

The distribution of kHz QPO frequencies in bright low mass X-ray binaries

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Received 31 May 2004 / Accepted 8 January 2005

Abstract. We analyzed all published frequencies, ν_1 and ν_2 , of the twin kilohertz quasi-periodic oscillations (kHz QPOs) in bright neutron star low-mass X-ray binaries. The two frequencies are well correlated but, contrary to recent suggestions, the frequency–frequency correlation is significantly different from a $\nu_2 = (3/2)\nu_1$ relation. To check whether, although not following the the $3/2$ relation, the QPO frequencies cluster around a region where $\nu_2/\nu_1 \approx 3/2$, we re-analyzed the Sco X-1 data that were used to report that ratio and show that, because the distribution of ratios of linearly correlated measurements is intrinsically biased, although the significance of the clustering around $\nu_2/\nu_1 \approx 3/2$ previously reported in the case of Sco X-1 is formally correct, it does not provide any useful information about a possible underlying resonance mechanism in this source. Using the same data, we then show that the (unbiased) distribution of QPO frequencies is consistent with a uniform distribution at a 2.4σ level. To investigate this further, we analyzed a larger data set of Sco X-1 and four other sources, 4U 1608–52, 4U 1636–53, 4U 1728–34 and 4U 1820–30. We find that for all five sources the distribution of the kHz QPO frequencies is not uniform and has multiple peaks, which have no analogy in the distribution of points in the spectral color–color diagrams of these sources. Finally, we demonstrate that a simple random walk of the QPO frequencies can reproduce qualitatively the observed distributions in frequency and frequency ratio. This result weakens the support for resonance models of kHz QPOs in neutron stars.

Key words. accretion, accretion disks – stars: neutron – binaries: close – X-rays: binaries

1. Introduction

The launch of the Rossi X-Ray Timing Explorer (RXTE) satellite has led to the discovery of high-frequency quasi-periodic oscillations (QPOs) in bright low mass X-ray binaries (LMXBs) containing neutron stars (see van der Klis 2000 for a review). These QPOs provide a probe into the accretion flow around neutron stars very close to the compact object, where effects of general relativity might be observable. They often appear in pairs at frequencies of a few hundred Hz to more than 1 kHz, from which the name *kHz QPOs* was derived. Their frequencies follow rather tight correlations with other timing features of the X-ray emission (see Ford & van der Klis 1998; Psaltis et al. 1999; Belloni et al. 2002).

There is currently no consensus as to the origin of these QPOs, nor on what physical parameters determine their frequencies, which have been identified with various characteristic frequencies in the inner accretion flow (see e.g. Stella & Vietri 1999; Osherovich & Titarchuk 1999; Lamb & Miller 2003).

Recently, pairs of 30–450 Hz QPOs have been observed from a few black-hole candidates and their frequencies tend to appear in certain ratios (3:2, 5:3, see e.g. Strohmayer 2001a,b; Miller et al. 2001), although there are exceptions (see e.g. Homan et al. 2001). Unfortunately, the detection of such QPOs in black-hole candidates is rather rare, which makes it difficult to assess the significance of this clustering around definite ratios. Abramowicz et al. (2003) reported that the ratio of the frequencies in the kHz QPOs from the brightest LMXB in the sky, Sco X-1, tend to cluster around a value of 1.5, which they interpret as evidence for a 3:2 resonance. A mathematical approach to the resonance model was presented by Rebusco (2004) in order to explain the Sco X-1 results; the discrepancies of the data with a pure 3:2 ratio were attributed to the action of an additional ad-hoc force.

In this paper, we present the results of an analysis of all published values of kHz QPO frequencies from neutron-star systems, including those from Sco X-1. We also extracted and analyzed long series of QPO frequencies from a few selected systems: Sco X-1, 4U 1608–52, 4U 1636–53, 4U 1728–34 and

4U 1820–30. From these data, we conclude that the presence of a single fixed ratio between the frequencies of these QPOs can be excluded. We do, however, find evidence of a clustering of QPO frequencies around specific values that might be associated with resonances. Additional independent observations are needed to exclude a random origin of this clustering.

2. The sample of published kHz QPO frequencies

We searched the literature for all published instances of kHz QPO frequencies. In some cases, no tables are provided and the numbers could only be obtained from figures. We did extract numbers from such figures only when no other frequencies were available from that source. We restricted our sample to the detection of *two* simultaneous kHz QPO peaks and discarded all data obtained via a shift-and-add technique (see Méndez et al. 1998a), as this procedure does not allow to reconstruct the distribution of pairs of frequencies (see Abramowicz et al. 2003). The adopted sources with the range of lower kHz QPO frequency and references are reported in Table 1. For Sco X-1, we used the same data as in Abramowicz et al. (2003).

2.1. Correlation between frequencies

Figure 1 shows a linear plot of the frequency of the upper kHz QPO ν_2 versus the frequency of the lower kHz QPO ν_1 . Stars indicate Atoll sources, circles are for Z sources (filled circles for Sco X-1). It is evident that a strong linear correlation exists between the two frequencies, although slightly different for the two classes of systems. A linear fit, $y = ax + b$, yields $a = 0.94 \pm 0.02$, $b = 350 \pm 15$ for Atoll sources and $a = 0.85 \pm 0.01$, $b = 383 \pm 8$ for Z sources without Sco X-1 (see dashed and dotted lines in Fig. 1). Errors are at a 1σ significance level. These relations, as well as the $a = 0.73 \pm 0.01$, $b = 469 \pm 7$ found for Sco X-1 alone, are clearly not compatible with the two frequencies having a constant 3:2 ratio ($a = 1.5$, $b = 0$; continuous line in Fig. 1).

Following Abramowicz et al. (2003), we plot the histogram of the ν_2/ν_1 ratio distribution. A clear peak around 1.5, as reported by Abramowicz et al. (2003), can be seen (see Fig. 2). However, this peak should be interpreted with some caution. The distribution of the ratio of two quantities that follow a linear correlation with $b \neq 0$ is largely determined by the distribution of the observed values of one of the two variables. Consider that for a linear relation

$$y/x = a + b/x. \quad (1)$$

For high values of x (i.e. $x > b$), which are the majority in Fig. 1, the ratio will be asymptotically close to a , while for low values of x the ratio will be higher. To illustrate this, we simulated a sample of frequency pairs around the correlation for atoll sources, but with the lower kHz frequencies uniformly distributed in the 150–900 range. The corresponding histogram can be seen in the inset of Fig. 2: a rapid increase toward low values of the ratio is clearly visible. This increase is the combined effect of the fact that the two frequencies are correlated and of the range of observed frequencies.

Table 1. List of sources considered in this work, with range of observed ν_1 frequency and references.

Source name	Frequency range (Hz)	References
Z sources		
GX 17+2	475–830	[1]
Sco X-1	565–829	[2]
GX 5-1	156–627	[3]
GX 340+0	197–565	[4]
Cyg X-2	532	[5]
GX 349-2	712–715	[6, 7]
Atoll sources		
4U 1728-34	308–876	[8, 9]
4U 0614+09	153–823	[10]
4U 1705-44	776	[11]
KS 1731-260	903	[12]
4U 1735-44	641–726	[13]
4U 1608-52	550	[14]
4U 1636-53	865–920	[15, 16]
4U 1820-30	790	[17]
4U 1915-05	224–707	[18]
XTE J2123-058	849–871	[19]

[1] Homan et al. (2002); [2] van der Klis et al. (1997); [3] Jonker et al. (2002a); [4] Jonker et al. (2000); [5] Wijnands et al. (1998); [6] Zhang et al. (1998); [7] O’Neill et al. (2002); [8] Migliari et al. (2003); [9] Di Salvo et al. (2001); [10] van Straaten et al. (2000); [11] Ford et al. (1998a); [12] Wijnands & van der Klis (1997); [13] Ford et al. (1998b); [14] Méndez et al. (1998b); [15] Wijnands et al. (1997); [16] Di Salvo et al. (2003); [17] Smale et al. (1997); [18] Boirin et al. (2000); [19] Homan et al. (1999).

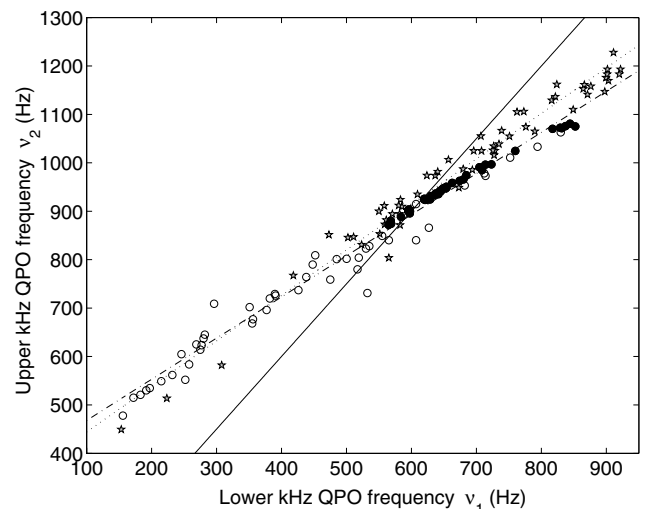


Fig. 1. Plot of upper kHz QPO frequency (ν_2) vs. lower kHz QPO frequency (ν_1) for the Atoll and Z sources for which double kHz QPOs have been published (see Table 1). Empty circles correspond to Z sources, stars to Atoll sources and filled circles to the Sco X-1 data (courtesy of M. van der Klis). The dotted line is the best linear fit to the Atoll points, the dot-dashed line is the best linear fit to the Z points (excluding Sco X-1), and the thick line represents a fixed 3:2 ratio. Errors of the frequencies are typically around a few Hz.

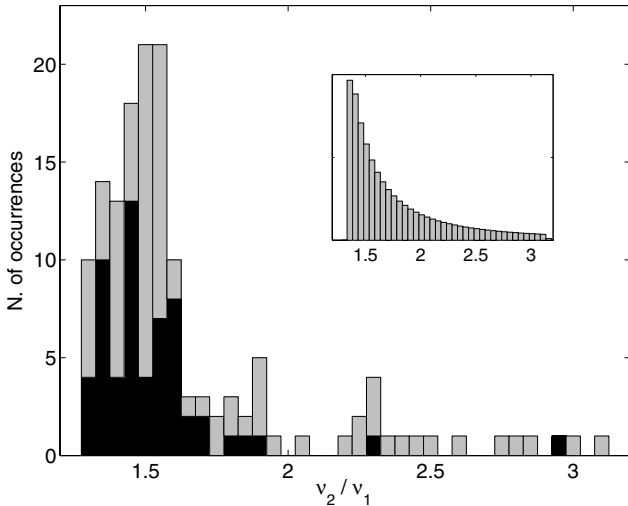


Fig. 2. Distribution of the ν_2/ν_1 values for points in Fig. 1. Black corresponds to Atoll sources, gray to Z sources (including Sco X-1). The inset shows the histogram corresponding to a simulated uniform distribution of points around the correlation for atoll sources (see text).

2.2. A preferred 3:2 ratio in Sco X-1?

While the effect described in the previous paragraph can (practically) account for the observed peak in Fig. 2, it is still possible that the points in Fig. 1 intrinsically cluster around the intersection with the 3:2 line, even though their own correlation significantly differs from that. This cannot be tested for most of the sources in Fig. 1, as Abramowicz et al. (2003) point out, as the real distribution in occurrence of the QPO pairs in these sources is hidden by the typical selection procedures applied by the respective authors. It can however be tested from the published Sco X-1 data, which have been selected in an unbiased way (see Abramowicz et al. 2003).

First, we produced a histogram of the distribution of one of the frequencies, in this case ν_2 . This histogram can be seen in Fig. 3 and shows a broad excess around 900–950 Hz. A fit of the distribution in Fig. 3 with a Gaussian model yields a centroid frequency of $\nu_2^c = 930 \pm 8$ Hz (1σ error). Then, we fitted the ν_1 vs. ν_2 relation (the inverse relation of that shown in Fig. 1) with a linear model, obtaining $a = 1.363 \pm 0.019$ and $b = -635 \pm 18$. Using this relation, we can assign a corresponding ν_1 frequency to ν_2^c and compute their ratio. The resulting ratio is $R = 1.47 \pm 0.04$. Notice however that a Kolmogorov-Smirnov test shows that the hypothesis that the ν_2 distribution is constant can only be rejected at a 2.4σ confidence level.

There is therefore some marginal evidence for a clustering around a frequency corresponding to a ratio of 1.5, marginal because of the small number of points in this dataset. We note, however, that although we compared the distribution of ν_2 against a uniform distribution, there is no reason to expect a constant distribution in QPO frequency. Also notice that in Sco X-1 twin kHz QPOs are observed only when the source is in the upper Normal Branch in the color-color diagram (van der Klis et al. 1996) and therefore, by selecting only data with two peaks, we introduce a bias toward low

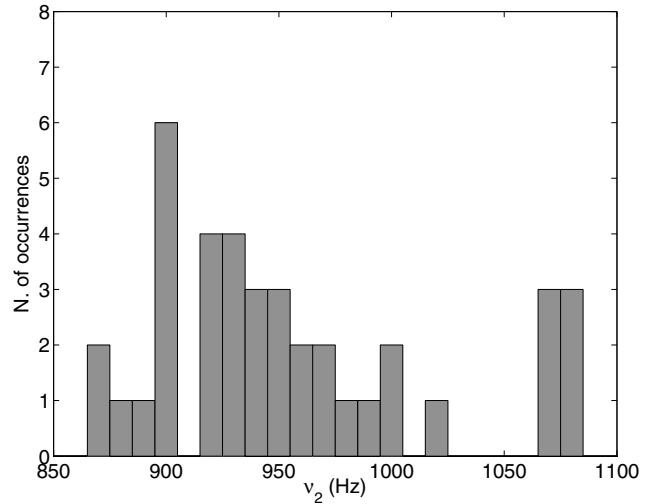


Fig. 3. Distribution of the frequency of the upper kHz QPO ν_2 for the Sco X-1 data analyzed by Abramowicz et al. (2003).

frequencies (since QPO frequencies correlate with the position of the source in a color-color diagram). Indeed, van der Klis et al. (1996) report single-peak detections above 1090 Hz. In a number of sources, a second low-frequency kHz QPO is found when the Power Density Spectra are shifted according to the high-frequency QPO and then summed, indicating that the second QPO might always be present, although at a lower rms.

In order to investigate the actual distribution of kHz QPO frequencies in bright LMXB we examined the high-frequency QPO detections of four additional sources, as well as from an expanded set of data from Sco X-1.

3. Data analysis

The selected data and the procedures used are described below.

- *Sco X-1*: we used the data analyzed by Méndez & van der Klis (2000). Power density spectra (PDS) were accumulated from data stretches 16 s long, then 8 consecutive PDS were averaged and searched for high-frequency (>250 Hz) peaks. If two peaks were found, only the highest one was considered. In all cases, the upper kHz QPO was detected (see Méndez & van der Klis for more details). The lower kHz peak was then recovered via a shift-and-add technique using narrow (5 Hz) frequency bins: this procedure allows one to obtain an averaged ν_2 vs. ν_1 relation (see Fig. 1 in Méndez & van der Klis 2000; also reported by Miller 2003). The output of these procedures are therefore a time series of ν_2 frequencies and a general correlation ν_2 vs. ν_1 . We fitted this correlation with a linear model, obtaining $a = 0.786 \pm 0.002$, $b = 433 \pm 2$.
- *4U 1608–52 and 4U 1728–34*: we used the data analyzed by Méndez et al. (2001). The extraction procedure was very similar to that for Sco X-1, with the following differences: PDS were accumulated from 64s data stretches, a variable number of consecutive PDS were averaged (in all cases less than 20) in order to obtain a significant detection, and the

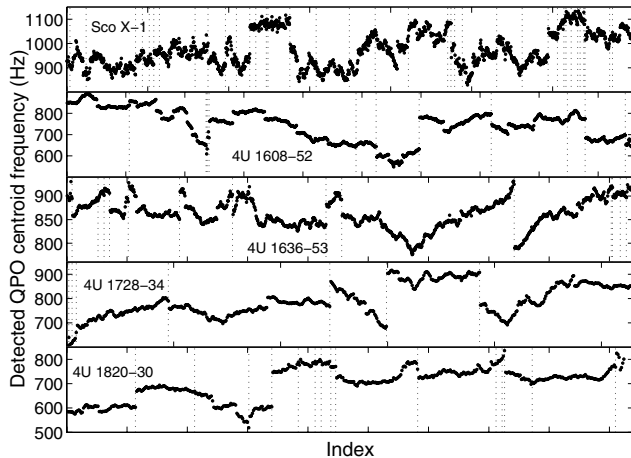


Fig. 4. The series of measured QPO centroid frequencies for the five sources analyzed. The X axis is simply the PDS number, i.e. the frequencies are plotted in sequence. All discrete jumps in frequency correspond to gaps in the data. Gaps longer than one hour are marked with dotted lines. Typical errors on the frequencies are 1–5 Hz.

lower kHz QPO peak was selected, obtaining the ν_2 vs. ν_1 relation through shift and add. A linear fit gives $a = 0.810 \pm 0.017$, $b = 418 \pm 10$ for 4U 1608–52 and $a = 0.897 \pm 0.024$, $b = 420 \pm 17$ for 4U 1728–34.

- 4U 1636–53: we used the data analyzed by Di Salvo et al. (2003). The extraction procedure was the same as for 4U 1608–52 and 4U 1728–34. To derive $[a, b]$, we also used the points by Jonker et al. (2002b). The linear fit for this source gives $a = 0.673 \pm 0.017$, $b = 540 \pm 14$.
- 4U 1820–30: The extraction procedure was the same as for 4U 1608–52 and 4U 1728–34. Preliminary results were shown in Méndez (2002). We used all RXTE/PCA data from 1996 October 15th to 1999 March 1st. For this source, we obtained a fit with $a = 0.741 \pm 0.039$, $b = 466 \pm 29$.

The resulting frequencies can be seen in Fig. 4, with vertical markers indicating gaps longer than one hour.

For each source, we repeated the analysis described in Sect. 2.2. The distributions of ν_2 (for Sco X-1) and ν_1 (for the other sources) can be seen in Fig. 5. As all of the QPO frequency distributions seem to be multi-peaked, we fitted them with a sum of Gaussians. Two or three Gaussians were needed to fit the observed peaks. The resulting centroids ν_α , ν_β and ν_γ are reported in Table 2. The multi-Gaussian models used are also plotted in Fig. 5.

For each source we then used the best-fit $\nu_2 - \nu_1$ relation to compute the frequency ratios R_α , R_β and R_γ corresponding to the Gaussian peaks, with corresponding errors. The results are shown in Table 2 and will be discussed in Sect. 4. Notice that the errors on R_α , R_β and R_γ are dominated by the propagated errors on the coefficients a and b from the linear fits to the $\nu_2 - \nu_1$ relations. The errors on the ratios are therefore correlated, i.e. even though the signs of the errors on the ratios are not known, they are most likely the same for all ratios of each individual source.

As the frequency of QPOs in these systems is known to correlate with the position on the color-color diagram (see e.g.

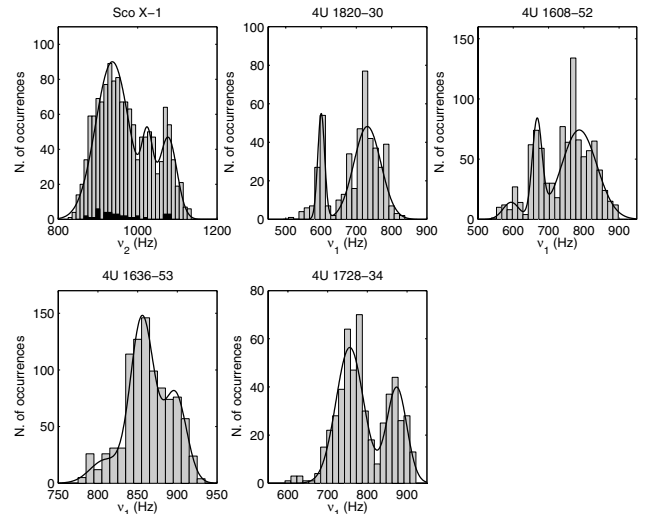


Fig. 5. Distribution of the frequency of the upper kHz QPO ν_2 for Sco X-1, and of the lower kHz QPO ν_1 for the other sources in our sample (see text). For Sco X-1, the black histogram represents the frequency distribution of the upper kHz QPO in the data by Abramowicz et al. (2003).

Di Salvo et al. 2003), it is possible that the non-uniformity of the distributions shown in Fig. 5 is the result of an analogous non-uniform distribution of points along the color-color diagram of the source. In order to check for such an effect for each of the sources, except Sco X-1, we produced a color-color diagram with each point corresponding to one of our QPO detections, using the procedures described in Di Salvo et al. (2003). In the case of Sco X-1, this proved not to be feasible as the strong dead-time effects caused by the high count rate and the different offset pointings to this source prevented the production of a single homogeneous color-color track. We parametrized the position on the color-color track following Di Salvo et al. (2003) and obtained for each point an S_a value, i.e. an index that characterizes the position along the color-color track. The distribution of the points in the $\nu - S_a$ plane is shown in Fig. 6, together with the marginal distribution along the two axes. It is clear from the figure that the multi-peaked distributions of QPO frequency do not have a corresponding multi-peaked distribution in S_a . For instance, in the case of 4U 1820–30 there is a gap in the distribution of QPO frequencies between $\nu_1 \sim 600$ and $\nu_1 \sim 650$ Hz, but there is no gap in the S_a distribution at the corresponding values between $S_a \sim 0.9$ and $S_a \sim 1.0$ (see upper left panel in Fig. 6).

Having ascertained that the distribution of QPO frequencies is not constant, but shows multiple peaks, we need to establish whether these peaks are simply the result of the random walk of the QPO frequencies. It is known that the frequencies of kHz QPOs do not jump in frequency, but show a sort of random walk in time (see Fig. 4). In fact, this behavior is not a simple random walk, as frequencies are only observed within a certain range. In order to test whether such a random movement of frequencies in time could produce peaks in the frequency distribution, we simulated a simple random walk in frequency. We produced a series of 10 000 frequencies starting at 700 Hz and adding a random value between -6 and $+6$ Hz to the previous

Table 2. Parameters of the linear fits and centroids of the Gaussian distributions fitted to the histograms in Fig. 5 and corresponding frequency ratios (see text). All errors are 1σ .

Source name	a	b	ν_α (Hz)	ν_β (Hz)	ν_γ (Hz)	R_α	R_β	R_γ
Sco X-1	0.786 ± 0.002	432.5 ± 1.5	936 ± 2	1025 ± 3	1077 ± 3	1.46 ± 0.01	1.36 ± 0.01	1.32 ± 0.01
4U 1608-52	0.810 ± 0.017	418.3 ± 10.3	594 ± 4	668 ± 2	787 ± 2	1.51 ± 0.02	1.44 ± 0.02	1.34 ± 0.02
4U 1636-53	0.673 ± 0.017	539.8 ± 14.1	809 ± 10	856 ± 2	897 ± 3	1.34 ± 0.03	1.30 ± 0.02	1.27 ± 0.02
4U 1728-34	0.897 ± 0.024	419.7 ± 16.8	756 ± 2	874 ± 3		1.45 ± 0.03	1.38 ± 0.03	
4U 1820-30	0.741 ± 0.039	466.2 ± 29.0	601 ± 1	731 ± 2		1.52 ± 0.06	1.38 ± 0.06	

value in the sequence. The resulting histograms for two realization of the random walk are shown in Fig. 7. Adopting a linear relation between ν_1 and ν_2 as the one shown in Table 2 for Sco X-1, for each simulation we also produced the distribution of ν_2/ν_1 . This very simplified model, which does not include a “re-call” force to keep the frequency between fixed bounds, can clearly produce multiple peaks in the frequency distribution, which for the reasons outlined above results in a sharp peak in the distribution of ratios.

4. Discussion

We investigated the possibility of a 3:2 resonance in the frequencies of the kHz QPOs in bright accreting LMXBs from all available data in the literature. We found that the strong linear correlation observed between the lower and upper kHz QPO frequencies both in Atoll and Z sources is *not* compatible with a single constant ratio, which can be excluded with high significance (see Fig. 1). This conclusion is also evident from Rebusco (2004), who suggests an ad hoc modification to the resonance model in order to explain the observed correlation. We showed that even a constant distribution of the frequency of one of the QPOs, together with an observed correlation between the two QPO frequencies, would result in a peaked histogram of frequency ratios. Since resonance models of the QPOs predict that the frequency ratios as well as the frequencies themselves should cluster around specific values, and since the distribution of QPO frequencies does not suffer from the bias of the distribution of frequency ratios, we used the same Sco X-1 data analyzed by Abramowicz et al. (2003) to check whether there is evidence for a clustering around a particular QPO frequency. We found marginal evidence for an excess in the distribution at a frequency consistent with a 3:2 ratio, whose significance is difficult to assess given the scarcity of available data, and whose nature might be the result of observational biases. We conclude that although the significance of the narrow peak in the distribution of QPO frequency ratios around a value of 3/2 reported by Abramowicz et al. (2003) in Sco X-1 is formally correct, since the distribution of frequency ratios is biased, their result has no power in testing the predictions of their resonance models; the peak in the distribution of ratios that they find is probably due to the distribution of QPO frequencies in Sco X-1 being mostly above the intersection with the line of a constant 3:2 ratio.

The analysis of a sample of high-frequency oscillations in five systems (including a more extensive coverage of Sco X-1) shows that the $\nu_2 - \nu_1$ relation is rather similar between different systems, and that the distribution of the observed frequencies is not flat in the observed range, but shows two or three peaks, depending on the source. Abramowicz et al. (2003) reported the presence of a second peak, although at low significance, corresponding to the six points above $\nu_2 = 1050$ Hz in Figs. 1 and 3. From Fig. 5, it is evident that those points correspond to our high frequency component ν_γ . Using the measured $\nu_1 - \nu_2$ relations, it is possible to associate a frequency ratio to the centroids of these peaks. The lowest peak corresponds to a ratio close to 1.5 in four systems, while for 4U 1636–34 it does not.

There is no a priori reason for such a distribution to have a flat shape, since it is known that the fractional rms of these oscillations decreases at high and at low frequencies (see Méndez et al. 2001; and Di Salvo et al. 2003). However, the frequency dependence of fractional rms does not show any evidence of being multi-peaked, and its maximum does not coincide with any of the maxima of the distributions in Fig. 5.

We have shown that the multi-peaked nature of the distribution of QPO frequencies is not due to a combination of the source spending more time on certain positions in the color–color diagram, and of a correlation between position on the color–color diagram and QPO frequency (Fig. 6). The lack of correspondence of the multi-peaked distribution of QPO frequency with the distribution of the S_a values contradicts scenarios in which the QPO frequency and the source spectrum are both determined only by the radius of the inner edge of the disk (van der Klis 2001). Notice that in particular 4U 1820–30, while showing a good correlation between ν_1 and S_a , clearly features a gap in the distribution of ν_1 not associated to a corresponding gap in S_a .

From the few instances of double high-frequency QPOs in black-hole systems there is some evidence for a preferred fixed ratio. The data are sparse and rather stable frequencies have been observed in these pairs. In the case of BH binaries the preferred ratio can also be the result of sampling, i.e. the few detections available to date could come from the upper region of a correlation similar to that in Fig. 1, in which case the observed ratio would not have a simple physical interpretation in terms of a resonance mechanism. For neutron-star systems, where a large number of detections of pairs with variable frequencies is available, a single fixed ratio can be excluded. The presence of

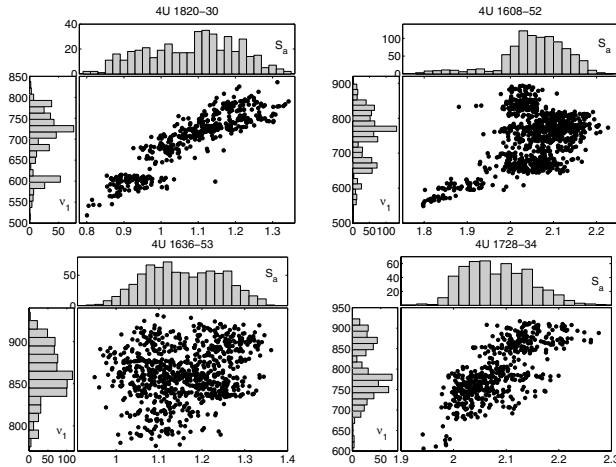


Fig. 6. Frequency of the lower kHz QPO ν_1 plotted versus the position of the source along the color-color diagram, parametrized by S_a (see text), for the four atoll sources in our sample. The histograms represent the marginal distributions of the single parameters. Notice that the ν histograms are not identical to those in Fig. 5, because not for all points we could extract a color.

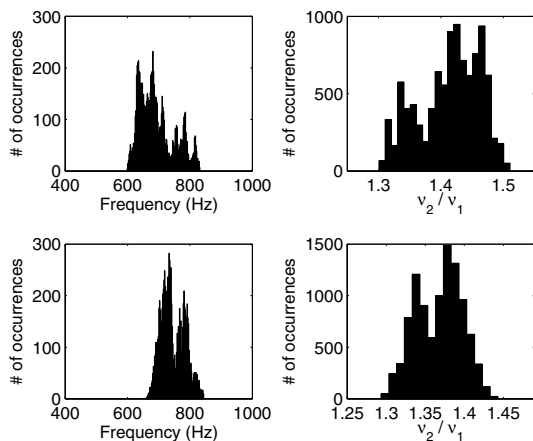


Fig. 7. *Left panels:* distribution of the frequencies of two random walks in frequency (see text). *Right panels:* corresponding distribution of frequency ratios assuming the $\nu_1 - \nu_2$ relation for Sco X-1 (see Table 2).

an additional external force as in the model by Rebusco (2004) (see also Abramowicz et al. 2003b) could explain the observed deviation from a fixed 3:2 ratio, at the expense of a considerable complication of the model, with the addition of an unknown parameter which biases the observed values away from a direct measurement of a 3:2 ratio. However, the presence of multiple peaks as those shown in Fig. 5 would complicate the interpretation further, as the presence of other preferred frequencies (or ratios) would also have to be explained.

If the kHz QPOs are due to a resonance mechanism in the accretion disk, the multi-peaked distribution of QPO frequencies (Fig. 5) suggests that not just the 3/2, but more than one resonance is at work. Using the $\nu_2 - \nu_1$ relations in Sect. 3, we can assign a frequency ratio to each of the peaks of the QPO distributions in Fig. 5. The inferred ratios are close to 3/2, 7/5, and 4/3 in the cases of Sco X-1 and 4U 1608-52, 4/3,

9/7 and 5/4 in the case of 4U 1636-53, and 3/2 and 7/5 in 4U 1728-34 and 4U 1820-30. However, the errors are sufficiently large that more than one identification is possible.

From the ratios shown in Table 2, it is apparent that 4U 1636-53 is different from the other sources in that the frequency ratios in this source are systematically lower than in the other four sources. While for all other sources the largest ratio is close to 1.5, the largest ratio in 4U 1636-53 is 1.34 ± 0.03 , significantly smaller than 1.5. From the sample of sources in Table 2, 4U 1636-53 is the only one that during the RXTE observations used for our analysis appeared always in a region of the color-color diagram characteristic of sources at high inferred mass accretion rates, and the QPOs appeared always at high frequencies (Di Salvo et al. 2003). Assuming there is a relation between QPO frequency and radius of the inner edge of the disk (e.g., Miller et al. 1998), this suggests that during these observations the inner edge of the disk in 4U 1636-53 was always relatively close to the neutron star. On the other hand, based on their intensity, color-color diagrams, and range of QPO frequencies, the other sources appeared to cover a larger range of mass accretion rate, which suggests that in those sources the inner radius of the disk covered a larger range than in 4U 1636-53.

It has already been shown (Stella & Vietri 1999) that the identification of the QPOs with the azimuthal and the periastron precession frequencies, ν_θ and $\nu_p = \nu_\theta - \nu_r$, respectively, reproduces qualitatively the QPO frequency-frequency correlations. (We note in passing that the alternative identification of the QPO frequencies with the vertical and radial epicyclic frequencies, e.g., Kluzniak et al. 2004, implies frequency-frequency correlations in contradiction with the observed data. For instance, unless another mechanism is at work, this interpretation predicts that the frequency difference between the kHz QPOs, $\nu_2 - \nu_1$, increases as the frequencies increase, opposite to what is observed in several sources; see e.g., Méndez et al. 1998a). The ν_θ, ν_p identification can also account for the range of QPO frequency ratios seen in 4U 1636-53, compared to those seen in the other sources. The ratio $R = \nu_\theta/\nu_p$ increases monotonically with r , and hence decreases as the frequencies increase (notice that, on the contrary, the ratio ν_θ/ν_r increases toward higher frequencies). Since in our observations of 4U 1636-53 the kHz QPOs never reached values as low as those seen in the other sources, the frequency ratio did not reach high values either. We expect that when the mass accretion rate in 4U 1636-53 decreases, and the kHz QPO move to lower frequencies, the ratio of QPO frequencies will increase and cluster around 1.5 as in the other sources.

There is in principle no a priori reason why higher frequency ratios such as 2/1, 3/1, 4/1, etc. should not appear. With the identification of the observed frequencies with those of Stella & Vietri (1999), for a neutron star mass in the range $1.4-2.2 M_\odot$, a frequency ratio $R \geq 2/1$ would only occur when the frequency of the lower kHz QPO is $\lesssim 300-500$ Hz, while a ratio $R \geq 3/1$ requires the frequency of the lower kHz QPO to be $\lesssim 150-200$ Hz. In none of the observations presented in this paper did the lower kHz QPO frequency reach such a low value. However, there are some reports in the literature of lower kHz QPO being detected in this frequency range. For instance,

the lowest frequencies reached by the kHz QPO in the atoll source 4U 0614+09 are $\nu_1 = 153.4 \pm 5.6$ and $\nu_2 = 449.4 \pm 19.5$ (van Straaten et al. 2000), such that $R = 2.9 \pm 0.2$. In the Z source GX 5–1, the lowest frequencies so far observed for a pair of kHz QPOs are $\nu_1 = 156 \pm 23$ Hz and $\nu_2 = 478 \pm 15$ Hz (Jonker et al. 2000), for which $R = 3.1 \pm 0.4$.

Finally, we showed that a simple frequency random walk could in principle reproduce the observed multi-peaked frequency distributions as well as the distribution of frequency ratios. Although we have used a very simple approach to simulate the drift of QPO frequency with time (e.g. the real frequencies do not follow a pure random walk, the observations sample many separate intervals of the real frequency evolution, etc.), it is clear that a multi-peaked distribution of QPO frequencies (or frequency ratios) could also result from purely random processes. A possible test of the idea that the observed QPO-frequency distribution in these neutron star systems is indeed produced by an underlying resonance mechanism would be to accumulate new (independent) samples of frequencies of similar size to the ones presented here, and check whether the distribution of QPO frequencies in these new samples show peaks with centroid frequencies that are consistent with those observed here (although some peaks could be missing while some new peaks may appear, depending on the range of frequencies sampled by the sources during the new observations). If this is not observed, we would have to conclude that the peaks are due to the random motion of the frequency. Since at present we cannot discard this possibility, our results weaken the support for resonance models of the kHz QPOs in neutron stars.

5. Summary and conclusions

We showed that:

- The relation between the two simultaneous kHz QPO frequencies in a large sample of sources, including Sco X-1, is significantly different from a $\nu_2 = 1.5\nu_1$ relation.
- Since in all these sources the two QPO frequencies are linearly correlated, the distribution of the ratios of the QPO frequencies would be peaked even if individually the frequencies are distributed more or less uniformly. Since resonance models predict that *both* the ratio of frequencies as well as the frequencies themselves should cluster around specific values, the distribution of frequency, and not of frequency ratios, should be used to test the predictions of those models.
- The hypothesis that in the same Sco X-1 data used by Abramowicz et al. (2003) the distribution of QPO frequencies is significantly different from constant can only be rejected at a 2.4σ confidence level.
- The distribution of QPO frequencies in Sco X-1, 4U 1608–52, 4U 1636–53, 4U 1728–34, and 4U 1820–30 is multi-peaked, with the peaks occurring at frequencies compatible with ν_2/ν_1 ratios of $3/2$, $4/3$, $5/4$, $7/5$, and $9/7$, not all ratios appearing in all sources. This is considerably different from the claim by Abramowicz et al. (2003) that the frequency ratio for Sco X-1 shows a narrow peak

at 1.5, which we showed in Sect. 2 not to be statistically founded.

- The multi-peaked nature of the frequency distribution appears difficult to explain by parametric resonance models, since the perturbation mechanism that drives the ratio away from the main, $3/2$, resonance would have to be able, at the same time, to produce the other resonances as well.
- A random walk of the QPO frequencies qualitatively reproduces the observed QPO frequency distribution as well as the frequency ratio distribution. This means that, until the presence of the same peaks at the same locations is confirmed by (large) independent datasets, it is premature to speculate further about the presence of preferred ratios.

Acknowledgements. We thank Michiel van der Klis for providing the Sco X-1 frequencies used in Abramowicz et al. (2003), and Gabriele Ghisellini, Sergio Campana, Rudy Wijnands, Michiel van der Klis, Peter Jonker and Tom Maccarone for useful discussions.

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