A&A 432, 355–367 (2005)
DOI: 10.1051/0004-6361:20041908

Astronomy & Astrophysics

Improvement of the IAU 2000 precession model

N. Capitaine[1], P. T. Wallace[2], and J. Chapront[1]

[1] Observatoire de Paris, SYRTE/UMR 8630 – CNRS, 61 avenue de l'Observatoire, 75014 Paris, France
e-mail: capitaine@syrte.obspm.fr
[2] H. M. Nautical Almanac Office, Space Science and Technology Department, CLRC / Rutherford Appleton Laboratory, UK
e-mail: ptw@star.rl.ac.uk

Received 26 August 2004 / Accepted 3 November 2004

Abstract. The IAU 2000 precession consists of the IAU 1976 ecliptic precession (Lieske et al. 1977, A&A, 58, 1) and the precession part of the IAU 2000A equator adopted by IAU 2000 Resolution B1.6 (Mathews et al. 2002, J. Geophys. Res., 107, B4, 10.1029/2001JB000390). In this paper we provide a range of new expressions as possible replacements for the IAU 2000 precession. The new expressions are based upon the so-called P03 solution of Capitaine et al. (2003b, A&A, 412, 567) for the equator and the ecliptic. In addition an improved model for the precession of the equator is discussed. This improved solution was obtained in exactly the same way as P03 but using a refined model for the contributions of the non-rigid Earth (Mathews 2004, private communication) and revised integration constants for the precession rates resulting from fits to the most recent VLBI data. The paper reports on the procedure that was used for improving the P03 solution and on the comparisons of this solution with the MHB 2000, IAU 2000 and P03 solutions. It also discusses the choices for the solution to be put forward as a replacement for IAU 2000. We concluded that the existing VLBI data were insufficient to provide convincing evidence that the improved solutions would deliver better accuracy than the existing P03 solution, and we recommend retaining P03 as the replacement for IAU 2000. P03, which unlike the IAU 2000 precession is dynamically consistent, has the advantage of already having been used experimentally by a number of groups; the model is recalled in Tables 3−5. Due to the strong dependence of the precession expressions on the precession rates and of the precession in longitude (or equivalently the celestial CIP X coordinate) on the J_2 rate model, we also provide a parameterized P04 solution for these quantities as functions of those parameters. The expressions include the quantities to be used in both the equinox-based and CIO-based (i.e. referred to the Celestial Intermediate Origin) transformations.

Key words. astrometry – reference systems – ephemerides – time

1. Introduction

The current IAU precession that is implemented in the IERS Conventions 2003 consists of (i) the IAU 1976 precession (Lieske et al. 1977) and (ii) the IAU 2000A precession components (Mathews et al. 2002, denoted MHB in the following) adopted by Resolution B1.6., which are merely corrections to the precession rates in longitude and obliquity of the IAU 1976 precession. The IAU 2000 precession, although of practical utility for the next few years, is in fact dynamically inconsistent and suffers, except for the improvements in the precession rates, from the same limitations as the IAU 1976 precession in the precision of the coefficients and compliance with up to date models for the ecliptic motion. As recommended by IAU Resolution B1.6, new precession models consistent with IAU 2000A have been studied, and this work has been reported in recent papers (Bretagnon et al. 2003; Fukushima 2003; Capitaine et al. 2003b). An IAU Division I Working Group on "precession and the ecliptic" was established at the 2003 General Assembly in order to look into these issues and select an improved, dynamically consistent, precession model to be proposed as a replacement for the precession component of IAU 2000A.

The solution by Capitaine et al. (2003b) (denoted P03 in the following) made a clear distinction between (i) the precession of the ecliptic due to planetary perturbations and (ii) the precession of the equator due to the luni-solar and planetary torques on the oblate Earth, both motions being expressed with respect to inertial space.

The P03 ecliptic precession was derived using both the analytical solution VSOP87 (Bretagnon & Francou 1988), which modeled the periodic terms, and the JPL numerical ephemeris DE406 (Standish 1998) fitted over a 2000-yr interval, which provided the secular portion of the model. Capitaine et al. (2004) tested the various 2003 solutions against the best numerical ephemerides and, after investigating the deficiencies in the other models, concluded that P03 was the most accurate ecliptic model currently available. This solution (the accuracy of which is estimated to be about 0.05 mas/cy over a two-millennium interval centered on J2000) is thus proposed as a replacement for the current IAU 1976 model (Lieske et al. 1977) for the precession of the ecliptic.

Article published by EDP Sciences and available at http://www.aanda.org or http://dx.doi.org/10.1051/0004-6361:20041908

The P03 equator precession solution was obtained by solving the dynamical precession equations, with (i) the P03 ecliptic precession, (ii) theoretical contributions to precession rates for a non-rigid Earth model from Williams (1994) and Mathews et al. (2002) and (iii) integration constants that were derived from the MHB estimates of precession rates by applying corrections for perturbing effects on the observed quantities (Capitaine et al. 2003b). One of these effects is due to the fact that the "observed" quantity for the precession in longitude has been shown to be not the precession in longitude itself, as generally considered, but instead its projection parallel to the equator on the conventional ecliptic with the obliquity used in the VLBI software (i.e. the IAU 1976 value) resulting in a dependence of the "observed" quantity for a given model on the obliquity value. Other spurious effects have been shown to be produced by the pre-2003 VLBI procedures, such as applying the fixed "frame bias" corrections to the moving pole as if they were nutation components. Although the P03 solution for the equator was shown in Capitaine et al. (2004) to be dynamically consistent and based upon both the most accurate model for the ecliptic precession and the best available models for effects of the non-rigid Earth, it was pointed out that (i) there were large uncertainties in some of the models, especially for the J_2 rate (i.e. the time derivative of the second degree zonal coefficient of the geopotential J_2) and (ii) VLBI records were at that stage unable to discriminate between the different solutions. It was therefore concluded that improvements were still needed both in the models and the fit to observations (for the integration constants) before a new model for the precession of the equator could be adopted that would represent a significant practical improvement with respect to IAU 2000.

Adopting the improved P03 ecliptic with the existing precession for the equator has been discussed as one of the possible options for replacing the current IAU precession. However, (i) retaining the IAU 2000A equator would be dynamically inconsistent due to the small dependence of the equator precession on a model for the ecliptic and (ii) new expressions for all of the angles would in any case be required. The reasons for the latter are (a) the change in the obliquity of the ecliptic at epoch, ϵ_0, affects the expressions for the basic quantities for the precession of the equator, ω_A and ψ_A, referred to the ecliptic of epoch and (b) the direct effect of the change in the ecliptic precession on the expressions for ϵ_A and χ_A, referred to the ecliptic of date (see Table 7 of Capitaine et al. 2004) and for p_A and GST as well, which are derived from the former expressions. Therefore, the option that was retained was to adopt both an improved ecliptic precession and the best dynamically consistent solution for the precession of the equator, albeit without any noticeable improvement in predictive power so far as the CIP is concerned. During subsequent numerical consistency tests (see Appendix B of the quoted paper) it emerged that the corrections for the effects of the pre-2003 VLBI procedure should in fact be applied to the MHB values with the opposite sign than that proposed in Paper P03. Furthermore, the MHB theory had by then been slightly revised (Mathews 2004). These factors suggested that a revision of the P03 solution for the equator would be worthwhile. This view was supported by the IAU Division I Working Group on "precession and the ecliptic" that recommended (April 2004) that the P03 solution be selected to be proposed as the new IAU precession model once its component for the precession of the equator had been revised.

This paper reports our attempts to refine the P03 solution for the equator and provides expressions for an improved model. However, the existing (unimproved) P03 solution, from Capitaine et al. (2003b, 2004), was found to have differences with respect to this improved model smaller than realistic uncertainties in the model and to fit the existing VLBI series distinctly better than the MHB 2000 or IAU 2000 models. The agreement with the improved model as well as with the VLBI results is evidence that the P03 model is trustworthy in itself. Moreover, further significant gains in accuracy would require revision of the IAU 2000A nutation model.

In view of these comparisons and of the fact that several groups already have experience in using P03 experimentally we propose that the P03 solution for both the ecliptic and equator be adopted as the replacement for the IAU 2000 precession. It is recalled in Tables 3−5.

2. Improving the P03 solution for the precession of the equator

2.1. The different steps for improving the solution

The improvement of the P03 solution for the precession of the equator was carried out in three steps, based on the set of procedures developed in the P03 paper for generating precession models but using improved observational inputs along with added refinements in the model.

As the first step, we updated the integration constants of the precession equations by using a revised MHB model (Mathews 2004) for the effects of the non-rigid Earth on precession and revised corrections for perturbing effects on the observed quantities (Capitaine et al. 2004). We then solved the P03 precession equations based on these revised integration constants and model and on the P03 solution for the ecliptic precession. This provided revised theoretical expressions for ψ_A and ω_A which, except for the linear terms, differ from the P03 solution by at most a few tens of microarcseconds.

At this stage, comparisons between the precession models and VLBI data showed clearly that: (i) the addition of four years of accurate VLBI data to the 20 years available at the time of the MHB work significantly modifies the linear fit of the models to VLBI, and (ii) the difference of about -7 mas/cy^2 in the quadratic term of the P03 expression in ψ_A with respect to IAU 2000 due to the J_2 rate effect considered in the P03 model has a non-negligible contribution on the estimated precession rate in longitude. We concluded that because of the strong dependence of the linear fit of the theoretical expressions to VLBI on both the span of the observations and the adopted theoretical t^2 term, the MHB precession rates, even when corrected for the improvement in the model and for the perturbing effects on the observed quantities mentioned above, may not be appropriate for providing the most reliable precession solution.

Therefore, as a second step, the final precession rates corresponding to the revised P03 solution were fitted to VLBI data, using the longest available interval. The quantities fitted were the residuals in the X and Y CIP coordinates, which correspond to what VLBI actually determines (as opposed to $\Delta\psi$ and $\Delta\epsilon$). This provided the final values for the precession rates for X and Y and therefore for ψ_A and ω_A.

In a third step, the finished solution was obtained by a final integration of the precession equations based on the rates found by fitting. As changes of 1 mas in the precession rates produce changes smaller than 5 microarcseconds in the coefficients of the higher-degree terms of these expressions, this last step changed the solution by no more than a few microarcseconds (apart from in the linear terms).

2.2. Upgrading the model

For solving the precession equations (i.e. Eqs. (24) and (26) of the P03 paper), we used the revised values for the MHB "non-linear" contributions to the precession rates (Mathews et al. 2002), recently computed by P. M. Mathews and made available to us for this purpose (Mathews 2004). These effects, denoted here by the subscript "nl", are, in longitude and obliquity, respectively,

$$\mathrm{d}r_{\psi_{\mathrm{nl}}} = -960\,\mu\mathrm{as/cy};\quad \mathrm{d}r_{\epsilon_{\mathrm{nl}}} = +340\,\mu\mathrm{as/cy} \tag{1}$$

replacing the corresponding MHB values for these contributions of $-21\,050$ μas/cy and 0 μas/cy, respectively (cf. Table 3 of the P03 paper). This modified the quadratic term (by 10 microarcseconds) in the solution for ψ_A and ω_A and changed the MHB value for the dynamical flattening of the Earth and consequently the MHB correction to the precession rate in longitude, which, according to P. M. Mathews has become:

$$(\psi_1)_{\mathrm{MHBrev}} = (-299\,110 \pm 710)\,\mu\mathrm{as/cy} \tag{2}$$

replacing the MHB value of $(-299\,650 \pm 400)\,\mu$as/cy.

2.3. Upgrading the integration constants of the theoretical solution

In paper P03 (Capitaine et al. 2003b), we evaluated the corrections to be applied to the estimated MHB precession rates for some perturbing effects on the observed quantities (see expressions (32) and (33) of the quoted paper). As mentioned in the introduction, subsequent to the P03 work being carried out a re-examination of the VLBI procedures made us change our interpretation of the spurious effects of the non-rigorous pre-2003 VLBI procedure. This meant that the corrections for these effects (expression (33) of the quoted paper) had in fact to be applied to the estimated MHB values with the opposite sign than that proposed in the P03 paper (see Appendix B of Capitaine et al. 2004). The correct relationship between the integration constants in longitude and obliquity, r_0 and u_0, and the MHB estimates is (in μas/cy):

$$\begin{aligned} r_0 &= (r_0)_{\mathrm{MHB}} + 2366 - 384 = +1982 \\ u_0 &= (u_0)_{\mathrm{MHB}} + 514. \end{aligned} \tag{3}$$

The values for the precession rates derived from (i) the IAU 1976 precession rates, (ii) the revised MHB correction to these precession rates (cf. Sect. 2.2) and (iii) the revised P03 corrections to the MHB estimates, rounded off to a 5 μas level, are, in arcseconds per century:

$$\begin{aligned} r_0 &= 5038.7784 - 0.299110 + 0.001980 = 5038.481270 \\ u_0 &= -0.02524 + 0.000515 = -0.024725. \end{aligned} \tag{4}$$

2.4. Revised theoretical expressions for ψ_A and ω_A

The revised expressions obtained by solving the P03 precession equations with the updated values (4) for the precession rates and the changes (1) in the model for the theoretical non-linear contributions to these precession rates are:

$$\begin{aligned} \psi_A &= 5038''.481270\,t - 1''.0789969\,t^2 - 0''.00114038\,t^3 \\ &\quad + 0''.000132851\,t^4 - 0''.0000000951\,t^5 \\ \omega_A &= \epsilon_0 - 0''.024725\,t + 0''.0512621\,t^2 - 0''.00772501\,t^3 \\ &\quad - 0''.000000467\,t^4 + 0''.0000003337\,t^5 \end{aligned} \tag{5}$$

with $\epsilon_0 = 84381''.406$, and:

$$\begin{aligned} X &= -\,0''.016617 + 2004''.191804\,t - 0''.4297558\,t^2 \\ &\quad -0''.19861829\,t^3 + 0''.000007575\,t^4 \\ &\quad + 0''.0000059285\,t^5 \\ Y &= -\,0''.006951 - 0''.024867\,t - 22''.4072727\,t^2 \\ &\quad +0''.00190026\,t^3 + 0''.001112525\,t^4 \\ &\quad +0''.0000001358\,t^5. \end{aligned} \tag{6}$$

This solution will be denoted $\mathrm{P03}_{\mathrm{rev1}}$.

Note that the unit of time used in all the expressions is Julian century after J2000 (i.e. 1 century of TDB, or TT in practice), denoted cy.

The differences larger than 1 μas of $\mathrm{P03}_{\mathrm{rev1}}$ with respect to the P03 solution are, in longitude and obliquity, respectively (in μas):

$$\mathrm{d}\psi_A = -237\,t + 10\,t^2;\quad \mathrm{d}\omega_A = +1029\,t. \tag{7}$$

The above expressions show that, except from the linear terms, the revised solution differs from the P03 solution by no more than a few tens of microarcseconds. Note that, as compared with realistic uncertainties in the precession rates (i.e. of the order of 500 μas/cy in longitude and 200 μas/cy in obliquity), only the difference in the linear term in obliquity can be considered as being significant.

2.5. Checks against VLBI data

Checks of four different precession models, namely MHB 2000, IAU 2000A, P03 of Capitaine et al. (2003) and $\mathrm{P03}_{\mathrm{rev1}}$ (i.e. expression (5) of this paper) against several series of VLBI data were carried out. The checks were based on the X and Y expressions of the components of the CIP unit vector in the Geocentric Celestial Reference System (GCRS), these being the parameters to which VLBI is directly sensitive. The coordinates X, Y were computed as the $(1, 3)$ and $(2, 3)$ elements, respectively, of the nutation × precession × bias matrix, using the precession model in question and the

IAU 2000A nutation and frame bias. The "MHB 2000" model corresponds to a straightforward use (similar to the original MHB model) of the MHB 2000 precession rates and frame biases as components of the MHB 2000A nutation. The other models correspond to a rigorous use of the MHB 2000 frame biases, precession expressions and MHB 2000A nutation in the individual rotations for biases, precession and nutation, respectively.

The observed X, Y were derived in the same way, namely by using the IAU 1976 precession and adding the VLBI celestial pole offsets to the IAU 1980 nutation. We verified that the fits of the models against observations were identical when based on either (a) the residuals in the X and Y parameters obtained as described above or (b) the residuals of the differences between the models for longitude $\times \sin \epsilon_0$ and obliquity with respect to IAU 1976/1980 and the series of VLBI observed "celestial pole offsets".

We used in these fits individual solutions from the Goddard Space Flight Center (GSFC), US Naval Observatory (USNO) and Institute of Applied Astronomy (IAA) and also the C04 combined solution of the IERS Product Center. These series were used for various intervals from 1980.0 to 2004.3.

Preliminary checks revealed that:

- the IAU 2000A precession-nutation model, not surprisingly, gives a much better fit to the data than IAU 1976/1980;
- there is no significant difference in the overall rms corresponding to the fit of the above four models against VLBI observations;
- fitting straight lines to the residuals between the models and the observations give results that are significantly different according to which precession model is used;
- the time span of the data is too short to allow a reliable quadratic fit in the residuals between the models and the observations;
- changing the quadratic term gives a significant difference in the estimated linear fit because the data interval is asymmetric about $t = 0$;
- the VLBI data before 1987 are excessively noisy;
- the free core nutation (FCN) must be corrected to avoid systematic errors in the linear fits;
- the C04 solution shows large structures and noise that do not appear in the individual series.

We concluded that:

- the limited span of the VLBI data prevents us estimating the corrections to both the precession rate and the quadratic term in longitude;
- as the quadratic term in longitude is strongly dependent on the J_2 rate model, the linear fits are dependent on the value chosen for this parameter and this effect has to be taken into account;
- even if VLBI cannot at present allow us to discriminate between these precession models, linear fits of the models against observations might yield interesting information;
- when performing the fits, it is essential to weight the observations according to their formal errors, in order to benefit from both the longest time spans and the most precise and recent observations;
- the inhomogeneous properties of the C04 IERS solution for the celestial pole offsets, which is a combination of several individual VLBI series of these parameters, makes this series less appropriate for such fits than the individual series.

The first step was to remove from the observations the FCN, which appears as a term of about 430 days period with a varying amplitude and phase. We fitted sine and cosine terms at the FCN period to data series of (in most cases) two years, verified that the results agreed with those supplied in the MHB_2000 Fortran code up to 2000 and then extended the model for a further four years with our own results (see Table 2). We used the extended model to remove from the observations the FCN effects, prior to performing a linear least-squares fit. Each observation, i, was weighted according to its formal error provided in the VLBI series, $(\sigma_\psi)_i$, $(\sigma_\omega)_i$ in longitude and obliquity, respectively. The weights in X and Y were computed as: $(W_X)_i = 1/(\sigma_\psi \sin \epsilon_0)^2_i$, $(W_Y)_i = 1/(\sigma_\omega)^2_i$.

We selected two different spans of data: (i) the interval 1980.0−2000.0, which was the interval used for the original MHB fit, and (ii) the interval 1980.0−2004.3, which is the longest available and has an additional four years of low-noise data. We also made some complementary linear fits for the interval 1990.0−2004.3. Results for these various fits are set out in Table 1.

Figures 1 to 10 plot the differences between the MHB 2000, P03$_{\rm rev1}$ or P03 solutions and the GSFC, IAA or C04 VLBI series, over the interval 1980.0 to 2004.3 (Figs. 1 to 5 for X (i.e. $\Delta\psi \sin \epsilon_0$) and Figs. 6 to 10 for Y (i.e. $\Delta\omega$)).

These figures and Table 1 show the following results.

- Fits of MHB 2000 to the GSFC and USNO VLBI series that were used in the original MHB fits (i.e. identified as GSF1122 and usn9901 IVS/IERS series according to Herring et al. (2002) and denoted here GSFC$_{\rm MHB}$ and USNO$_{\rm MHB}$), are in very good agreement with the MHB results, giving constant differences less than 50 μas and linear differences less than 110 μas/cy in both X and Y.
- There is a similar agreement of the fits of MHB 2000 to revised GSFC and USNO series for the same interval and good agreement with all the VLBI series for the period 1980−2000: a linear fit to the residuals of any of these series provides a slope smaller than 100 μas/cy for X and 400 μas/cy for Y.
- There are significant differences between results for the intervals 1990.0−2004.3 and 1980.0−2004.3 with respect to those for the same models and same VLBI series, but for the interval 1980.0−2000.0.
- GSFC and USNO series provide results that are equivalent within the formal errors of the fits; therefore retaining one of them will be sufficient for the present work.
- The differences between the fits of IAU 2000 and MHB 2000 correspond, as expected, to the difference

Table 1. Linear fits ($dX_0 + dX_1 t$; $dY_0 + dY_1 t$) of precession models for the X, Y coordinates of the CIP in the GCRS against VLBI data; unit: microarcsecond.

VLBI series	Precession model	Interval	dX_0	σ_{X0}	dX_1	σ_{X1}	dY_0	σ_{Y0}	dY_1	σ_{Y1}	rms_X	rms_Y
C04	MHB 2000	1980.0–2000.0	0	5	–32	71	27	6	71	86	257	305
GSFC	MHB 2000	1980.0–2000.0	37	5	61	94	6	5	–86	94	219	226
IAA	MHB 2000	1980.0–2000.0	0	5	–14	100	15	6	397	99	208	216
USNO	MHB 2000	1980.0–2000.0	8	5	66	94	–10	5	8	95	228	233
GSFC_{MHB}	MHB 2000	1980.0–2000.0	–13	5	–100	93	28	5	108	89	526	469
USNO_{MHB}	MHB 2000	1980.0–2000.0	–28	7	39	106	46	7	36	109	318	326
C04	IAU 2000	1980.0–2000.0	–3	5	129	71	28	6	–439	86	257	305
GSFC	IAU 2000	1980.0–2000.0	34	5	213	93	7	5	–580	94	219	227
IAA	IAU 2000	1980.0–2000.0	–3	6	138	100	16	6	–96	99	208	216
USNO	IAU 2000	1980.0–2000.0	5	5	218	94	–9	5	–482	95	228	233
GSFC_{MHB}	IAU 2000	1980.0–2000.0	–16	5	51	93	29	5	–387	89	526	469
USNO_{MHB}	IAU 2000	1980.0–2000.0	–31	7	190	106	47	7	–461	109	318	326
C04	P03	1980.0–2000.0	5	5	666	71	28	6	–948	86	257	305
GSFC	P03	1980.0–2000.0	41	5	710	94	7	5	–1090	94	220	227
IAA	P03	1980.0–2000.0	4	6	631	100	16	6	–606	99	209	216
USNO	P03	1980.0–2000.0	12	5	707	94	–9	5	–993	95	229	233
GSFC_{MHB}	P03	1980.0–2000.0	–8	5	554	93	29	5	–897	89	526	469
USNO_{MHB}	P03	1980.0–2000.0	–23	7	704	106	47	7	–972	109	318	326
C04	P03_{rev1}	1980.0–2000.0	5	5	565	71	27	6	57	86	258	305
GSFC	P03_{rev1}	1980.0–2000.0	40	5	564	94	6	5	–80	94	220	227
IAA	P03_{rev1}	1980.0–2000.0	3	6	481	100	15	6	405	99	209	216
USNO	P03_{rev1}	1980.0–2000.0	10	5	555	94	–10	5	20	95	229	233
GSFC_{MHB}	P03_{rev1}	1980.0–2000.0	–9	5	410	93	28	5	112	89	526	469
USNO_{MHB}	P03_{rev1}	1980.0–2000.0	–23	7	571	106	45	7	35	109	318	326
C04	MHB 2000	1980.0–2004.3	–43	3	–567	51	46	4	328	61	242	285
GSFC	MHB 2000	1980.0–2004.3	–6	3	–700	55	73	3	1059	57	195	206
IAA	MHB 2000	1980.0–2004.3	–42	3	–734	59	39	3	802	59	187	191
USNO	MHB 2000	1980.0–2004.3	–32	3	–669	56	47	3	1038	58	201	208
C04	IAU 2000	1980.0–2004.3	–44	3	–386	51	47	4	–174	61	242	285
GSFC	IAU 2000	1980.0–2004.3	–7	2	–511	55	74	3	570	57	195	206
IAA	IAU 2000	1980.0–2004.3	–42	3	–544	59	40	3	313	59	187	191
USNO	IAU 2000	1980.0–2004.3	–33	2	–479	56	48	3	551	58	201	208
C04	P03	1980.0–2004.3	–44	3	47	51	47	4	–685	61	243	285
GSFC	P03	1980.0–2004.3	–9	3	–161	55	74	3	58	57	196	206
IAA	P03	1980.0–2004.3	–44	3	–199	60	40	3	–199	59	188	191
USNO	P03	1980.0–2004.3	–35	2	–134	56	48	3	38	58	202	208
C04	P03_{rev1}	1980.0–2004.3	–45	3	–67	51	47	4	329	61	243	285
GSFC	P03_{rev1}	1980.0–2004.3	–10	3	–307	55	73	3	1080	57	196	206
IAA	P03_{rev1}	1980.0–2004.3	–45	3	–345	60	40	3	823	59	188	191
USNO	P03_{rev1}	1980.0–2004.3	–36	2	–281	56	48	3	1062	58	202	208
GSFC	MHB 2000	1990–2004.3	–6	3	–936	64	73	3	1414	66	151	153
IAA	MHB 2000	1990–2004.3	–41	3	–972	70	40	3	869	71	148	152
GSFC	IAU 2000	1990–2004.3	–7	3	–740	64	74	3	933	66	148	152
IAA	IAU 2000	1990–2004.3	–42	3	–776	70	41	3	387	71		
GSFC	P03	1990–2004.3	–8	3	–430	64	74	3	420	66	150	153
IAA	P03	1990–2004.3	–43	3	–466	70	40	3	–126	71	148	152
GSFC	P03_{rev1}	1990–2004.3	–10	3	–590	64	74	3	1449	66	151	153
IAA	P03_{rev1}	1990–2004.3	–44	3	–624	70	40	3	902	71	149	152

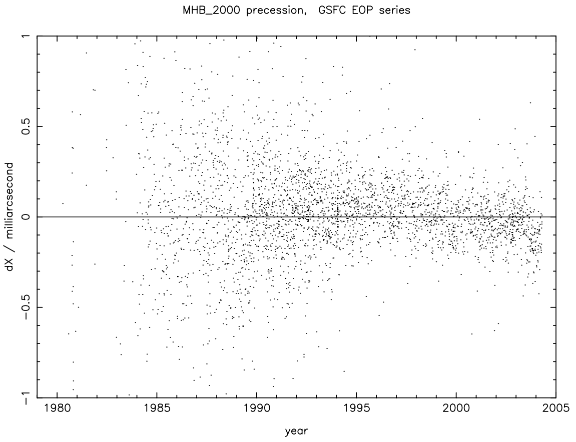

Fig. 1. X differences, MHB 2000 minus GSFC.

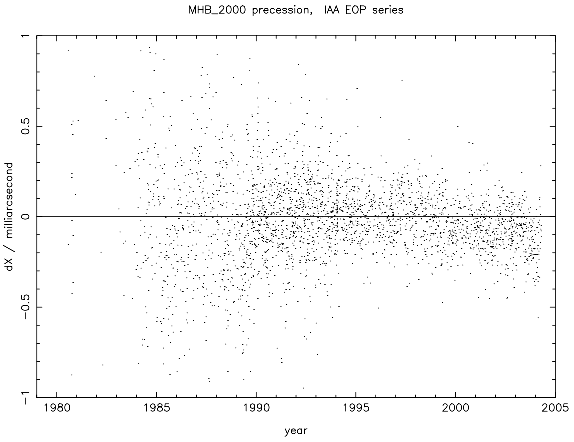

Fig. 2. X differences, MHB 2000 minus IAA.

between the pre-2003 and post-2003 VLBI procedures (i.e. $+152\,\mu$as/cy for X and $-514\,\mu$as/cy for Y).

- The differences between the fits of P03 and IAU 2000 include, as expected, (i) the correction applied in P03 for correcting the effect of the pre-2003 VLBI procedure, (ii) the effect[1] of the difference in the quadratic term in X between P03 and IAU 2000.
- The $\mathrm{P03_{rev1}}$ solution in obliquity gives exactly the same results as MHB 2000, which proves that this solution is reproducing a precession rate that, when based on an implementation that is different from MHB (due to the change in obliquity at epoch, frame bias handling, etc.), reproduces the MHB model.
- The $\mathrm{P03_{rev1}}$ solution in longitude does not give similar results to MHB 2000, as might be expected given that (i) it is based on the $\mathrm{MHB_{rev}}$ precession rate which differs by $540\,\mu$as/cy from the MHB value and (ii) there is the additional effect due to the difference in the t^2 term between the two solutions.
- The linear fit of MHB 2000 with respect to VLBI over this interval 1980.0−2004.3 is of the order of $700\,\mu$as/cy (for X) and $900\ \mu$as/cy for Y, using several series of VLBI data. Note that such large residual linear terms in the VLBI fit of MHB 2000 over the longest available span of data is consistent with comparisons reported by Malkin (2004).
- The P03 solution fits the VLBI series distinctly better than the MHB 2000, IAU 2000 and $\mathrm{P03_{rev1}}$ models.

We also performed similar fits for the Bretagnon et al. (2003) model and saw the expected discrepancies with respect to the fits of the IAU 2000 model: i.e. the use of a value for ϵ_0 which is not appropriate for the MHB precession rate in longitude has produced a -0.9 mas t effect in X while the use of a rigid Earth model has led to a -1.3 mas t effect in Y. The additional discrepancy with respect to the fits of the P03 model comes from the fact that this model contains a -7 mas/cy^2 contribution for the $J2$ rate.

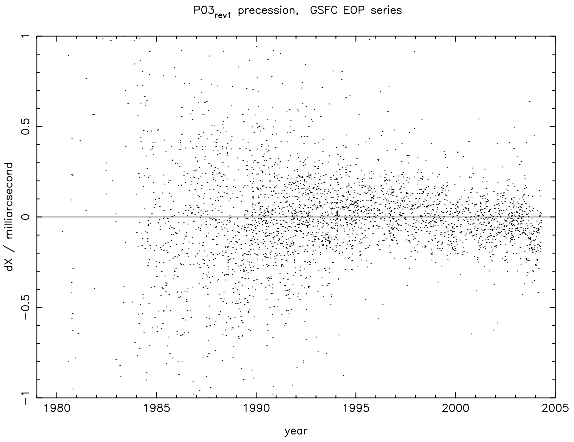

Fig. 3. X differences, $\mathrm{P03_{rev1}}$ minus GSFC.

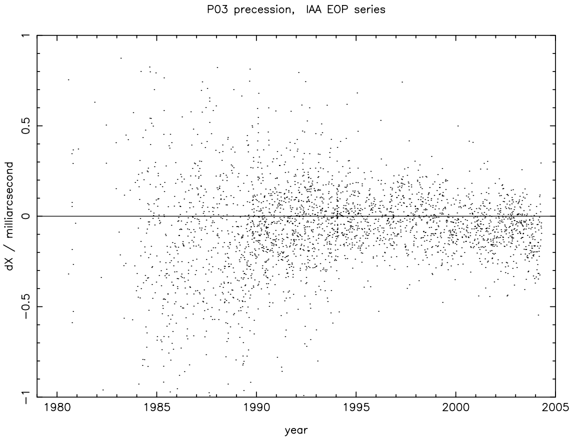

Fig. 4. X differences, P03 minus IAA.

2.6. Revised expressions based on precession rates fitted to VLBI

We have concluded from the various VLBI checks that the MHB values do not fit well with the most recent and accurate VLBI data. Therefore it appeared that it might be preferable that the final precession solution rely on more recent VLBI fits (1980.0−2004.3) than applying theoretical corrections to the MHB precession rates (that result from a fit to VLBI data on the interval 1980−2000). For the final estimates of the

---

[1] Which has been evaluated to be of 1.451 mas/cy in ψ_A for the interval 1980−2000, corresponding to $557\,\mu$as/cy for X.

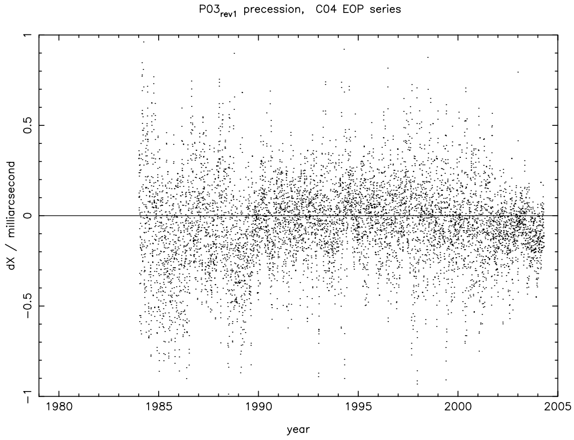

Fig. 5. X differences, P03$_{\rm rev1}$ minus C04.

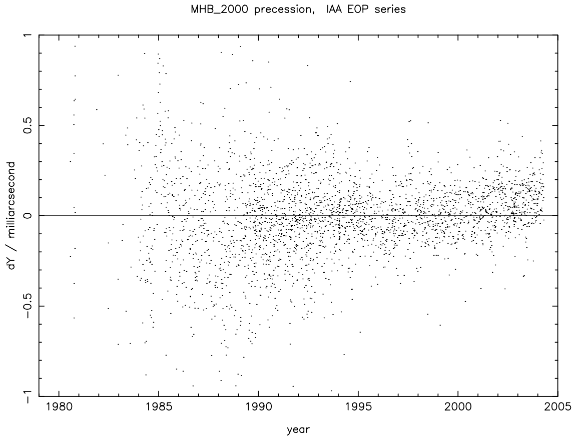

Fig. 6. Y differences, MHB 2000 minus IAA.

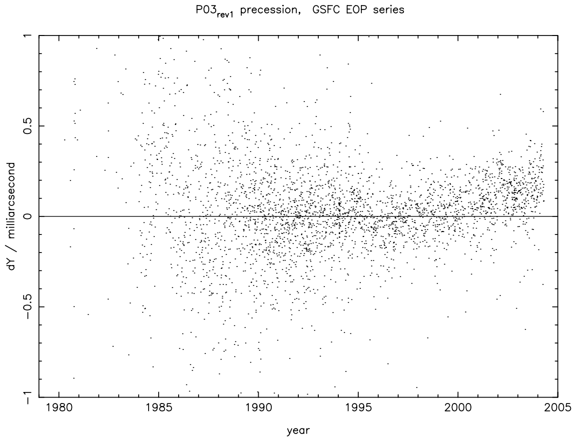

Fig. 7. Y differences, P03$_{\rm rev1}$ minus GSFC.

correction to the precession rates of the models, we have retained only the GSFC and IAA series which (i) both cover the longest time span and (ii) are based upon independent reduction software (viz. CALC for GSFC and OCCAM for IAA).

The estimated correction to the P03$_{\rm rev1}$ solution is:

$$\mathrm{d}X = +326\,\mu\mathrm{as/cy}; \quad \mathrm{d}Y = -950\,\mu\mathrm{as/cy} \qquad (8)$$

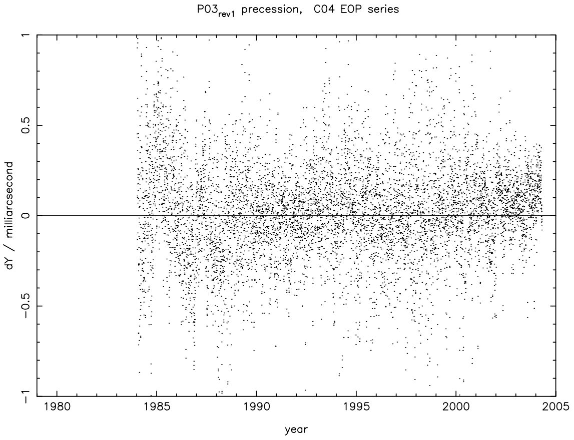

Fig. 8. Y differences, P03$_{\rm rev1}$ minus C04.

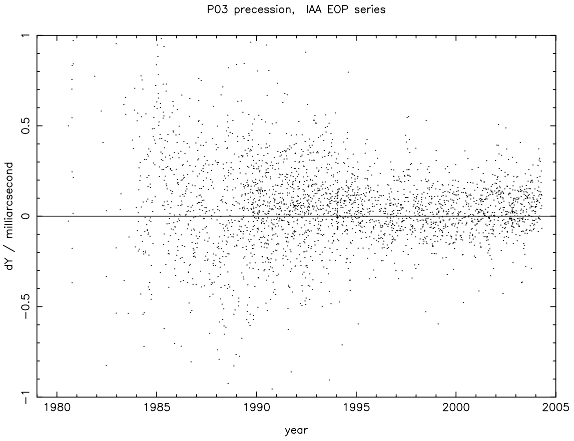

Fig. 9. Y differences, P03 minus IAA.

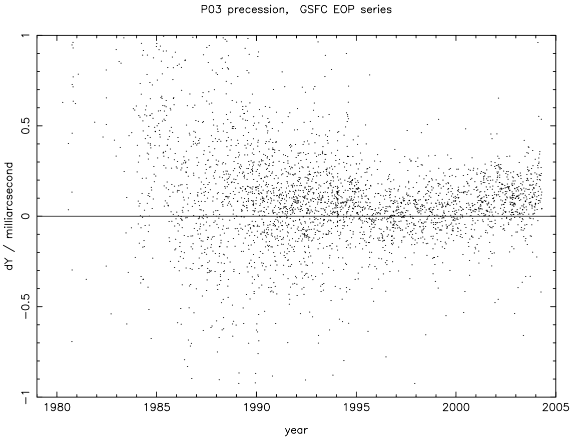

Fig. 10. Y differences, P03 minus GSFC.

from which we can derive the correction to the precession rates in ψ_A and ω_A corresponding to the IERS 2003 value for $\mathrm{d}\alpha_0$ of -14.6 mas:

$$\mathrm{d}\psi_A = +820\,\mu\mathrm{as/cy}; \quad \mathrm{d}\omega_A = \mathrm{d}\epsilon_A = -950\,\mu\mathrm{as/cy} \qquad (9)$$

Table 2. Amplitudes of the FCN mode fitted to VLBI data; unit: milliarcsecond. For the fits made in this paper, the MHB model was used up to and including 2000/01/01, and the fitted coefficients after this date.

	MHB_2000				This paper			
Node date	cos	±	sin	±	cos	±	sin	±
1979/01/01	−0.0620	0.1256	−0.1346	0.1293	0.042	0.015	−0.021	0.015
1984/01/01	0.0447	0.0302	−0.1679	0.0309	0.022	0.017	0.005	0.017
1986/01/01	0.2406	0.0163	−0.2759	0.0159	0.153	0.016	−0.202	0.016
1988/01/01	0.1183	0.0127	−0.2163	0.0128	0.075	0.017	−0.247	0.017
1990/01/01	0.0479	0.0084	−0.1965	0.0083	0.051	0.013	−0.168	0.013
1992/01/01	−0.0796	0.0071	−0.1321	0.0071	−0.038	0.010	−0.134	0.010
1994/01/01	−0.0075	0.0057	−0.1150	0.0057	−0.031	0.011	−0.087	0.011
1996/01/01	−0.0128	0.0058	−0.0998	0.0058	−0.010	0.010	−0.084	0.010
1998/01/01	−0.0263	0.0059	−0.1122	0.0059	−0.052	0.012	−0.090	0.012
2000/01/01	0.0519	0.0071	0.0081	0.0070	0.078	0.011	−0.030	0.011
2001/01/01					0.148	0.010	0.046	0.010
2001/06/01	0.2100	0.0162	0.1401	0.0163				
2002/01/01					0.149	0.009	0.101	0.009
2003/01/01					0.046	0.009	0.120	0.009

and therefore:

$$\begin{aligned} \psi_1 &= 5038''.482090/\mathrm{cy}; \quad \omega_1 = -0''.025675/\mathrm{cy}, \\ \epsilon_1 &= -46''.836690/\mathrm{cy}. \end{aligned} \tag{10}$$

We developed a solution like $\mathrm{P03_{rev1}}$ in Sect. 2.2 by solving again the P03 precession equations but this time using as integration constants the precession rates in longitude and obliquity derived from VLBI fits given by Eq. (10). The final expressions, that we will denote “the $\mathrm{P03_{rev2}}$ solutions”, are:

$$\begin{aligned} \psi_A &= 5038''.482090\,t - 1''.0789921\,t^2 - 0''.00114040\,t^3 \\ &\quad + 0''.000132851\,t^4 - 0''.0000000951\,t^5 \\ \omega_A &= \epsilon_0 - 0''.025675\,t + 0''.0512622\,t^2 - 0''.00772501\,t^3 \\ &\quad - 0''.000000467\,t^4 + 0''.0000003337\,t^5 \end{aligned} \tag{11}$$

with $\epsilon_0 = 84381''.406$, and:

$$\begin{aligned} X &= -\,0''.016617 + 2004''.192130\,t - 0''.4297752\,t^2 \\ &\quad -0''.19861839\,t^3 + 0''.000007576\,t^4 \\ &\quad +0''.0000059285\,t^5 \\ Y &= -0''.006951 - 0''.025817\,t - 22''.4072801\,t^2 \\ &\quad +0''.00190046\,t^3 + 0''.001112526\,t^4 \\ &\quad +0''.0000001358\,t^5. \end{aligned} \tag{12}$$

The linear fits of the $\mathrm{P03_{rev2}}$ X, Y expressions to VLBI over the time range of 1980.0−2004.3 are:

(i) using the IAA series:
$\mathrm{d}X = (-45 \pm 3) + (+21 \pm 60)\,t$
$\mathrm{d}Y = (+39 \pm 3) + (-137 \pm 60)\,t$;
(ii) using the GSFC series:
$\mathrm{d}X = (-8 \pm 3) + (+72 \pm 55)\,t$
$\mathrm{d}Y = (+73 \pm 3) + (+124 \pm 60)\,t$;

and the differences larger than 1 μas of $\mathrm{P03_{rev2}}$ with respect to the P03 solution are, in μas:

$$\mathrm{d}\psi_A = +583\,t + 15\,t^2; \quad \mathrm{d}\omega_A = +79\,t, \tag{13}$$

or equivalently:

$$\mathrm{d}X = +232\,t + 8\,t^2; \qquad \mathrm{d}Y = +79\,t - 5\,t^2. \tag{14}$$

We notice that this solution (i) fits well to the longest VLBI series and (ii) is very close to the P03 solution both in X and Y.

2.7. Precision of the coefficients

Regarding the precession of the ecliptic, we recall from Sect. 1 that the accuracy of the secular variations of the P03 solution in P_A and Q_A was estimated to be about 0.05 mas/cy over a two-millennium interval centered on J2000.0.

Regarding the precession of the equator we have already mentioned that, although the precision in computing the precession solution has been significantly improved compared with that of the IAU 2000 precession by (i) using the best available model for the ecliptic precession, (ii) using the best available model for the non-rigid Earth effects and (iii) providing a solution that is dynamically consistent, this model is dependent on the model for some parameters of the non-rigid Earth, such as the J_2 rate, that have large uncertainties. The accuracy of the expression for the precession in longitude is therefore strongly limited by the uncertainty in the model for the J_2 time variations, which is if the order of 20% (Bourda & Capitaine 2004), resulting in uncertainties of the order of 1.5 mas/cy^2 in the quadratic term in ψ_A. As noted in that paper, the J_2 rate value used for the P03 solution (i.e. of the order of -3×10^{-9}/cy) is in good agreement with the J_2 rate

Table 3. The P03 primary precession quantities compared with the IAU 2000 solution (IAU); unit: milliarcsecond.

Source		t^0	t	t^2	t^3	t^4	t^5
IAU 2000	P_A		4197.6	194.47	−0.179		
P03			4199.094	193.9873	−0.22466	−0.000912	0.0000120
IAU	Q_A		−46815.0	50.59	0.344		
P03			−46811.015	51.0283	0.52413	−0.000646	−0.0000172
IAU 2000	ψ_A		5 038 478.750	−1072.59	−1.147		
P03			5 038 481.507	−1079.0069	−1.14045	0.132851	−0.0000951
IAU 2000	ω_A	84381448.0	−25.240	51.27	−7.726		
P03		84381406.0	−25.754	51.2623	−7.72503	−0.000467	0.0003337

evaluation of $(-3.4 \pm 0.6) \times 10^{-9}$/cy by Morrison & Stephenson (1997) from long term studies of the Earth rotation variations, based upon eclipse data over two millennia.

Moreover, the linear fit to VLBI is still spanning too small an interval to provide a realistic uncertainty better than about 150 μas/cy for X and Y despite the smaller formal uncertainty provided by the fit.

2.8. Choice of the precession solution for the equator

There are three choices for replacing the IAU 2000 precession of the equator, all dynamically consistent: (i) use P03$_{\rm rev1}$, based on the MHB precession rates but corrected for various theoretical effects; (ii) use P03$_{\rm rev2}$, based on the latest VLBI estimated precession rates; and (iii) retain the existing P03 solution (the small change in the t^2 term from P03 to P03$_{\rm rev2}$ being in fact smaller than the uncertainty in this term). Given the results of Sects. 2.5 and 2.6, and the evaluation of the precision provided in Sect. 2.7, choice (iii) (i.e. retain P03) seems the most appropriate as long as it is associated with a parameterized solution as a function of the parameters (i) to which the precession expressions are the most sensitive and (ii) which are expected to be improved in the future.

3. Final expressions for the new precession model

The final expressions proposed here as replacements for the IAU 2000 precession quantities include (i) P03 expressions for the ecliptic precession; (ii) P03 expressions for the primary precession quantities for the equator; (iii) precession quantities for classical use derived from the previous P03 quantities including revised sidereal time and (iv) expressions for precession quantities for use with the new paradigm based on the positions of the celestial intermediate pole (CIP) and celestial intermediate origin (CIO)[2], respectively, in the GCRS. These P03 numerical expressions are associated with a parameterized extension comprising functions of the $J2$ rate model and of the changes to the P03 precession rates.

Note that the coefficients of the P03 expressions are given in the tables with more digits than the real uncertainty, for the purpose of internal consistency.

3.1. P03 expressions for the primary precession quantities

The most "fundamental" parameters for the precession of the equator are those (either angular variables or Cartesian coordinates) providing the "dynamically derived" position of the equator (or equivalently the CIP) relative to the fixed ecliptic. These are indeed the most basic variables due to the fact that the most recent analytical theories for the solar system bodies, upon which the Earth's rotation theory is based, are referred to the fixed ecliptic. The dynamics of Earth rotation do not involve the concept of an ecliptic, but are directly based on the dynamics of the solar system bodies in an inertial frame, which means that using the ecliptic as a reference is only an approximate intermediate stage in obtaining simplified expressions for the precession-nutation of the equator. Suitable precession parameters for the ecliptic are the (x, y) coordinates of the ecliptic pole in the mean ecliptic frame of epoch. Note that the x, y coordinates (either of the ecliptic pole or the equatorial pole) are more basic quantities than individual angles that are determined by their trigonometric relations.

The primary precession quantities selected for the P03 solution, from which all the other quantities can be derived, are:

(i) the expressions for the ecliptic quantities P_A, Q_A which may be regarded as, respectively, the x and $-y$ components of the secularly-moving ecliptic pole vector in a (right-handed) coordinate system that has its x-axis through the J2000 (inertial) mean equinox and its z-axis through the J2000 ecliptic pole;

(ii) the expressions for the quantities ψ_A, ω_A for the equator, which are the polar coordinates of the CIP with respect to the J2000 ecliptic pole and J2000 (inertial) mean equinox.

The developments for these primary quantities are provided in Table 3 together with the corresponding IAU 2000 expressions.

3.2. P03 expressions for the derived precession quantities for classical use and GMST

The P03 expressions for the ecliptic precession angles π_A and Π_A were derived from the developments for the basic ecliptic quantities P_A and Q_A, which can be written as

---

[2] Formerly known as the celestial ephemeris origin or CEO.

Table 4. The P03 expressions for the derived precession quantities for classical use and Greenwich mean sidereal time expressed in terms of the Earth rotation angle θ; unit: milliarcsecond.

Source	t^0	t	t^2	t^3	t^4	t^5
π_A		46 998.973	−33.4926	−0.12559	0.000113	−0.0000022
Π_A	629 546 793.6	−867 957.58	157.992	−0.5371	−0.04797	0.000072
ζ_A	2650.545	2 306 083.227	298.8499	18.01828	−0.005971	−0.0003173
θ_A		2 004 191.903	−429.4934	−41.82264	−0.007089	−0.0001274
ϵ_A	84 381 406.0	−46 836.769	−0.1831	2.00340	−0.000576	−0.0000434
χ_A		10 556.403	−2381.4292	−1.21197	0.170663	−0.0000560
p_A		5 028 796.195	1105.4348	0.07964	−0.023857	−0.0000383
z_A	−2650.545	2 306 077.181	1092.7348	18.26837	−0.028596	−0.0002904
GMST(UT1)-θ(UT1)	14.506	4 612 156.534	1391.5817	−0.00044	−0.029956	−0.0000368

Table 5. The P03 expressions for the GCRS components of the CIP and angular position of the CIO, and for the rotation vector components; unit: milliarcsecond.

Source	t^0	t	t^2	t^3	t^4	t^5
X	−16.617	2 004 191.898	−429.7829	−198.61834	0.007578	0.0059285
Y	−6.951	−25.896	−22 407.2747	1.90059	1.112526	0.0001358
$s + XY/2$	0.094	3.80865	−0.12268	−72.57411	0.02798	0.01562
x_r	6.8192	26.0106	0.0236	−3.8564	−0.0004	
y_r	−16.6171	2 004 191.9789	−429.4924	−0.0697	0.0092	
z_r	−14.6000	−4 612 160.3744	−1391.5844	0.0006	0.0300	

$P_A = \sin\pi_A \sin\Pi_A$ and $Q_A = \sin\pi_A \cos\Pi_A$. Similarly, the P03 expressions for the two first equatorial precession angles ζ_A and θ_A were derived from the P03 developments for ψ_A and ω_A.

The expressions for all the precession parameters that refer to the ecliptic of date (i.e. the third equatorial precession angle, z_A, the obliquity of the equator on the moving ecliptic, ϵ_A, the planetary precession along the equator of date, χ_A and the general precession in longitude, p_A) were derived from both the P03 ecliptic and equator primary expressions.

Table 4 provides expressions for all the precession quantities for classical use, along with the expression for GMST(UT1,TT) the IAU 2000 expression for which (Capitaine et al. 2003a) must be revised in order to take account of the changes in the expressions for the precession quantities ψ_A, χ_A and ω_A. Note that, in contrast, the expressions for the complementary terms in the equation of the equinoxes are unchanged.

3.3. P03 expressions for precession quantities for use with the new paradigm

A replacement for the IAU model must provide improved precession in both the classical and new paradigms. The new paradigm provides the CIP directly, without any concept of "mean pole". If "precession" is to be a meaningful and useful concept in the new paradigm, we should consider precession of the equator as being the secular part of the "orientation parameters" of the CIP equator with respect to either J2000 mean equatorial system or the GCRS. The precession expressions for X and Y are thus identified with the polynomial part of the GCRS CIP direction cosines, based on the P03 expressions for ψ_A and ω_A.

The P03 expressions for these quantities as well as for the quantity $s + XY/2$ that provides the GCRS position of the CIO are given in Table 5. Note that the only significant change with respect to the IAU 2000 expression for the GCRS position of the CIO is of 2.7 μas in the quadratic term, the other changes all being less than 0.5 μas. Table 5 also provides the components x_r, y_r, z_r of the "rotation vector" representing bias plus the P03 precession (see Capitaine et al. 2003b). The rotation vector is a concise and direct way to represent the rotation of the coordinate system, in this case from GCRS axes to mean equator and equinox of date. Its direction is the pole of the rotation (the Euler axis) and its magnitude is the amount of rotation (the Euler angle). In this case, to first order, the rotation vector points at the ecliptic pole and its magnitude increases steadily by about 50 arcsec per year.

3.4. The parameterized P04 precession expressions

Due to the strong dependence of (i) the precession expressions on the precession rates r_0 and u_0 in longitude and obliquity, respectively and (ii) the precession in longitude (or equivalently in the GCRS CIP X coordinate) on the J_2 rate model, both of which are expected to be improved in the future, we have developed a parameterized solution for these quantities as a function of those parameters, calling it P04$_{\rm par}$. Such a solution is intended to be used to produce (or check) future precession models based on extended VLBI records and improved geophysical models. It is based on the P03 precession expressions (Capitaine et al. 2003b) and on the theoretical relationship of their linear, quadratic, and cubic coefficients, ψ_i, ω_i, ϵ_i, χ_i (with i = 1, 2, 3 for the linear, quadratic and cubic coefficients, respectively), with constant and linear contributions, r_0, u_0, r_1, u_1 to the precession rates, r_ψ in longitude and r_ϵ in obliquity. The theoretical relationships were derived from Tables 3 and 7 of the P03 Paper using the following expressions:

$$\begin{aligned}\psi_1 &= r_0;\quad \omega_1 = u_0\\ \psi_2 &= \frac{1}{2}\left(r_1 + r_0 c_1 \cot\epsilon_0 - \frac{u_0 s_1}{\sin^2\epsilon_0}\right)\\ \omega_2 &= \frac{1}{2}(u_1 + r_0 s_1)\end{aligned}\tag{15}$$

and:

$$\begin{aligned}\chi_1 &= s_1/\sin\epsilon_0;\quad \epsilon_1 = c_1 + u_0\\ \chi_2 &= \frac{1}{\sin\epsilon_0}(s_2 + r_0 c_1 - s_1\cot\epsilon_0(u_0 + c_1))\\ \epsilon_2 &= c_2 + \frac{1}{2}(u_1 - r_0 s_1 + s_1^2\cot\epsilon_0)\end{aligned}\tag{16}$$

s_1, c_1 and s_2, c_2 being the linear and quadratic coefficients of the precession ecliptic quantities P_A and Q_A, respectively, and:

$$\begin{aligned}r_0 &= r_{01} + r_{02} = f_{01}\cos\epsilon_0 + r_{02}\\ u_0 &= u_{01} + u_{02} = g_{01}\cos\epsilon_0 + u_{02},\end{aligned}\tag{17}$$

$$\begin{aligned}r_1 &= -f_{01}(u_0 + c_1)\sin\epsilon_0 + f_{11}\cos\epsilon_0 + r_{12}\\ u_1 &= -g_{01}(u_0 + c_1)\sin\epsilon_0 + g_{11}\cos\epsilon_0 + u_{12}.\end{aligned}\tag{18}$$

$(f_{01} + f_{11}\,t)\cos\epsilon_0$ and $(g_{01} + g_{11}\,t)\cos\epsilon_0$ being the first-order luni-solar contributions to r_ψ and r_ϵ, respectively. Note that these components are such that f_{01} and g_{01} are proportional to J_2 and f_{11} and g_{11} contain a part proportional to $\dot{J}_2$.

This gives for ψ_A and ω_A:

$$\begin{aligned}\psi_2 &= \frac{1}{2}\Big[-f_{01}u_0\sin\epsilon_0 + f_{11}\cos\epsilon_0 + r_{12} - \frac{u_0 s_1}{\sin^2\epsilon_0}\\ &\quad + f_{01}c_1(\cos^2\epsilon_0 - \sin^2\epsilon_0)/\sin\epsilon_0 + r_{02}c_1\cot\epsilon_0\Big]\\ \omega_2 &= \frac{1}{2}\Big[-g_{01}(u_0 + c_1)\sin\epsilon_0 + r_0 s_1\\ &\quad + g_{11}\cos\epsilon_0 + u_{12}\Big],\end{aligned}\tag{19}$$

and similar relationships for the other precession quantities.

The parameterized expressions of the precession quantities as functions of $\dot{J}_2$ and of corrections dr_0, du_0 to the P03 precession rates r_0 and u_0, respectively, should retain only the parameterized terms that, given the expected values for the parameters considered, can contribute to the expressions with amplitudes larger than one microarcsecond. The time derivative $\dot{J}_2$ of the coefficient J_2 contributes to r_1 as follows:

$$r_1(J2d) = f_{11}(J2d)\cos\epsilon_0 = (\dot{J}_2/J_2)f_{01}\cos\epsilon_0.\tag{20}$$

According to Table 7 of the P03 Paper and to expected values lower than 1 mas/cy for the corrections to the precession rates and 5×10^{-6} for the term $\dot{J}_2/J_2$ (see Sect. 3.6), this requires considering the following terms in the partials of the coefficients of the precession expressions:

$$\begin{aligned}\partial\psi_1/\partial r_0 &= 1;\ \partial\omega_1/\partial u_0 = \partial\epsilon_1/\partial u_0 = 1,\\ \partial\psi_2/\partial u_0 &\approx -\frac{1}{2}r_{01}\tan\epsilon_0;\ \partial\psi_2/\partial r_1 = \frac{1}{2},\\ \partial\psi_3/\partial r_1 &= \frac{1}{3}c_1\cot\epsilon_0; \partial\chi_2/\partial r_0 = 2\partial\chi_3/\partial r_1 = \frac{c_1}{\sin\epsilon_0}\cdot\end{aligned}\tag{21}$$

The resulting P04$_{\rm par}$ expressions for classical use are, with dr_0, du_0 in arcseconds:

$$\begin{aligned}\psi_A(P04_{\rm par}) &= \psi_A(P03) + {\rm d}r_0\,t - 0.0053\,{\rm d}u_0\,t^2\\ &\quad + \left[0''.007000\,t^2 - 0''.000002\,t^3\right]\\ &\quad + (\dot{J}_2/J_2)\times\left(2520''.4\,t^2 - 0''.9\,t^3\right)\\ \omega_A(P04_{\rm par}) &= \omega_A(P03) + {\rm d}u_0\,t,\end{aligned}\tag{22}$$

$$\begin{aligned}\chi_A(P04_{\rm par}) &= \chi_A(P03) - 0.0006\,{\rm d}r_0\,t^2\\ &\quad - [0''.000004\,t^3] - (\dot{J}_2/J_2)\,(1''.4\,t^3)\\ \epsilon_A(P04_{\rm par}) &= \epsilon_A(P03) + {\rm d}u_0\,t.\end{aligned}\tag{23}$$

And the expression for Greenwich mean sidereal time is:

$$\begin{aligned}GMST(P04_{\rm par}) &= GMST(P03)\\ &\quad + {\rm d}r_0\cos\epsilon_0\,t - 0.0098\,{\rm d}u_0\,t^2\\ &\quad + \left[0''.006422\,t^2 + 0''.000002\,t^3\right]\\ &\quad + (\dot{J}_2/J_2)\times\left(2312''.4\,t^2 + 0''.6\,t^3\right).\end{aligned}\tag{24}$$

The additional t^2 and t^3 terms that appear between square brackets in the expressions for the differences (P04$_{\rm par}$–P03) for the precession quantities ψ_A, χ_A and *GMST* (and also X, Y below) are for removing the $\dot{J}_2/J_2$ contribution to the P03 solution. Taking into account additionally the effects of the corrections dξ_0, dη_0 and d(dα_0) to the IAU 2000 frame biases, we get the parameterized P04 expressions for the GCRS X and Y quantities for use with the new paradigm:

$$\begin{aligned}X(P04_{\rm par}) &= X(P03) + {\rm d}\xi_0 + 0.0001\,{\rm d}({\rm d}\alpha_0)\,t^2\\ &\quad + {\rm d}X_1\,t + 0.0203\,{\rm d}u_0\,t^2\\ &\quad + \left[0''.002784\,t^2 - 0''.000001\,t^3\right]\\ &\quad + (\dot{J}_2/J_2)\times(1002''.5\,t^2 - 0''.4\,t^3)\end{aligned}$$

$$\begin{aligned}Y(P04_{\rm par}) &= Y(P03) + {\rm d}\eta_0 + X_1\,{\rm d}({\rm d}\alpha_0)\,t\\ &\quad + {\rm d}u_0\,t - 0.0224\,{\rm d}X_1\,t^2\\ &\quad - [0''.000062\,t^3] - (\dot{J}_2/J_2)\times(22''.5\,t^3)\end{aligned}\tag{25}$$

where dX_1 (=d$r_0\sin\epsilon_0$) is the correction to the linear term X_1 of the P03 X expression.

3.5. Discussion on the choice of the basic parameters for precession-nutation

Suitable precession-nutation parameters would integrate the computation of bias, precession and nutation and provide a transformation between celestial and terrestrial coordinates that involves a minimum number of variables and coefficients.

Suitable candidates for replacing the classical precession angles ψ_A, ω_A, χ_A and nutation angles ϵ_A,$\Delta\psi$,$\Delta\epsilon$, which are usually considered separately, are the combined precession-nutation angles (Aoki & Kinoshita 1983) $(\psi_A+\Delta\psi_1)$, $(\omega_A+\Delta\epsilon_1)$ referred to the ecliptic of epoch, which have the advantages:

(i) of being the "fundamental parameters" mentioned above for precession alone, associated with the parameter $(\chi_A + \Delta\chi_A)$ for positioning the true equinox along the equator; note that the last quantity is not in fact necessary if it is omitted both in the PN matrix and in the expression for GST; note also that the combination of precession and nutation could easily include the frame biases as well;

(ii) of being referred to an inertial frame which is more in agreement with the IAU adoption of the International Celestial Reference System (ICRS) than referring to a moving ecliptic that involves taking additional precession effects into account.

The use of such parameters would mean using solutions for nutation that are directly referred to the mean ecliptic at epoch instead of ecliptic of date. Note that such an approach was already followed by Woolard (1953) and Bretagnon et al. (1997) and would be the natural choice for any numerical integration.

In the new paradigm, the basic quantities are the CIP coordinates X, Y in the GCRS, that appear directly as the (1, 3) and (2, 3) elements of the celestial-to-terrestrial transformation matrix. The polar coordinates E and d in the GCRS can easily be derived from X and Y.

A third option is to use the "rotation vector" approach (see Sect. 3.3), once it has been extended to include nutation as well.

3.6. Changes in the IAU nutation corresponding to the adoption of the P03 precession

Adopting a new precession model requires slight adjustment in the amplitudes of the nutation model that is used. The effects to be considered are the following:

(i) a change of the amplitudes of nutation in longitude due the change in the obliquity of the ecliptic in the precession model as compared with its value when estimating the nutation amplitudes of the model, similar to the effect considered for the P03 precession as compared with the IAU 2000 precession; in order to take this change into account, it is necessary to multiply the amplitudes of the nutation in longitude by $\sin\epsilon_{\rm IAU2000}/\sin\epsilon_{\rm P03} = 1.000000470$;

(ii) a change of the amplitudes of nutation (both in longitude and obliquity) due to the secular variation of the Earth's dynamical flattening (or equivalently J_2) which is used in the P03 precession model, whereas it was not considered in the IAU 2000 model; the amplitudes of nutation being proportional to J_2, this J_2 rate effect gives rise to additional Poisson terms that are proportional to $\dot{J}_2/J_2$. The J_2 rate contribution to the linear precession rate in longitude, which was considered in the P03 precession to be -14 mas t, corresponds to a value for $\dot{J}_2/J_2$ of -2.7774×10^{-6}, or equivalently (with $J_2 = 1.0826358 \times 10^{-3}$) a value of -0.3001×10^{-9}/cy.

The corrections larger than 1 μas to be added to the IAU 2000 nutation for these above effects are, respectively, in μas:

$$d_1\psi = -8.1\ \sin\Omega - 0.6\ \sin(2F - 2D + 2\Omega) \tag{26}$$

$$\begin{aligned} d_2\psi &= +47.8\,t\sin\Omega + 3.7\,t\sin(2F - 2D + 2\Omega) \\ &\quad +0.6\,t\sin(2F + 2\Omega) - 0.6\,t\sin 2\Omega \\ d_2\varepsilon &= -25.6\,t\cos\Omega - 1.6\,t\cos(2F - 2D + 2\Omega). \end{aligned} \tag{27}$$

Expressions (27) can also be provided in arcseconds in the following parameterized form:

$$\begin{aligned} d_2\psi &= (\dot{J}_2/J_2)\,t\,[-17.2\sin\Omega - 1.3\sin(2F - 2D + 2\Omega) \\ &\quad -0.2\sin(2F + 2\Omega) + 0.2\sin 2\Omega] \\ d_2\varepsilon &= (\dot{J}_2/J_2)\,t\,[9.2\cos\Omega + 0.6\cos(2F - 2D + 2\Omega)]. \end{aligned} \tag{28}$$

The above corrections can be written as functions of $\Delta\psi_{\rm IAU2000}$ and $\Delta\varepsilon_{\rm IAU2000}$, the IAU 2000 nutation angles in longitude and obliquity respectively:

$$d_1\psi = [(\sin\epsilon_{\rm IAU2000}/\sin\epsilon_{P03}) - 1]\,\Delta\psi_{\rm IAU2000} \tag{29}$$

$$\begin{aligned} d_2\psi &= (\dot{J}_2/J_2)\,t\,\Delta\psi_{\rm IAU2000} \\ d_2\varepsilon &= (\dot{J}_2/J_2)\,t\,\Delta\varepsilon_{\rm IAU2000}. \end{aligned} \tag{30}$$

4. Summary

In this paper we have provided expressions for precession as possible replacements for the current IAU precession (adopted by IAU 2000 Resolution B1.6, Mathews et al. 2002).

The proposed expressions are the P03 solution of Capitaine et al. (2003b) for the ecliptic and the equator. We have in addition provided a parameterized P04 solution for the equator that could be used for future improvement. Before recommending the P03 solution for the equator, we compared it with a revised version of the solution, which is based on the P03 ecliptic and was obtained in exactly the same way as P03 but using a refined model for the contributions of the non-rigid Earth (Mathews 2004) and revised integration constants for the precession rates that came from fits to the latest and most reliable VLBI data. After considering several options for the final precession expressions, we concluded that retaining the P03 solution, which was already in experimental use, was the preferred option.

In this paper we have described the different steps in obtaining the improved solution and in making comparisons, and we have summarized the expressions for all the usual precession quantities as well as for sidereal time and for the quantities to be used in the CIO-based transformation. We have also provided the corresponding corrections to be applied to the IAU 2000 nutation.

Acknowledgements. We are grateful to P. M. Mathews for having provided his latest pre-publication results for the effects of the non-rigid Earth on precession, and for valuable discussion. We thank J. Hilton, G. Kaplan and J. Vondrák for their interest and encouragement and V. Dehant for helpful comments and suggestions for improving the manuscript.

References

Aoki, S., & Kinoshita, H. 1983, Celest. Mech., 29, 335

Bourda, G., & Capitaine, N. 2004, A&A, 428, 691

Bretagnon, P., & Francou, G. 1988, A&A, 202, 309

Bretagnon, P., Rocher, P., & Simon, J.-L. 1997, A&A, 319, 305

Bretagnon, P., Fienga, A., & Simon, J.-L. 2003, A&A, 400, 785

Capitaine, N., Wallace, P. T., & McCarthy, D. D. 2003a, A&A, 406, 1135

Capitaine, N., Wallace, P. T., & Chapront, J. 2003b, A&A, 412, 567

Capitaine, N., Wallace, P. T., & Chapront, J. 2004, A&A, 421, 365

Fukushima, T. 2003, AJ, 126, 1

Herring, T. A., Mathews, P. M., & Buffett, B. A. 2002, J. Geophys. Res., 107, B4, 10.1029/2001JB000165

IERS Conventions 2003, IERS Technical Note 32, ed. D. D. McCarthy, & G. Petit, Frankfurt am Main: Verlag des Bundesamts für Kartographie und Geodäsie, 2004

Lieske, J. H., Lederle, T., Fricke, W., & Morando, B. 1977, A&A, 58, 1

Malkin, Z. 2004, in Proc. of the Journées 2003, ed. A. Finkelstein, & N. Capitaine, St. Petersburg, 24

Mathews, P. M. 2004, private communication

Mathews, P. M., Herring, T. A., & Buffett, B. A. 2002, J. Geophys. Res., 107, B4, 10.1029/2001JB000390

Morrison, L., & Stephenson, R. 1997, Contemporary Physics, 38, 1

Standish, E. M. 1998, JPL Planetary and Lunar Ephemerides, DE405/LE405, JPL IOM 312.F-98-048

Williams, J. G. 1994, AJ, 108 (2), 711

Woolard, E. W. 1953, Astr. Pap. Amer. Ephem. Naut. Almanac XV, I, 1