A&A 437, 1017–1021 (2005)
DOI: 10.1051/0004-6361:20042306

Astronomy
&
Astrophysics

RR Lyrae variables in Galactic globular clusters

V. The case of M 3 pulsators

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Received 22 October 2004 / Accepted 21 April 2005

Abstract. We use our synthetic Horizontal Branch (HB) procedure to approach the often debated problem of how adequate canonical HB stellar models are to account for the observed peaked distribution of RR Lyrae fundamentalised periods in the globular cluster M 3. We find that by assuming a suitable bimodal mass distribution, canonical models do account for the observed period distribution. In particular, the best fit model, out of nine random extractions, reaches a 99.9% Kolmogorov-Smirnov (KS) probability. We also attempt to predict the relative distribution of variables in fundamental and first overtone pulsators, reaching a satisfactory agreement. However, one finds that canonical models outnumber the observed number of red HB stars by roughly a factor of two. Possible solutions for this discrepancy are outlined. Alternative evolutionary scenarios are also briefly discussed.

Key words. instabilities – stars: interiors – stars: low-mass, brown dwarfs – stars: evolution – stars: Hertzsprung-Russell (HR) and C–M diagrams – stars: oscillations

1. Introduction

In a previous paper of this series (Cassisi et al. 2004, Paper IV) we presented the overall scenario of pulsational predictions based on our synthetic Horizontal Branch (HB) procedure, and discussed the satisfactory agreement between predictions and selected properties of RR Lyrae pulsators in globular clusters with various metallicities and/or HB types. In this paper we will approach a more detailed investigation, by discussing the case of the RR Lyrae period frequency histogram in M 3, which was recently claimed to be at variance with current predictions of stellar evolution theory.

To briefly recall the history, Castellani & Tornambé (1981) first drew attention to the peaked distribution of fundamental periods in M 3 as evidence of a peculiar distribution of stars within the instability strip. The problem was revisited by Rood & Crocker (1989), who pointed out the difficulty of accounting for the observed periods on the basis of smooth mass distributions. More recently, the same problem was addressed by Catelan (2004), who reached the conclusion that the period distribution of RR Lyrae variables in M 3 is at odds with canonical HB model predictions.

The structure of this paper is the following: in the next section we discuss results based on our synthetic procedure, showing that canonical HB models can closely reproduce the period frequency histogram of RR Lyrae in M 3, if a suitable bimodal distribution of HB masses is assumed. In Sect. 3 we will attempt to predict the relative distribution of fundamental and first overtone pulsators, while Sect. 4 deals with the distributions of stars along the HB, discussing a troublesome disagreement between the observed and predicted number of red HB stars. A few comments concerning alternative evolutionary scenarios will close the paper.

2. M 3: The models

For the evolutionary framework, we rely on the stellar models already presented in Paper IV, adopting a metallicity $Z = 0.001$ for the cluster, as in Catelan (2004). However, as outlined in Paper IV, one has to bear in mind that similar canonical models are still affected by uncertainties, due to current limits on the physical inputs. For example, changes in the adopted $^{12}\mathrm{C}(\alpha, \gamma)^{16}\mathrm{O}$ nuclear cross section (Caughlan & Fowler 1985) affect both the morphology of the evolutionary track and the core He-burning lifetime (Dorman 1992). In addition, all stellar models whose evolutionary tracks are located in the cooler portion of the HB are affected by current uncertainty in the efficiency of superadiabatic convection.

The procedure adopted for computing synthetic RR Lyrae pulsators has been exhaustively described in the previous papers of this series so it will not be repeated here. We only note that this work is based on up-to-date relations for both

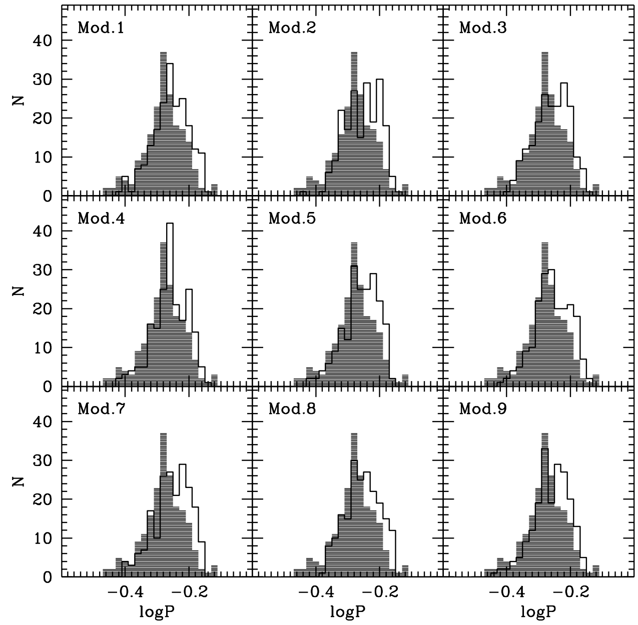

Fig. 1. Synthetic fundamentalised period frequency histograms as obtained for a mean HB mass $M = 0.68\ M_{\odot}$ with a dispersion $\sigma_M \sim 0.005$, and for nine different random extractions. The shaded area shows the actual M 3 distribution.

periods and instability boundaries, as derived and discussed in Paper III (Di Criscienzo et al. 2004). As a result, our blue instability boundary (BE), is located around $T_{\rm e} \sim 7100$–7200 K, i.e., about 200 K cooler than in Case A in Catelan (2004). Moreover, the width of the instability strip is not a free parameter, but is fixed by pulsational constraints at a temperature cooler than the BE temperature by about $\Delta T_{\rm e} \sim 1200$ K.

However, the results presented in the following appear only marginally dependent on the adopted pulsational scenario. As a matter of fact, since Van Albada & Baker's (1971) formulation of the relation connecting pulsation periods with the structural parameters M, L, and $T_{\rm e}$, it was already clear that a smooth distribution of stars within the instability strip will produce a smooth distribution of periods, independent of any plausible assumption about the instability boundaries and/or the star luminosity level (see e.g. Caputo & Castellani 1975). As a consequence, it was also clear that any smooth distribution of HB star masses, as produced by the often adopted mass dispersion $\sigma_M \sim 0.02\ M_{\odot}$, was inadequate to account for the peaked period distribution observed in M 3.

To explore the predictions of canonical HB evolutionary tracks, we decided – as a first step – to keep the assumption of a normal deviate mass distribution, progressively decreasing the adopted mass dispersion σ_M and exploring the predicted period frequency distribution with the mean mass as a free parameter. In the case that such an approach fails, we were ready to test different mass distributions. However, one finds that the period distribution in M 3 can be nicely reproduced when assuming a mean HB mass $M = 0.68\ M_{\odot}$ with a dispersion $\sigma_M \sim 0.005$.

Table 1. Values of KS probability for 9 runs, related to different samples: (a) RR Lyrae stars from the "red HB" only; (b) like (a) but after the shift in luminosity, and (c) adding the RR Lyrae from the "blue HB". The last column gives the number of red HB.

Run	KS (a)	KS (b)	KS (c)	N(red)
1	9.7 E−3	98.55	92.24	246
2	1.5 E−2	13.24	22.62	302
3	1.7 E−1	77.87	69.50	243
4	2.9 E−2	91.54	99.93	240
5	1.2 E−1	44.86	41.93	219
6	1.8 E−1	85.37	61.49	242
7	9.7 E−3	20.78	21.52	213
8	8.2 E−2	77.87	83.79	240
9	6.1 E−3	69.59	60.78	201

Figure 1 shows the results of our first nine random extractions as performed under the quoted assumption concerning the mass distribution and by populating the HB till the number of RR Lyrae observed in M 3 was reached (see Castellani et al. 2003, Paper I). Comparison with the observed distribution, as given in the same figure, unambiguously demonstrate that at least two out of the nine experiments (i.e. simulations 1 and 4) give a period frequency distribution which closely resembles the observed one.

However, one easily recognizes that the predicted period distribution appears marginally shifted toward higher values, by an amount on the order of $\Delta \log P \sim 0.02$. Such a shift is detected by the Kolmogorov-Smirnov test which gives quite a low probability between observed and predicted period distributions (Col. 2 in Table 1). According to the pulsation theory (see Paper III), one can account for such a shift by simply decreasing the adopted HB luminosity by $\Delta \log L \sim 0.02$ or, alternatively, by increasing the HB masses by $\Delta M \sim 0.04\ M_{\odot}$. We recall here that in Paper IV the HB luminosity level was calibrated to reproduce the period interval observed in M 3. In this context, the above correction appears well inside the uncertainty of that calibration.

By applying such a correction one eventually finds that the Kolmogorov-Smirnov test gives probabilities for the similarity of observed and predicted distributions that sensitively increases, ranging from 13.2% (case n.2) to 98.6% (case n.1). Column 3 in Table 1 gives the results of such a test, while Fig. 2 shows the comparison with the observed period distribution and the two "best" predictions, as given by models 1 and 4. We conclude that canonical evolutionary tracks can account for the peaked period histogram by adopting a suitable mass distribution.

According to this simulation, RR Lyrae should be HB stars crossing the instability strip during their blueward evolution from the original red Zero Age Horizontal Branch (ZAHB) location. As a consequence, the simulation is producing mainly red and RR Lyrae stars, with very few stars hotter than the instability strip. The evidence for a rich population of blue HB stars in M 3 thus requires the additional contribution of a separate population of less massive HB stars, located on the blue side of the instability strip.

One may easily predict that such an additional population should marginarly contribute to the bulk of RR Lyrae

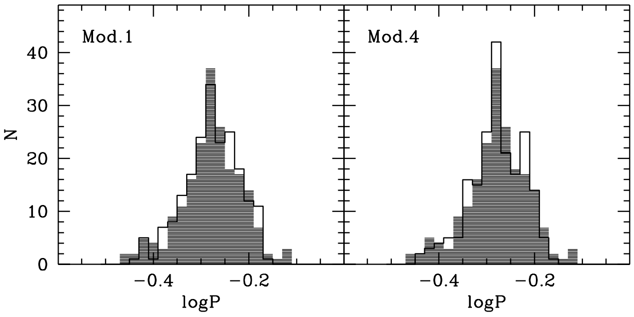

Fig. 2. The comparison between the observed period distribution and the two best synthetic predictions (models 1 and 4) as obtained from a Gaussian distribution of the HB masses after the correction discussed in the text and before addition of the blue component.

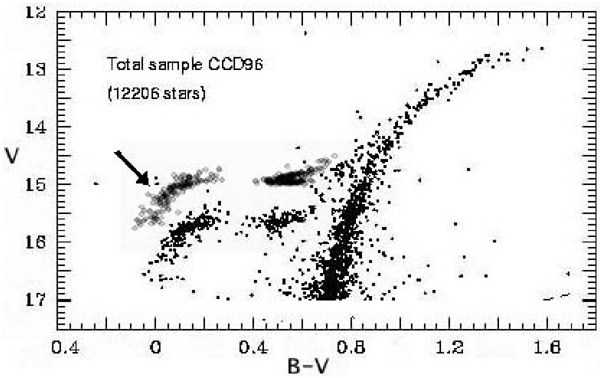

Fig. 3. The V, $B - V$ color–magnitude diagram of M 3 (Ferraro et al. 1997) compared with the predicted distribution of HB stars (arrow) from the best model (model 4), as arbitrarily shifted in magnitude.

pulsators, since only stars in the later phases of HB evolution will cross the instability strip. However, we will follow the suggestion of our referee by discussing this point in more detail. As already done by Catelan, we took the number of blue HB stars by stellar counts, populating the blue HB portion with 206 stars uniformly distributed over the range of masses 0.65 to 0.61 $M_{\odot}$. This means that we account for an HB population whose ZAHB temperatures range from $\log T_{\rm e} = 3.859$ to $\log T_{\rm e} = 4.045$, i.e., from a temperature close to the BE up to the temperature at the hot end of the bulk of M 3 blue HB stars. Note that the adopted distribution maximizes the contribution of the blue HB population to the variables. The same number of blue stars with, e.g., a Gaussian distribution around the mean mass 0.63 $M_{\odot}$ and a dispersion $\sigma_M \sim 0.01$ covers a similar range of temperatures and gives a lower number of pulsators, since a large fraction of HB stars located close to the blue instability boundary moved towards hotter colors.

Once again using a set of 9 different random extractions, we found that the number of RR Lyrae from this population ranges from 7 to 20, i.e., making a marginal but not negligible contribution to the pulsator population. Figure 3 compares the predicted color magnitude diagram distribution and observational data from Ferraro et al. (1997). The simulations account for an observational Gaussian error with $\sigma = 0.02$ in both magnitudes. Column 4 in Table 1 shows that the inclusion of such a contribution in the best case raises the KS probability up to 99.9%, thus supporting the occurrence of a bimodal distribution of HB masses in M 3.

Such a bimodal distribution is of course an unexpected feature. However, we notice that within the galactic globular cluster family this feature is far from unusual, since Harris (1974) brought forward the striking bimodal distribution of HB stars in the galactic globular NGC 2808 (see, e.g., Catelan 2004; D'Antona & Caloi 2004, and references therein). According to the result of the present investigation, one should conclude that even in the M 3 case we are faced with a bimodal HB, though one with a hidden bimodality that becomes evident only when the distribution of RR Lyrae periods is properly taken into account.

3. Fundamental and first overtone pulsators

By relying on the overall agreement between observed and predicted fundamentalised periods, one can go deeper in the comparison, thus testing the predictions of the theoretical scenario concerning the relative distribution of fundamental and first overtone pulsators. To this purpose we adopted the topology of the instability strip from pulsational theories, with first overtone pulsators in the hotter portion of the strip, fundamental pulsators in the cooler portion, and an intermediate range of temperatures where both modes can be stable (the OR zone).

Figure 4 shows that, using the recipe of the hysteresis mechanism (Van Albada & Baker 1971), i.e., by assuming that in the OR zone the variables pulsate in their previous pulsation mode, the synthetic models can reach a reasonable similarity with observations, provided that theoretical estimates for the fundamental blue boundary are decreased by about 100 K, i.e., down to $\log T_{\rm e} = 3.832$. On the contrary, the third panel in Fig. 4 shows that assuming a fixed transition temperature gives a sharp separation in the fundamentalised periods, which is not observed in M 3.

This evidence is supported by the Kolmogorov-Smirnov test for the distribution of c-type periods, as listed in Cols. 2 and 3 of Table 2. The hysteresis case, out of the 9 random extractions, reaches a maximum probability of 93.3% against 61.3% for a fixed transition. The issue is, however, open to other possible assumptions, and we present data in Fig. 4 only as a first step of a long difficult investigation that will require more efforts.

4. The problem of red HB

The main goal of this investigation has been reached by showing that there is not an intrinsic and unavoidable incompatibility between the M 3 period distribution and canonical evolutionary tracks. However, one may go deeper by discussing theoretical predictions in connection with the observed HB distribution. Synthetic models in the previous sections have been constructed by directly relying on the observed number of variables (V) and blue (B) HB stars. In contrast, the number of red (R) HB stars is a computational result, linked by evolution to the V-number.

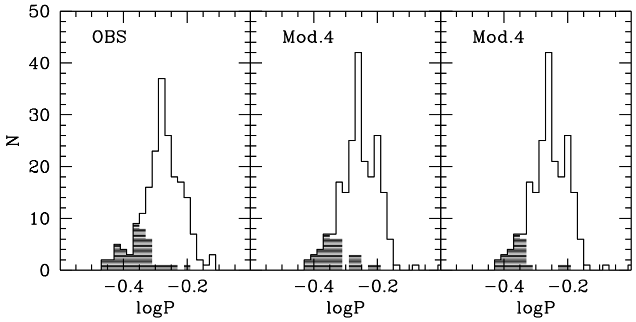

Fig. 4. The observed distribution of fundamentalised periods in M 3 (*left panel*) as compared with the best synthetic theoretical distributions obtained assuming an hysteresis mechanism (*middle panel*) or a fixed transition temperature (*right panel*). The shaded areas show the contribution of first overtone (RRc) pulsators.

Table 2. Values of KS test for the 9 runs considering the distribution of only RRc variables, and assuming the efficiency of an hysteresis mechanism (d) or a fixed transition temperature (e) (see text).

Run	*KS* (d)	*KS* (e)
1	93.35	61.29
2	7.69	19.37
3	18.98	8.44
4	91.57	36.70
5	81.22	39.61
6	89.20	54.91
7	43.99	12.90
8	7.67	14.08
9	19.05	6.16

The last column in Table 1 discloses that present computations in all cases give a number of red HB stars higher than of RR Lyrae variables. On the contrary, Catelan (2004) quotes unpublished data for which the 530 HB stars in M 3 should be distributed according to the ratios B:V:R = 0.39:0.40:0.21. These ratios, once confirmed, give the contradictory evidence for which evolutionary models produce an almost perfect fitting of the period distribution, but do not account for the observed number of 111 red HB stars.

In this context, a firm evaluation of the HB distribution appears highly relevant in assessing the adequacy of the synthetic procedure. However, one has to notice that on the theoretical side the V:R ratio can be modulated in several ways by modulating the shape of the mass distribution and/or the temperature interval covered by the evolutionary tracks. This in turn depends on several assumptions for red stars, in particular the ^{12}C$(\alpha,\gamma)^{16}$O nuclear cross section and the free parameter α governing the efficiency of convection in stellar envelopes.

As an example, one can modify the adopted Gaussian distribution by cutting away the tail of most massive stars, in this way decreasing the number of red stars but leaving the RR Lyrae distribution substantially unchanged. The number of red HB stars predicted by assuming a truncated Gaussian, i.e., neglecting all the HB masses larger than the mean mass, are listed in the last column of Table 3. It turns out that, over the 9 random extractions, the number of red HB stars reaches a minimum value of 138 stars, against the 111 given by Catelan, keeping in that case a robust 82% of KS probability. The semi-Gaussian distribution is of course an "ad hoc" assumption, but here we are just exploring whether such ad hoc assumptions can reconcile canonical models with observations. In this context, one may note that the largely adopted Gaussian mass distribution appears a useful and reasonable assumption. However, firm constraints on the plausibility of such a distribution should await new insights on the still unknown mechanism driving mass loss.

Table 3. Values of KS test for 9 runs with the truncated Gaussian assumption. The third column gives the number of red HB stars for the various cases.

Run	*KS* (f)	*N*(red)
1	61.07	165
2	37.67	169
3	13.24	149
4	4.75	164
5	31.25	204
6	61.07	180
7	10.40	159
8	81.99	138
9	37.67	145

An overabundance of red stars could also be taken as evidence for the occurrence of shorter but still canonical evolutionary tracks, as produced by passing from the adopted cross section for the ^{12}C$(\alpha,\gamma)^{16}$O reactions (Caughlan & Fowler 1985) to the revised values presented by Kunz et al. (2002; see also Dorman 1992). In principle, the same effect could be produced by an increase in the mixing length parameter α (see, e.g., Brocato et al. 1999). However, in our models the mixing length has been calibrated on the temperature of the RGB branch, and a difference of mixing length between RGB and HB stars can barely be acceptable. Even more interesting, following the recently suggested revised solar metallicity ($Z_\odot$ = 0.0122: Asplund et al. 2004), we also found that a decrease of the metallicity below Z = 0.001 would solve the problem.

In this context, one has to notice first that such a new value for the global solar metallicity should be not simply scaled according to the available values of [Fe/H] for M 3 in the absence of suitable 3D non-LTE models for globular clusters metal poor stars. However, as a first order approximation one may take [Fe/H] = −1.5 from Kraft & Ivans (2003), thus deriving Z(M3) ∼ 0.0006 where an enhancement of α elements by [α/Fe] = 0.3 has been taken into account. As a matter of fact, one finds that the nine synthetic models with Z = 0.0006 and a truncated Gaussian predict a mean number of red stars as given by 133 with smaller KS probabilities, but still in the best case reaching a number of red stars as low as 98 with a KS probability still on the order of 30%.

We are not in the position of discriminating among the various possibilities, since both the cluster metallicity scale, the ^{12}C$(\alpha,\gamma)^{16}$O cross section, and perhaps the cluster HB type are far from firmly established. However, the discussion so far shows that the theory does not appear to firm predict the V:R ratio, and different paths can be followed to reconcile this number with observations by relying on canonical evolutionary models.

5. Discussion and conclusions

As already mentioned, the canonical scenario leaves some degree of freedom for the computed evolutionary tracks, as a consequence of uncertainties in various physical ingredients. On the contrary, it appears difficult to move HB models beyond their canonical paths. The growth of convective cores and the efficiency of semiconvection appear the major assumptions which could be debated, at least in principle. As is well known (see, e.g., Demarque & Sweigart 1976) both these mechanisms act to increase the amount of mixing in the central region and, in turn, the width in temperature of the loop experienced by HB stars during their central He-burning phase. Thus a decreased amount of central convection or of semiconvection appears just as an additional, alternative way to decrease to range of effective temperatures covered by HB evolutionary tracks.

However, one cannot safely decrease the canonical efficiency of these mechanisms without running into severe observational constraints. A decrease in the efficiency of mixing will indeed cause a decrease in the HB lifetime with a corresponding increase in time spent on the Asymptotic Giant Branch. We find that in the extreme case of no mixing, the number ratio $N_{\rm AGB}/N_{\rm HB}$ of stars in the two quoted evolutionary phases will rise towards untenable values up to $N_{\rm AGB}/N_{\rm HB} \sim 0.26$, whereas the canonical value $N_{\rm AGB}/N_{\rm HB} \sim 0.15$ has been already proved to be in excellent agreement with observations (Cassisi et al. 2003).

On the other hand, the supposed occurrence of mass loss during the HB phase (Wilson & Bowen 1984) can hardly be of help, since in the case of M 3 ($Z \sim 0.001$) this will further increase the range of effective temperatures covered by evolving HB models and, in turn, the range of RR Lyrae periods. Neither can the problem be solved by the conjectured efficiency of an “evolutionary trapping” (Koopmann et al. 1994) at the transition line between RRc and RRab, as discussed by Catelan, since it would produce a peaked distribution for both ab and c-type pulsators, not observed in M 3.

Hence the supposed failure of canonical models would affect the basis of an evolutionary theory which has already proved to finely account for many observational features of stars evolving in our Galaxy as well as in nearby ones. In this paper we have shown that canonical HB models might account for the observed distribution of fundamentalised periods of RR Lyrae in the globular cluster M 3. We assumed a bimodal mass distribution with a sharply peaked mass mode just to the red side of the instability strip, which closely resembles the one discussed by Catelan (2004) in the framework of his “trimodal” scenario.

Such a bimodal mass distribution is clearly different from the semi-empirical one derived by Rood & Crocker (1989) on the basis of the color distribution of the cluster stars, which is to our knowledge the only one that has appeared in the literature. Further studies to derive the mass distribution of the HB stars in M 3 by using more recent data could provide an important test of the bimodal mass distribution hypothesis.

One may finally notice that the peaked period distribution discussed in this paper for the M 3 case is not a common feature of RR periods in galactic globular clusters. We plan to discuss this point in a subsequent paper, which will be devoted to a synthetic approach to RR Lyrae period distributions in both Oosterhoff I and Oosterhoff II RR Lyrae rich clusters. Here we only notice that inspection of Fig. 1 in Paper I (Castellani et al. 2003) reveals that several Oosterhoff I globular clusters show much flatter period distributions. This is in particular the case for the two RR Lyrae rich clusters M 5 and M 62. This occurrence requires much less severe constraints on the mass distribution of HB stars, supporting perhaps the Catelan (2004) suggestion that M 3 might be “a pathological case that cannot be considered representative of the OoI class”.

Acknowledgements. This research made use of NASA’s Astrophysics Data System Abstract Service and SIMBAD database operated at the CDS, Strasbourg, France. This work was partially supported by MURST (PRIN2002, PRIN2003). We acknowledge Giuseppe Bono for a critical reading of the manuscript. We thank our anonymous referee for his/her quite detailed comments and suggestions that stimulated a relevant widening of the original investigation.

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