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Research Note

A new non-linear limb-darkening law for LTE stellar atmosphere models III

Sloan filters: Calculations for –5.0 ≤ log [M/H] ≤ +1, 2000 K ≤ $T_{\rm eff}$ ≤ 50 000 K at several surface gravities[⋆]

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Abstract. Continuing our studies on stellar atmospheres (Claret 2000, 2004), we present in this paper the limb-darkening coefficients for the Sloan photometric system. The calculations cover a wide range of values of log g, $T_{\rm eff}$, metallicities and microturbulent velocities. The atmosphere models used are ATLAS and PHOENIX. In addition to the traditional applications of the limb-darkening coefficients, the present ones will be useful since the Sloan filters are now being used in the interpretation of light curves of extrasolar transiting planets, for example. The present calculations may also be useful, as a complement, for the Kepler mission, dedicated to the search for terrestrial planets.

Key words. stars: atmospheres – stars: binaries: eclipsing – stars: low mass, brown-dwarfs – stars: planetary systems

1. Introduction

The calculations of the limb-darkening coefficients (LDC) that we have done previously cover the needs of several fields of Astrophysics. However, less usual photometric systems are now being used or are planned to be used. That is the case of the space mission Kepler. As a complement and as an exploratory work, the Sloan filters are being used to monitor candidates of extrasolar planetary transits (Torres 2004). In this case, the need of the LDC for Sloan filters is clear. The aim of this short paper is to present such calculations.

Despite the field of application of the limb-darkening coefficients, the adopted law and the numerical method used should provide a good description of the distribution of the specific intensities. Simple laws such as the linear one are not accurate (Kinglesmith & Sobieski 1970; Manduca et al. 1977; Claret & Giménez 1990; Van Hamme 1993). Quadratic or root-square approximations provide better fittings only for a narrow range of spectral types (Díaz-Cordovés 1990). In order to overcome these limitations, a new non-linear law was introduced (Claret 2000, Paper I). Among the advantages of this new law, we can quote: a) a single law is valid for the whole HR diagram; b) it is capable of reproducing very well the intensity distribution; c) the flux, computed using such a law, is conserved within an acceptable tolerance; d) it can be applied to different filters, log g, effective temperatures, metallicities and microturbulent velocities; e) it can be applied with success even in irradiated atmospheres (Claret 2004); f) it can be applied also in models with spherical symmetry (Claret & Hauschildt 2003). We adopted the new non-linear law and the Least-Square Method (LSM) throughout this paper.

2. The calculations and table organization

The numerical procedure and methodology used here, as well as the terrestrial transmission, detector sensitivity and reflectivity of the aluminium coated mirror, were described in detail in Paper I. The only difference is the filter transmissions for the Sloan system, which were taken from Fukugita et al. (1996, their Fig. 1). We have adopted the ATLAS (Kurucz 2000) and PHOENIX (Hauschildt 2000) atmosphere models. In the case of PHOENIX models the version of the plane-parallel approximation was used in order to make it easier to compare with ATLAS outputs. We have performed calculations of LDC for

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[⋆] Tables 2 to 11 are only available in electronic form at the CDS via anonymous ftp to `cdsarc.u-strasbg.fr (130.79.128.5)` or via `http://cdsweb.u-strasbg.fr/cgi-bin/qcat?J/A+A/428/1001`
Additional data are available on CD ROMs.

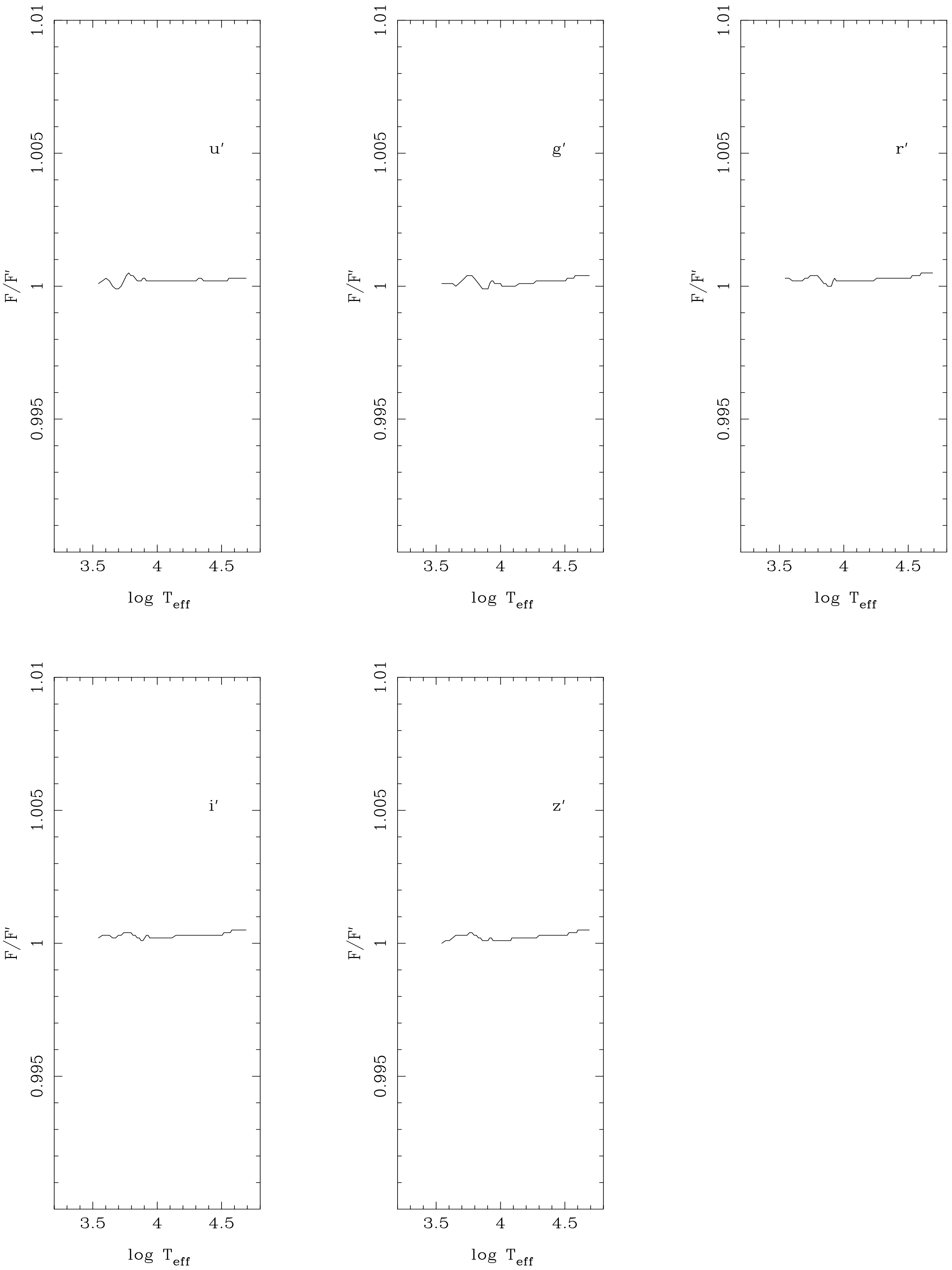

Fig. 1. The ratio between the actual flux (F) and that computed using Eq. (5) (F'). Sloan filters, ATLAS models, $\log g$ = 4.5, solar abundance, microturbulet velocity = 2 km s^{-1}.

Table 1. Limb-Darkening coefficients for u' g' r' i' z' bands (Least-Squares Method).

Name	Source	Range $T_{\rm eff}$	Range log g	Range log [M/H]	Range Vel Turb.	Kind of fitting
Table 2	ATLAS	3500 K–50 000 K	0.0–5.0	[+1]–[−5.0]	0–2 km s^{-1}	Eq. (5), 5 filters
Table 3	ATLAS	3500 K–50 000 K	0.0–5.0	[+1]–[−5.0]	0–2 km s^{-1}	Eq. (1), 5 filters
Table 4	ATLAS	3500 K–50 000 K	0.0–5.0	[+1]–[−5.0]	0–2 km s^{-1}	Eq. (2), 5 filters
Table 5	ATLAS	3500 K–50 000 K	0.0–5.0	[+1]–[−5.0]	0–2 km s^{-1}	Eq. (3), 5 filters
Table 6	ATLAS	3500 K–50 000 K	0.0–5.0	[+1]–[−5.0]	0–2 km s^{-1}	Eq. (4), 5 filters
Table 7	PHOENIX	2000 K–9800 K	3.5–5.0	[0]	2 km s^{-1}	Eq. (5), 5 filters
Table 8	PHOENIX	2000 K–9800 K	3.5–5.0	[0]	2 km s^{-1}	Eq. (1), 5 filters
Table 9	PHOENIX	2000 K–9800 K	3.5–5.0	[0]	2 km s^{-1}	Eq. (2), 5 filters
Table 10	PHOENIX	2000 K–9800 K	3.5–5.0	[0]	2 km s^{-1}	Eq. (3), 5 filters
Table 11	PHOENIX	2000 K–9800 K	3.5–5.0	[0]	2 km s^{-1}	Eq. (4), 5 filters

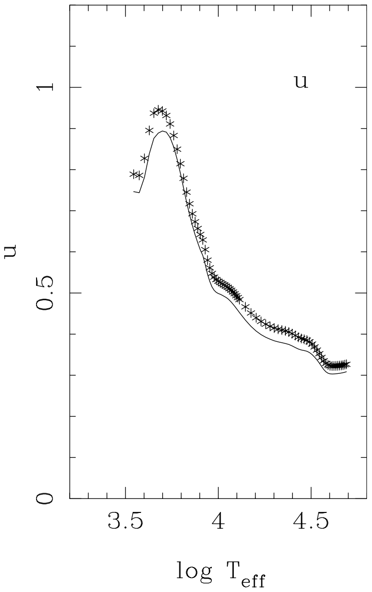

Fig. 2. The linear coefficient u for ultraviolet bands. Continuous lines represent Strömgren u and asterisks denote Sloan u'. ATLAS models. $\log g$ = 4.5, solar abundance, microturbulet velocity = 2 km s^{-1}.

19 metallicities ranging from 10^{-5} up to 10^{+1} solar abundances, $0 \le \log g \le 5$ and 2000 K $\le T_{\rm eff} \le$ 50 000 K as well as for 5 values of the microturburlent velocities (0, 1, 2, 4, 8 km s^{-1}). The LDC for the Sun and Vega are also available. We reproduce below the adopted LDC laws in order to facilitate the identification of the coefficients contained in the tables:

Linear

$$\frac{I(\mu)}{I(1)} = 1 - u(1-\mu). \qquad (1)$$

Quadratic

$$\frac{I(\mu)}{I(1)} = 1 - a(1-\mu) - b(1-\mu)^2. \qquad (2)$$

Square root

$$\frac{I(\mu)}{I(1)} = 1 - c(1-\mu) - d(1-\sqrt{\mu}). \qquad (3)$$

Logarithmic

$$\frac{I(\mu)}{I(1)} = 1 - e(1-\mu) - f\mu\ln(\mu) \qquad (4)$$

where $I(1)$ is the specific intensity at the center of the disk, u, a, b, c, d, e, f are the corresponding LDC and $\mu = \cos(\gamma)$, γ is the angle between the line of sight and the emergent intensity. The new law introduced in Paper I is written as

$$\frac{I(\mu)}{I(1)} = 1 - a_1\left(1-\mu^{1/2}\right) - a_2(1-\mu) - a_3\left(1-\mu^{3/2}\right) - a_4\left(1-\mu^2\right). \qquad (5)$$

This superiority of the LSM over the Flux Conservation Method (FCM) was again detected in this series of calculations for all the filters, effective temperatures, $\log g$, metallicities and microturburlent velocities. We refer readers interested in details of the comparison LSM versus FCM to Paper I and for irradiated atmospheres to Claret (2004). As an example of the goodness of the new non-linear law and the LSM, we show in Fig. 1 the ratio F/F' for ATLAS models with solar metallicity and microturbulent velocity = 2 km s^{-1}. As it can be seen from the figure, the flux is conserved with very good accuracy and simultaneously the specific intensities are well described by Eq. (5).

Since these are the first calculations of LDC for the Sloan filters, it is convenient to compare the results with those derived for better known photometric systems. However, the only filter which can be really compared with a Sloan counter-part is the Strömgren u. Figure 2 shows such a comparison for ATLAS models. Note that the LDC derived for u' are systematically larger than those derived for the Strömgren u band, the maximum difference being around 10% at lower effective temperatures. The reason for these differences are in the effective wavelength of the two ultraviolet bands; the effective wavelength of the Sloan u' is slightly larger than its counterpart of Strömgren u. Of course, the general feature of the spectra near that spectral region and the sensitivity function also plays an important role.

Another interesting comparison that can be done is to investigate how the ATLAS and PHOENIX models depend on

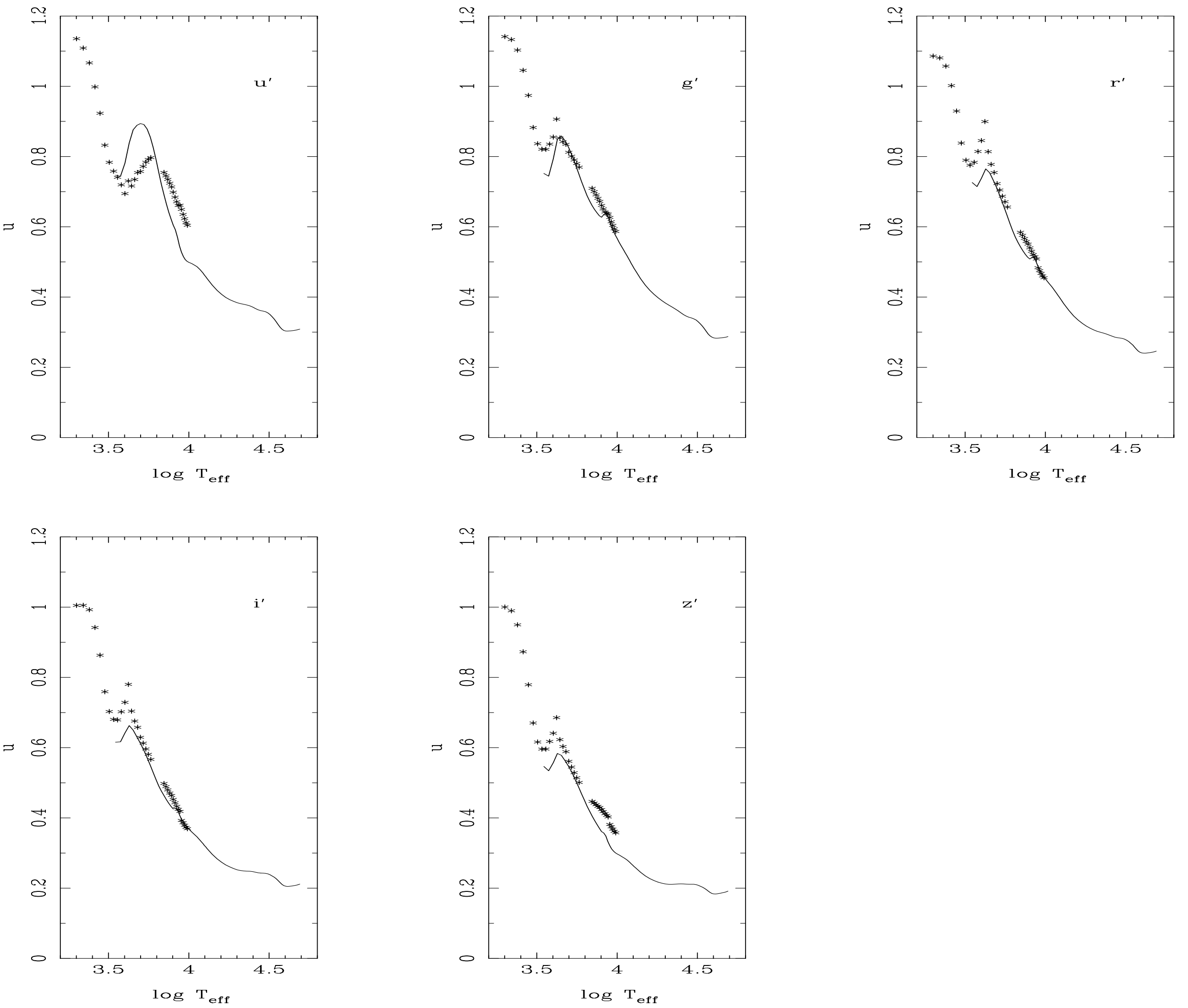

Fig. 3. The linear coefficient u for $u'g'r'i'z'$ following PHOENIX models (asterisks) and ATLAS (continuous line). log g = 4.5, solar abundance, microturbulet velocity = 2 km s^{-1}.

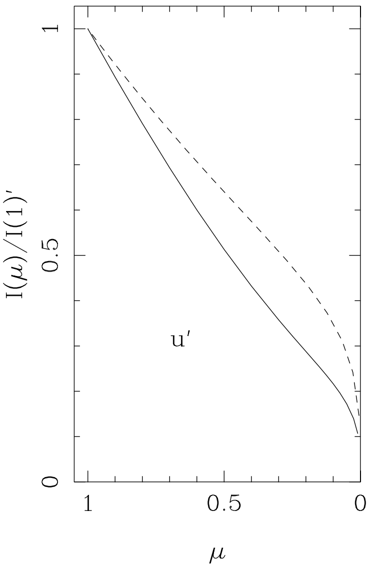

Fig. 4. The profiles of the specific intensities following a non-linear law (Eq. (5)) for a model with $T_{\rm eff}$ = 5000 K. PHOENIX models (dashed line) and ATLAS (continuous line). log g = 4.5, solar abundance, microturbulet velocity = 2 km s^{-1}.

the effective temperatures. Such a comparison can only be done for the models that present similar input physics, such as log g, metallicity, etc. In Fig. 3 are displayed the linear LDC for ATLAS (continuous lines) and PHOENIX (asterisks) as a function of the effective temperature, for fixed values of log g (4.5), metallicity (0.0) and microturbulent velocity (2 km s^{-1}). The agreement can be considered as acceptable but for lower effective temperatures (particularly at log $T_{\rm eff}$ around 3.5) and for u' band the comparison shows worse agreement. Such differences can be attributed to the different input physics in both codes of stellar atmospheres. In order to illustrate how both models differ, we display in Fig. 4 the profiles of the specific intensities for a model with $T_{\rm eff}$ = 5000 K, log g = 4.5, solar metallicity and 2 km s^{-1} of microturbulent velocity at filter u'.

Table 1 summarizes the main characteristics of the computed LDC. We encourage the reader to use the new non-linear law since it provides a very good description of $I(\mu)$ (with merit functions about 10^3–10^4 times smaller than those for linear and bi-parametric approximations (see Paper I). However, if one is interested in using the linear or bi-parametric approximations, additional tables containing the respectives merit functions are also available by request to evaluate the errors.

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