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Letter to the Editor

On the nature of the hydrodynamic stability of accretion disks★

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Abstract. The linear stability of accretion disks is revisited. The governing equations are expanded asymptotically and solved to first order in the expansion parameter ϵ defined by the ratio of the disk's vertical thickness to its radial extent. Algebraically growing solutions are found for global perturbations on the radial accretion flow of thin inviscid compressible Keplerian disks. The algebraic temporal behavior is exhibited in the vertical velocities and the thermodynamic variables and has the form $t \sin \Omega_0 t$ locally in the disk where Ω_0 is the Keplerian rotation rate. The physical implications and relations to the Solberg-Hoiland stability criteria are discussed.

Key words. hydrodynamics – accretion disks – transient growth

1. Introduction

It is well known that the pure Keplerian shear flow (KSF in this letter) is linearly stable, i.e. no exponentially growing solutions are known. This stability analysis is implemented in accretion disks (AD) to also imply their linear stability. However, the governing equations for AD entail the existence of meridional circulation in the disk (Kluzniak & Kita 2000; Regev & Gitelman 2002) and here we are interested in how weak meridional flow affects the linear stability of ADs.

A classical approximation in ADs is to integrate over the thin vertical extent of the disk. However, since the dominant energy transfer is in this direction, this may result in a loss of some salient physical features. Furthermore, it is also known that the meridional circulation has a significant component in this direction. Hence, the present work includes the vertical dimension explicitly.

To the point: the possibility that transient growth (TG) phenomena may play a significant role in astrophysical disks has gained recent attention with the work of Iaonnou & Kakouris (2001) and Yecko (2004). The essence of the TG phenomenon is that significant growth of an initial perturbation develops (up to orders of magnitude) before final decay (for a review see Schmid & Henningson 2001). Indeed, linear stability analysis of KSFs show that asymptotic stability is promoted because (i) the Rossby number of the flow is ∼1 and; (ii) there are no inflection points in the KSF profile (Balbus 2003), except in the star-disk boundary layer (Bertout & Regev 1996).

Even though linear disturbances decay in the long run, the fate and character of short time transient responses (and its nonlinear consequences) in an AD may be significant and deserves deeper exploration. Two observations provide circumstantial support to the idea of TG of KSFs: (a) the analogy to laboratory flows like Couette-Taylor experiments (Richard & Zahn 1999; Longaretti 2002) which have many common features with KSFs; and (b) linear stability analysis of shearing flows in localized sections of KSFs show significant TG (Chagelelishvili et al. 2003; Tevzadze et al. 2003; Yecko 2004; Mukhopadhyay et al. 2004; Afshordi et al. 2004) leading to significant nonlinear ramifications (e.g. 2D simulations of Umurhan & Regev 2004). Also, Iaonnou & Kakouris (2001) demonstrate TG for global disk perturbations but their considerations were restricted to two-dimensional (radial-azimuthal) disturbances of an incompressible KSF. Studies of viscous compressible boundary layer shear flows have been shown by Hanifi & Henningson (1998) to have solutions which grow like t in the inviscid limit. What is most fascinating is that these algebraically growing solutions are present in the limit where the streamwise perturbations are absent. These trends beget the question: is there some sort of analogous compressible TG effect in accretion disks in the inviscid limit?

In this work we show that algebraically growing solutions indeed do exist for global perturbations of a geometrically thin AD. The steady state is assumed to be a general barotrope and we consider relatively fast adiabatic perturbations. The equations studied here emerge from an asymptotic analysis of the equations appropriate for such a disk environment (Regev 1983; Kluzniak & Kita 2000; Regev & Gitelman 2002) in the zero viscous limit.

2. Equations and general solution

We assume that what governs the evolution of the geometrically thin ADs are the Euler equations (inviscid flow).

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By "geometrically thin" we mean that

$$\epsilon \equiv \tilde{H}/\tilde{R} \ll 1, \tag{1}$$

where the vertical and radial scales are $\tilde{H}$ and $\tilde{R}$ respectively. The governing equations are asymptotically expanded in ϵ and this is detailed in Appendix A. We assume the expansion converges fast for sufficiently small ϵ. The physical advantage of the asymptotic expansion is that the procedure draws out of the governing equations terms with equal physical importance, namely the terms with the same ϵ power have the same order of magnitude effect on the flow. Further, we assume here that the perturbations can be treated adiabatically, namely we restrict the discussion to time scales shorter than the dissipation timescales in the AD. The three momentum conservation equations governing the evolution of disturbances in cylindrical coordinates are (to the lowest non vanishing power in ϵ),

$$\partial_t u_1' = 2\Omega_0 r \Omega_2' \tag{2}$$
$$r\partial_t \Omega_2' = -u_1' \tfrac{1}{r}\partial_r r^2 \Omega_0, \tag{3}$$
$$\rho_0 \partial_t v_2' = -\partial_z P_2' - \rho_2' g(r,z), \tag{4}$$

where the dynamical radial (r), azimuthal (ϕ) and vertical (z) velocities are denoted by u_1', Ω_2', v_2' respectively. The mass continuity and energy conservation equations are

$$\partial_t \rho_2' = -\tfrac{1}{r}\partial_r \rho_0 r u_1' - \partial_z \rho_0 v_2', \tag{5}$$
$$\partial_t P_2' = -u_1'\partial_r P_0 - v_2'\partial_z P_0 - \gamma P_0 \left(\tfrac{1}{r}\partial_r r u_1' + \partial_z v_2'\right). \tag{6}$$

The above are nondimensionalized according to the procedure described in Appendix A. We stress that the undisturbed state contains the possibility of *steady* meridional circulation only for non-zero viscosities. The pressure and density disturbances are P_2', ρ_2' while their steady state configurations are P_0, ρ_0. The vertical component of gravity (which points towards $z = 0$) is expressed generally as a function of r and z; for thin rotationally supported Keplerian disks it is given by, $g(z,r) = z/r^3$, up to lowest nontrivial order in the ϵ expansion. The steady rotation rate for Keplerian flow is given as $\Omega_0 = r^{-3/2}$. The ratio of specific heats is γ.

In the spirit of this general discussion, it is assumed that the steady state thermodynamic quantities behave as barotropes which means that there is a unique function relating $P_0 = P_0(\rho_0)$. For general barotropes the spatially dependent *barotropic index*, $n(r,z)$, is given by $\frac{n+1}{n} \equiv \frac{d\ln P_0}{d\ln \rho_0}$. The steady state relationship satisfies the vertical hydrostatic equilibrium, namely, $\partial_z P_0 = -g(z,r)\rho_0$. It should be noted that integration of this equation introduces an arbitrary function of the variable r which is usually taken to be the *height* of the disk, $h(r)$, which is assumed to be well behaved. Finally, we define the adiabatic sound speed as $c_0^2 \equiv \gamma P_0/\rho_0$. Once the form of the barotrope is known then so is the sound speed. Equations (2) and (3) may be reduced to the simple PDE,

$$\left(\partial_t^2 + \Omega_e^2\right) u_1' = 0, \quad \jmath \equiv r^2\Omega_0, \quad \Omega_e^2 = \tfrac{1}{r^3}\partial_r \jmath^2. \tag{7}$$

If as an initial condition we have an arbitrary meridional flow field and zero azimuthal flow perturbation, i.e. at $t = 0$, $u_1' = \bar{u}(r,z)$, $\Omega_2' = 0$, then the solution to Eq. (7) is

$$u_1' = \bar{u}(r,z)\cos\Omega_e t, \qquad \Omega_2' = \bar{\Omega}\sin\Omega_e t, \tag{8}$$

with $\bar{\Omega} = -\bar{u}(r,z)/2r^{1/2}$. The *epicyclic frequency* Ω_e is equal to Ω_0 in KSF's. Note that because Ω_e is a function of r, inspection of the solution at this order reveals its inseperability. Combining Eqs. (4–6) reveals the single PDE,

$$\left(\partial_t^2 + \mathcal{L}\right)\rho_0 v_2' = \partial_z\left(u_1'\partial_r P_0 + \tfrac{\gamma P_0}{r}\partial_r r u_1'\right) + \tfrac{g}{r}\partial_r \rho_0 r u_1', \tag{9}$$

where the differential operator $\mathcal{L}$ is given as

$$\left[\partial_z\left(g\tfrac{n+1-n\gamma}{n+1}\right)\right] - \tfrac{n-1}{n+1}\gamma g\partial_z - c_0^2\partial_z^2. \tag{10}$$

We notice immediately that the evolution of v_2' is explicitly driven by the solution obtained for u_1'. With $\rho_2' = P_2' = 0$ at $t = 0$ we have the solutions,

$$\rho_0 v_2' = M_v^{(1)} t\sin\Omega_e t + M_v^{(0)}\cos\Omega_e t,$$
$$\rho_2' = \rho^{(1)} t\cos\Omega_e t + \rho^{(0)}\sin\Omega_e t,$$
$$P_2' = P^{(1)} t\cos\Omega_e t + P^{(0)}\sin\Omega_e t. \tag{11}$$

The structure vector $\boldsymbol{M} = (M_v^{(1)}, M_v^{(0)})$ satisfies

$$\left[\mathcal{L} - \Omega_e^2\right] M_v^{(i)} = \Phi_i \qquad i = 0, 1 \tag{12}$$

where the source vector Φ, the driver, is given by:

$$\Phi = \left\{-\phi_1, -\left(\phi_0 + 2\Omega_e M_v^{(1)}\right)\right\}, \tag{13}$$

in which

$$\phi_0 = \partial_z\left(\bar{u}\partial_r P_0\right) + \partial_z\left(\gamma P_0 \tfrac{1}{r}\partial_r r\bar{u}\right) + \tfrac{g}{r}\partial_r \rho_0 r\bar{u},$$
$$\phi_1 = -\left(\partial_z \gamma P_0 \bar{u} + g\rho_0\bar{u}\right)\partial_r\Omega_e. \tag{14}$$

The remaining structure functions are given to be

$$\rho^{(1)} = \tfrac{1}{\Omega_e}\left(\partial_z M_v^{(1)} - \rho_0\bar{u}\partial_r\Omega_e\right),$$
$$\rho^{(0)} = -\tfrac{1}{\Omega_e}\left(\tfrac{1}{r}\partial_r r\rho_0\bar{u} + \partial_z M_v^{(0)} + \rho^{(1)}\right),$$
$$P^{(1)} = -\tfrac{1}{\Omega_e}\left[g M_v^{(1)} - \gamma P_0\partial_z(M_v^{(1)}/\rho_0 + \gamma P_0\bar{u}\partial_r\Omega_e\right],$$
$$\Omega_e P^{(0)} = -P^{(1)} - \bar{u}\partial_r P_0 - \gamma P_0\left(\tfrac{1}{r}\partial_r r\bar{u} + \partial_z\tfrac{M_v^{(0)}}{\rho_0}\right) + g M_v^{(0)}.$$

For $\partial_r\Omega_e \neq 0$ the expression for ϕ_1 is in general not zero. The above family of solutions is completely determined once the initial perturbation in the radial (meridional) velocity, $\bar{u}$ (or equivalently $\bar{m} = \rho_0\bar{u}$), is given (in contrast to a density or a pressure perturbation). Next, the structure vector $\boldsymbol{M}$ must be determined subject to the boundary conditions. The classical assumption is the vanishing of the Lagrangian pressure perturbation on the moving surface at $z_s = \pm h(r,t)$ (e.g. Korycansky & Pringle 1995). We also study the homogeneous solutions of Eq. (9). These acoustic modes (i) oscillate stably and; (ii) have no frequencies which are resonant with $\sin\Omega_e t$ (for n constant).

3. Nature of the solution

Classical stability analysis of linear systems assumes solutions of the simple $\exp(i\omega t)$ form and searches for a dispersion relation for ω. Here we were able to find a complete solution (i.e. $\sim t\sin\Omega_e t$) that cannot be represented by a *finite* number of exponential functions. The basic reason for this behavior lies in the structure of the properties of the ϵ expansion as reflected in the the set (7–9) and revealed in Eq. (12). The equation for the first component is linear in the perturbation. The second equation is also linear however, the second equation has as a source the solution to the first equation. This is a system of staggered linear equations where at each level the solutions to the previous level appear as source terms.

The physical effect responsible for the algebraic growth can be traced from the driver vector Φ to arise from the basic shear, i.e. $\partial_r \Omega_{\rm e}$. Solutions growing with t depend on ϕ_1. This growth vanishes if there is no shear.

The main physical consequence of the solution is an algebraic growth on the shortest time scale in the problem, namely the local dynamical time scale $\tau = 1/\Omega_{\rm e}$. The growth continues at least till the next order terms in ϵ become important. The exact point of saturation (in this inviscid limit) is under investigation now. However it is clear that the smaller $\epsilon = H/R$ is the longer the growth will continue. The effect is inherent to the thinness of the disk (or equivalently how cold it is). Also, the solution found here is in the limit of vanishing viscosity. In reality, viscous terms would enter at some stage to cause a decay.

The existence of algebraically growing solutions is insensitive to the global entropy gradient, i.e. whether or not it is stable to buoyancy oscillations, or to the sign of the Rayleigh criterion, i.e. $\partial_r \jmath$. This may then give the impression that the solution found here violates the Solberg-Hoiland criteria (SHc) (e.g. Tassoul 2000; Rudiger et al. 2002) which predicts the instability of linear infinitesimal axisymmetric disturbances of steady rotating gas flows. The criteria pertain to the dynamics of infintesimal perturbations of the steady solution of the full Euler Eqs. (A.1) and the adiabatic condition (A.2) without any assumption about the size of ϵ. We refer to this perturbation procedure as *classical-linearization* (CL). The resulting equations are subsequently analyzed and, following some simplifying assumptions, imply the SHc.

On the other hand, the approach we have taken here is different in philosophy: we instead seek to develop a finite amplitude non-linear solution to the problem of dynamical disturbances of the full equations. In this approach we start with the apriori assumption that ϵ is small. This is followed by assuming that solutions of the equations, both steady and dynamical, may be ϵ expanded. Solutions are determined by iteratively solving the resulting equations at each order of ϵ. Unlike the CL procedure, in which disturbances are infinitesimal, disturbances here are introduced with amplitudes comparable with the characteristic parameter of the system, namely ϵ. It is in this sense that these perturbations are considered finite-amplitude.

Because the two approaches yield different evolution operators (see below) it is not surprising that the algebraic growth we discover is not predicted by the SHc. The SHc are not valid for finite amplitude perturbations as these but, instead, they are valid for infinitesimal ones.

To put this in somewhat more concrete terms let $\boldsymbol{V}_L = (u'_L, \Omega'_L, v'_L, \rho'_L, P'_L)^T$, represent a column vector of the perturbed quantities resulting from the CL procedure. In CL the governing equations for the evolution of the perturbed quantities are,

$$\partial_t \boldsymbol{V}_L + \mathcal{P}\boldsymbol{V}_L = 0, \qquad \mathcal{P} = \begin{pmatrix} \boldsymbol{H} & \tilde{\boldsymbol{O}}_{\rm e} \\ \boldsymbol{D} & \boldsymbol{L} \end{pmatrix}, \tag{15}$$

with the following operator submatrices defined:

$$\boldsymbol{H} = \begin{pmatrix} 0 & -2\frac{\jmath}{r} \\ \frac{1}{r^3}\jmath_r & 0 \end{pmatrix}, \quad \boldsymbol{L} = \begin{pmatrix} 0 & \frac{g_z}{\rho_{\rm s}} & \frac{1}{\rho_{\rm s}}\partial_z \\ \partial_z \rho_{\rm s} & 0 & 0 \\ \partial_z P_{\rm s} + \gamma P_{\rm s}\partial_z & 0 & 0 \end{pmatrix},$$

in which $\jmath_r = \partial_r \jmath$, and,

$$\boldsymbol{D} = \begin{pmatrix} 0 & 0 \\ \frac{1}{r}\partial_r \rho_{\rm s} r & 0 \\ \partial_r P_{\rm s} + \gamma P_{\rm s}\frac{1}{r}\partial_r r & 0 \end{pmatrix}, \quad \tilde{\boldsymbol{O}}_{\rm e} = \epsilon^2 \begin{pmatrix} 0 & \frac{g_r}{\rho_{\rm s}} & \frac{1}{\rho_{\rm s}}\partial_r \\ 0 & 0 & 0 \end{pmatrix},$$

with $g_r = -\frac{1}{\rho_{\rm s}}\partial_r P_{\rm s}$. The quantities $P_{\rm s}$ and $\rho_{\rm s}$ are the steady state pressure and density solutions to Eq. (A.1) for arbitrary ϵ. The SHc refers to the perturbation matrix $\mathcal{P}$ of the *linearized* system. No reference to the ϵ parameter is made. We can go one step further and expand the matrix operator $\mathcal{P}$ into powers of ϵ and when doing so we discover that the $\tilde{\boldsymbol{O}}_{\rm e}$ operator, since it is proportional to ϵ^2, can be neglected for thin accretion disks (see below).

By contrast, having started from Eq. (A.1) and expanded in ϵ, we arrived upon a different system: one which has taken advantage of the extreme geometry of the system (i.e. $\epsilon \ll 1$) but also one which has bypassed the assumption of linearization and all of its host implications. In particular, the finite-amplitude expansion procedure implemented has to lowest order lead to Eqs. (2–6), which is expressed compactly, $\partial_t \boldsymbol{V}' + \mathcal{P}_{\boldsymbol{0}}\boldsymbol{V}' = 0$, with $\boldsymbol{V}' = (u'_1, \Omega'_2, v'_2, \rho'_2, P'_2)^T$. This equation is similar to (15) in which $\boldsymbol{V}_\ell \to \boldsymbol{V}'$ and where $\mathcal{P}_0$ is $\mathcal{P}$. The differences between these are (i) $P_{\rm s} \to P_0$ and $\rho_0 \to \rho_{\rm s}$ and; (ii) $\tilde{\boldsymbol{O}}_{\rm e}$ is replaced by $\boldsymbol{O}$, a 2×3 zero matrix. Evidentally $\mathcal{P}$ and $\mathcal{P}_0$ differ in quality due to the absence of $\tilde{\boldsymbol{O}}_{\rm e}$.

The algebraic growth solution discovered results from this finite-amplitude approach. The obvious question to ask is: how do the two approximations relate to each other? The approximation carried out here is not restricted to infinitesimal perturbations but instead is one that exploits the geometrical nature of the object under discussion. It is therefore clear that the implications of the SHc analysis and the analysis presented in this work refer to two disjoint perturbation spaces which evolve according to different operators.

It is easily seen that the work performed by the solution (8) per cycle $(2\pi/\Omega_{\rm e})$ is equal to $2\pi\gamma P_0 \bar{u}(\partial_r \Omega_{\rm e}/\Omega_{\rm e}^2)$. Evidentally this work is zero if the shear vanishes. It is mainly this extra work which drives the algebraic growth.

4. Physical meaning and consequences

ADs are subject to continuous noise. The consequences of the effect discovered here is a fast rise in the vertical direction followed, presumably, by decay. The process takes place continuously. The disk is never "quiet". The ability to observe such a rise depends on the radial extent of the initial perturbation. It is clear that the perturbation should be observed to propagate from small to large radii.

The main result of this work shows that both the vertical velocities and the density/pressure fluctuations show algebraic growth proportional to $t \sin \Omega_{\rm e} t$ where $\Omega_{\rm e}$ is the disk rotation rate while the radial velocity (and $\dot{m}$) show no growth in this approximation – again a result of the staggered nature of the system of asymptotic expansion. Physically the result holds so long as $t < 1/\epsilon$ for otherwise the asymptotic orderings loses their validity. Finally the results apply to any rotation law provided the object has $\epsilon \ll 1$.

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Appendix A: Equations, scalings and expansion

We consider the equations of circumstellar flow under the influence of a central gravitating point source and subject axisymmetric perturbations. Self gravity is ignored. The scalings here are identical to those used in the germinal investigations of the these problems, (e.g. Regev 1983; Kluzniak & Kita 2000; and Regev & Gitelman 2002). The Keplerian rotation speed (Ω_k) is set by the Keplerian value at the fiducial radius, i.e. $\Omega_\mathrm{k}^2 \sim GM/\tilde{R}^3$ with corresponding Keplerian speed $\tilde{V}_\mathrm{k}^2 \sim GM/\tilde{R}$. The gas pressure is given by $P = c^2\rho$ where c is the speed of sound and the pressure scale is given by $\tilde{P} \sim \tilde{\rho}\tilde{c}^2$.

Since $H \sim \tilde{c}^2/g$, it also happens that $\epsilon^2 = \tilde{c}^2/V_\mathrm{k}^2$. The choice for the temporal scale is the sound crossing time in the vertical direction, which happens to be equivalent to the rotation time at the given radius $\tilde{R}$. We assume that the radial (u) and vertical (v) velocities are scaled by the sound speeds and the rotation rate (Ω) is scaled by Ω_k. Substituting all of the above into the governing equations yields the following,

$$\begin{aligned}
&\epsilon\partial_t u + \epsilon^2 u\partial_r u + \epsilon v\partial_z u - \Omega^2 r = -\epsilon^2 \tfrac{1}{\rho}\partial_r P - \tfrac{1}{r^2}\left[1 + \epsilon^2 \tfrac{z^2}{r^2}\right]^{-\frac{3}{2}}\\
&\rho\partial_t\Omega + \epsilon\tfrac{\rho u}{r^2}\partial_r r^2\Omega + \rho v\partial_z\Omega = 0,\\
&\partial_t v + \epsilon u\partial_r v + v\partial_z v = -\tfrac{1}{\rho}\partial_z P - \tfrac{z}{r^3}\left[1 + \epsilon^2 \tfrac{z^2}{r^2}\right]^{-\frac{3}{2}},\\
&\partial_t\rho + \epsilon\tfrac{1}{r}\partial_r r\rho u + \partial_z\rho v = 0.
\end{aligned} \tag{A.1}$$

We assume here short time scale dynamic perturbations so that it is safe to assume adiabatic perturbations, $\mathrm{d}S/\mathrm{d}t = 0$, where S is the specific entropy of the gas given by $C_v \ln P/\rho^\gamma$, where γ is the usual ratio of specific heats and C_v is the specific heat at constant volume. Under these assumptions the energy equation becomes:

$$(\partial_t + \epsilon u\partial_r + v\partial_z)\, P + \gamma P\left(\epsilon\tfrac{1}{r}\partial_r u + \partial_z v\right) = 0. \tag{A.2}$$

We consider the following asymptotic expansions of the solutions

$$\begin{aligned}
\Omega &= \Omega_0(r) + \epsilon^2\left[\Omega_2(r,z) + \Omega_2'(r,z,t)\right] + \cdots\\
u &= \epsilon\left[u_1(r,z) + u_1'(r,z,t)\right] + \cdots\\
v &= \epsilon^2\left[v_2(r,z) + v_2'(r,z,t)\right] + \cdots\\
P &= P_0(r,z) + \epsilon^2\left[P_2(r,z) + P_2'(r,z,t)\right] + \cdots\\
\rho &= \rho_0(r,z) + \epsilon^2\left[\rho_2(r,z) + \rho_2'(r,z,t)\right] + \cdots
\end{aligned} \tag{A.3}$$

To lowest order we find the standard thin disk solutions which do not depend at all on any viscous mechanism:

$$\Omega_0 = r^{-3/2}, \quad \partial_z P_0 = -\rho_0 g(r,z), \quad g(r,z) \equiv z/r^3, \tag{A.4}$$

together with barotropic equation of state. The equations governing the dynamics of the temporally evolving quantities (i.e. $\rho_2', P_2', v_2', u_1', \Omega_2'$) are quoted in the body of the text. The solutions to the steady quantities $\rho_2, P_2, v_2, u_1, \Omega_2$ are not sought here but are detailed in Umurhan et al (2005, in preparation). We do note that in the absence of viscosity v_i and u_i vanish to all orders unless we externally impose some amount of radial mass flux into the system.

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