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Precession from Hipparcos and FK5 proper motions compared with current values: Reasons for discrepancies

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Abstract. The comparison of Hipparcos and FK5 proper motions points to an inconsistency with the correction, Δp, of the luni-solar precession derived from VLBI and LLR observations with unprecedented accuracy. An attempt is made to explain this inconsistency of approximately −1.3 mas/yr by rotational offsets of the Hipparcos and FK5 proper motion systems. In terms of right ascension and declination, these offsets give rise to proper motion offsets in the range of ±1 mas/yr on average which is not exceptional given the FK5 error budget. In the case of Hipparcos it is proven that the proper motions are not affected by rotational offsets larger than those indicated by the errors of the proper motion link to the ICRF. This result is obtained by analysing the Hipparcos proper motions in view of the existence of additional systematic motions other than those caused by galactic rotation and the parallactic motion of stars due to the solar motion with respect to the LSR. It is concluded that the Hipparcos proper motions are nearly free of unmodelled rotations, confirming that the Hipparcos frame is inertial at the accuracy level of the proper motion link to the ICRF. The gap between the estimated precessional corrections is bridged primarily by minor changes in the FK5 proper motions of the order of their errors, and only to a small extent by the elimination of a bias in the Hipparcos proper motions.

Key words. astrometry – reference systems

1. Introduction

From Very Long Baseline Interferometry (VLBI) observations acquired during the last two decades and Lunar Laser Ranging (LLR) data, the value of the luni-solar precession has been determined with unprecedented precision and accuracy (Charlot et al. 1995; IERS 1996, p. 25; Dehant et al. 1999; Capitaine et al. 2004). However, the precession correction determined from comparing Hipparcos and FK5 proper motions (e.g., Mignard & Froeschlé 2000; Schwan 2001) reveals a significant discrepancy compared to the VLBI-determined value. The causes of this discrepancy could be manifold, and some of them are subject to speculation. At first, it seems evident to blame the proper motion systems involved for the disagreement when, in particular, the latest precession determination by VLBI is not called into question. On this basis we discuss some possible sources of uncertainty that may have influenced the proper motion systems and, thus, the determination of corrections of precession from proper motions of stars:

- To what extent does a systematic error of the FK5 proper motions affect the precession correction derived from comparing Hipparcos and FK5 proper motions?
- Is it acceptable to suspect a systematic error in the catalogued Hipparcos proper motions (ESA 1997) caused, possibly, by an uncertainty in the Hipparcos proper motion link to the International Celestial Reference Frame (ICRF) (Lindegren & Kovalevsky 1995; Kovalevsky et al. 1997; Ma et al. 1998)?
- Does the selection of stars influence the analysis of the FK5 and Hipparcos proper motions with respect to the determiation of the precession correction?

2. Formulation of the problem

First let us recall the determination of the final Hipparcos positions and proper motions from those related to the original Hipparcos working frame, H_0, established prior to the linking to the ICRF. Once the orientation and spin matrices, M_0 and $\dot{M}_0$, had been determined (Kovalevsky et al. 1997), the stellar positions and proper motions of the Hipparcos Catalogue (HCAT) were derived by

$$\boldsymbol{r}(\mathrm{HCAT}) = M_0\, \boldsymbol{r}(\mathrm{H}_0), \quad (1)$$

$$\dot{\boldsymbol{r}}(\mathrm{HCAT}) = \dot{M}_0\, \boldsymbol{r}(\mathrm{H}_0) + M_0\, \dot{\boldsymbol{r}}(\mathrm{H}_0), \quad (2)$$

where M_0 and $\dot{M}_0$ are structured as the matrices given in Eq. (6) below.

Some insight into the exactness of the correction of the luni-solar precession adopted in 1976 (IAU 1977) may be expected from the comparison of the FK5 proper motions (Fricke et al. 1988) and the Hipparcos proper motions (ESA 1997) since the latter ones refer to a basically non-rotating coordinate frame. The problem, therefore, consists of finding out whether

Table 1. Angular rates of rotation and internal errors obtained from the comparison of FK5 and Hipparcos proper motions for different sets of stars (d stands for stellar distance).

Author	Number of stars	ω_x [mas/yr]	ω_y [mas/yr]	ω_z [mas/yr]
Mignard & Froeschlé (Mignard & Froeschlé 2000)	1233	-0.30 ± 0.10	0.60 ± 0.10	0.70 ± 0.10
Schwan (Schwan 2001)	1151	-0.34 ± 0.08	0.74 ± 0.08	0.89 ± 0.08
Walter & Hering (this paper)	1151	-0.30 ± 0.07	0.65 ± 0.07	0.77 ± 0.08
	529 ($d > 100$ pc)	-0.39 ± 0.10	0.58 ± 0.11	0.62 ± 0.12
	208 ($d > 200$ pc)	-0.61 ± 0.16	0.70 ± 0.18	0.59 ± 0.19

a residual rotation of the FK5 proper motions exists relative to the Hipparcos proper motions.

The basic principles for establishing the orientation and spin differences between reference frames are well known and have been discussed in several papers (e.g., Froeschlé & Kovalevsky 1982; Lindegren & Kovalevsky 1995). Applying these principles to the FK5 and Hipparcos catalogues we get for small angles of rotation that, by analogy to Eqs. (1) and (2), the FK5 and Hipparcos positions and proper motions are related through the matrices R and $\dot{R}$ by the following formulae:

$$\boldsymbol{r}(\mathrm{HCAT}) = R\,\boldsymbol{r}(\mathrm{FK5}), \tag{3}$$

$$\dot{\boldsymbol{r}}(\mathrm{HCAT}) = \dot{R}\,\boldsymbol{r}(\mathrm{FK5}) + R\,\dot{\boldsymbol{r}}(\mathrm{FK5}), \tag{4}$$

or to a first order approximation

$$\dot{\boldsymbol{r}}(\mathrm{HCAT}) - \dot{\boldsymbol{r}}(\mathrm{FK5}) = \dot{R}\,\boldsymbol{r}(\mathrm{FK5}), \tag{5}$$

where

$$R = \begin{pmatrix} 1 & \varepsilon_z & -\varepsilon_y \\ -\varepsilon_z & 1 & \varepsilon_x \\ \varepsilon_y & -\varepsilon_x & 1 \end{pmatrix}, \quad \dot{R} = \begin{pmatrix} 0 & \omega_z & -\omega_y \\ -\omega_z & 0 & \omega_x \\ \omega_y & -\omega_x & 0 \end{pmatrix}, \tag{6}$$

and ε and ω are rotation angles and rates, respectively, about the x, y and z axes of the rectangular FK5 coordinate frame.

The well known equations in right ascension (α) and declination (δ) can be deduced from Eq. (5)

$$\{\mu_\alpha(\mathrm{HCAT}) - \mu_\alpha(\mathrm{FK5})\}\cos\delta = \omega_x \sin\delta\cos\alpha + \omega_y \sin\delta\sin\alpha - \omega_z\cos\delta, \tag{7}$$

$$\{\mu_\delta(\mathrm{HCAT}) - \mu_\delta(\mathrm{FK5})\} = -\,\omega_x\sin\alpha + \omega_y\cos\alpha, \tag{8}$$

which allow an estimate of the angular rates of rotation, ω_x, ω_y and ω_z, that convert the FK5 proper motions into the Hipparcos proper motions. These two proper motion systems are expected to differ only by the combined effects of precessional corrections and the FK5 equinox motion. Some more recent determinations of the angular rates of rotation for different sets of basic FK5 stars are listed in Table 1. Visual double stars and astrometric binaries are excluded since their FK5 and Hipparcos proper motions may be different due to the individual determination procedures (Wielen et al. 1999). Information on the luni-solar precession is embedded in ω_y and ω_z because of the relationship (e.g., Fricke 1977):

$$\omega_x = 0, \tag{9}$$

$$\omega_y = -\,\Delta p\,\sin\varepsilon, \tag{10}$$

$$\omega_z = +\,\Delta p\,\cos\varepsilon - (\Delta\lambda + \Delta e). \tag{11}$$

Here, p stands for the luni-solar precession, λ for the planetary precession and e for the motion of the equinox; ε denotes the obliquity of the ecliptic.

If, in Table 1, we take account only of the sets with more than 1100 stars the average values of ω_y and ω_z become 0.66 and 0.78 mas/yr, respectively, thus yielding $\Delta p = -1.66$ mas/yr and $\Delta\lambda + \Delta e = -2.30$ mas/yr, while the VLBI-determined precession correction is approximately $\Delta p = -3.0$ mas/yr. This quantity is in close agreement with the current optimum value of the precession correction of $\Delta p = (-2.997 \pm 0.008)$ mas/yr (McCarthy & Capitaine 2002). The most recent comparisons of high precision precession models (Capitaine et al. 2004) indicate marginal improvements of the value adopted above. They are, however, of no significant influence on the results inferred below. Using this value as a yardstick (Sôma 2000) we infer from Eqs. (9) to (11) the corresponding nominal values for the angular rates of rotation. They are

$$\omega_x(\mathrm{nom}) = 0.0\,\mathrm{mas/yr}, \tag{12}$$

$$\omega_y(\mathrm{nom}) = 1.2\,\mathrm{mas/yr}, \tag{13}$$

$$\omega_z(\mathrm{nom}) = -1.5\,\mathrm{mas/yr}, \tag{14}$$

provided we put $\Delta e = (-1.2 \pm 0.3)$ mas/yr as determined by Miyamoto & Sôma (1993) for the FK5 equinox motion, and omit $\Delta\lambda$ which has been proven to be negligibly small.

One way of reconciling the solutions of Eqs. (7) and (8) given in Table 1 with the nominal values above consists of tentatively adding offsets to the angular rotations so that the nominal values are obtained. These offsets, for example, are in the first case of Table 1 as follows:

$$\omega_x(\mathrm{nom}) - \omega_x = \Delta\omega_x = (\;0.3 \pm 0.1)\,\mathrm{mas/yr}, \tag{15}$$

$$\omega_y(\mathrm{nom}) - \omega_y = \Delta\omega_y = (\;0.6 \pm 0.1)\,\mathrm{mas/yr}, \tag{16}$$

$$\omega_z(\mathrm{nom}) - \omega_z = \Delta\omega_z = (-2.2 \pm 0.3)\,\mathrm{mas/yr}, \tag{17}$$

It is the objective of this paper to discuss the reality of such offsets and, thus, to give possible reasons for the discrepancy of the two aforementioned values of Δp. Clearly, systematic errors of the FK5 proper motions could be responsible for the difference. On the other hand this way of looking at the problem does not apply to the Hipparcos proper motions because they are essentially free of systematic errors due to the method of determination. It may be suspected, however, that a systematic effect has been propagated into the proper motions through the Hipparcos link to the ICRF (Kovalevsky et al. 1997; Ma et al. 1998). The next section deals with the question of whether deformations of the spin matrix, $\dot{M}_0$, exist, which is the decisive item in establishing the Hipparcos proper motions.

3. Assessment of the spin matrix

3.1. Overview

For the assessment of the spin matrix we propose a method based exclusively on Hipparcos data and on the assumption that the Hipparcos proper motions referring to the quasi-inertial frame, ICRF, are composed of the peculiar motion of the stars and the following systematic contributions:

- The parallactic motion due to the solar motion.
- The motion due to the rotation of the assembly of stars (galactic rotation).
- The deformation of the velocity field of the stars due to non-rigid rotation of the galaxy (differential galactic rotation).

Below we investigate the Hipparcos proper motions for the existence of possible surplus angular rotations that are not explained by the galactic rotation.

3.2. Do the Hipparcos proper motions comprise unmodelled rotations?

On the basis of the Oort-Lindblad model of galactic rotation the formulae of our approach read as follows (Green 1985, p. 346; Fricke 1977):

$$\begin{aligned}\mu_\alpha \cos\delta &= \mathrm{P1(solar\ motion)}_\alpha \\ &\quad - \omega_x \cos\alpha \sin\delta - \omega_y \sin\alpha \sin\delta + \omega_z \cos\delta \\ &\quad + \mathrm{P2(different.galact.rot.)}_\alpha, \qquad (18)\\ \mu_\delta &= \mathrm{P1(solar\ motion)}_\delta \\ &\quad + \omega_x \sin\alpha - \omega_y \cos\alpha \\ &\quad + \mathrm{P2(different.galact.rot.)}_\delta. \qquad (19)\end{aligned}$$

P1 and P2 are abbreviations standing for the contributions of solar motion and differential galactic rotation, respectively. The calculations below have taken full account of them; their explicit expressions, however, are not needed for the subsequent discussion. As the Hipparcos proper motions refer to the ICRF, the angular rates, $\omega_x, \omega_y, \omega_z$, are supposed to be caused by the galactic rotation only in the absence of other disturbing influences. To indicate that these angular rates originate from galactic rotation they are labelled by the superscript "G"; they are related to Oort's constant Q by the following expressions valid for the equinox 2000 (ESA 1997, Vol. 1, p. 92):

$$\omega_x^G = -0.8677\,Q, \qquad (20)$$

$$\omega_y^G = -0.1981\,Q, \qquad (21)$$

$$\omega_z^G = +0.4560\,Q. \qquad (22)$$

In a least squares estimate we apply Eqs. (18) and (19) to the proper motions of selected subsets of stars that define the HCRF, the Hipparcos Celestial Reference Frame (IAU 2001, p. 29). In selecting the stars according to the size of the residuals provided by Eqs. (18) and (19) we were guided by the conception that those stars leaving large residuals are subject to large peculiar motions, thus jeopardizing the search for systematic effects. In selecting the stars we aimed at avoiding stars of large peculiar motions. For this purpose the parameter estimates in Eqs. (18) and (19) have been restricted to HCRF stars having distances greater than 100 pc. From this subset we removed, step by step, those stars for which the least squares fit of the parameters produced residuals larger than 10, 8 or even 5 mas/yr, since we suspect that stars with large residuals are not compatible with the model adopted in Sect. 3.1. This model assumes peculiar motions of random distribution about zero. Stars producing large residuals, however, are liable to give rise to a bias and, therefore, are omitted. The resulting least squares estimates of the unknowns are listed in Table 2.

In all four cases of Table 2 the components X, Y, Z of the solar motion relative to stars in the solar neighberhood are of the same order of magnitude. For Case 3 we obtain 19.79 ± 0.05 km s^{-1} for the total velocity relative to the local standard of rest (LSR), compatible with values derived independently (cf. Cox 2000, p. 493). Expressed in galactic coordinates, Case 3, Table 2 results in $X_G = 10.68 \pm 0.05, Y_G = 15.35 \pm 0.04, Z_G = 6.47 \pm 0.04$ km s^{-1} in fair agreement with Feast & Whitelock (1997) and Mignard (2000). With regard to the angular rates of rotation, Cases 2 to 4 give evidence of robustness and stability of the results, which applies even more to Cases 3 and 4 yielding errors of unit weight tolerable in the present circumstances. In view of Eqs. (20) to (22), Case 3, for instance, yields for Oort's constant Q, in turn $Q_x = -2.59 \pm 0.04$ mas/yr, $Q_y = -3.13 \pm 0.15$ mas/yr and $Q_z = -4.08 \pm 0.09$ mas/yr. Only the z component falls distinctly outside the range of the widely accepted value of Q by Kerr & Lynden-Bell (1986), which is $Q = -2.53 \pm 0.59$ mas/yr and will be called the "standard value" in the following text. Possible reasons for this deviation in z are discussed in Sect. 3.3. For Oort's constant P we get without exception acceptable values between 2.56 and 2.82 mas/yr ; the widely accepted value is 3.04 ± 0.25 mas/yr (Kerr & Lynden-Bell 1986). Recent determinations of the Oort constants based on more modern data (e.g., Feast & Whitelock 1997) including those of the Hipparcos Catalogue are in close agreement with the above standard values.

When we adopt in Eqs. (20)–(22) the standard value of Q as a benchmark, we conclude from Table 2 that in these circumstances the Hipparcos proper motions cause surplus angular rotations given by:

$$\Delta\omega_x = \omega_x - \omega_x^G, \qquad (23)$$

Table 2. Angular rates of rotation estimated for 4 different sets of HCRF proper motions referred to the ICRF. X, Y, Z denote the components of solar motion in equatorial coordinates, "P" is Oort's constant related to differential galactic rotation, "d" stands for stellar distance, "elim" for elimination of residuals and "no" refers to the number of HCRF stars.

	Case 1	Case 2	Case 3	Case 4
	d > 100 pc	d > 100 pc	d > 100 pc	d > 100 pc
	elim: none	elim: >10 mas/yr	elim: >8 mas/yr	elim: >5 mas/yr
	no: 78741	no: 28084	no: 22204	no: 12750
X [km s^{-1}]	1.71 ± 0.10	1.37 ± 0.04	1.39 ± 0.04	1.47 ± 0.04
Y [km s^{-1}]	−17.57 ± 0.11	−17.52 ± 0.05	−17.44 ± 0.05	−17.45 ± 0.04
Z [km s^{-1}]	10.43 ± 0.10	9.00 ± 0.05	9.25 ± 0.04	9.72 ± 0.04
ω_x [mas/yr]	2.09 ± 0.10	2.23 ± 0.03	2.25 ± 0.03	2.27 ± 0.03
ω_y [mas/yr]	0.83 ± 0.10	0.64 ± 0.03	0.62 ± 0.03	0.63 ± 0.03
ω_z [mas/yr]	−1.82 ± 0.13	−1.91 ± 0.04	−1.86 ± 0.04	−1.78 ± 0.03
P [mas/yr]	2.82 ± 0.14	2.61 ± 0.04	2.56 ± 0.04	2.68 ± 0.03
Error of unit weight	30.9	6.0	5.0	3.4

$$\Delta\omega_y = \omega_y - \omega_y^G, \tag{24}$$

$$\Delta\omega_z = \omega_z - \omega_z^G. \tag{25}$$

If, for instance, Case 3, Table 2 is chosen, we obtain the surplus angular rates of rotations listed in Table 3 based on the standard value of Q, and a few variations of it lying within the error range of Q. At first glance the surplus angular rates of rotations are modest as long as we consider the vicinity of the standard value of Q only and disregard the z component whose size may have been affected by a weakness of the link to the adopted zero point of right ascension of the ICRF. Essentially, the estimated surplus rates lie within the errors of the Hipparcos proper motion link that are ±0.25 mas/yr per axis. This result means that the Hipparcos proper motions are free of significant unmodelled rotations and that the assessment of the spin matrix treated in this section confirms the reliability of the Hipparcos proper motion link despite the estimated surplus angular rates that will be ananlysed in the next section.

3.3. Interpretation of the surplus angular rates of rotation

For further analysis we suggest the hypothesis that the Hipparcos proper motions, at least partly, are responsible for the surplus angular rates of rotation and that the source of these rotational rates are associated with the spin matrix, $\dot{M}_0$, of the Hipparcos link. In other words, we suppose that the surplus angular rates, which in the ideal case are expected to be zero, are indicators of a bias of the spin matrix.

The surplus angular rotations depend on the Oort constant Q, the value of which, however, is not known with great precision. We assume the true value of Q within the error range of its standard value and we believe it to be on the safe side if we extend the estimate of $\Delta\dot{M}_0$ over the whole error range of Q, as illustrated in Table 3 by the variety of surplus rotations as a function of Q. Whatever way we choose Q in Table 3, the modification of the Hipparcos proper motions by the corresponding surplus rotations and the application of Eqs. (18) and (19) to these modified proper motions results in iterated surplus angular rates that tend to approach zero. This is outlined by the following procedure:

Table 3. Surplus angular rates of rotation produced by the subset of Hipparcos proper motions of Case 3 based on the standard value of the Oort constant, $Q = -2.53$ mas/yr, and its variations for comparison (errors are only given for the standard value of Q as those of the comparative cases are almost identical).

Q [mas/yr]	$\Delta\omega_x$ [mas/yr]	$\Delta\omega_y$ [mas/yr]	$\Delta\omega_z$ [mas/yr]
−2.0	0.52	0.22	−0.95
−2.2	0.35	0.18	−0.86
−2.4	0.17	0.14	−0.77
−2.53⋆	0.06 ±0.52	0.12 ±0.12	−0.71 ±0.28
−2.8	−0.18	0.07	−0.59
−3.0	−0.35	0.03	−0.50

⋆ Kerr & Lynden-Bell (1986).

In a first step it is shown that the surplus angular rates approach zero if the Hipparcos proper motions are subjected to rotations by $\Delta\omega_i, i = x, y, z$. By analogy to Eqs. (7) and (8) the corrected proper motions are related to the original ones through

$$\{\mu_\alpha(\text{HCAT})_{\text{corr}} - \mu_\alpha(\text{HCAT})\}\cos\delta = \Delta\mu_\alpha\cos\delta$$
$$= \Delta\omega_x \sin\delta\cos\alpha + \Delta\omega_y \sin\delta\sin\alpha - \Delta\omega_z\cos\delta, \tag{26}$$
$$\mu_\delta(\text{HCAT})_{\text{corr}} - \mu_\delta(\text{HCAT}) = \Delta\mu_\delta$$
$$= -\Delta\omega_x\sin\alpha + \Delta\omega_y\cos\alpha. \tag{27}$$

Indeed, if the estimation process described in Eqs. (18) and (19) is performed by using the Hipparcos proper motions corrected according to Eqs. (26) and (27), one obtains, e.g., for Case 3, Table 2 with $Q = -2.53$ mas/yr

$$\Delta\omega_x = -0.06 \pm 0.52 \text{ mas/yr}, \tag{28}$$

$$\Delta\omega_y = -0.05 \pm 0.12 \text{ mas/yr}, \tag{29}$$

$$\Delta\omega_z = \quad 0.21 \pm 0.28 \text{ mas/yr}, \tag{30}$$

underlining the tendency to vanish. The results obtained for $Q = -2.0$ mas/yr and $Q = -3.0$ mas/yr using the same data are listed below for comparison:

$\Delta\omega_x$ [mas/yr] :	−0.20	0.08
$\Delta\omega_y$ [mas/yr] :	−0.07	−0.03
$\Delta\omega_z$ [mas/yr] :	0.31	0.13 .

Note that the iterated surplus angular rates fall almost entirely inside the error range of the Hipparcos proper motion link.

It remains to trace back, in a second step, the correction of the Hipparcos proper motions given in Eqs. (26) and (27) to a bias of the spin matrix. Since $\dot{M}_0$ requires the correction $\Delta\dot{M}_0$, the corrected proper motions, to a first approximation, follow from Eq. (2) and are given by

$$\dot{\boldsymbol{r}}(\text{HCAT})_{\text{corr}} - \dot{\boldsymbol{r}}(\text{HCAT}) = \Delta\dot{M}_0 \, \boldsymbol{r}(\text{HCAT}), \tag{31}$$

where $\Delta\dot{M}_0$ is structured like $\dot{M}_0$ in Eq. (2) and the numerical values of its components are listed in Table 3.

Starting from Eq. (31) and by analogy to Eq. (5) as well as Eqs. (7) and (8) one arrives at Eqs. (26) and (27) provided that $\Delta\omega_i, i = x, y, z$ are chosen as elements of $\Delta\dot{M}_0$. Thus, we conclude that the surplus angular rates are caused by a bias of the spin matrix of the Hipparcos link. The components of the correction matrix, $\Delta\dot{M}_0$, are of the order of the surplus angular rates of rotation. For example, for Case 3, Table 2 with $Q = -2.53$ mas/yr the correction matrix becomes

$$\Delta\dot{M}_0 = \begin{pmatrix} 0 & -0.71 & -0.12 \\ 0.71 & 0 & 0.06 \\ 0.12 & -0.06 & 0 \end{pmatrix}; \tag{32}$$

units are mas/yr.

3.4. Side effects of the correction matrix

From Eqs. (26) and (27) follows, in the case of the standard value of Q, that the suspected corrections of the spin matrix, $\dot{M}_0$, cause proper motion changes in right ascension and declination of less than 0.9 mas/yr and less than 0.2 mas/yr in absolute value, respectively. Lying within the error ranges of the Hipparcos proper motions, these changes are tolerable. Incidentally, the changes in declination do not exceed the error level of the Hipparcos proper motion link, which is ±0.25 mas/yr in all three axes. The large proper motion offset in right ascension possibly has been caused by a minor estimation error of ω_z in $\dot{M}_0$. It reduces to the level of the proper motion link if the z component of the spin matrix is corrected by a quantity of the order of $\Delta\omega_z = -0.7$ mas/yr, see Eq. (30).

When the corrections above are applied to the Hipparcos proper motions, the estimates by means of Eqs. (18) and (19) yield $Q_x = -2.47$, $Q_y = -2.27$, $Q_z = -2.06$ mas/yr in good agreement with the standard value of the Oort constant Q and its error bars. Good agreement is also obtained for Q_x, Q_y and Q_z with the variations of the standard values of Q when the cases $Q = -2.0$ mas/yr and $Q = -3.0$ mas/yr are treated.

Further discussions require us to know how $\Delta\dot{M}_0$ propagates into the estimates of the angular rates of rotation of the HCAT relative to the FK5 as formally expressed by Eqs. (5)–(8). Taking account of corrected Hipparcos proper motions we obtain from Eq. (5)

$$\dot{\boldsymbol{r}}(\text{HCAT})_{\text{corr}} - \dot{\boldsymbol{r}}(\text{FK5}) = (\dot{R} + \Delta\dot{R}) \, \boldsymbol{r}(\text{FK5}), \tag{33}$$

where $\Delta\dot{R}$ is the correction matrix caused by the Hipparcos proper motions due to an assumed imperfection of the link. After some matrix algebra one finds

$$\Delta\dot{R} \approx \Delta\dot{M}_0. \tag{34}$$

This result enables us to assess the influence of offsets of the spin matrix on the rotation matrix, $\dot{R}$, see Eqs. (6) and (33), and finally to apply Eqs. (9) to (11). As a consequence of this result the surplus rotation angles valid for the standard value of Q referred to in Table 3, affect only negligibly the x and y components of the rotation matrix, $\dot{R}$. Therefore, the assumption of a biased spin matrix of the Hipparcos link is untenable and unsuitable to explain the discrepancy of the precession correction presented in Sect. 2. Let us recall that $\Delta\omega_y \approx 0.6$ mas/yr would be required to bridge the gap between the Hipparcos and the latest estimates of the precession correction. A surplus rate of rotation about the y axis of 0.6 mas/yr, however, is totally incompatible with the standard value of the Oort constant, Q, and its variations in Table 3. On the contrary, the method treated here relying on the standard value of Q confirms the correctness of the Hipparcos proper motion link with a minor reservation concerning the z axis.

Moreover, the correction of the z component in $\dot{M}_0$ has repercussions on the correction of the motion of the equinox, Δe. By comparing FK5 and Hipparcos proper motions based on Eq. (33) we infer, following the analogy of Eqs. (10) and (11),

$$\omega_y + \Delta\omega_y = -\Delta p \, \sin\varepsilon, \tag{35}$$

$$\omega_z + \Delta\omega_z = +\Delta p \, \cos\varepsilon - (\Delta\lambda + \Delta e). \tag{36}$$

Neglecting $\Delta\lambda$ and putting $\Delta\omega_y = 0.12$ and $\Delta\omega_z = -0.71$ mas/yr as suggested by Table 3 we obtain with the figures of Case 1, Table 1 $\Delta p = -1.8 \pm 0.4$ mas/yr and $\Delta e = -1.6 \pm 0.5$ mas/yr to be compared to the $\Delta e = -1.2 \pm 0.3$ mas/yr as derived by Miyamoto & Sôma (1993).

Having set up the correction of the spin matrix one could speculate about the share of the FK5 proper motions that is required to explain the discrepancy of the corrections, Δp, referred to in Sect. 2. In pursuing this goal the inclusion of the equinox motion is inevitable. We adopt for it the plausible correction $\Delta e = -1.6$ mas/yr suggested by the preceding considerations. As a consequence, the z component of the nominal angular rates of rotation in Eqs. (12) to (14) reads now $\omega_z(\text{nom}) \approx -1.1$ mas/yr. To estimate the share of the FK5 proper motions we employ Eq. (33) where $\dot{R}$ is the matrix introduced in Eq. (6). Note that one part of $\Delta\dot{R}$ is made up of $\Delta\dot{M}_0$; another part will be ascribed to the FK5 proper motions as outlined below.

In the ideal case the elements of $\dot{R} + \Delta\dot{R}$ would consist of the nominal angular rates

$$\omega_x(\text{nom}) = \quad 0.0 \text{ mas/yr}, \tag{37}$$

$$\omega_y(\text{nom}) = \quad 1.2 \text{ mas/yr}, \tag{38}$$

$$\omega_z(\text{nom}) = -1.1 \text{ mas/yr}, \tag{39}$$

Table 4. Rotational offsets of Hipparcos and FK5 proper motions required to constrain the comparison of the two proper motion systems to the VLBI-determined correction of the luni-solar precession, $\Delta p = -3.0$ mas/yr.

	[mas/yr]	[mas/yr]	[mas/yr]
$\dot{R}$:	$\omega_x = -0.30$	$\omega_y = 0.60$	$\omega_z = 0.70$
$\Delta\dot{R}$:	$\Delta\omega_x = 0.06$	$\Delta\omega_y = 0.12$	$\Delta\omega_z = -0.71$
$\Delta\dot{R}$(FK5):	$\Delta\omega_x$(FK5) = 0.24	$\Delta\omega_y$(FK5) = 0.48	$\Delta\omega_z$(FK5) = −1.10

because in these circumstances Eq. (33) would result in the desirable value $\Delta p = -3.0$ mas/yr. Failing this, we add a further correction matrix, $\Delta\dot{R}$(FK5), which is ascribed to rotations of the FK5 proper motions. This matrix is structured like $\Delta\dot{R}$ and is composed of angular rates of rotation, i.e. $\Delta\omega_x$(FK5), $\Delta\omega_y$(FK5) and $\Delta\omega_z$(FK5), chosen so that the nominal angular rates of rotation are met by $\dot{R} + \Delta\dot{R} + \Delta\dot{R}$(FK5). To estimate the changes of the FK5 proper motions due to the rotation, $-\Delta\dot{R}$(FK5), we proceed as follows. By analogy to Eq. (5) one obtains

$$\dot{\boldsymbol{r}}(\mathrm{FK5})_{\mathrm{corr}} - \dot{\boldsymbol{r}}(\mathrm{FK5}) = -\Delta\dot{R}(\mathrm{FK5})\, \boldsymbol{r}(\mathrm{FK5}), \quad (40)$$

and in accordance with Eqs. (7) and (8) one gets the corrections

$$\begin{aligned} C_\alpha &= \{\mu_\alpha(\mathrm{FK5})_{\mathrm{corr}} - \mu_\alpha(\mathrm{FK5})\}\cos\delta = \Delta\mu_\alpha \cos\delta \\ &= -[\Delta\omega_x \sin\delta\cos\alpha + \Delta\omega_y \sin\delta\sin\alpha - \Delta\omega_z\cos\delta], \quad (41) \\ C_\delta &= (\mu_\delta(\mathrm{FK5})_{\mathrm{corr}} - \mu_\delta(\mathrm{FK5})) = \Delta\mu_\delta \\ &= -[-\Delta\omega_x \sin\alpha + \Delta\omega_y\cos\alpha]. \quad (42) \end{aligned}$$

From Eqs. (41) and (42) it is concluded that the FK5 proper motions are subject to changes of the order of ±1 mas/yr for FK5 rotations suggested in Table 4.

For a better insight into the combined corrections we show, in Table 4, the aforementioned procedure by turning to the numerical values in Table 1, first case, and Table 3, case $Q = -2.53$ mas/yr. The first line of Table 4 refers to the angular rates of rotation resulting from comparisons of the catalogued FK5 and Hipparcos proper motions, see Eq. (5). Rotational offsets caused by a bias of the spin matrix of the Hipparcos link are taken from Table 3 and are given in the second line. The third line of Table 4 contains the hypothetical offsets of the FK5 proper motions that are needed to reach agreement with the precessional correction adopted here of $\Delta p = -3.0$ mas/yr, see Eqs. (41) and (42). The sizes of these offsets are within the errors of the FK5 proper motions and the systematic differences between the FK5 and Hipparcos proper motions (Schwan 2001).

4. Conclusions

As a benchmark we adopted the correction of the luni-solar precession determined by VLBI and LLR techniques, i.e. $\Delta p = -3.0$ mas/yr, a value which is also supported by the latest high precision precession models; furthermore, we used as a guideline the widely accepted value of the Oort constant Q, i.e. $Q = -2.53$ mas/yr. On this basis we have shown that the discrepancy between this correction of Δp and the one derived from the comparison of FK5 and Hipparcos proper motions cannot be explained by a systematic rotation of the Hipparcos system of proper motions relative to the extragalactic reference frame (ICRF). In particular, it could be confirmed that the proper motion link of Hipparcos to the ICRF is responsible for this discrepancy only to a small extent. On the contrary, the results indicate that the Hipparcos proper motions referring to the Hipparcos Celestial Reference Frame (HCRF) are aligned to the ICRF within the order of the errors of 0.25 mas/yr per axis, perhaps with a minor exception in the direction of the z axis. Instead, we conclude that the comparison of the FK5 and Hipparcos proper motions is impaired by systematic errors of the FK5 proper motions amounting in extreme cases to about ±1 mas/yr depending on the region of the sky.

As a by-product we obtained results on the solar motion relative to the LSR as well as on Oort's constants A and B that compare well with widely accepted values. Furthermore, the study supports evidence for the amendment of the recent correction of the equinox motion (Miyamoto & Sôma 1993) which apparently should be changed from −1.2 mas/yr to about −1.6 or even −2 mas/yr, which has the desirable consequence of diminishing the unusually large offset of the angular rotation about the z axis given in Eq. (17). This seems to be plausible for the z axis because it provides more compatibility with the undisputed FK5 error budget of proper motions and, which should not be overlooked, the systematic differences between FK5 and Hipparcos proper motions would easily absorb this smaller offset. On these grounds it could be proved that the corrections to the luni-solar precession obtained by VLBI and LLR on the one side and by comparing FK5 and Hipparcos proper motions on the other are not contradictory. They are explained primarily by errors of the FK5 proper motions caused by rotational effects within the error ranges of the FK5, and to a lesser extent by slightly biased Hipparcos proper motions that originate from a rotational imperfection about the z axis of the spin matrix of the Hipparcos link to the ICRF.

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