Trade-off between angular resolution and straylight contamination in the PLANCK Low Frequency Instrument

I. Pattern simulations

M. Sandri^{1,2}, F. Villa¹, R. Nesti³, C. Burigana¹, M. Bersanelli^{4,5}, and N. Mandolesi¹

¹ CNR - INAF/IASF, Sezione di Bologna, via P. Gobetti, 101, 40129 Bologna, Italy
e-mail: [sandri;villa;burigana;mandolesi]@bo.iasf.cnr.it

² Dipartimento di Astronomia, Università degli Studi di Padova, Vicolo dell'Osservatorio, 2, 35100 Padova, Italy

³ INAF - Osservatorio Astrofisico di Arcetri, L.go Fermi, 5, 50100 Firenze, Italy
e-mail: nesti@arcetri.astro.it

⁴ Dipartimento di Fisica, Università degli Studi di Milano, via G. Celoria, 16, 20133 Milano, Italy
e-mail: marco.bersanelli@fisica.unimi.it

⁵ CNR - INAF/IASF, Sezione di Milano, via E. Bassini, 15, 20133 Milano, Italy
On behalf of the LFI Consortium

Received 8 June 2004 / Accepted 27 July 2004

Abstract. The study of cosmic microwave background (CMB) anisotropies represents one of the most powerful cosmological tools. After the great success of the two NASA satellite missions COBE and WMAP, PLANCK is the third generation of mm-wave instruments designed for space observations of CMB anisotropies within the new Cosmic Vision 2020 ESA Science Programme. PLANCK will map the whole sky with unprecedented sensitivity, angular resolution and frequency coverage, using two instruments that share the focal region of a 1.5 m off-axis dual reflector telescope: the Low Frequency Instrument (LFI) and the High Frequency Instrument (HFI). In the optimisation of the optical interfaces of the LFI two concurrent demands have to be satisfied: the very good angular resolution (which affects the ability to reconstruct the angular power spectrum of the CMB anisotropies at high multipoles) and a very low level of straylight contamination (which may be one of the most serious sources of systematic effects). We present the results of the optical simulations aimed at establishing the trade-off between angular resolution and straylight rejection, carried out for the 100 GHz channel of the PLANCK LFI. Antenna patterns of different models of dual profiled corrugated conical feed horns have been simulated using advanced simulation techniques, considering the whole spacecraft geometry in order to obtain reliable sidelobe predictions. We show the optical computation accuracy necessary to provide strong straylight evaluation in reasonable computational time and demonstrate the inadequacy of a Gaussian feed model in realistic far beam predictions.

Key words. cosmology: cosmic microwave background – telescopes – instrumentation: detectors –
space vehicles: instruments

1. Introduction

The study of cosmic microwave background (CMB) anisotropies represents one of the most powerful cosmological tools. The shape of the angular power spectrum of CMB anisotropies sensitively depends on fundamental cosmological parameters, so that an accurate measure of the spectrum provides a unique method to establish the value of these parameters with high precision. After the discovery of the CMB anisotropies by the NASA satellite COBE (Smoot et al. 1992), several ground-based and balloon-borne experiments have been set up with the purpose of measuring the

anisotropies at sub-degree angular scales, by using dedicated reflecting telescopes and interferometer techniques (see e.g., Bersanelli et al. 2003, for a review). Recently the NASA satellite mission WMAP (Wilkinson Microwave Anisotropy Probe) equipped with a pair of 1.4 m back-to-back telescopes (Page et al. 2003a) reconstructed the power spectrum with high precision at multipoles up to ~ 800 (Bennett et al. 2003; Page et al. 2003b). Within the Horizon 2000 Scientific Programme, ESA planned PLANCK as the third generation of mm-wave instruments designed for space observations of CMB anisotropies. PLANCK will be launched in 2007 and will carry state-of-the-art of microwave radiometers (Low

Frequency Instrument, Mandolesi et al. 1998) and bolometers (High Frequency Instrument, Puget et al. 1998) coupled with a 1.5 m telescope and working between 30 and 900 GHz in nine frequency channels.

Owing to the small amplitude of the CMB anisotropies (about 10–100 μK rms), accurate control of systematic effects is mandatory to perform high precision CMB measurements, with experiments either from space or on ground. Specifically the optics (composed by the telescope-feed horn assembly) is one of the major limiting factors for high precision CMB measurements, as aberrations of the main beam and straylight are two of the main sources of systematic errors. The angular resolution, eventually degraded by the optical aberrations, limits the reconstruction of the anisotropy power spectrum at high multipoles (Mandolesi et al. 2000). In addition, fluctuations of the straylight signal (Polegre et al. 2000; Burigana et al. 2001) contaminate the measurements mainly at large and intermediate angular scales (i.e. at multipoles ℓ less than ≈ 100), and must be kept below a level of few μK .

Accurate predictions and measurements of the antenna pattern are essential both during the instrument development phase and for an in-depth knowledge of the whole-instrument response in the development of the data reduction pipeline, for all high precision CMB experiments. Robust optical simulations are of primary importance in the understanding of the straylight rejection capability of the instrument and telescope, in particular for the far sidelobe region (at large angles from the main beam), where the power levels are extremely low and direct measurements become difficult and uncertain.

In the PLANCK LFI, the antenna response features at large angles from the beam centre are determined mainly by the rays coming from the feed (considered as the source of radiation) that are reflected by the lower part of the subreflector and that are not intercepted by the main reflector. These rays fall in the so called spillover region with a power level up to -60 dB. Also the energy coming directly from the feed horn, without any interaction with the reflecting structures, increases the field and helps to increase the spillover. Moreover, as explained in Sect. 4, other important contributions that increase the far sidelobes are due to the diffraction from the edge of the mirrors and from the shielding structure. Therefore the spillover (and consequently the straylight contamination) can be reduced by decreasing the illumination near the edge of the mirrors, i.e. by increasing the Edge Taper (ET), defined as the ratio of the power per unit area incident on the centre of the mirror to that incident on the edge¹. On the other hand, increasing the ET has a negative impact on the angular resolution, which reduces the ability to reconstruct the anisotropy power spectrum of the CMB at high multipoles. With a given telescope the ET can be varied by changing the feed horn design, since the feed horn pattern determines the illumination function of the telescope. Clearly, an appropriate choice of feed design have a large effect on the angular resolution and the straylight rejection.

In the framework of the PLANCK LFI optical interface optimisation, the trade-off between angular resolution and straylight rejection has been derived. The work is presented in this paper, Paper I, and in an associated paper (Burigana et al. 2003, hereafter Paper II). Three steps have been taken.

1. Computation of realistic feed horn patterns. Electromagnetic design of feed horns has been performed in order to simulate realistic radiation patterns with different edge tapers.
2. Computation of the full optical response. Each feed horn pattern has been propagated through the PLANCK optical system to obtain the complete 4π beam. Although several powerful electromagnetic simulation methods, like Physical Optics (PO), Physical Theory of Diffraction (PTD), Geometrical Optics (GO), and Geometrical Theory of Diffraction (GTD), are well understood and widely used, their applicability to real optical systems can be very difficult particularly when many reflecting structures have to be simulated. Multiple diffractions and reflections between optical elements (reflectors, one or more shields, supporting structures) have to be considered and this leads to unacceptable computational time. To overcome this difficulty, an advanced simulation technique based on GTD has been used and it is described in this paper.
3. Straylight evaluation. This has been calculated by convolving the full pattern with the sky signal by considering the observational strategy. This step is described in Paper II.

We present, as a working case, the optimisation activity carried out on two representative feed horns of the PLANCK LFI, aimed at reaching the best angular resolution in view of the straylight requirements. By means of accurate beam simulations, the details of the antenna response have been computed for four different models of feed horn #4 and three models of feed horn #9 (see the right panel of Fig. 1). Through the comparison between the level of Galactic straylight contamination described in Paper II the best horn design has been identified.

We are currently applying the optimisation procedure presented here to the other LFI frequency channels but the method and basic results reported in this work can in principle also be applied to similar experiments aimed at measuring CMB anisotropies. Specifically, the assessment of the optical computation accuracy necessary to provide robust straylight evaluation – in reasonable computational time – is the basis of the optimisation procedures of each CMB anisotropy experiment.

In Sect. 2 the PLANCK optical configuration (telescope, shields and focal plane unit) is described. In Sect. 3 the electromagnetic design of the feed horns analysed is presented, as well as the major electromagnetic characteristics of each model. Section 4 reports a description of simulation methods used to compute the beams. In Sect. 5 the simulation results are reported, divided in main beam, intermediate beam, and far beam. The correlation between the computed integrated power, normalized to the total power, from the far and intermediate beams and the spillover is reported in Sect. 6, which is relevant for future optimisation activities. Finally, the conclusions are reported in Sect. 7.

¹ Since the ET is not constant but varies along the reflector rim, for convenience we define the ET at feed level as a dB level below the peak at a certain angle from the feed boresight.

Fig. 1. On the left, the satellite geometry and the major reference frames are shown: the spacecraft coordinate system (SC), the telescope coordinate system (TEL), the reference detector plane coordinate system (RDP), and the line of sight (LOS). The LFI field of view is shown ($\theta \leq 5^\circ$ around the LOS). On the right side, the PLANCK/LFI Focal Plane Unit configuration is represented in the RDP. The two feed horns under study (LFI4 and LFI9 at 100 GHz) are identified with a circle.

2. Electromagnetic model of the optics

Accurate beam simulations require a realistic representation of the optics in the electromagnetic model. Although for the main beam simulations only the telescope geometry needs to be considered, for the far beam the structures surrounding the reflecting mirrors also need to be considered in detail. These structures could be the mirror mounting structure, the shields and, in a general context, all the surfaces that could emit, reflect, or scatter radiation from the sky to the feed (i.e. detectors) (Mennella et al. 2002).

In the case of the PLANCK spacecraft, the geometry can be divided into four groups (see Fig. 1): the dual reflector telescope, the telescope main baffle (a shield surrounding the reflectors), the cryo-structure (three V-grooves under the telescope), and the Service Module at the bottom of the satellite. However, the PLANCK optical environment is dominated by a large baffle surrounding the telescope and, below this, the higher of the three thermal shields (V-grooves) used to thermally decouple the telescope and the focal assembly from the service module (Bard 1987). Then, for the simulations reported in this paper only the telescope, the baffle and the first V-groove have been considered. The Service Module and the other two V-grooves could contribute to the pattern after diffraction on the first V-groove which then dominates the pattern at large angles and low levels (≤ 100 dB).

The PLANCK telescope (Dubruel et al. 2000; Villa et al. 2002a; Tauber 2001) is designed as an off-axis tilted system offering the advantage of accommodating instruments on a large focal surface with an unblocked aperture, keeping diffraction by the secondary mirror and supporting structures at very low levels. The baseline configuration has been selected from among several different optical designs in 1999. The current

configuration has been obtained by optimising the telescope performance for a set of representative feed horns equally distributed in frequency (from 30 to 857 GHz) and space (within the focal plane box). Both mirrors have an ellipsoidal shape (aplanatic configuration). The conic constants, the focal length, the tilting, and the offset of the mirrors have been combined to reduce the main beam aberrations, the curvature of the focal surface, and the spillover. The physical dimensions of the primary mirror are about 1.9×1.5 m, allowing a projected circular aperture of 1.5 m diameter. The secondary reflector has been oversized up to approximately 1 m diameter to avoid under-illumination of the primary. The telescope field of view is $\pm 5^\circ$ centred on the line of sight (LOS) which is tilted at about 3.7° with respect to the main reflector axis, and forms an angle of 85° with the satellite spin axis, typically oriented in the anti-Sun direction during the survey.

The telescope is surrounded by a so-called main baffle that protects it from straylight and provides the optimum radiative surface that passively cools the telescope at approximately 50 K. The baffle is interfaced with the coldest of three V-groove shields, located at the bottom of the telescope, that ensure the passive thermal control design.

3. The focal plane unit and the feed horns

The Low Frequency Instrument is coupled to the PLANCK telescope by an array of corrugated feed horns. Dual-profiled corrugated horns have been selected as the best design in terms of shape of the main lobe, very low level of cross polarization, level of sidelobes, control of the phase centre location, low weight and compactness (Clarricoats & Olver 1984; Olver & Xiang 1988; Villa et al. 2002b; Maffei et al. 2000; Murphy et al. 2001).

The focal plane layout considered in this work foresees 12 feed horns at 100 GHz, 6 at 70 GHz, 3 at 44 GHz and 2 at 30 GHz, distributed around the HFI, as reported in the right panel of Fig. 1. In this study our attention is focused on feed horns #4 and #9 (hereafter, LFI4 and LFI9), both at 100 GHz. LFI4 is located at $(X, Y, Z)_{\text{RDP}} = (80.77 \text{ mm}, 65.82 \text{ mm}, -16.03 \text{ mm})$ with respect to the Reference Detector Plane Coordinate System (RDP, the frame represented in the centre of the LFI clock shown in the right panel of the Fig. 1) and its orientation and polarization are defined by the three angles $(\theta, \phi, \psi)_{\text{RDP}} = (12.17^\circ, -142.31^\circ, -153.6^\circ)$. LFI9 is located at $(X, Y, Z)_{\text{RDP}} = (-40.95 \text{ mm}, 90.94 \text{ mm}, 7.48 \text{ mm})$ with respect to the RDP and its orientation and polarization are defined by the three angles $(\theta, \phi, \psi)_{\text{RDP}} = (11.81^\circ, -66.93^\circ, -62.26^\circ)$. LFI4 is located in the highest position of the Focal Plane Unit (FPU), and it is the worst feed horn in terms of angular resolution and straylight contamination, whereas LFI9 is the best one. They represent the two extreme cases for the 100 GHz channel: for this reason we have decided to present the study carried out on these two horns, as evidence of the different optical response of the PLANCK telescope due to the different location of the horns on the FPU. Analogous considerations hold for feed horns #10 and #15, located symmetrically on the opposite side of the FPU.

Four models of the LFI4 (4A, 4B, 4C, 4D) and three models of the LFI9 (9A, 9B, and 9C) have been analysed. In Fig. 2 the radiation pattern of each model is shown, plotted in the feed horn's own reference frame. The seven different patterns have been computed by Modal Matching/MoM models (Harrington 1968; Harrington & Mautz 1969) for dual profile corrugated horns in 72 azimuthally equidistant cuts, in which θ (angle from the boresight) ranges from 0° to 180° .

4A (ET 28.3 dB @ 24°) and 9A (ET 25.5 dB @ 24°) represent two models, for LFI4 and LFI9 respectively, whose edge taper values ensure good straylight rejection (lower than $2 \mu\text{K}$ peak-to-peak), to the detriment of the angular resolution ($12.85'$ and $10.53'$ for the baseline models 4A and 9A, respectively). To reach a better angular resolution ($12'$ is the requirement for the LFI 100 GHz channel and $10'$ is the goal), the edge taper of the horns has been degraded to improve the main reflector illumination. In terms of mechanical constraints, a degradation of the edge taper involves a smaller feed horn aperture. The length and the nominal phase centre position remain unchanged. Decreasing the edge taper leads to a better angular resolution, but also to a higher sidelobe level. In fact, the off-axis response is largely due to reflections near the sub-reflector edge that is more illuminated, as should be noted in Fig. 2.

Three other designs have been tried out for the LFI4 (4B, 4C, and 4D) with the same edge taper (ET 19.0 dB @ 24°) that should lead to an angular resolution of about $12'$, on the basis of optical simulations carried out using a Gaussian feed as a source. Since it has been ascertained that further edge taper degradation does not lead to an angular resolution improvement for this horn, because of the strong illumination of the mirrors that increase the aberrations on the main beam, we tried to improve the angular resolution by changing the horn design,

Fig. 2. Co-polar pattern of the four models for the LFI4: 4A (ET 28.3 dB @ 24°), 4B (ET 19.0 dB @ 24°), 4C (ET 19.0 dB @ 24°), 4D (ET 19.0 dB @ 24°) (left panel) and co-polar pattern of the three models for the LFI9: 9A (ET 25.5 dB @ 24°), 9B (ET 19.0 dB @ 24°), 9C (ET 15.0 dB @ 24°) (right panel). Only the E-plane is reported ($\phi = 0^\circ$ in the feed horn coordinate system). The nominal edge taper has been indicated with a plus sign. For both feed horns, two vertical dashed lines are reported at the angle corresponding to the mirror edge (35° for LFI4 and 30° for LFI9): these angles are different for the two horns due to their different location and orientation in the focal plane unit. Decreasing the edge taper leads to a higher sidelobe level since the mirror edges are more illuminated: -27.34 dBi (-49.63 dB) for the 4A model and -8.37 dBi (-29.2287 dB) for the 4B model; -17.71 dBi (-38.50 dB) for 9A, -10.51 dBi (-30.15 dB) for 9B, and -3.73 dBi (-22.70 dB) for 9C. The models 4C and 4D have the same edge taper as 4B but present a lower illumination of the edge: -11.75 dBi (-32.46 dB) and -10.89 dBi (-31.58 dB) respectively, as evidence that the inner corrugation profile has a significant impact on the mirror illumination. Obviously, the mirror illumination is fairly elliptical and the power at the reflector edge is not the same along the rim: the values reported above are those in the E-plane of the feed pattern, corresponding to the cuts reported in the figure.

the edge taper remaining the same. Two different designs with two different ET values were calculated for the LFI9, instead: 9B (ET 19.0 dB @ 24°) and 9C (ET 15.0 dB @ 24°), which lead to angular resolutions of $10'$ and $9.5'$, respectively.

Through a simple ray-tracing the spillover was evaluated for each feed model and the results are reported in Table 1. This is a first approximation to the real spillover since it takes into account only the rays reflected by the subreflector that do not hit the main reflector. A more precise and time-consuming computation of the spillover has been done using Physical Optics and the results are very similar, as reported in Table 1. Neither of these estimates takes the place of a true 4π beam prediction.

Astrophysical simulations reported in Paper II, based on the beams computed using these feed models coupled with the PLANCK telescope, tell us which is the best choice in terms both of angular resolution and straylight contamination, for the two feed horns under study. In principle more electromagnetic models should be analysed but the huge computational time for

Table 1. Spillover computed using a simple ray tracing procedure (RT) and the physical optics (PO) approach, for each feed horn model. In the first case, the computation is very quick and the spillover can be evaluated in a few seconds; in the second case the result is more realistic to the detriment of the computational time, which can be a thousand times as high as in the first case. Differences of about 5% (on the average) appear.

	LFI4				LFI9		
	A	B	C	D	A	B	C
Edge Taper (dB @ 24°)	28.3	19.0	19.0	19.0	25.5	19.0	15.0
Spillover RT (%)	0.09	1.34	0.88	1.16	0.29	0.61	1.77
Spillover PO (%)	0.10	1.31	0.87	1.15	0.31	0.68	1.90

each beam compared to the LFI schedule constraints allow us to consider only a limited, albeit reasonable, set of cases.

4. Simulation techniques

The simulations are performed using GRASP² by considering the feed as a source and computing the pattern scattered by both reflectors on the far field. To predict the radiation pattern, different techniques can be applied: Physical Optics (PO); Physical Theory of Diffraction (PTD); Geometrical Optics (GO); and Geometrical Theory of Diffraction (GTD). PO is the most accurate method and may be used in all regions of the space surrounding the reflector antenna system. The field of the source is propagated on the reflector and the current distribution on the surface is calculated. Then the currents are used for evaluating the radiated field from the reflector. The calculation of the currents close to the edge of the scatterer is modeled by PTD. Unfortunately, as the frequency increases the reflectors have to be more and more precisely sampled. As a consequence, the density of the integration grids in which currents have to be computed on the reflectors must be finer and the computation time becomes huge. For a two-reflector antenna system like PLANCK, the computation time increases with the fourth power of the frequency³. Moreover, proper evaluation of the effect of shields is crucial for the optimisation of the edge taper, since these structures redistribute the power that is radiated by the horns and is not reflected by the telescope.

Although a full PO computation would be required to predict accurately the antenna pattern of our model of the telescope, this is not the case for the whole-spacecraft simulations since the PO approach cannot be correctly applied when multiple diffractions and reflections between scatterers are involved. For this reason, in our simulations we use a new GRASP8 technique, named Multi-reflector GTD (MrGTD), that computes the scattered field from the reflectors performing a backward ray tracing, and represents a suitable method for predicting the full-sky radiation pattern of complex mm-wavelength optical systems in which the computational time is

frequency-independent. When many scatterers are involved, the amount of ray tracing contributions may lead to unacceptable computational time even with MrGTD. Therefore, it is crucial to identify those and only those contributions (i.e. sequences of diffractions and/or reflections on each scatterer, see Sandri et al. (2002) for a detailed description of the method applied to the PLANCK case) which produce significant power levels in the resulting radiation pattern. We set a threshold equal to -100 dB, -50 dBi at 100 GHz, since this is the required straylight rejection level in this frequency channel. Lower power levels do not produce significant straylight contamination from the diffuse Galactic emission, as reported in Burigana et al. (2001) and this can also be inferred from the results presented in this work and in Paper II.

In our simulations we have considered the two reflectors, the baffle and the first V-groove as blocking structures. The simplest (1st order) optical contributions producing significant power levels are reflections onto the subreflector, onto the main reflector, and onto the baffle, as well as diffractions by the subreflector, by the main reflector, and by the baffle. Other non-negligible contributions are found considering two interactions with the reflectors (2nd order – for example, rays reflected on the subreflector and then diffracted by the main reflector), three interactions (3rd order – for example, rays reflected on the subreflector, diffracted by the main reflector, and then diffracted by the baffle) and so on. In this framework, we have not considered reflections or diffractions on the V-groove, as well as higher order contributions, since we expect them at very low levels (less than -100 dB). As example, for the feed horn LFI9 B, in Fig. 3 the contribution due to the radiation directly coming from the feed and not intercepted by the reflecting structures is shown, together with the 1st order contribution from the rays reflected by the subreflector and not intercepted by the main reflector. This latter contribution falls in the main spillover region with significant power levels (at about -60 dB) and it is the brighter contribution in the sidelobe region. In Fig. 4 the 1st order contribution from the rays reflected by the baffle is shown, with a maximum power level of about -66 dB, and in Fig. 5 the 2nd order contribution from the rays reflected by the subreflector and then diffracted by the main reflector is shown, with a maximum power level of about -67 dB.

Figure 6 shows the beam of LFI9 B in the plane $\phi_{bf} = 54^\circ$ passing through the power peak of the main spillover region, computed at the 1st order, at the 2nd order, and up to the 3rd order. In this cut, differences between the 2nd and the 3rd optical contribution are evident at $\theta_{bf} \simeq 115^\circ$. The optical contribution of the direct rays from the feed is also plotted, together with the pattern of the feed computed without blocking structure. It is evident that in the region with θ_{bf} between few degrees and 60° the feed sidelobes prevail. Therefore, using a Gaussian feed model (accurate enough for the main beam computation) is completely misleading for a realistic far pattern prediction.

5. Full beam simulations

For each LFI feed horn we have defined a corresponding *beam frame* on the sky (O_{bf} , X_{bf} , Y_{bf} , Z_{bf}). This local coordinate system is defined starting from the Line Of Sight (LOS)

² GRASP8 is a software developed by TICRA (Copenhagen, DK) for analysing general reflector antennas (www.ticra.com).

³ Without considering the effect of the shields, this means more than 800 h to perform a full pattern computation at 100 GHz, using a 550 MHz Pentium II machine.

Fig. 3. In the left panel the 4π map of the field due to the rays coming from the feed (direct contribution) is shown, together with the field due to the rays reflected on the subreflector and not intercepted by the main reflector (Rs contribution). Most of the map is empty (grey colour) since most of the rays are blocked by the baffle. The power peak of the direct contribution is about -10.25 dBi whereas the power peak of the Rs contribution is about -1.18 dBi. This latter is the brightest contribution in the far sidelobes. In the central panel a sketch of the optics in the symmetry plane is shown together with the ray-tracing of these two contributions in the plane at $\phi_{bf} = 45^\circ$ (these rays lie in the white dashed cut indicated in the map, where the field is present). In the right panel the co- and x-polar components of the field due to the rays plotted in the central panel are shown.

Fig. 4. In the left panel the 4π map of the field due to the rays reflected by the baffle (Rb contribution) is shown. The power peak of this contribution is about -5.73 dBi. In the central panel a sketch of the optics in the symmetry plane is shown together with the ray-tracing of this contribution in the plane at $\phi_{bf} = 45^\circ$ (these rays lie in the white dashed cut indicated in the map, where the field is present). In the right panel the co- and x-polar components of the field due to the rays plotted in the central panel are shown.

coordinate system (O_{LOS} , X_{LOS} , Y_{LOS} , Z_{LOS}). Z_{LOS} is tilted 85° from the satellite spin axis (X_{SC} in Fig. 1), O_{bf} coincides with O_{LOS} , and Z_{bf} is the pointing direction of the beam to which the coordinate system refers. All the coordinate systems are right-handed cartesian systems. The orientation of each *beam frame* with respect to the LOS is defined by the three angles (θ_{MB} , ϕ_{MB} , ψ_{MB})⁴. According to Fig. 7, the Z_{LOS} axis is tilted at an angle θ_{MB} by rotating the (X_{LOS} , Y_{LOS} , Z_{LOS}) coordinate system around the axis, lying in the (X_{LOS} , Y_{LOS}) plane, which is orthogonal to the line forming the angle ϕ_{MB} with the X_{LOS} vector. This rotation generates an intermediate coordinate system (O_{int} , X_{int} , Y_{int} , Z_{int}). The beam frame coordinate system is then obtained rotating the angle ψ_{MB} around the Z_{int} axis (see Appendix A of Paper II for the explicit transformation rules). Since Z_{bf} is equal to the intermediate Z_{int} , the angle ψ_{MB} defines the local polarization direction.

We have divided the far-field pattern into three regions at different angular distances from the beam centre: the main

beam region ($\theta_{bf} < 1.2^\circ$), the intermediate beam region ($1.2^\circ < \theta_{bf} < 5^\circ$), and the far beam region ($\theta_{bf} > 5^\circ$). The choice of these angles has been driven by the fact that, for the PLANCK telescope, at LFI frequencies the first minimum of the beam pattern falls at about 1.2° , whereas 5° roughly divides pattern regions where significant response variations occur on angular scales less than 1° from those where they occur on degree or larger scales (Burigana et al. 2001). In each of the three regions, simulations are performed using different methods, as explained below.

5.1. Main beam

The main beam simulations have been performed using PO analysis on each reflector. Far-field radiation patterns have been computed in the co- and x-polar basis according to the Ludwig's third definition (Ludwig 1973) on UV-spherical grids with 301×301 points ($\Delta U = \Delta V \approx 10^{-4}$), in order to compute the main beam angular resolution of each feed model analysed, as well as all major characteristics reported in Table 2. The contour plot of the main beam computed with the B model of LFI9 is shown in Fig. 8, as example.

⁴ $U_{peak} = \sin \theta_{MB} \cos \phi_{MB}$ and $V_{peak} = \sin \theta_{MB} \sin \phi_{MB}$, where U_{peak} and V_{peak} are the coordinates of the power peak of the main beam computed in a UV-grid with respect to the LOS frame.

Fig. 5. In the left panel the 4π map of the field due to the rays reflected on the subreflector and then diffracted by the main reflector (RsDm contribution) is shown. The power peak of this contribution in the far beam is about -6 dBi; note that this contribution raises the power in the intermediate beam region (near sidelobes, originating mainly from the diffractions by the edge of the reflector). In the central panel a sketch of the optics in the symmetry plane is shown together with the ray-tracing of this contribution in the plane at $\phi_{bf} = 45^\circ$ (these rays lie in the white dashed cut indicated in the map, where the field is present). The bundle of rays that cause the bright annular region at $\theta_{bf} \approx 90^\circ$ is indicated in the central panel with a darker colour. In the right panel the co- and x-polar components of the field due to the rays plotted in the central panel are shown.

Table 2. Main beam characteristics at the central frequency. U_{peak} and V_{peak} are computed with respect to the telescope line of sight. The Full Widths (FW , minimum and maximum) at -3 , -10 , -20 dB are also reported since the main beams are strongly distorted by the optics, due to the off axis location of the feeds in the focal plane unit. The Full Width Half Maximum ($FWHM$) is the average value between the minimum and maximum of the beam width at -3 dB. The main beam directivity (Dir), the cross polar discrimination factor (XPD), and the main beam depolarization parameter (d) are reported.

Feed model	U_{peak}	V_{peak}	FW at -3 dB		FW at -10 dB		FW at -20 dB		FWHM	Dir	XPD	d
			(arcmin)	(arcmin)	(arcmin)	(arcmin)	(arcmin)	(arcmin)				
LFI4 A	0.05142	0.03850	11.87	13.84	21.49	26.11	29.93	37.31	12.85	59.05	26.41	0.62
LFI4 B	0.05142	0.03851	11.63	13.01	21.76	25.01	29.75	36.54	12.32	59.14	24.75	0.95
LFI4 C	0.05143	0.03851	11.39	12.78	21.10	24.20	29.56	36.92	12.08	59.34	25.17	0.90
LFI4 D	0.05141	0.03852	11.63	13.21	21.10	25.13	29.27	36.24	12.42	59.28	24.91	0.93
LFI9 A	-0.01767	0.05553	9.19	11.87	16.45	21.62	22.27	31.85	10.53	60.69	27.46	0.45
LFI9 B	-0.01767	0.05552	8.88	11.14	15.56	19.86	20.96	33.14	10.01	61.13	27.23	0.56
LFI9 C	-0.01767	0.05554	8.56	10.62	15.20	20.28	20.56	33.58	9.59	61.19	26.09	0.67

Fig. 6. Far-field power pattern of LFI9 B in the plane $\phi_{bf} = 54^\circ$, passing through the main spillover. Differences between simulations to the 1st, 2nd, and up to the 3rd order are evident. Note the 3rd order peak at θ_{bf} between 110° and 120° . The contribution of the direct rays from the feed is indicated, and the feed sidelobes appear at about -20 dBi, between the main beam direction and the main spillover.

Fig. 7. The orientation of the *beam frame* coordinate system relative to the LOS reference system specified by the angles θ_{MB} , ϕ_{MB} , and ψ_{MB} .

Fig. 8. Contour plot in the UV-plane ($-0.015 < U, V < 0.015$) of the main beam computed for the B model of LFI9 at 100 GHz (*left panel*) and contour plot of the beam obtained averaging the response in the 20% of the operational bandwidth (*right panel*). Note that some beam features shown in the left panel disappear in the right one due to the finite bandwidth that smears the rings at the lower power levels (at about -50 dBi). The lines of the contour plots, normalized at the peak gain (reported in Table 2), are at -3 , -6 , -10 , -20 , -30 , -40 , -50 , -60 , and -70 dB, whereas the colour scale goes from -80 dB to 0 dB. The beam is centred on the relative beam axis.

5.2. Intermediate beam

Intermediate beams have been computed in spherical polar cuts with $-5^\circ < \theta_{bf} < 5^\circ$ ($\Delta\theta = 0.1^\circ$) and $0^\circ < \phi_{bf} < 180^\circ$ ($\Delta\phi = 0.5^\circ$), using MrGTD up to the 2nd order for all models of LFI4 and for 9A and 9C, and up to the 3rd order for 9B⁵. However, we have found that the straylight level evaluated using a beam computed with the PO/PTD (neglecting the effect due to the shields) is very similar to that computed with MrGTD: differences of about 1% appear (see Table 2 of Paper II). In Fig. 9 the polar cuts at $\phi = 0^\circ$ for the four models of LFI4 and the three models of LFI9 are shown. In the left panel, note the good performance of 4C with respect to 4B and 4D, ET being equal. In the right panel, differences of about 6 dB between 9A and 9B, and 4 dB between 9B and 9C appear, reflecting the differences between horn edge tapers.

5.3. Far beam

Far beam simulations have been carried out using MrGTD up to the 2nd order for 4B, 4C, and 4D, and up to the 3rd order for 4A, 9A, 9B, and 9C. Figure 10 shows the antenna response at large angles from the beam centre for the azimuthal cut corresponding to the main spillover, for all considered designs. As expected the radiation passing through the higher part of the primary reflector, the so-called main spillover, at about $\phi_{bf} = 50^\circ$ and $\theta_{bf} = 90^\circ$ for LFI9 and at about $\phi_{bf} = 130^\circ$ and $\theta_{bf} = 90^\circ$ for LFI4, increases the illumination at the edge of the primary mirror (-6 dBi for 9A, -2 dBi for 9B, and about 1 dBi for 9C; -14 dBi for 4A and about -2 dBi for 4B, 4C, and 4D).

The resulting 4π map of the best feed horn model analysed for LFI9 (9B) in terms of angular resolution and straylight rejection is presented in Fig. 11, as function of the two polar coordinates $\theta_{bf} \in [-180^\circ; +180^\circ]$ and $\phi_{bf} \in [0^\circ; +180^\circ]$. The beam

Fig. 9. Intermediate beams computed using the four feed models for the LFI4 and the three feed models for LFI9: 4A, 4B, 4C, 4D (*left panel*) and 9A, 9B, 9C (*right panel*), all computed using MrGTD up to the 2nd order except for 9B for which also the 3rd order interaction has been simulated. Polar cuts at $\phi_{bf} = 0^\circ$ are shown. In the left panel, note the good performances of 4C with respect to 4B and 4D, ET being equal. In the right panel, differences of about 6 dB between 9A and 9B, and 4 dB between 9B and 9C appear, reflecting the differences between horn edge tapers.

boresight direction is at the top of the map. Each polar cut with a constant ϕ_{bf} value is a meridian of the map. The θ_{bf} angle runs, on each meridian, from $\theta_{bf} = 0^\circ$ (towards the main beam direction) to $\theta_{bf} = 180^\circ$ in the left side of the map, and from $\theta_{bf} = 0^\circ$ to $\theta_{bf} = -180^\circ$ in the right side of the map. The ϕ_{bf} angle goes from $\phi_{bf} = 0^\circ$ (centre of the map) to $\phi_{bf} = 180^\circ$ (left side of the map) on the left side of the map, and from $\phi_{bf} = 0^\circ$ (right side of the map) to $\theta_{bf} \rightarrow 180^\circ$ (centre of the map) on the right side of the map. Therefore the centre of the map has $\theta_{bf} = 90^\circ$ and $\phi_{bf} = 0^\circ$.

⁵ Not all beams have been computed considering 3rd order optical contributions since it takes about one month per beam. As reported in Sect. 6, we have found that neglecting 3rd order optical contributions implies errors of only a few per cent in the straylight evaluation.

Table 3. Integrated power in the intermediate and far beams, normalised to the total power ($f\%$), for each simulated feed horn. Galactic straylight contamination (GSC, in μK rms and peak-to-peak antenna temperature) computed in Paper II considering contributions from dust, diffuse free-free emission, diffuse synchrotron emission, and HII regions, is also reported for each pattern region. For the far beam region, when the 3rd order MrGTD contribution has been computed, the corresponding GSC has been reported (n.a. means *not available*).

Feed Model	Spillover (%)	$f\%$	Intermediate beam		Far beam (3rd order)			Far beam (2nd order)		
			GSC _{RMS}	GSC _{p-p}	$f\%$	GSC _{RMS}	GSC _{p-p}	$f\%$	GSC _{RMS}	GSC _{p-p}
LFI4 A	0.10	0.002	1.86×10^{-3}	6.51×10^{-2}	0.041	1.48×10^{-2}	1.10×10^{-1}	0.036	1.36×10^{-2}	1.11×10^{-1}
LFI4 B	1.31	0.057	4.88×10^{-2}	$1.65 \times 10^{+0}$	n.a.	n.a.	n.a.	0.459	2.09×10^{-1}	$1.63 \times 10^{+0}$
LFI4 C	0.87	0.042	3.67×10^{-2}	$1.27 \times 10^{+0}$	n.a.	n.a.	n.a.	0.292	1.40×10^{-1}	$1.12 \times 10^{+0}$
LFI4 D	1.15	0.053	4.57×10^{-2}	$1.58 \times 10^{+0}$	n.a.	n.a.	n.a.	0.398	1.92×10^{-1}	$1.56 \times 10^{+0}$
LFI9 A	0.31	0.010	8.38×10^{-3}	2.94×10^{-1}	0.185	7.56×10^{-2}	6.38×10^{-1}	0.175	7.33×10^{-2}	6.16×10^{-1}
LFI9 B	0.68	0.034	2.78×10^{-2}	8.84×10^{-1}	0.378	1.99×10^{-1}	$1.92 \times 10^{+0}$	0.351	1.90×10^{-1}	$1.87 \times 10^{+0}$
LFI9 C	1.90	0.099	8.08×10^{-2}	$2.26 \times 10^{+0}$	1.186	7.05×10^{-1}	$6.35 \times 10^{+0}$	1.086	6.72×10^{-1}	$6.18 \times 10^{+0}$

Fig. 10. Far-field antenna response for the azimuthal direction, reported in each panel, corresponding to the main spillover. Beams computed using the four feed models for LFI4 and the three feed models for LFI9 are shown: 4A, 4B, 4C, 4D (*left panel*) and 9A, 9B, 9C (*right panel*), all computed using MrGTD up to the 2nd order. In the case of LFI9 B we show the beam computed including also the 3rd order optical interaction, for comparison.

Figure 12 shows the 4π map of the difference⁶ (in dB) between the full antenna pattern response computed including 1st, 2nd and 3rd order optical interactions and that obtained neglecting the 3rd order optical interaction, in the beam reference frame and for model B of the LFI9. These contributions would seem not to be negligible in the far sidelobes.

Since the computation of high-order optical interactions is very time-consuming it is useful for optical optimisation studies to quantify the loss of accuracy in the straylight contamination evaluation introduced by neglecting the 3rd order optical interactions, and this has been done in Sect. 6.

⁶ The shown quantity is $10 \times \text{Log}_{10}(R_{1+2+3} - R_{1+2})$, where R_{1+2+3} (R_{1+2}) is the linear telescope response computed up to the 3rd (2nd) order.

6. Discussion

In Table 3 the fractional contributions to the integrated beam, $f\%$, in each considered pattern region, are summarised for each feed horn model analysed. Differences of about 9% appear in $f\%$ for the beams computed with the MrGTD up to the 2nd order compared to the beams computed with the MrGTD up to the 3rd order. The Galactic straylight contamination (GSC, in μK antenna temperature, rms and peak-to-peak), as computed in Paper II considering contributions from dust, diffuse free-free emission, diffuse synchrotron emission, and HII regions, is also reported for each pattern region. Differences in $f\%$ derived from different feed horn designs are about 2% (5%) in the GSC peak to peak (rms). Note that neglecting 3rd order optical contributions on the average implies errors of only a few per cent in the straylight evaluation. This is very important in optimisation since tell us that the accuracy of an optical simulation computed using the MrGTD up to the 2nd order is sufficient to provide robust straylight results, saving about 75% of computational time.

In Fig. 13, $f\%$ versus the spillover is shown, as computed for the four models of the LFI4 and the three models of the LFI9 at 100 GHz. Dashed, dotted, and solid lines are the fitted curves (forcing the obvious zero crossing) of these points for the intermediate beam computed up to the 2nd order, the full pattern computed up to the 2nd order, and, only for the LFI9, the full pattern computed up to the 3rd order, respectively.

Finally, we have found linear approximations describing the dependence of the fractional contribution on the integrated antenna pattern, $f\%$, by the spillover (%), from the considered pattern regions:

$$f\% \simeq 0.051 \times \text{spillover} \quad (1)$$

for the intermediate beam region, and

$$f\% \simeq 0.62 \times \text{spillover} \quad \text{considering 3rd order optical contributions} \quad (2)$$

$$f\% \simeq 0.57 \times \text{spillover} \quad \text{considering 2nd order optical contributions} \quad (3)$$

for the far beam region.

Fig. 11. Beam of the B model of LFI9 at 100 GHz, computed with the MrGTD up to the 2nd order, except in the main beam region that has been computed using the PO/PTD analysis in order to avoid caustic artifacts originated by the GTD approach. In the upper side of the figure, three polar cuts are reported: ϕ_{bf} equal to 0° (left panel), ϕ_{bf} equal to 54° (central panel; cut passing through the main spillover), and ϕ_{bf} equal to 110° (right panel; cut passing through one of the two wings originated by the rays reflected on the inner part of the baffle). At the bottom, the 4π map is shown. In the uniform grey region around $\theta_{bf} \approx -100^\circ$ and $\phi_{bf} \approx 67.5^\circ$ no ray coming from the source has been found and thus the total field is null.

These equations concern LFI9 at 100 GHz, for which the simulated feed models had three different values of the edge taper. The same has been done for LFI4 and the resulting relations are

$$f_{\%} \approx 0.045 \times \text{spillover}$$

for the intermediate beam region, and

$$f_{\%} \approx 0.346 \times \text{spillover} \quad \text{considering 2nd order optical contributions} \quad (5)$$

for the far sidelobes. An upper limit to a realistic uncertainty is computed as the maximum difference between the fit value

(4)

Fig. 12. Map of the difference (in dB) between the full antenna pattern response computed by including 1st, 2nd and 3rd order optical interactions and that obtained neglecting the 3rd order optical interaction, in the beam reference frame and for model B of LFI9. The uniform grey region at about $\theta_{bf} \sim -100^\circ$, $\phi_{bf} \sim 67.5^\circ$ is where the contributions up to the 3rd order optical interaction are null, while the region immediately below is the region where the 3rd order optical interaction is the only one that does not give a null contribution. Therefore, the reported values in the latter region are equal to the full antenna pattern response (in dBi) due to the 3rd optical interaction alone). It is also interesting to note the *spot* at $\theta_{bf} \sim 115^\circ$, $\phi_{bf} \sim 54^\circ$ corresponding to the significant contribution from the 3rd order optical interaction clearly visible in the right panel of Fig. 10 (dotted line). The 3rd order optical contributions considered are those in which the maximum power level is greater than -40 dBi (-100 dB at 100 GHz) in at least one point of the map. In particular, we have indicated in the map two relevant contributions which increase the spillover region: RsRbDb (rays reflected onto the subreflector and then reflected onto the baffle and then diffracted by the baffle) and DsRbRr (rays diffracted by the subreflector and then reflected onto the baffle and then reflected onto the subreflector).

and the real value, normalized to the real value (without consider the lower spillover values, $<0.01\%$, for which the uncertainty would be somewhat high because of the normalization): 4% (6%) and 10% (3%) for the far (intermediate) beam of LFI4 and LFI9, respectively. The fractional contribution to the integrated beam has been related to the Galactic straylight contamination in Paper II (see Eqs. (3)–(6)).

7. Conclusions

One of the major sources of systematic errors in CMB experiments is the non-ideal response of the optics. Main beam aberrations and sidelobes may degrade the reconstruction of the power spectrum of the CMB anisotropies at high and low multipoles, respectively. While for the main beam and intermediate beam regions Physical Optics gives accurate and well established results taking into account only the feed horn and the telescope alone, for the sidelobe regions the optics should also include the structures surrounding the telescope. This complicates the electromagnetic model, increasing the computational time, and may preclude the ability to perform full pattern calculations. On the other hand, for a satellite mission like PLANCK the knowledge of the beam is a decisive factor for guaranteeing the high performances of the instruments onboard (LFI and HFI). For this reason we studied in detail the way to predict the beam at LFI frequencies using GRASP8.

Fig. 13. Fractional contribution (in per cent) to the integrated beam as computed for the four models of LFI4 and the three models of LFI9 at 100 GHz. Dashed, dotted, and solid lines are the fitted curves (forcing the obvious zero crossing) of these points for the intermediate beam computed up to the 2nd order, the full pattern computed up to the 2nd order, and, only for feed horn #9, the full pattern computed up to the 3rd order, respectively. For LFI4 the fitted angular coefficients are $0.346 (\pm 0.003)$ and $0.045 (\pm 0.001)$ in the far and intermediate beam, respectively; whereas for LFI9 they are $0.62 (\pm 0.02)$ for the far beam computed up to 3rd order, $0.57 (\pm 0.01)$ for the computation up to the 2nd order, and $0.051 (\pm 0.002)$ for the intermediate beam. The values reported between round brackets are the 1σ uncertainty estimates for the fit angular coefficients. A more realistic uncertainty is reported in the text.

An electromagnetic model of PLANCK LFI has been set up. The model includes the feed pattern, the telescope optics, the first V-groove and the shield surrounding the telescope. Moreover, for this geometry, the sequences of reflections/diffractions which give a significant contribution (>100 dB below the beam peak) have been identified. This assures a reasonable beam estimation for robust straylight evaluation. It has been found that the contribution of more than 2 optical interactions is not required in the optimisation activity, since neglecting the 3rd order optical contributions involve differences on average less than 3% in the straylight evaluation, saving about 75% of the computational time.

The inadequacy of a pure Gaussian feed model in realistic far beam predictions is demonstrated: relevant features in the beam are due to the sidelobes in the feed horn pattern. Not only the realistic pattern needs to be considered, but the detail of the corrugation design could also affect the beam characteristics. The edge taper being equal, different corrugation profiles involve differences of about 3% in the main beam angular resolution and about 40% in the straylight signal, although it remains below $2 \mu\text{K}$ in all cases for LFI4. The 4C model (with an angular resolution of $10'$ and a straylight contamination of $1.12 \mu\text{K}$ peak-to-peak) and the 9B model (with an angular resolution of $12'$ and a straylight contamination of $1.87 \mu\text{K}$

peak-to-peak) represent the best choice for LFI feed horn #4 and #9, reaching the goal and the requirement, respectively.

A linear fit of the integrated power normalised to the total power ($f\%$) versus the spillover has been obtained. Although these fits have been specifically derived from the channel studied here, they could be used to set to the first order the edge taper of the other LFI channels with feed horns located near those studied, without any pattern simulations. Of course, sophisticated pattern simulations are mandatory to accurately quantify the beam aberrations and the straylight contamination, since not only the spillover level is crucial, but also how the spilled radiation is distributed in the sky.

Acknowledgements. We wish to thank people of the Herschel/PLANCK Project, ALCATEL Space Industries, and the LFI Consortium that are involved in activities related to optical simulations. Some of the results in this paper have been derived using the HEALPix (Górski et al. 1999). Thanks to Per Nielsen (TICRA Engineering Consultants, Copenhagen), for his clarifying correspondence about MrGTD and support with GRASP8. Special thanks to L. Terenzi and G. Morgante for the useful support. We warmly thank J. M. Lamarre for constructive comments and suggestions, and for the careful reading of the first version of this work.

References

- Bard, S. 1987, *J. Spacecraft Rockets* (ISSN 0022-4650), 24, 193
- Bennett, C. L., Halpern, M., Hinshaw, G., et al. 2003, *ApJS*, 148, 1
- Bersanelli, M., Maino, D., & Mennella, A. 2002, *Riv. Nuovo Cimento*, 25 [arXiv:astro-ph/0209215]
- Burigana, C., Maino, D., Górski, K. M., et al. 2001, *A&A*, 373, 345
- Burigana, C., Sandri, M., Villa, F., et al. 2004, *A&A*, 428, 311
- Clarricoats, P. J. B., & Olver, A. D. 1984, *Corrugated horns for microwave antennas* (London: IEE)
- De Zotti, G., Toffolatti, L., Argüeso Gómez, F., et al. 1999, *AIP Conf. Proc.*, 476, ed. L. Maiani, F. Melchiorri, & N. Vittorio, 204 [arXiv:astro-ph/9902103]
- Dubruel, D., Cornut, M., Fargant, G., et al. 2000, *ESA Conf. Proc.*, SP-444, ed. D. Danesy, & H. Sawaya, CD-ROM
- Górski, K. M., Hivon, E., & Wandelt, B. D. 1998, *Proc. of the MPA/ESO Conference on Evolution of Large-Scale Structure: from Recombination to Garching*, ed. A. J. Banday, R. K. Sheth, & L. Da Costa, 37 [arXiv:astro-ph/9812350]
- Harrington, R. F. 1968, *Field Computation by Moment Methods* (New York: MacMillan)
- Harrington, R. F., & Mautz, J. R. 1969, *Appl. Sci. Res.*, 20, 405
- Ludwig, A. C. 1973, *The Definition of Cross Polarization*, *IEEE Trans. Antenn. Propag.*, 116
- Maffei, B., Ade, P. A. R., Tucker, C. E., et al. 2000, *Int. J. Infrared Millimetre waves*, 21, 2023
- Mandolesi, N., et al. 1998, *PLANCK LFI, A Proposal Submitted to the ESA*
- Mandolesi, N., Bersanelli, M., Burigana, C., et al. 2000, *A&AS*, 145, 323
- Mennella, A., Villa, F., Sandri, M., & Burigana, C. 2002, *PL-LFI-PST-TN-034, LFI Optical Interfaces*
- Murphy, J. A., Colgan, R., Gleeson, E., et al. 2001, *AIP Conf. Proc.*, 616, 282
- Olver, A. D., & Xiang, J. 1988, *IEEE Trans. Antenn. Propag.*, 36, No. 7, 936
- Page, L., Jackson, C., Barnes, C., et al. 2003a, *ApJ*, 585, 566
- Page, L., Nolte, M. R., Barnes, C., et al. 2003b, *ApJS*, 148, 233
- Polegre, A. M., de Maagt, P., Tauber, J., & Passvogel, T. 2000, *Astrophys. Lett. Commun.*, 37, 239
- Puget, J. L., et al. 1998, *HFI for the PLANCK Mission, A Proposal Submitted to the ESA*
- Tauber, J. A. 2001, *Proc. IAU Symp.* 204, ed. M. Harwit, 493
- Villa, F., Bersanelli, M., Burigana, C., et al. 2002a, *AIP CP616*, ed. M. De Petris, & M. Gervasi, 224 [arXiv:astro-ph/0112173]
- Villa, F., Sandri, M., Mandolesi, N., et al. 2002b, *Exp. Astron.*, 14, 1
- Sandri, M., Villa, F., Bersanelli, M., et al. 2002, *25th ESA Antenna Workshop on Satellite Antenna Technology*, *ESA Conf. Proc.*, WPP-202, 621 [arXiv:astro-ph/0304142]
- Smoot, G. F., Bennett, C. L., Kogut, A., et al. 1992, *ApJ*, 396