termination shock presumably reach down to almost $u_{\text{min}} \approx 1$ because the speeds of the waves causing these tails are much smaller than the speed $U_{SW} = 1$ of the supersonic bulk solar wind. Population 2) is presumably negligible at the termination shock. For the protons at the Jovian bow shock it is already hard to identify population 2) below the suprathermal tails (Fig. 5), and the flux of the tails scales as ρ^{-1} while the flux of the freshly ionized pick-up ions scales as ρ^{-2} . In a very idealized picture the cross-shock potential is characterized by $u_S = V_S/V_{SW} = 1$ which stops the bulk protons to zero speed and conserves the number of suprathermal ions at $u > u_{\text{min}}$. Of course, in reality u_S is less than unity because the downstream plasma does not have zero speed.

We define a normalized transmission function $T_{S;\mathcal{R}}$ for the suprathermal tails which yields the downstream distribution function when multiplied with the upstream distribution function. The upstream distribution function is assumed to be a power-law above the minimum speed u_{min} with the same spectral index α as upstream. The data of Fig. 5 suggest that this is a valid approach. Therefore, the normalized transmission function $T_{S;\mathcal{R}}$ for the suprathermal tails at speeds $u > u_{\text{min}}$ is:

$$\int_{u_{\text{min}}}^{\infty} u^{-\alpha+2} du = T_{n;\mathcal{R}} \int_{\frac{u_S}{\sqrt{\mathcal{R}}}}^{\infty} \left(1 - \frac{u_S^2}{\mathcal{R}u^2} \right) u^{-\alpha+2} du \Rightarrow T_{n;\mathcal{R}} = \frac{\alpha - 1}{2} \left(\frac{u_S}{\sqrt{\mathcal{R}}u_{\text{min}}} \right)^{\alpha-3}; \quad T_{S;\mathcal{R}} = T_{n;\mathcal{R}} \left(1 - \frac{u_S^2}{\mathcal{R}u^2} \right). \quad (48)$$

The normalizing factor $T_{n;\mathcal{R}}$ indicates the increase of the phase space density at high speeds downstream, and, hence, can yield an increase in the injection efficiency. For protons and $u_S \approx u_{\min} \approx 1$, the factor $T_{S;\mathcal{R}=1} (u \rightarrow \infty) \rightarrow 2$ for $\alpha = 5$ ($T_{n;\mathcal{R}=1} \approx 2$).

The data of Fig. 5, however, suggest $T_{n;\mathcal{R}=1} \approx 5$. This presumably is due to the fact that some of the bulk protons are also injected into the suprathermal tails. We include this “injection efficiency” into the ratio $u_S/u_{\min}$. The data of the Jovian bow shock give $u_S^2/u_{\min}^2 \approx 2.5$. This ratio is probably smaller at the termination shock because the ratio of the bulk proton flux to the flux in the suprathermal tails is probably lower, possibly by a factor 5/85.

4. Acceleration of pick-up ions in the subsonic solar wind

Subsequent stochastic acceleration in the subsonic heliosheath plasma further increases the phase space density of the suprathermal ions transmitted through the termination shock. Ions with energies above the injection threshold diffuse back to the upstream solar wind plasma. We derive a model for the suprathermal ion population created by stochastic acceleration in the heliosheath plasma. We compare the theoretical flux with ENA flux data measured with CELIAS/HSTOF at 1 AU. This section very closely follows the work by Kallenbach et al. (2004).

The transport Eq. (18) also applies for the heliosheath, although the parameters are different. The convection speed is only 45 km s^{-1} according to Voyager 1 data (Krimigis et al. 2003), if it really has entered the heliosheath. This is a factor $\xi_V = V_{\text{SW;up}}/V_{\text{SW;ds}} \approx 10$ less compared to the region far upstream from the termination shock i.e. at a few AU. Of course, this is a controversial number which may have to be modified in the future, but we take it as baseline here. The Alfvén speed in the heliosheath is probably about the same as in the upstream solar wind. The pick-up ion source term diminishes to a lower speed scale because the ions are picked up by a much slower plasma. The Alfvénic turbulence power is increased by a factor ranging from $a_{\text{TS}} \approx 4$ to $a_{\text{TS}} \approx 400$ or even higher, depending on the efficiency of upstream Alfvén wave generation. Both, the increase of the convection time scale τ_{conv} as well as the increase of the relative turbulent power ζ_A increase the importance of stochastic acceleration in the heliosheath compared to the situation of the supersonic solar wind. The momentum diffusion parameter D_A is increased by a factor of $\xi_V^3 \approx 10^3$ if the transport Eq. (18) is rewritten in speed units of $V_{\text{SW;ds}}$, $w = v/V_{\text{SW;ds}}$. We do not treat the effect of statistical acceleration in large-scale magnetosonic waves and in ion-acoustic waves because we do not know the speed fluctuations in the heliosheath. Therefore, the efficiency of momentum diffusion in Alfvénic turbulence gives a lower limit of the minimum contribution of ENAs originating in the heliosheath to the ENAs observed by HSTOF (Hilchenbach et al. 1998).

The source of the population accelerated by Alfvén waves is assumed to be the suprathermal pick-up ion distribution f_{UP} which is convected to the heliosheath from the upstream solar wind region. The population f_{UP} is described by a power law with spectral index $\alpha \approx 5 \dots 6$. As already mentioned, we take as baseline for slow solar wind conditions the numbers $f_{\text{UP;p}}(u, \rho) \approx g_{\text{UP;p}}(u)\rho^{-1} \approx 50 \rho^{-1} u^{-5} \text{ s}^3 \text{ km}^{-6}$ for H^+ and $f_{\text{UP;He}}(u, \rho) \approx g_{\text{UP;He}}(u)\rho^{-1} \approx 4 \rho^{-1} u^{-5} \text{ s}^3 \text{ km}^{-6}$ for He^+ , which are close to observations (Gloeckler 2003). We define $g_{\text{p;0;u}} = 50 \text{ s}^3 \text{ km}^{-6}$, $g_{\text{He;0;u}} = 4 \text{ s}^3 \text{ km}^{-6}$, $g_{\text{p;0;w}} = \xi_V^\alpha g_{\text{p;0;u}} \approx 5 \times 10^6 \text{ s}^3 \text{ km}^{-6}$, and $g_{\text{He;0;w}} = \xi_V^\alpha g_{\text{He;0;u}} \approx 4 \times 10^5 \text{ s}^3 \text{ km}^{-6}$. The pick-up ions freshly ionized in the heliosheath plasma are neglected.

To keep the problem treatable, we assume that D_A scales with $\rho^{-1}\rho_{\text{TS}}^{-1}$ in the heliosheath. As the heliopause is perhaps at $\rho_{\text{HP}} \approx 2\rho_{\text{TS}}$, the scaling with ρ^{-1} is similar to a ρ^{-1} -dependence over the range of heliocentric distance in the heliosheath. The transport equation is then:

$$\frac{\partial f}{\partial \rho} = \frac{2w}{3\rho} \frac{\partial f}{\partial w} + \frac{D_{\text{A;H}}}{\rho w^2} \frac{\partial}{\partial w} \left[w^{8/3} \frac{\partial f}{\partial w} \right] + Q(\rho, w); \quad D_{\text{A;H}} = D_{\text{A;E}} \xi_V^3 \frac{a_{\text{TS}}}{\rho_{\text{TS}}} = 2.5 \times 10^4 \mathcal{R}^{-1/3} \frac{a_{\text{TS}}}{\rho_{\text{TS}}} \zeta_{\text{A;E}}. \quad (49)$$

The homogeneous solution ($Q = 0$) for the above equation is

$$f_{\text{HS;hom}}(\rho, w) \approx \rho^{-8/9} w^{-5/3} \exp\left(-\frac{w^{4/3}}{2D_{\text{A;H}}}\right). \quad (50)$$

The solution f_{HS} in the heliosheath, however, must be a continuation of the distribution f_{trans} transmitted from the upstream solar wind at the termination shock.

Without stochastic acceleration, the function

$$f_{\text{trans}}(w) = T_{n;\mathcal{R}} g_{0;\text{w}} w^{-\alpha} \rho_{\text{TS}}^{(\beta'-1)} \rho^{-\beta'} - T_{n;\mathcal{R}} \frac{w_S^2}{\mathcal{R} w^2} g_{0;\text{w}} w^{-\alpha} \rho_{\text{TS}}^{(\beta''-1)} \rho^{-\beta''} \quad (51)$$

solves the transport equation in the downstream plasma ($\rho > \rho_{\text{TS}} \approx 85$), where we have made use of the transmission function (48). The spectral index α in speed is assumed to be the same upstream and downstream. The parameters β' and β'' are $\beta' = 2\alpha/3$ and $\beta'' = 2\alpha'/3$, where $\alpha' = \alpha + 2$.

Including the term describing the stochastic acceleration the function

$$q_{\text{inj}}(w) = T_{n;\mathcal{R}} g_{0;\text{w}} D_{\text{A;H}} \left[\alpha(\alpha - 5/3) \frac{w^{-\alpha-4/3}}{\rho_{\text{TS}}^2} \left(\frac{\rho}{\rho_{\text{TS}}}\right)^{-\beta'-1} - \alpha'(\alpha' - 5/3) \frac{w_S^2 w^{-\alpha'-4/3}}{\mathcal{R} \rho_{\text{TS}}^2} \left(\frac{\rho}{\rho_{\text{TS}}}\right)^{-\beta''-1} \right] \\ \approx q_{\text{inj;0}} D_{\text{A;H}} w^{-\alpha-4/3} \rho^{-17/9} - q_{\text{inj;1}} D_{\text{A;H}} w^{-\alpha'-4/3} \rho^{-17/9};$$

$$q_{\text{inj};0} = T_{\text{n};\mathcal{R}} g_{0;\text{w}} \alpha (\alpha - 5/3) \rho_{\text{TS}}^{-1/9} A'; \quad q_{\text{inj};1} = T_{\text{n};\mathcal{R}} w_{\text{S};\mathcal{R}}^2 \mathcal{R}^{-1} g_{0;\text{w}} \alpha' (\alpha' - 5/3) \rho_{\text{TS}}^{-1/9} A''; \\ A' = \frac{8}{9\beta'} \frac{1 - (\rho_{\text{HP}}/\rho_{\text{TS}})^{-\beta'}}{1 - (\rho_{\text{HP}}/\rho_{\text{TS}})^{-8/9}}; \quad A'' = \frac{8}{9\beta''} \frac{1 - (\rho_{\text{HP}}/\rho_{\text{TS}})^{-\beta''}}{1 - (\rho_{\text{HP}}/\rho_{\text{TS}})^{-8/9}}, \quad (52)$$

remains when entering f_{trans} into the transport Eq. (49). The term $q_{\text{inj}}(w)$ has been approximated by the power $\rho^{-17/9}$ for the range $\rho_{\text{TS}} \leq \rho \leq \rho_{\text{HP}}$. This does not introduce a large error in the upstream (apex) direction of the interstellar gas flow because the heliopause may be located at $\rho_{\text{HP}} \approx 2\rho_{\text{TS}}$. For the heliotail direction the error is larger. The factors A' (and A'' correspondingly) are introduced such that the term $A'\rho^{-17/9}$ has the same value in average over the range $\rho_{\text{TS}} \leq \rho \leq \rho_{\text{HP}}$ as the term $\rho^{-\beta'-1}$. The transport equation must be solved including the function $q_{\text{inj}}(w)$. As it looks mathematically equivalent to a source term for the homogeneous solution (Eq. (50)), we have chosen the denotation q_{inj} for this function. To ease the following book-keeping we introduce the variable $w_{\text{S};\mathcal{R}} := w_{\text{S}}/\sqrt{\mathcal{R}}$.

We try to solve the inhomogeneous equation again by the “variation of the constant”. Multiplying $f_{\text{HS;hom}}(w)$ by another function $h(w)$, and entering the product into the transport equation leads to

$$0 = w \frac{d^2 h}{dw^2} - \frac{2}{3} \frac{dh}{dw} - \frac{2}{3D_{\text{A;H}}} w^{4/3} \frac{dh}{dw} + \left(w^{-\alpha+2/3} q_{\text{inj};0} - w^{-\alpha'+2/3} q_{\text{inj};1} \right) \exp\left(\frac{w^{4/3}}{2D_{\text{A;H}}}\right) \Rightarrow \\ h(w) = q_{\text{inj};0} \frac{w_{\text{S};\mathcal{R}}^{-\alpha+5/3} - w^{-\alpha+5/3}}{(\alpha - 5/3)(\alpha - 4/3)} - q_{\text{inj};1} \frac{w_{\text{S};\mathcal{R}}^{-\alpha'+5/3} - w^{-\alpha'+5/3}}{(\alpha' - 5/3)(\alpha' - 4/3)} \quad \text{if } w_{\text{S};\mathcal{R}} < w \ll D_{\text{A;H}}^{3/4}. \quad (53)$$

We have defined $h(w)$ such that $h(w_{\text{S};\mathcal{R}}) = 0$ because the distribution function that describes the suprathermal particles transmitted through the termination shock vanishes at $w < w_{\text{S};\mathcal{R}}$. At the speed $w_{\text{S};\mathcal{R}}$ the process of stochastic acceleration presumably operates because the momentum diffusion parameter $D_{\text{A;H}}$ applies for super-Alfvénic particles, i.e. for $w > 2$ as $V_{\text{SW;ds}} \approx V_{\text{A}}$. Note, that for a minimum amplification of the turbulence $a_{\text{TS}} \approx 4$ at the termination shock, the diffusion parameter is about $D_{\text{A;H}} \approx 7$. Therefore the assumption $w_{\text{S};\mathcal{R}} \ll D_{\text{A;H}}^{3/4}$ is not well justified at $a_{\text{TS}} \approx 4$, but better justified at any larger a_{TS} .

For speeds $w \gg D_{\text{A;H}}^{3/4}$, it can be shown that $h(w) f_{\text{HS;hom}}(w) \ll f_{\text{trans}}(w)$ despite the exponential term $\exp[w^{4/3}/(2D_{\text{A;H}})]$ in the differential Eq. (53) for $h(w)$. This term is compensated by the exponential term $\exp[-w^{4/3}/(2D_{\text{A;H}})]$ of $f_{\text{HS;hom}}(w)$. For $w \gg [3D_{\text{A;H}}(\alpha - 5/3)/2]^{3/4}$, the function $f_{\text{trans}}(w)$ approximately fulfills the transport equation of the heliosheath.

Therefore, the sum of the functions f_{trans} and $h f_{\text{HS;hom}}$ fulfills fairly well both the transport equation for the heliosheath and the boundary condition at the termination shock. For speeds $w \gg w_{\text{S};\mathcal{R}}$ and using $(\rho_{\text{HP}}/\rho_{\text{TS}})^{-\beta'} \ll 1$, the total distribution function is finally expressed as

$$f_{\text{HS}}(w, \rho) \approx g_{0;\text{w}} T_{\text{n};\mathcal{R}} \left[w^{-\alpha} \rho^{-\beta'} \rho_{\text{TS}}^{\beta'-1} + \frac{4\rho_{\text{TS}}^{-1/9} \rho^{-8/9} w^{-5/3} w_{\text{S};\mathcal{R}}^{5/3-\alpha}}{(3\alpha - 4) [1 - (\rho_{\text{TS}}/\rho_{\text{HP}})^{8/9}]} \exp\left(-\frac{w^{4/3}}{2D_{\text{A;H}}}\right) \right]. \quad (54)$$

Note, that the damping of the Alfvén waves has not been included into the derivation. The damping of Alfvén waves in the bulk plasma is very weak similar to the case of the solar wind. However, the energetic particles themselves damp the Alfvén waves with which they resonate and through which they gain energy. Without solving the full self-consistent problem, the damping of Alfvén waves can be described by replacing ρ_{HP} by $\rho_{\text{TS}} + \Delta\rho_{\text{A}}$, where $\Delta\rho_{\text{A}}$ is the damping length of the Alfvén waves.

The differential flux $j_{\text{ENA}}(v) = f_{\text{ENA}}(v)v^2/m$ of ENAs near Earth’s orbit in units of $\text{cm}^{-2} \text{s}^{-1} \text{keV}^{-1} \text{amu sr}^{-1}$ is calculated from the phase space density $f_{\text{PUI}}(v, \rho)$ in $\text{s}^3 \text{km}^{-6}$ of heliospheric suprathermal pick-up ions (Gruntman et al. 2001) as

$$j_{\text{ENA}}(v, \rho_{\text{E}}) \approx \frac{r_{\text{E}}}{2} \int_1^{\rho_{\text{HP}}} \sigma(v) n_{\text{n}} f_{\text{PUI}}(v, \rho) p_{\text{surv}}(v, \rho_{\text{E}}, \rho) v^2 v_{\text{I}}^2 \text{keV} d\rho \\ \approx 4.5 \times 10^{-6} \text{cm}^{-2} \text{s}^{-1} (\text{keV/amu})^{-1} \text{sr}^{-1} g_{0;\text{w}} \text{s}^{-3} \text{km}^6 [A_{\text{ST}}(w) + A_{\text{HS}}(w) + A_{\text{SA}}(w)], \\ A_{\text{ST}}(w) = w^{2-\alpha} \ln \rho_{\text{TS}}, \\ A_{\text{HS}}(w) = w^{2-\alpha} \frac{3(\alpha - 1)}{(4\alpha - 6)} \left(\frac{w_{\text{S};\mathcal{R}}}{w_{\text{min}}}\right)^{\alpha-3} \left[1 - \left(\frac{\rho_{\text{TS}}}{\rho_{\text{HP}}}\right)^{2\alpha/3-1}\right], \\ A_{\text{SA}}(w) = \frac{18(\alpha - 1)}{(3\alpha - 4)} \left(\frac{w_{\text{S};\mathcal{R}}}{w_{\text{min}}}\right)^{\alpha-3} \frac{(\rho_{\text{HP}}/\rho_{\text{TS}})^{1/9} - 1}{1 - (\rho_{\text{TS}}/\rho_{\text{HP}})^{8/9}} w^{1/3} w_{\text{S};\mathcal{R}}^{5/3-\alpha} \exp\left(-\frac{\mathcal{R}^{1/3} \rho_{\text{TS}} w^{4/3}}{5 \times 10^4 a_{\text{TS}} \zeta_{\text{A;E}}}\right), \quad (55)$$

where, for example, $g_{\text{p};0;\text{w}} \approx 5 \times 10^6 \text{s}^3 \text{km}^{-6}$. The parameter $\sigma(v)$ is the velocity-dependent charge exchange cross section between the suprathermal pick-up ions and neutral atoms. For protons at energies of 58–88 keV this cross section is about 10^{-26}km^2 for both the charge exchange with hydrogen atoms and with helium atoms. The neutral density has been taken to be $n_{\text{n}} \approx 5 \times 10^{13} \text{km}^{-3}$. The survival probability p_{surv} of hydrogen atoms with energies of more than 55 keV to reach Earth’s orbit from the heliosheath is close to unity (Gruntman et al. 2001). At energies of 58–88 keV/amu the dimensionless parameter w is of order $w \approx 80$.

Fig. 6. Model functions of Eq. (55) for four situations: typical suprathermal tails of the slow solar wind fed into the heliosheath in the apex direction (*upper left*) and into the heliotail (*upper right*), and typical suprathermal tails of the fast solar wind fed into the heliosheath in the apex direction (*lower left*) and into the heliotail (*lower right*). The numbers 1, 2, 3, 4, and 5 denote the contributions to the ENA flux from the region outside the termination shock without stochastic acceleration in the heliosheath (1), from the region inside and outside the termination shock without stochastic acceleration in the heliosheath (2), from stochastic acceleration with turbulence parameters (see text) $a_{TS\zeta_{A,E}} = 0.012$ (3), $a_{TS\zeta_{A,E}} = 0.12$ (4), and $a_{TS\zeta_{A,E}} = 1.2$ (5). The parameters $w_{S,R} = 5$ and $w_{min} = 3$, which reproduce the injection efficiency of the bulk solar wind protons at the Jovian bow shock, have been used. Data of the IMAGE/HENA instrument are only upper limits for the flux (Ed Roelof, private communication). HSTOF data distinguish spectra of ENAs detected in the apex direction and in the heliotail direction. Highest flux has been detected during solar activity minimum.

Equation (55) provides a simple analytical formula to estimate the contribution of suprathermal particles inside and outside the termination shock to the energetic neutral atom production. The first term of j_{ENA} , A_{ST} , is the convected power law matching SWICS observations in the slow solar wind between 1 AU and 5 AU (Gloeckler 1999) and the low-energy part of the Voyager 1 spectra at 85 AU (Krimigis et al. 2003). The second term A_{HS} describes the power-law distribution of suprathermal ions transmitted through the termination shock and neutralized in the heliosheath. The term A_{HS} includes the acceleration of the ions in the motional electric field, but not stochastic acceleration in the heliosheath. The third term A_{SA} describes the stochastic acceleration by Alfvén waves in the heliosheath. Note that again the self-consistent damping of Alfvén waves by energetic ions in the heliosheath can be introduced to first order by replacing ρ_{HP} by $\rho_{TS} + \Delta\rho_A$. If $\Delta\rho_A \ll \rho_{TS}$ the contribution to the ENA flux $A_{SA}(w)$ from the heliosheath is reduced by a factor of about 15 compared to the case of $\Delta\rho_A \approx \rho_{TS}$.

If the injection threshold at the termination shock is above about 100 keV/amu, the ENA flux detected by HSTOF near Earth’s orbit must come from the population described by Eq. (55). Figure 6 shows the model functions and spacecraft data for four stationary situations: suprathermal tails of the slow or fast solar wind fed into either the upwind heliosheath or into the heliotail, respectively. Charge exchange cross sections $\sigma(v)$ and survival probabilities p_{surv} are incorporated numerically from (Gruntman et al. 2001). We note that the ENA flux detected by HSTOF near Earth’s orbit is compatible with the stationary situation when the typical suprathermal tails of the slow solar wind are fed into the heliosheath for both the apex and the anti-apex direction. Only moderate Alfvén wave amplification by ACRs and/or pick-up ions upstream from the termination shock is necessary to reconcile the final ENA flux near Earth’s orbit. The field of view of the HSTOF sensor is always near the ecliptic plane, so that HSTOF may preferentially detect ENAs from heliosheath regions that are fed by slow solar wind. In fact, the ENA flux detected by HSTOF was considerably higher during solar activity minimum, when fairly stable slow solar wind streams but also co-rotating interaction regions (CIRs) were present in the ecliptic plane. Both, the slow solar wind and CIRs generate strong suprathermal

Fig. 7. Injection efficiencies of H^+ , He^+ , and O^+ ACR ions as a function of the injected phase space density of the proton suprathermal tails characterized as in Fig. 3. Same as Fig. 4 except that injected abundances of He and O are modified by the processes of acceleration in the electric potential of the termination shock and by stochastic acceleration in Alfvénic fluctuations in the heliosheath.

tails. Modest acceleration of pick-up ions in the electric field of the termination shock and in the turbulence of the heliosheath would both explain HSTOF data.

However, enhanced suprathermal particle flux in the inner and middle heliosphere would do so as well (Kóta et al. 2001), which makes it difficult to decide which fraction of ENAs detected by HSTOF originate in the heliosheath, and which fraction in the supersonic solar wind. The spectra marked by label 1 in the upper panels of Fig. 6 denote the ENAs originating from suprathermal ions neutralized in the supersonic slow solar wind. Their flux is not much below the flux detected by HSTOF.

Data of the IMAGE/HENA instrument – detecting neutrals at even lower energies than HSTOF i.e. possibly further below the injection threshold – are compatible with the situation when fast solar wind is fed into the heliosheath. The field of view of the HENA instrument very often includes regions of the heliosheath that are fed by the fast solar wind. The seeming discrepancy between the HENA data and the HSTOF data – HSTOF has detected ENAs, while HENA has not – may be caused by the different view directions of the instruments. Furthermore, HENA has only operated during solar activity maximum, while HSTOF data are dominated by times during solar activity minimum.

Regarding the observed abundances of Anomalous Cosmic Rays i.e. the observed relative enrichment of He over H by a factor 10 and of O over He by another factor 10, we note that Eq. (54) predicts an enhancement of the abundance in the suprathermal tails that scales with $\mathcal{R}^{2/3}$. For singly charged ions this means an ordering by atomic mass $A^{2/3}$. This factor comes from the product of $T_{n;\mathcal{R}} \propto \mathcal{R}^{(3-\alpha)/2}$ with $w_{S;\mathcal{R}}^{5/3-\alpha} \propto \mathcal{R}^{(\alpha-5/3)/2}$. This product is proportional to $\mathcal{R}^{2/3}$. If we multiply the injected abundances of He and O by these factors we arrive at the injection efficiencies plotted in Fig. 7. These injection efficiencies match observations (Cummings et al. 2002).

5. Discussion of the Voyager 1 data

There are, in our view, three possibilities to explain the Voyager 1 observations near 85 AU at 34° N of long-lasting energetic particle enhancements below 1 MeV/amu.

1. Voyager 1 has crossed the termination shock in 2002. As has been mentioned in Sect. 2.4, the low-energy part of the spectra observed with Voyager 1 at 85 AU (Fig. 4 of Krimigis et al. 2003, below a few MeV/amu and Fig. 2, *left*) may represent suprathermal tails convected from the slow solar wind into the heliosheath being accelerated in the electric field of the termination shock and in the turbulence of the heliosheath. The ENA flux created by these ion spectra approximately corresponds to the ENA spectra marked by label 1 in the upper panels of Fig. 6. The rise in energetic particle flux observed by Voyager 1 near 85 AU may mark the crossing of a magnetic boundary separating regions where the heliosheath is fed by fast solar wind and by slow solar wind. The modulated high-energy ACR spectra may represent a more distant source in regions where the termination shock is more quasi-parallel.
2. Voyager 1 has not crossed the termination shock in 2002. The Voyager 1 particle enhancements may simply mark the crossing of a solar wind boundary between fast and slow streams. In that case the low-energy part of the Voyager 1 data represent extensions of the suprathermal tails in the slow supersonic solar wind of the outer heliosphere. In the fast solar wind the flux level of the suprathermal tails is much lower than in the slow solar wind. This interpretation would explain, why the magnetic

field data do not look like a shock transition during the rise of energetic particles, and it would explain, why the spectra above a few MeV/amu look like those of a distant source (McDonald et al. 2003).

3. An alternative interpretation is that Voyager 1 had not reached the heliosheath yet in 2002, and that there is magnetic connection to the near-by quasi-perpendicular termination shock which generates approximately the ACR flux level as observed below 1 MeV/amu and as modelled in Fig. 3 (*left*). The modulated high-energy ACR spectra above a few MeV/amu may represent a more distant source in regions where the termination shock is more quasi-parallel and therefore more efficient in accelerating ACR ions. This is again concordant with the view that these high-energy spectra look like those of a distant source (McDonald et al. 2003). This interpretation would also be concordant with the observation that the magnetic field data do not look like a shock transition during the rise of energetic particles.

6. Conclusions

Our model gives the following results:

1. The question whether there is an “injection problem” at the solar wind termination shock may rather be answered by “no”. In the absence of ACRs slightly super-Alfvénic ions are easily accelerated by the first-order Fermi process. The injection threshold in the absence of ACRs is derived from the large parallel mean free paths unless the termination shock is strictly perpendicular. With increasing ACR intensity the injection threshold increases if the ACRs amplify Alfvén waves upstream of the termination shock. These Alfvén waves are further amplified when they pass the shock. They cause a reduction of the parallel mean free path. In that sense the efficiency of the first-order Fermi process adjusts itself self-consistently. Possibly, the parallel mean free path is reduced so much that the first-order Fermi process is rather dominated by perpendicular diffusion. Perpendicular diffusion has increasing acceleration time in high levels of Alfvénic fluctuations (Eq. (34)), which also limits the efficiency of ACR production.
2. Regarding the pre-acceleration processes in the upstream supersonic solar wind, we conclude that the compressional fluctuations of the slow supersonic solar wind are more efficient in creating or maintaining suprathermal tails in the ion distribution functions than the predominantly Alfvénic fluctuations in the fast supersonic solar wind. This agrees with observations. The turbulence levels of compressional fluctuations necessary to reconcile the observed spectral indices are consistent with observations on both magnetosonic and ion-acoustic waves in the slow solar wind, although more experimental work remains to be done. In particular there is no full characterization of compressional and non-compressional fluctuations in the supersonic solar wind in the outer heliosphere. From the theoretical point of view, the analytical estimates require compromises and assumptions that need to be kept in mind when comparing the model to spacecraft data. However, the analytical results give easy-to-estimate baseline values to be refined by numerical models.
3. Stochastic acceleration in the heliosheath plasma downstream of the termination shock may be very efficient because of the increased convection time scale, which simply leaves more time for acceleration. We only can present some predictions for stochastic acceleration in Alfvénic turbulence because there are some estimates on the amplification of Alfvénic turbulence at the termination shock (McKenzie & Westphal 1969; Bamert et al. 2004). The level of compressional fluctuations in the heliosheath plasma are even less known. Pre-acceleration of ions by Alfvén waves in the heliosheath would prefer ions with high mass-to-charge ratio (Eq. (49)) to reach the injection threshold for the first-order Fermi process at the termination shock. This would be consistent with observations on the elemental abundance of the Anomalous Cosmic Rays (Cummings et al. 2002).
4. Equation (55) gives baseline values for the energetic neutral atom flux in the energy range below the threshold for injection into first-order Fermi acceleration at the termination shock from three origins: 1) from suprathermal ion populations inside the termination shock in a) the regions of the supersonic fast solar wind and b) the regions of the compressional supersonic slow solar wind; 2) from suprathermal ions directly accelerated in the motional electric field at the termination shock; and 3) the suprathermal ions stochastically accelerated in the subsonic heliosheath region. We conclude from this equation and the comparison of its terms to spacecraft data that it could possibly be difficult to image the region outside the termination shock by energetic neutral atom (ENA) flux at energies below 100 keV/amu near Earth’s orbit. If there is no efficient stochastic acceleration by turbulent waves in the heliosheath plasma, most of the ENA flux at 1 AU originating from the region near and outside the quasi-perpendicular termination shock is obscured by ENA flux originating from charge exchange of suprathermal ions inside the termination shock. This needs to be considered with caution in particular if observations are restricted to energies below 16 keV/amu as planned for the IBEX mission. However, the presented model predicts that there is efficient stochastic acceleration near and outside the termination shock.

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