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Magnetic helicity and cosmological magnetic field

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Abstract. The magnetic helicity has paramount significance in the nonlinear saturation of the galactic dynamo. We argue that magnetic helicity conservation is violated at the lepton stage in the evolution of the early Universe. As a result, a cosmological magnetic field which can be a seed for the galactic dynamo obtains from the beginning a substantial magnetic helicity which has to be taken into account in the magnetic helicity balance at a later stage of the galactic dynamo.

Key words. magnetic fields – the early Universe

1. Introduction

Magnetic fields of galaxies are believed to be generated by a galactic dynamo based on the joint action of the so-called α-effect and differential rotation. The α-effect is connected with a violation of mirror symmetry in MHD-turbulence and therefore caused by rotation. For a weak galactic magnetic field, the mirror asymmetry is associated with helicity of the velocity field and is proportional to the linkage of vortex lines.

The magnetic helicity is an inviscid integral of motion and its conservation strongly constrains the nonlinear evolution of the galactic magnetic field. The helicity density $\boldsymbol{B} \cdot \boldsymbol{A}$ of a galactic large-scale magnetic field is enhanced by galactic dynamo (here $\boldsymbol{B}$ is a large-scale magnetic field, $\boldsymbol{A}$ is its vector-potential). Because of magnetic helicity conservation, this income must be compensated by magnetic helicity of a small-scale magnetic field. Note that the magnetic helicity density is bounded from above by $b_l^2 l$, where b_l is the magnetic field strength at the scale l. Hence the supply of a small-scale magnetic helicity occurs to be insufficient for the compensation required. This fact strongly constrains the galactic dynamo action (see Brandenburg 2001a,b, and references therein).

On the other hand, a weak almost homogeneous cosmological magnetic field B_c can introduce a new element in this scheme. If we adopt a thickness of the gaseous galactic disc $l_{gal} \approx 1$ kpc as a typical scale for galactic dynamo action and $B_c = 10^{-9}$ G, then the magnetic helicity supplied via the cosmological magnetic field ($h_c \simeq B_c^2 l_H \sim 10^{10}$ G^2 cm, where l_H is the horizon size) can be of the same order as that one for the galactic magnetic field ($h_{gal} \simeq B_G^2 l_{gal} \sim 3 \times 10^9$ G^2 cm, $B_G \approx 10^{-6}$ G).

The cosmological magnetic field B_c if it exists must be substantially weaker than the galactic magnetic field (see Beck et al. 1996, for review). According to the analysis of rotation measures of remote radio sources, $B_c \leq 10^{-10...-11}$ G. This estimate however is based on the assumption that a substantial part of charged particles in the Universe is in form of thermal electrons in the intergalactic medium. A more robust estimate, $B_c < 10^{-9}$ G is based on the isotropy constrains.

We estimated above that if the cosmological magnetic field is of the order $B_c = 10^{-9}$ G its magnetic helicity density can be comparable with the magnetic helicity density of the galactic magnetic field. Of course, the estimate $B_c = 10^{-11}$ gives a much lower value for the magnetic helicity density. However it is more than natural to expect a magnetic helicity concentration in the processes of galactic formation. The question is whether a mechanism of magnetic helicity production can be suggested for physical processes in the early Universe.

Here we suggest a mechanism for magnetic helicity generation by a collective neutrino-plasma interactions in the early Universe after the electroweak phase transition.

2. Weak force contribution to dynamo action in early Universe

Let us consider the electron-positron plasma as a two-component medium, for which the strong correlation between opposite charges due to Coulomb forces gives $\boldsymbol{V}_\pm \approx \boldsymbol{V}$. Here $\boldsymbol{V}$ is the common fluid velocity of *electroneutral* conducting gas while its positively and negatively charged components have different velocities, $\boldsymbol{V}_{\pm} = \boldsymbol{V} \pm \delta\boldsymbol{V}$, where a small difference $\delta\boldsymbol{V} \ll \boldsymbol{V}$ gives the separation of charges at small scales and enters the electromagnetic current $\boldsymbol{j}^{(\mathrm{em})} = 2|e|n_{\mathrm{e}}\gamma_{\mathrm{e}}\delta\boldsymbol{V}$ obeying in MHD the Maxwell equation, $4\pi\boldsymbol{j}^{(\mathrm{em})} = \nabla\times\boldsymbol{B}$ ($\hbar = c = 1$).

The electric field $\boldsymbol{E}$ derived from Euler equations for plasma components takes the form (Semikoz 2004) which includes the contribution of weak interactions $\boldsymbol{E}_{\mathrm{weak}}$ taken in the collisionless Vlasov approximation. We do not consider other known terms which describe weak interaction collisions (Dolgov & Grasso 2002), Biermann battery effects, etc., and which do not play substantially in favor of helicity generation. In turn, we keep in $\boldsymbol{E}_{\mathrm{weak}}$ only the axial vector term which violates the parity:

$$\boldsymbol{E}^{(\mathrm{A})}_{\mathrm{weak}} = -\frac{G_{\mathrm{F}}}{\sqrt{2}\,|e|\,n_{\mathrm{e}}}\sum_{\nu_a} c^{(a)}_{\mathrm{A}}\left(n_0^{(-)} + n_0^{(+)}\right)\hat{\boldsymbol{b}}\,\frac{\partial\delta n_{\nu_a}(\boldsymbol{x},t)}{\partial t} + \left(N_0^{(-)} + N_0^{(+)}\right)\nabla\left(\hat{\boldsymbol{b}}\cdot\delta\boldsymbol{j}_{\nu_a}(\boldsymbol{x},t)\right). \quad (1)$$

Here $G_{\mathrm{F}} = 10^{-5}/m_{\mathrm{p}}^2$ is the Fermi constant, m_{p} is the proton mass; $c^{(a)}_{\mathrm{A}} = \mp 0.5$ is the axial weak coupling, upper (lower) sign is for electron (muon or tau) neutrinos, $\delta n_{\nu_a} = n_{\nu_a} - n_{\bar{\nu}_a}$ is the neutrino density asymmetry, $\delta\boldsymbol{j}_{\nu_a} = \boldsymbol{j}_{\nu_a} - \boldsymbol{j}_{\bar{\nu}_a}$ is the neutrino current asymmetry; $\hat{\boldsymbol{b}}$ is the unit vector along the mean magnetic field; n_0 is the lepton number density at the main Landau level given by the equilibrium Fermi distribution in a hot plasma, $n_0^{(-)} \approx n_0^{(+)} = (|e|B/2\pi^2)T\ln 2$, where at equilibrium the temperature obeys $T_+ = T_- = T \gg m_{\mathrm{e}}$. In the non-relativistic limit, $v \ll 1$, the relativistic polarization terms in Eq. (1) tend to the lepton densities at the main Landau level, $N_0^{(\sigma)} \to n_0^{(\sigma)}$.

Accounting for the first line in Eq. (1) we obtain the axial vector term $\boldsymbol{E}^{(\mathrm{A})}_{\mathrm{weak}} = -\alpha\boldsymbol{B}$, where the helicity coefficient α is the *scalar* in the standard model (SM) with neutrinos instead of the pseudoscalar $\langle\boldsymbol{v}\cdot(\nabla\times\boldsymbol{v})\rangle$ in standard MHD:

$$\alpha = \frac{G_{\mathrm{F}}}{2\sqrt{2}\,|e|\,B}\sum_a c^{(\nu_a)}_{\mathrm{A}}\left[\left(\frac{n_0^{(-)} + n_0^{(+)}}{n_{\mathrm{e}}}\right)\frac{\partial\delta n_{\nu_a}}{\partial t}\right] \simeq \frac{\ln 2}{4\sqrt{2}\pi^2}\left(\frac{10^{-5}T}{m_{\mathrm{p}}^2\lambda^{(\nu)}_{\mathrm{fluid}}}\right)\left(\frac{\delta n_\nu}{n_\nu}\right). \quad (2)$$

Here we substituted $n_\nu/n_{\mathrm{e}} = 0.5$, and assumed a scale of neutrino fluid inhomogeneity $t \sim \lambda^{(\nu)}_{\mathrm{fluid}}$, that is small compared with a large Λ-scale of the mean magnetic field.

Let us stress that instead of the *difference* of electron and positron contributions in axial vector terms entering the pair motion equation (Semikoz 2004) and given by the polarized density asymmetries $\sim(n_0^{(-)} - n_0^{(+)})$ we obtained here the *sum* of them $\sim(n_0^{(-)} + n_0^{(+)})$ that can lead to a significant effect in a hot plasma.

The admixture of the pseudovector $\alpha\boldsymbol{B}$ to the pure vector $\boldsymbol{E}$, e.g. for the constitutive relations $\boldsymbol{D} = \varepsilon\boldsymbol{E} + \beta\boldsymbol{B}$, $\boldsymbol{H} = \gamma\boldsymbol{E} + \mu^{-1}\boldsymbol{B}$ due to the same neutrino-plasma weak interaction described by the constants β, γ, has been already discussed in literature (see Nieves & Pal 1994, Eqs. (3.5), (3.6)). In a forthcoming paper we show that such unusual coefficients appear in *chiral media* and are simply connected with α given by Eq. (2), $(\varepsilon - 1)\alpha = \beta + \gamma$, where ε is the dielectric permittivity of plasma.

Thus, using Eq. (1) from the Maxwell equation $\partial_t\boldsymbol{B} = -\nabla\times\boldsymbol{E}$ one obtains the Faraday equation generalized in SM with neutrinos and antineutrinos:

$$\frac{\partial\boldsymbol{B}}{\partial t} = \nabla\times\alpha\boldsymbol{B} + \eta\nabla^2\boldsymbol{B}, \quad (3)$$

where we omitted the weak vector contribution $\nabla\times\boldsymbol{E}^{(V)}_{\mathrm{weak}} \sim \nabla\times\delta\boldsymbol{j}_{\nu_a}(\boldsymbol{x},t)$ suggested by Brizard et al. (2000) since we neglect any neutrino flux vorticity in the hot plasma of early universe. Because the early universe is almost perfectly isotropisc and homogeneous, we ignore here any contribution from large-scale motions as well. In the relativistic plasma the diffusion coefficient η takes the form $\eta = (4\pi\times 137\,T)^{-1}$.

The first term in r.h.s. of Eq. (3), $\nabla\times\alpha\boldsymbol{B}$, is associated with the parity violation in weak interactions in the early universe plasma.

We stress that the Eq. (3) is the usual equation for mean magnetic field evolution with α-effect based on particle effects rather on the averaging of turbulent pulsations. It is well-known (see e.g. Ruzmaikin et al. 1988) that Eq. (3) describes a self-excitation of a magnetic field with the spatial scale $\Lambda \approx \eta/\alpha$ and the growth rate $\alpha^2/4\eta$. Semikoz & Sokoloff (2004) estimated these values for the early universe to get

$$\frac{\Lambda}{l_{\mathrm{H}}} = 1.6\times 10^9\left(\frac{T}{\mathrm{MeV}}\right)^{-5}\left(\frac{\lambda^{(\nu)}_{\mathrm{fluid}}}{l_\nu(T)}\right)(|\,\xi_{\nu_{\mathrm{e}}}(T)\,|)^{-1}, \quad (4)$$

$$B(x) = B_{\max}\exp\left(25\int_x^1\left(\frac{\xi_{\nu_{\mathrm{e}}}(x')}{0.07}\right)^2 x'^{10}\mathrm{d}x'\right). \quad (5)$$

Here $l_{\mathrm{H}}(T) = (2\mathcal{H})^{-1}$ is the horizon size and $\mathcal{H} = 4.46\times 10^{-22}(T/\mathrm{MeV})^2$ MeV is the Hubble parameter; the variable $x = T/2\times 10^4$ MeV corresponds to the maximum temperature $T \simeq 20$ GeV $\ll T_{\mathrm{EW}} \simeq 100$ GeV for which the point-like Fermi approximation for weak interactions we rely on is still valid. Finally $l_\nu(T)$ is the neutrino free path and $\xi_{\nu_{\mathrm{e}}} = \mu_{\nu_{\mathrm{e}}}/T \ll 1$ is the small dimensionless electron neutrino chemical potential normalized in Eq. (5) on the maximum value $\xi_{\nu_{\mathrm{e}}} \le 0.07$ allowed by the Big Bang Nucleosynthesis (BBN) bound on light element abundance (Dolgov et al. 2002).

Thus, while in the temperature region $T_{\mathrm{EW}} \gg T \gg T_0 = 10^2$ MeV there are many small random magnetic field domains, a weak mean magnetic field turns out to be developed into the uniform *global* magnetic field at temperatures below T_0 (see Eq. (4). The global magnetic field can be small enough to preserve the observed isotropy of the cosmological model (Zeldovich 1965) while being strong enough to be interesting as a seed for galactic magnetic fields. This scenario was extensively discussed by experts in galactic magnetism (Kulsrud 1999), however until now no viable origin for the global magnetic field has been suggested. We believe that the dynamo based on the α-effect induced by particle physics solves this fundamental problem and opens a new and important option in galactic magnetism.

3. Magnetic helicity generation by collective neutrino-plasma interactions

Let us consider how the collisionless neutrino interaction with charged leptons can produce the primordial magnetic helicity $H = \int_v (\boldsymbol{A} \cdot \boldsymbol{B}) \mathrm{d}^3 x$, where v is the volume that encloses the magnetic field lines.

For that we should substitute into the derivative,

$$\frac{\mathrm{d} H}{\mathrm{d} t} = -2 \int_v (\boldsymbol{E} \cdot \boldsymbol{B}) \mathrm{d}^3 x, \tag{6}$$

the electric field $\boldsymbol{E}$ given by Eq. (1). Neglecting any rotation of primordial plasma given by the first dynamo term, or retaining the resistive term and the weak interaction term $\boldsymbol{E}^{(A)}_{\rm weak}$ given by Eq. (1) that is the main one in the absence of any vorticities, one finds from Eq. (6)

$$\frac{\mathrm{d} H}{\mathrm{d} t} = -2\eta \int_v \mathrm{d}^3 x (\nabla \times \boldsymbol{B}) \cdot \boldsymbol{B} + 2\alpha \int_v \mathrm{d}^3 x B^2. \tag{7}$$

Note that the second term in the r.h.s. violates parity: it is a pure *scalar* while other terms are pseudoscalars as it should be for the helicity H in standard MHD. Nevertheless, all terms in the generalized helicity evolution Eq. (7) obey CP-invariance as it should be for the electroweak interactions in SM since the new coefficient α (2) is CP-odd, $(CP)\, \alpha\, (CP)^{-1} = -\alpha$, as well as $\nabla \times$ ... This is due to the changes $n_{0-} \longleftrightarrow n_{0+}$ and $\delta n_{\nu_a} \to -\delta n_{\nu_a}$ in Eq. (2), provided by the well-known properties: particle helicities are P-odd and particles become antiparticles under the charge conjugation operation C, in particular, active left-handed neutrinos convert to the active right-handed neutrinos under CP-operation, $\nu_a \longrightarrow \overline{\nu}_a$. Obviously, the product $(\boldsymbol{E} \cdot \boldsymbol{B})$ entering the helicity evolution (6) is CP-odd too because both electric and magnetic fields are C-odd and have opposite P-parities.

First term in the r.h.s. of Eq. (7) gives conventional ohmic losses for magnetic helicity and usually is neglected in helicity balance.

4. Seed magnetic helicity in cosmology

Neglecting in evolution Eq. (7) the first diffusion term, we can calculate the magnetic helicity $H(t)$ using Eq. (5) to yield

$$H(t) = 2B^2_{\rm max} \int\limits_v \mathrm{d}^3 r \int\limits_{t_{\rm max}}^{t} \mathrm{d}t' \alpha(t')\, \mathrm{e}^{\int_{t_{\rm max}}^{t'} \left[\frac{\alpha^2(t'')}{4\eta(t'')}\right] \mathrm{d}t''} + H(t_{\rm max}), \tag{8}$$

where $H(t_{\rm max})$ is the initial helicity value at the moment $t_{\rm max}$ if it exists, and we present the helicity density entering the integrand as $h(x) = 2.4 \times 10^3 (B_{\rm max})^2 m_{\rm e}^{-1} J(x)$. Here we use the dimensionless variable $x = T/2 \times 10^4$ MeV. The maximum value $x = 1$ corresponds to the maximum temperature $T_{\rm max} \simeq 20$ GeV $\ll T_{\rm EW} \sim 100$ GeV (see motivation above), and we assumed that the neutrino gas inhomogeneity scale is of the order of the neutrino free path $l_\nu(T)$, $\lambda^{(\nu)}_{\rm fluid} \sim l_\nu$. The WKB value of the mean magnetic field amplitude $B_{\rm max}$ obeys $B_{\rm max} \ll T^2_{\rm max}/e = 4(T_{\rm max}/{\rm MeV})^2 B_{\rm c}$, where the Schwinger field $B_{\rm c} = m^2_{\rm e}/e = 4.41 \times 10^{13}$ G. Hence we may introduce a small WKB parameter $\kappa \ll 1$ for the mean field $B_{\rm max} = \kappa T^2_{\rm max}/e \ll T^2_{\rm EW}/e$, or substituting $T_{\rm max} = 20$ GeV one obtains $B_{\rm max} = \kappa \times 7 \times 10^{22}$ G $\ll B_{\rm EW} \sim 10^{24}$ G, where, let us say, $\kappa \sim 0.01$.

It is worth noting that such WKB value of $B_{\rm max}$ which is scaled being frozen-in as $B(T) = B_{\rm max}(T/T_{\rm max})^2$ does obey the BBN limit $B \leq 10^{11}$ G at the temperature $T_{\rm BBN} \simeq 0.1$ MeV, i.e. $B(T_{\rm BBN}) = \kappa \times (7/4) \times 10^{12}$ G. Note also that the sign of the first term in Eq. (8) is not well determined since it depends on the combined neutrino density asymmetry (Semikoz & Sokoloff 2004), $\delta n_\nu / n_\nu = \sum_a c^{(A)}_{{\rm e}\nu_a} \delta n_{\nu_a}/n_{\nu_a} \sim [\xi_{\nu_\mu} + \xi_{\nu_\tau} - \xi_{\nu_{\rm e}}]$, where the values of the dimensionless neutrino chemical potentials $\xi_{\nu_a} = \mu_{\nu_a}/T$ are given by the BBN limit (Dolgov et al. 2002): $-0.01 < \xi_{\nu_{\rm e}} < 0.07$, or by the CMBR/LSS bound (Hansen et al. 2002): $-0.01 < \xi_{\nu_{\rm e}} < 0.22$, $|\xi_{\nu_{\mu,\tau}}| < 2.6$. One can use, e.g., the conservation of the lepton number $L_{\rm e} - L_\mu$ that implies $\xi_{\nu_{\rm e}} = -\xi_{\nu_\mu}$, however, this does not guarantee the definite sign of the combined neutrino density asymmetry $\delta n_\nu / n_\nu$. The definite sign of the magnetic helicity (left-handed, $H < 0$), arising during electroweak baryogenesis (Vachaspati 2001) is another case connected with the CP-violation.

Let us emphasize that cosmological helicity production via the collective neutrino interaction with hot plasma ceases if the neutrino chemical potential $\mu_{\nu_{\rm e}}$ (hence the neutrino density asymmetry $\delta n_\nu / n_\nu$) vanishes, $\xi_{\nu_{\rm e}} = \mu_{\nu_{\rm e}}/T \to 0$, $\delta n_\nu / n_\nu \to 0$. Solely the inequality $\xi_{\nu_{\rm e}} \leq 0.07$ is known from the BBN bound on light elements abundance at $T < O$(MeV) (Dolgov et al. 2002), thereby substituting for a rough estimate $\xi_{\nu_{\rm e}} = 0.07$ we estimate the integral for $h(x)$ as $J(x) \sim 10$.

Thus, collecting numbers for $B_{\rm max}$, $J(x)$ and using the electron Compton length $m_{\rm e}^{-1} = 3.86 \times 10^{-11}$ cm, one finds the huge value of *cosmological helicity density* that could seed galactic magnetic helicity

$$h(x) \simeq 4.5 \times 10^{38} \kappa^2 J(x)\, {\rm G}^2\ {\rm cm} \sim 4.5 \times 10^{39} \kappa^2\, {\rm G}^2\ {\rm cm}. \tag{9}$$

5. Discussion

Traditional galactic dynamo considered galactic magnetic field produced from a very weak seed field. This implies that the magnetic helicity of the seed field is weak. We argue that the applicability of this viewpoint is limited. The seed field for galactic dynamo can be a field of substantial strength and substantial helicity. The first part of this statement is already quite well-accepted in modern galactic dynamo (see e.g. Beck et al. 1996) while the second one is new. In this letter we suggest a physical mechanism for the magnetic helicity production for the seed field of galactic dynamo. As far as we know, such mechanisms was not considered previously. Note that Field & Carroll (2000) pointed out in a general form the importance of the electroweak phase transition for magnetic helicity generation.

We stress that the epoch just after the electro-weak phase transition and that one of galaxy formation are quite remote in respect to their physical properties. We appreciate that the magnetic helicity evolution in the time interval between these epoches has to be addressed separately. In particular, large-scale magnetic helicity produced by galactic dynamo is antisymmetric in respect to the galactic equator while the magnetic helicity from any cosmological sources is obviously independent on the galactic equator position. It is far from clear how important this asymmetry is for nonlinear galactic dynamos and for the observed asymmetry of magnetic field in Milky Way. Note also that the strong cosmological magnetic field could prevent the inverse MHD cascade on the scale of galaxies (Milano et al. 2003; Brandenburg & Matthaeus 2004).

We should remark that the huge helicity value (9) exists only in hot ultrarelativistic ($T \gg m_{\rm e}$) early universe plasma where $\alpha(T)$ is sufficiently large. The evolution of magnetic helicity $H(t)$, or how cosmological magnetic helicity feeds protogalactic fields is a complicated task. In the nonrelativistic plasma, first, positrons vanish, then with the cooling for the frozen-in magnetic field $B \sim T^2 \to 0$ the electron density at the main Landau level drops, $n_{0-} \to 0$, resulting in $\alpha \to 0$, and the magnetic helicity production becomes impossible.

Let us note that the neutrino collision mechanism (Dolgov & Grasso 2002) can not produce magnetic helicity unlike in our collisionless mechanism. This immediately comes after the substitution of the electric field term stipulated by weak collisions $\boldsymbol{E} = - \mid J_{\rm ext} \mid \boldsymbol{V}/\sigma$ (σ is the electric conductivity in the ultrarelativistic plasma) and taken from Eq. (5) in (Dolgov & Grasso 2002), where the electric current $J_{\rm ext} \sim G_{\rm F}^2$ is caused by the friction force due to the difference of weak cross-sections for neutrino scattering off electrons and positrons. This current is directed along the fluid velocity. The generalized momentum $\boldsymbol{P} = w_{\rm e}\gamma_{\rm e}\boldsymbol{V}$, $w_{\rm e} = 4T$ is the enthalpy, $\gamma_{\rm e} \gg 1$ is the γ-factor in the ultrarelativistic plasma, obeys Euler equation (Semikoz 2004)

$$(\partial_t + \boldsymbol{V}\cdot\nabla)\,\boldsymbol{P} = -\frac{\nabla p}{n_{\rm e}} + \frac{{\rm rot}\,\boldsymbol{B}\times\boldsymbol{B}}{4\pi n_{\rm e}} + \text{ weak terms,} \qquad (10)$$

from which retaining the standard MHD terms only (the first and the second ones in the r.h.s. of Euler equation) one can obtain the velocity $\boldsymbol{V} \propto \int_{t_0}^{t}[-\nabla p/n_{\rm e} + ({\rm rot}\,\boldsymbol{B}\times\boldsymbol{B})/4\pi n_{\rm e}]$ that does not contribute (in the lowest approximation over $\sim G_{\rm F}^2$) to the helicity change.

Let us note that we rely here on homogeneous magnetic fields with a scale which is less (however comparable) than the horizon $l_{\rm H}$, hence the magnetic force lines are closed within the integration volume $\int_v {\rm d}^3r(...)$, or applying the Gauss theorem one can show that the contribution of the first term in the r.h.s. of Eq. (10) to the helicity production (6) vanishes. This is exactly like for the gauge transformation of the vector potential $\boldsymbol{A}$ in the helicity $H = \int_v {\rm d}^3r\boldsymbol{A}\cdot\boldsymbol{B}$, $\boldsymbol{A} \to \boldsymbol{A} + \nabla\chi$. On the other hand, there remains an open question how to define the gauge invariant helicity for superhorizon scales.

There are other astrophysical objects for which axial vector weak forces acting on electric charges and driven by neutrinos can lead to the amplification of mean magnetic field and its helicity as given in Eq. (7). For instance, the neutrino flux vorticity which is proportional to $\nabla\times\boldsymbol{j}_{(\nu)}$ can vanish for isotropic neutrino emission from supernovas in the diffusion approximation when neutrino flux $\boldsymbol{j}_{(\nu)}(\boldsymbol{r})$ is parallel to the radius $\boldsymbol{r}$. In such case the mechanism of collective neutrino-plasma interactions originated by the axial vector weak currents becomes more efficient to amplify magnetic field than the analogous mechanism based on weak vector currents (Brizard et al. 2000).

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