

## Research Note

# On the $z$ -distribution of pulsars

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**Abstract.** The  $z$ -distribution of pulsars in the vicinity of the Sun is investigated using data from the ATNF pulsar catalogue and the recent model for the Galactic distribution of free electrons (NE2001). It is found that the  $z$ -distribution of pulsars with  $L > 10$  mJy kpc<sup>2</sup> is exponential with a characteristic scale height of about 350 pc. Evidence of pulsar oscillations in the direction perpendicular to the Galactic plane is presented.

**Key words.** Galaxy: stellar content – stars: neutron – stars: pulsars: general

## 1. Introduction

It is accepted that the  $z$ -distribution of the Galactic Population I objects is approximately exponential (Binney & Merrifield 1998). This kind of the observed distribution can be explained by the dynamic equilibrium that exists in the Galaxy.

It is known that the Galactic gravitational potential causes oscillation of the Galactic Population I objects in the direction perpendicular to the Galactic plane. If Population I subsystem objects have a Gaussian  $\varphi_0(v_z)$  velocity distribution in the Galactic plane and if the subsystem is “well-mixed”, the  $z$ -distribution of the objects is approximately exponential (Oort 1965). The scale height of the  $z$ -distribution of pulsar progenitors is of the order 50–100 pc. On the other hand according to relatively recent data the  $z$ -distribution of pulsars is approximately an exponential with an estimated scale height of 600 pc (Lyne & Graham Smith 1998).

Today this discrepancy is explained by a suggestion that pulsars are runaway stars (Gunn & Ostriker 1970). Gunn & Ostriker (1970) analyzed statistically the data of 41 pulsars known at that time and found that the scale height in the  $z$ -distribution was about 120 pc, similar to that of A stars. The picture of the Galactic distribution of the pulsars depends strongly on the model of the free electron distribution in the Galaxy. Different models give different  $z$ -distributions. Gunn & Ostriker (1970) adopted a hypothesis according to which the pulsar luminosity should decay exponentially, with a time scale of about 4.55 Myr. As the pulsars are born near the Galactic plane as runaway objects, they should have moved some distance away from the Galactic plane before their luminosities fall below  $L_{\min}$  and they effectively “die”, i.e. they become unobservable. The combination of the pulsar motion away from the Galactic plane with a decay of the luminosity should determine the observed  $z$ -distribution. Therefore,

although pulsars and Population I subsystems show a similar exponential  $z$ -distribution, the mechanisms of the formation of the  $z$ -distribution should differ.

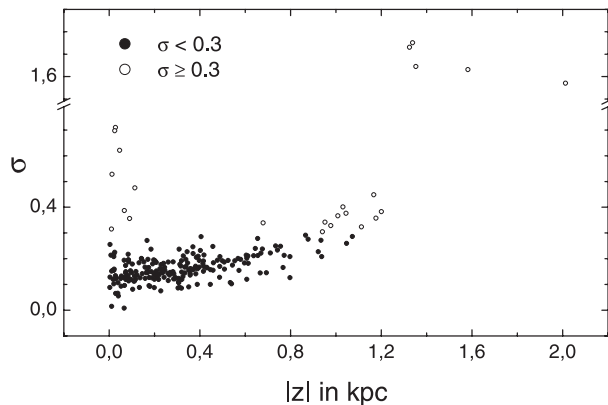
In the present paper we have used data from the ATNF pulsar catalogue version 1.15 ([www.atnf.csiro.au/research/pulsar/psrcat/](http://www.atnf.csiro.au/research/pulsar/psrcat/)) and the recent model for the Galactic distribution of free electrons (NE2001) by Cordes & Lazio (2002) to estimate the distances and their uncertainties. While studying the  $z$ -distribution of pulsars we have used pulsars located inside a cylinder with 3 kpc radius centered on the Sun, and with the axis along the  $z$ -direction. We discuss the problem of possible oscillation of the pulsars in the direction perpendicular to the Galactic plane.

## 2. Method of analysis

While studying the  $z$ -distribution of pulsars one should take into account various systematic errors caused by (1) the observational biases of the surveys; (2) instrumental selection effects (Lyne et al. 1985); and (3) random and systematic errors caused by the used distance model (Gunn & Ostriker 1970). Detailed analysis of these biases, which is beyond the scope of this paper, needs Monte Carlo simulations of a synthetic population of pulsars.

Nowadays, it is accepted that from most present pulsar surveys it is not possible to make reliable statements about the population of pulsars with luminosities lower than about 10 mJy kpc<sup>2</sup> (Lorimer et al. 1993; Lyne et al. 1998). Therefore, we limited ourselves to the sample of pulsars with  $L > 10$  mJy kpc<sup>2</sup> located inside a cylinder with 3 kpc radius centered on the Sun.

It is also accepted that the pulsar distance scale is statistically consistent with other methods of deriving galactic distances and that it has no systematic errors. Random errors in the



**Fig. 1.** Dependence of the distance uncertainties  $\sigma$  on  $|z|$ .

distance measurements affect the estimation of the scale height in the  $z$ -distribution. According to Gunn & Ostriker (1970) for a logarithmic-normal distribution law of the observed distances the scale height of the observed  $z$ -distribution is related to the real one as

$$\langle |z| \rangle = \langle |z| \rangle \exp(0.5\sigma^2). \quad (1)$$

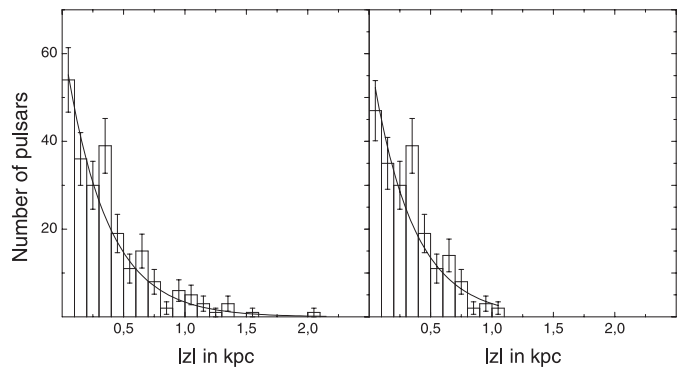
For a distance uncertainty  $\sigma < 0.3$  the multiplier  $\exp(\frac{1}{2}\sigma^2) < 1.05$  and can be neglected in the following analysis. Note that the distances to the OB stars (the pulsar progenitors) are measured with the same accuracy. We also used the method of testing hypotheses (student's  $t$ -test) while comparing two mean values from different samples (Devore 1995).

### 3. Sample statistics

According to the new distance model there are 377 pulsars within 3 kpc cylindrical radius from the Sun, excluding the millisecond pulsars and the pulsars that are members of the globular clusters. As mentioned above a reliable statistical analysis can be made for a luminosity-limited sample with  $L > 10$  mJy kpc<sup>2</sup>, where  $L = S_{400}r^2$ ;  $S_{400}$  is the flux (in mJy) at 400 MHz and  $r$  is the distance in kpc. With this restriction our sample comprises 234 pulsars. To filter the data we use the estimate of the distance uncertainty  $\sigma$ . To estimate the average  $\sigma$  16th and 84th percentiles the dithered distance estimates are used. They define a nominal 68% confidence range in the model distance for each pulsar. These percentiles are given in NE2001 (Lazio 2002).

First let us consider the dependence of the observed  $z = r \sin b$  ( $b$  is a Galactic latitude) on the distance uncertainty  $\sigma$ . Figure 1 shows the relation between  $z$  and  $\sigma$ . The uncertainty increases with increasing  $z$  and for a higher  $z$  the uncertainty greatly exceeds the criterium  $\sigma < 0.3$ . It should be mentioned that according to the new distance model the number of pulsars with  $\sigma \geq 0.3$  is not large; it comprises about 10% of the sample. Some pulsars (about 3% of the sample) located near the Galactic plane also have large uncertainties which must be caused by their location near the regions of irregularities in the free electron distribution.

Let us now consider the distribution of the absolute value of  $z$  for the sample with accurate distances ( $\sigma < 0.3$ ). Figure 2



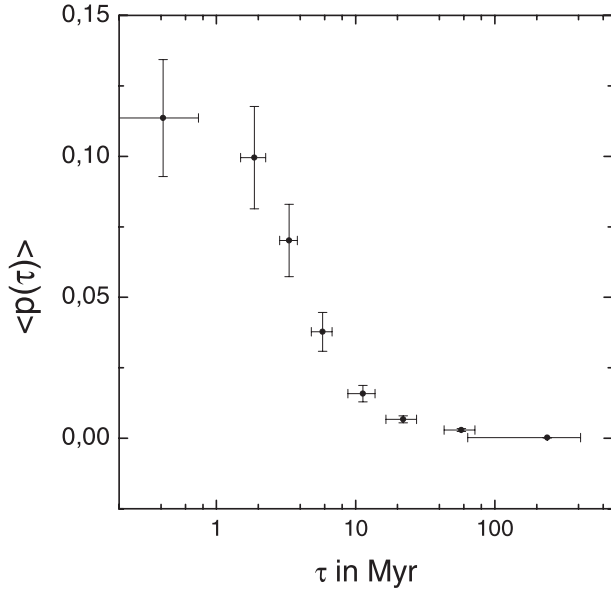
**Fig. 2.** Histograms of the  $|z|$ -distribution of the pulsars for the whole sample (left panel) and the sample with  $\sigma < 0.3$  (right panel).

(the right panel) shows that the distribution decays roughly exponentially, with scale height  $333 \pm 29$  parsecs and a median value of about 270 pc. The whole sample is located inside a layer with a thickness of about 2 kpc from the Galactic plane. The  $z$ -distribution of the whole sample can also be approximated by an exponentially decaying function (left panel in Fig. 2) with scale height  $400 \pm 26$  pc and a median value of about 285 pc. From these results we can argue that the scale height of the observed  $z$ -distribution is about 350 pc which is less than the accepted value of 600 pc (Lyne & Graham Smith 1998).

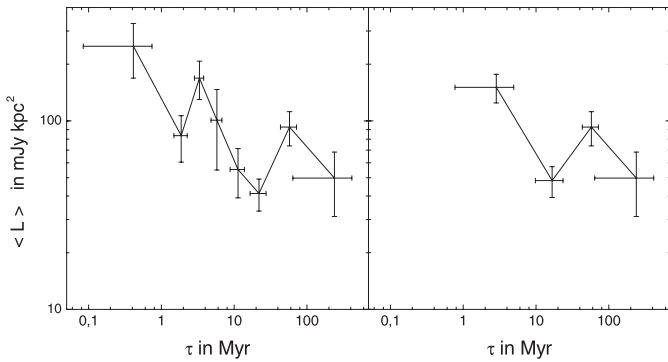
It is interesting to consider the characteristic age distribution of the pulsars from our sample of  $N = 234$  pulsars. To take advantage of the law of large numbers while comparing the mean values of the parameters, we have arranged the sample according to increasing age and then divided it into 8 subgroups, consisting of  $n_i = 30$  pulsars ( $i = 1 \dots 7$ ), except for the last subgroup ( $n_8 = 24$ ). The characteristic age varies in a wide range, from about 1000 year (the Crab pulsar) up to 600 Myr (J1834-0010). In Fig. 3 we show dependence of the relative frequency of pulsars per age interval for the subgroups  $\langle p(\tau_i) \rangle = n_i/N\Delta\tau_i$  vs.  $\log \tau_i$ , where  $\Delta\tau_i$  is the age interval and  $\tau_i$  is the mean age for the  $i$ th subgroup. One effect that may be able to alter the observed distribution is the instrumental bias against the detection of short period pulsars. These missing pulsars are likely to be young, luminous objects, staying near the Galactic plane and therefore particularly prone to the effects of pulse broadening in the interstellar medium (Lyne et al. 1985). Therefore, the observed value of  $\langle p(\tau) \rangle$  for young pulsars will be depressed relative to its unbiased value for the underlying population. So the age distribution for pulsars younger than 10 Myr seems to show an exponential decay.

Let us consider a dependence of the mean luminosity  $\langle L \rangle$  ( $L = S_{400}r^2$ ) on  $\tau$  for the sample. The dependence for the 8 age intervals is given in the left panel of Fig. 4. The error bars mainly overlap, so we need to use methods of testing these hypotheses. To compare the mean values  $\langle L \rangle$  of two different ( $i$ th and  $i + 1$ th) subgroups we use the following statistics

$$Z_{n_i, n_{i+1}} = (\langle L_i \rangle - \langle L_{i+1} \rangle) \left( \frac{S_i^2}{n_i} + \frac{S_{i+1}^2}{n_{i+1}} \right)^{-1/2}, \quad (2)$$



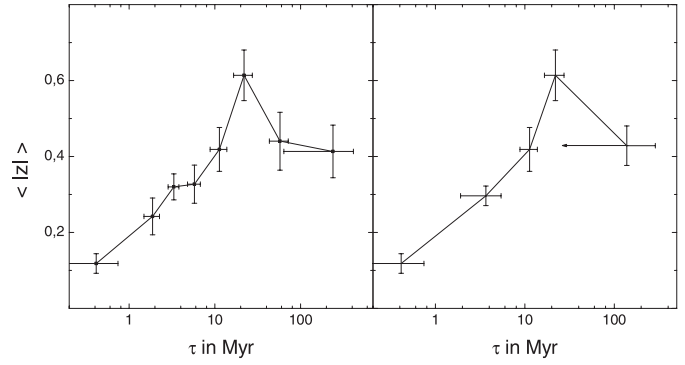
**Fig. 3.** The characteristic age distributions. The error bars represent the standard deviation for  $\langle p(\tau) \rangle$  and  $\tau$  for the corresponding subgroup.



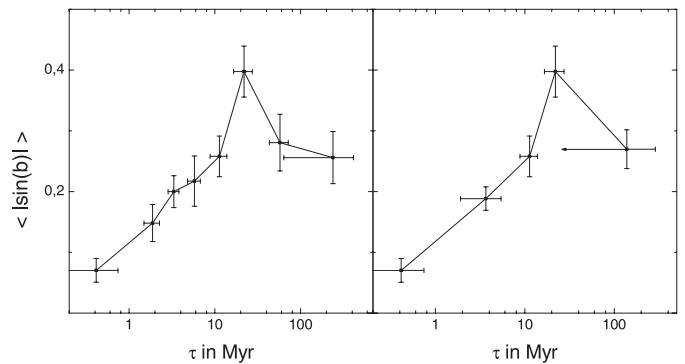
**Fig. 4.** The mean luminosity distribution, before (left panel) and after (right panel) using of the testing hypotheses method.

where  $\langle L_{i,i+1} \rangle$  is the mean value,  $S_{i,i+1}$  is the standard deviation and  $n_{i,i+1}$  is the number of pulsars in the corresponding subgroups. If both subgroups have the same expected means and  $n_i$  and  $n_{i+1}$  both are large numbers, then according to the central limit theorem and the law of large numbers it can be shown, that  $Z_{n_i, n_j}$  has an approximately standard normal distribution (Devore 1995). As a null hypothesis ( $H_0$ ) we accept that two expected means are equal and as for the alternative ( $H_1$ ) hypotheses we use the inequality between the computed means. We chose a 90% confidence level. If the null hypothesis is accepted for two subgroups ( $i$  and  $i+1$ ) we unite the subgroups and repeat the procedure from  $i=1$ . In the opposite case, when the null hypothesis is rejected, we accept the alternative hypothesis and continue the procedure for the second and the successive subgroups ( $i+1$  and  $i+2$ ) and so on. The results of the described procedure for the mean luminosity  $\langle L \rangle$  is shown in the right panel of Fig. 4. As one can see from the right panel of Fig. 4, the dependance of the mean luminosity on  $\tau$  does not show the expected exponential decay.

We used the same treatment for the dependance of  $\langle |z| \rangle$  vs.  $\tau$  for the sample. The results are shown in Fig. 5. It is clear



**Fig. 5.** Dependance of  $\langle |z| \rangle$  and on  $\log \tau$  for the whole sample, before (left panel) and after (right panel) using the testing hypotheses method.



**Fig. 6.** Dependance of  $\langle |\sin b| \rangle$  and on  $\tau$  for the whole sample, before (left panel) and after (right panel) using the testing hypotheses method.

that  $\langle |z| \rangle$  increases with age up to its maximum value which is reached near the age of 21 Myr, and then a significant reduction is observed, which is, in our opinion, evidence of oscillation in  $z$ -direction. One can argue that the obtained evidence of pulsar oscillations in the  $z$ -direction (perpendicular to the Galactic plane) might be caused by the biases in the distance model used. To avoid this argument we propose to use a method of revealing the oscillations that is independent of the distance model.

As known, the observer is located near the Galactic plane. Therefore, a stellar object moving in the  $z$ -direction ( $z = r \sin b$ ) should show a variation of the value of  $|\sin b|$  in accordance with its motion relative to the Galactic plane. The pulsars are apparently born near the Galactic plane with high peculiar velocities and, on the average, move to higher  $z$ -distances during their lifetime. In this case we should expect an increase of the average  $|\sin b|$  of the observed pulsars. If there exists an oscillation it should be seen from the behavior of  $|\sin b|$  with  $\tau$ . So we applied the method of testing hypothesis to find the dependance of  $\langle |\sin b| \rangle$  on  $\tau$ . The results are given in Fig. 6. It is clear that Figs. 5 and 6 show almost the same behavior. Thus we obtain further evidence of the oscillation of pulsars in the  $z$ -direction. It should be noted that the dependance of both  $\langle |z| \rangle$  and  $\langle |\sin b| \rangle$  on  $\tau$  shows the same quarter-period of oscillation which is about 21 Myr, near to the nowadays

accepted value of 25 Myr for the Population I objects (Moffat et al. 1998).

#### 4. Discussion and conclusion

There is a number of suggested mechanisms to explain the pulsar age distribution (see e.g. Lyne & Graham Smith 1998, and references therein). We believe that the simplest possible explanation of such a distribution in statistical terms is the following. Let us assume that characteristic ages correspond to real ages and that the creation rate of pulsars is constant during the whole interval of  $\tau$ . Let us also assume that in each stage of pulsar evolution, before a pulsar reaches the death line in the  $(P, \dot{P})$  diagram, there is a non-zero probability that a pulsar becomes undetectable in a time interval of 1 Myr. Appropriate mechanisms might be: luminosity decay with age (Gunn & Ostriker 1970) or constant luminosity until an abrupt cut-off presumably due to pulse nulling (Phinney & Blandford 1981), surface magnetic field evolution due to the plate movement (Ruderman et al. 1998; Gil et al. 2002), a pulsar beaming dependence on spin period which causes older pulsars to become difficult to detect (Biggs 1990), some unmodelled effects related to the selection criteria in the analysis etc. If this probability does not depend on the age of pulsars the distribution of pulsars in age can be approximated by an exponential decay function. The analysis of our sample shows that the observed distribution of pulsars at characteristic ages can be properly approximated by a third order exponential decay. We have found that the third order in exponential decay was conditioned due to young pulsars being missed because of pulse broadening, and by the existing long tail of older pulsars. We have estimated the fraction of missing pulsars according to the method proposed by Lyne et al. (1985) and found that the approximation of the age distribution can be reduced to a second order exponential decay. So the characteristic age distribution for our sample indicates that we have a composition of two exponential distributions with different scale-heights (i.e. with different “death-rates”).

As is shown in Fig. 4 dependence of  $\langle L \rangle$  on  $\tau$  does not show the same exponential decay as the whole sample. But some luminosity decay is observed in both the first six combined subgroups and the last two combined subgroups. We cannot make any statement on the character of the decay because for both sets of combined subgroups we only have two points of data. It seems that this decay is not exponential as was proposed by Gunn & Ostriker (1970), because in the presence of the field exponential decay the characteristic age varies non-linearly with real (chronological) age and the predicted distribution of  $\tau$  must be non-exponential  $p(\tau) \sim (1 + k\tau)^{-2}$ , where  $k = \text{Const.}$  (Lyne et al. 1985).

In our opinion the observed luminosity and  $z$ -distributions can be explained in terms of two distinct population of pulsars (see Harrison et al. 1993; Arzoumanian et al. 2002, and references therein); one born with low velocities and low magnetic fields, and the other born with high velocities and high magnetic fields.

In this case, if the characteristic ages correspond to true ages we expect that those pulsars that have velocities high enough to escape the gravitational potential of the Galaxy will move away so that they soon become unobservable. The remaining part of the sample should oscillate in the direction perpendicular to the Galactic plane. These pulsars should gradually become undetectable with age due to one of the mechanisms described above. Combination of these effects with with the presumably constant pulsar birth rate in the Galactic plane should give the observed  $z$ -distribution. The maximal separation of pulsars from the Galactic plane is about 2 kpc. But, as is clear from Fig. 2, the distances to the pulsars that have higher  $|z|$  are estimated with very low accuracy, some of them have  $\sigma > 1$ . Therefore, we have actually no information about their real location. They may be located much further and actually be escaping objects.

We think that the determining factor in the revealing of pulsar oscillations must be a proper motion study of older pulsars.

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#### References

- Arzoumanian, Z., Chernoff, D. F., & Cordes, J. M. 2002, ApJ, 568, 289
- Biggs, J. D. 1990, MNRAS, 245, 514
- Binney, J., & Merrifield, M. 1998, Galactic astronomy (Princeton: Princeton University Press)
- Devore, J. L. 1995, Probability and statistics for engineering and the sciences (Duxbury Press)
- Cordes, G. M., & Lazio, T. J. W. 2002 [arXiv:astro-ph/0207156]
- Gil, J. A., Melikidze, G. I., & Mitra, D. 2002, A&A, 388, 235
- Gunn, J. E., & Ostriker, J. P. 1970, ApJ, 160, 979
- Harrison, P. A., Lyne, A. G., & Anderson, B. 1993, MNRAS, 261, 113
- Lazio, T. J. W. 2002, [http://rsd-www.nrl.navy.mil/7213/lazio/ne\\_model/model.html](http://rsd-www.nrl.navy.mil/7213/lazio/ne_model/model.html)
- Lorimer, D. R., Bailes, M., Dewey, R. J., & Harrison, P. A. 1993, MNRAS, 263, 403
- Lyne, A. G., & Graham Smith, F. 1998, Pulsar Astronomy (Cambridge University Press)
- Lyne, A. G., Manchester, R. N., & Taylor, J. H. 1985, MNRAS, 213, 613
- Lyne, A. G., Manchester, R. N., Lorimer, D. R., et al. 1998, MNRAS, 295, 743
- Moffat, A. F. J., Manchester, S. K., Seggewiss, W., et al. 1998, A&A, 331, 949
- Ootr, J. H. 1965, in Stars & Stellar Systems V, Galactic structure, ed. A. Blaauw, & M. Schmidt (Chicago: The Univ. of Chicago press), 455
- Phinney, E. S., & Blandford, R. D. 1981, MNRAS, 194, 137
- Ruderman, M., Zhu, T., & Chen, K. 1998, ApJ, 492, 267