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The Integrated Sachs-Wolfe effect as a probe of non-standard cosmological evolution

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Abstract. The Integrated Sachs-Wolfe effect is studied in non-standard cosmologies. By considering flat universes, it is shown how the quadrupole power can in principle be effectively suppressed by atypical evolution of the scale factor at a high redshift. However, an efficient suppression requires models that are very difficult to realize. The effect on the overall normalization of the CMB power spectrum is also discussed as non-standard cosmologies can affect the overall normalization significantly and enhance the primordial fluctuations. The possibility of constraining such non-standard models with CMB and independent measures of σ_8, is discussed.

Key words. cosmology: theory – cosmology: cosmic microwave background – cosmology: large-scale structure of Universe

1. Introduction

There is mounting evidence that we are living in a universe dominated by a dark energy component, acting as a source of gravitational repulsion causing late-time acceleration of the expansion rate. Early hints came from the classical test of using the magnitude-redshift relationship with galaxies as standard candles (Solheim 1966), but the reality of cosmic acceleration was not taken seriously until the magnitude-redshift relationship was measured recently using high-redshift supernovae type Ia (SNIa) (Riess et al. 1998; Perlmutter et al. 1999). Cosmic acceleration requires a contribution to the energy density with negative pressure, the simplest possibility being a cosmological constant. Independent evidence for a non-standard contribution to the energy budget of the universe comes from e.g. the combination of CMB and large-scale structure: the position of the first peak in the CMB is consistent with the universe having zero spatial curvature, which means that the energy density is equal to the critical density. However, large-scale structure shows that the contribution of standard sources of energy density, whether luminous or dark, is only a fraction of the critical density. Thus, an extra, unknown component is needed to explain the observations (Efstathiou et al. 2002).

The primary anisotropies in the cosmic microwave background (CMB) radiation have their origin in effects in the recombination era when photons and baryons decoupled. In particular, the by now familiar pattern of peaks in the CMB power spectrum is interpreted as acoustic oscillations in the photon-baryon plasma prior to last scattering. In addition to the primary anisotropies, secondary anisotropies may arise as the photons travel from last scattering at a redshift $z \sim 1100$ to us. One such source of secondary anisotropies is CMB photons climbing in and out of evolving gravitational potential wells (Sachs & Wolfe 1967; Rees & Sciama 1968), this is the so-called Integrated Sachs-Wolfe (ISW) effect. In an Einstein-de Sitter universe, the gravitational potential is time-independent and hence there is no ISW effect. In contrast, in the ΛCDM model the gravitational potential will start to decay once the cosmological constant starts dominating the expansion. This will produce an extra contribution to the CMB anisotropies on large angular scales (Crittenden & Turok 1996). Additionally, large-scale anisotropies are caused by gravitational potential wells present at the last-scattering surface, the ordinary Sachs-Wolfe effect (Sachs & Wolfe 1967).

Working in the conformal Newtonian gauge, the perturbed metric can be written as

$$\mathrm{d}s^2 = a^2(\eta)\left[\mathrm{d}\eta^2(1+2\Psi) - (1+2\Phi)\mathrm{d}\boldsymbol{x}^2\right], \quad (1)$$

where η is the conformal time, $\mathrm{d}\eta = \mathrm{d}t/a$, and $\Psi = -\Phi$ in the absence of anisotropic stress, and can be interpreted as the Newtonian gravitational potential. Photons traveling to us from the last scattering surface obey the collisionless Boltzmann equation

$$\frac{\partial}{\partial\eta}(\Theta+\Phi) + n^i\frac{\partial}{\partial x^i}(\Theta+\Psi) = 0, \quad (2)$$

where $\Theta(\eta, \boldsymbol{x}, \boldsymbol{n})$ is the fractional temperature perturbation observed in the direction $\boldsymbol{n}$ on the sky at the conformal time η and position $\boldsymbol{x}$. After a Fourier and a Legendre transform,

$$\Theta(\eta, \boldsymbol{x}, \boldsymbol{n}) = \int \frac{\mathrm{d}^3k}{(2\pi)^3}\Theta(\eta, \boldsymbol{k}, \boldsymbol{n})\mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}} \quad (3)$$

$$\Theta(\eta, \boldsymbol{k}, \boldsymbol{n}) = \sum_{\ell=0}^{\infty}(-\mathrm{i})^{\ell}(2\ell+1)\Theta_{\ell}(\eta, k)P_{\ell}(\mu), \quad (4)$$

where $\mu = \hat{\boldsymbol{k}} \cdot \boldsymbol{n}$, and $P_\ell(x)$ is the Legendre polynomial of degree ℓ, one can show that the solution to the Boltzmann equation on large scales is given by

$$\Theta_\ell(\eta, k) = [\Theta_0(\eta_\mathrm{r}, k) + \Psi(\eta_\mathrm{r}, k)]\, j_\ell(k(\eta_0 - \eta_\mathrm{r})) + \int_{\eta_\mathrm{r}}^{\eta_0} \mathrm{d}\eta \mathrm{e}^{-\tau(\eta)}(\Psi' - \Phi')j_\ell(k(\eta_0 - \eta)), \quad (5)$$

where η_r is the conformal time at recombination, η_0 is the conformal time at the present epoch, τ is the optical depth, and the j_ℓ are the spherical Bessel functions. Primes denote derivatives with respect to conformal time. The CMB power spectrum is given by

$$C_\ell = 4\pi \int \frac{\mathrm{d}^3k}{(2\pi)^3}\langle|\Theta_\ell(\eta_0, k)|^2\rangle = 4\pi \int \frac{\mathrm{d}^3k}{(2\pi)^3}|\,[\Theta_0(\eta_\mathrm{r}, k) + \Psi(\eta_\mathrm{r}, k)]\, j_\ell(k(\eta_0 - \eta_\mathrm{r})) + \int_{\eta_\mathrm{r}}^{\eta_0} \mathrm{d}\eta \mathrm{e}^{-\tau(\eta)}(\Psi' - \Phi')j_\ell(k(\eta_0 - \eta))|^2. \quad (6)$$

The first term corresponds to the Sachs-Wolfe effect, the second term is the contribution from the Integrated Sachs-Wolfe effect.

Since the gravitational potential is related to the matter density through the Poisson equation, one should expect correlations between the ISW effect and the local matter distribution (Crittenden & Turok 1996). Several detections of this effect has been reported in the last year, based on correlating the WMAP measurements of the CMB anisotropies (Bennett et al. 2003; Hinshaw et al. 2003; Kogut et al. 2003; Page et al. 2003; Spergel et al. 2003; Verde et al. 2003) with various tracers of the mass distribution (Boughn & Crittenden 2003; Nolta et al. 2003; Fosalba & Gaztañaga 2003; Fosalba et al. 2003; Scranton et al. 2003; Afshordi et al. 2003). In Boughn & Crittenden (2003), the CMB was cross correlated with the hard X-ray background observed by the HEAO-1 satellite (Boldt 1987) and the NVSS survey of radio galaxies (Condon et al. 1998), and a 2–3σ detection of an ISW signal was reported. Nolta et al. (2003) also investigated the cross-correlation with the NVSS catalogue within the ΛCDM model, and found 2σ evidence for a non-zero cosmological constant. The WMAP data was cross-correlated with the APM Galaxy survey (Maddox et al. 1990) by Fosalba & Gaztañaga (2003), and a 98.8% detection at the largest angular scales was found. Furthermore, in (Fosalba et al. 2003; Scranton et al. 2003) a detection of the ISW signal was reported from the cross-correlation of WMAP with various samples of galaxies from the Sloan Digital Sky Survey (Abazajian et al. 2003). Finally, the cross-correlation with the 2MASS galaxy survey was investigated by Afshordi et al. (2003), resulting in a 2.5σ detection of the ISW signal, consistent with the expected value for the concordance ΛCDM model. These results give important evidence for the presence of a dark energy component in the universe, independent of the SNIa results, but are not at the level of precision where they can accurately pin down the properties of the dark energy.

A feature of the WMAP results (Bennett et al. 2003; Hinshaw et al. 2003; Kogut et al. 2003; Page et al. 2003; Spergel et al. 2003; Verde et al. 2003) which has attracted a lot of attention is the lack of power on large scales. The two-point correlation of the WMAP data shows an almost complete lack of signal on angular scales greater than 60 degrees, and according to Spergel et al. (2003) the probability of finding such a result in the overall best fitting ΛCDM model is about 1.5×10^{-3}. In the power spectrum, this lack of large-scale power is evident in the low value of the quadrupole and, to a lesser extent, of the octopole. This has spurred a great deal of interest, inspiring several authors to introduce exotic physics to explain this feature (Efstathiou 2003a; Contaldi et al. 2003; Cline et al. 2003; Feng & Zhang 2003; DeDeo et al. 2003). In this paper we investigate whether a novel dark energy component can explain the apparent puzzle in the CMB data through its influence on the Integrated Sachs-Wolfe effect. Note, however, that the statistical significance of the apparent lack of large-scale power in the WMAP data has been called into question, and the actual discrepancy with the concordance ΛCDM model may only be at the level of a few percent (Gaztañaga et al. 2003; de Oliveira-Costa 2003; Efstathiou 2004).

2. ISW effect in a flat universe

When no anisotropic stress is present, $\Psi = -\Phi$. For adiabatic, scale-invariant fluctuations, the Sachs-Wolfe contribution is given by $\Theta_0(\eta_\mathrm{r}, k) + \Psi(\eta_\mathrm{r}, k) = \Psi(\eta_\mathrm{r}, k)/3$ with $\Psi^2(\eta_\mathrm{r}, k) \propto k^{-3}$. Furthermore, on scales larger than the sound horizon where the scale and time dependence separate, $\Psi(\eta, k) \approx 5\Psi(\eta_\mathrm{r}, k)f(\eta)/3$, where the function f is defined as

$$f(t) \equiv 1 - \frac{a'(\eta)}{a^3}\int_0^\eta a^2(\eta')\mathrm{d}\eta' = 1 - \frac{\dot{a}}{a^2}\int_0^t \mathrm{d}t'\, a(t'). \quad (7)$$

Then, the power spectrum in a flat universe with a dark energy component that does not fluctuate is approximately described by (Kofman & Starobinsky 1985; Hwang & Vishniac 1990)

$$C_l = \frac{A^2}{100\pi l(l+1)}K_l^2, \quad (8)$$

where

$$K_l^2 = 200l(l+1)\int_0^\infty \frac{\mathrm{d}k}{k}\Big[\frac{1}{10}j_l(k\eta_0) + \int_0^{\eta_0}\mathrm{d}\eta\,\frac{\mathrm{d}f}{\mathrm{d}\eta}j_l\,(k(\eta_0 - \eta))\Big]^2 \equiv 200l(l+1)\widetilde{K}_l^2. \quad (9)$$

Here A is the overall normalization of the primordial fluctuations, j_l is the spherical Bessel function and $\eta = \int_0^t \mathrm{d}t/a(t)$ is the conformal time. In Eq. (9) we have approximated $\eta_\mathrm{r} = 0$, which for large multipoles is a reasonable approximation (Kofman & Starobinsky 1985). We have also ignored any effects arising from a finite optical depth. Looking at Eq. (9) and recalling that $\int_0^\infty \mathrm{d}k j_l(k\eta_0)^2/k = 1/2l(l+1)$, one sees that, if $\mathrm{d}f/\mathrm{d}\eta$ vanishes, K_l is a constant.

Note that the above equations hold only for a dark energy component that does not collapse gravitationally. The question of perturbations of the dark energy is very central when calculating the magnitude of the ISW effect. In dark energy models with $w > -1$, the inclusion of perturbations increase large scale power whereas for $w < -1$ the opposite occurs (Weller & Lewis 2003). The effect for the $w < -1$ models can be dramatic, where for $w = -2$ the inclusion of dark energy perturbations reduces the power by more than a factor of two (Weller & Lewis 2003). One should hence be cautious when calculating the ISW effect in a particular model to make sure that possible perturbations are correctly accounted for.

For a derivation of Eqs. (9), (7), see e.g. Hwang & Vishniac (1990). These equations are valid for perfect fluids, and on scales much larger than the sound horizon. In calculating the ISW effect, we need the derivative with respect to conformal time:

$$\frac{\mathrm{d}f}{\mathrm{d}\eta} = \frac{1}{a^2}\left(3\left(\frac{a'}{a}\right)^2 - \frac{a''}{a}\right)\int_0^\eta a^2\,\mathrm{d}\eta - \frac{a'}{a}, \tag{10}$$

where now $a = a(\eta)$.

From Eq. (10) it is straightforwardly seen that if $f = f_0 =$ const., then

$$a \sim t^{\frac{1}{f_0}-1}. \tag{11}$$

In an EdS universe, $a \sim t^{2/3}$, and hence $f = 3/5$ as can be seen directly from Eq. (7) or from Eq. (11). Therefore there is no ISW effect in a universe that expands as a power-law, e.g. a universe dominated by matter or radiation. A flat ΛCDM universe, with

$$a(t) = a_0\left(\frac{\Omega_\mathrm{M}}{\Omega_\Lambda}\right)^{\frac{1}{3}} \sinh^{\frac{2}{3}}\left(\frac{3}{2}\sqrt{\Omega_\Lambda}H_0 t\right), \tag{12}$$

on the other hand, does not expand as a power law and hence f varies with time leading, to a non-zero ISW effect.

By expanding the square in Eq. (9), we note that $\widetilde{K}_l^2$ is made of three terms: two positive terms and a term whose overall sign is indeterminate and depends on the evolution of the universe. It is precisely this term that allows one, at least in principle, to have less power at large multipoles compared to a ΛCDM universe (the possible significance of the cross term was also noted by Contaldi et al. 2003). Of the three different k-integrals the first one can be readily evaluated,

$$\int_0^\infty \frac{\mathrm{d}k}{k} j_l(k\eta_0)^2 = \frac{1}{2l(l+1)}, \tag{13}$$

whereas the other two are of the form

$$I_l(\eta_1,\eta_2) \equiv \int_0^\infty \frac{\mathrm{d}k}{k} j_l(k(\eta_0-\eta_1))\, j_l(k(\eta_0-\eta_2)) \tag{14}$$

and cannot evaluated as easily. For numerical calculations, it is useful to express Eq. (14) in terms of hypergeometric functions:

$$I_l(\xi) = 2^{2l-1}\xi^l \frac{l!(l-1)!}{(2l+1)!^2}\, F_1\left(-\frac{1}{2}, l, \frac{3}{2}+l, \xi^2\right), \tag{15}$$

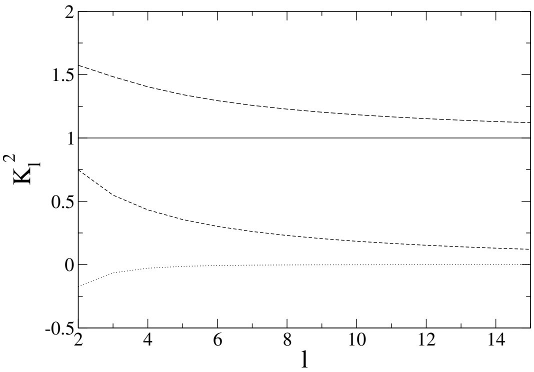

Fig. 1. K_l^2 in the ΛCDM model: the solid curve is the SW-contribution, dotted line is the cross term, dashed line is the square term and the long-dashed line is the sum.

where $\xi \equiv (\eta_0-\eta_1)/(\eta_0-\eta_2) < 1$ (since the integral is symmetric with respect to $\eta_1 \to \eta_2$, $\eta_2 \to \eta_1$, we can always choose ξ to be less than one). This allows us to express $\widetilde{K}_l^2$ in terms of I_l as

$$\begin{aligned}\widetilde{K}_l^2 \;=\; & \frac{1}{200}\frac{1}{l(l+1)} + \frac{1}{5}\int_0^{\eta_0}\mathrm{d}\eta\,\frac{\mathrm{d}f}{\mathrm{d}\eta} I_l(1-\frac{\eta}{\eta_0})\\ & + \int_0^{\eta_0}\mathrm{d}\eta_1\,\frac{\mathrm{d}f}{\mathrm{d}\eta_1}\int_0^{\eta_0}\mathrm{d}\eta_2\,\frac{\mathrm{d}f}{\mathrm{d}\eta_2} I_l(\xi).\end{aligned} \tag{16}$$

In this form the integrals are numerically simple to compute as the numerical approximations to hypergeometric functions are readily available. We have checked that we can successfully reproduce the shape of the large scale multipoles (up to the accuracy of our analytical approximation), in the case where the dark energy component does not fluctuate compared to results from a full CMB code (Weller & Lewis 2003).

Calculating the value of K_l^2 at different multipoles is straightforwardly done using Eq. (16). As an example, we have plotted the different contributions to the low multipoles in a ΛCDM model with $\Omega_\mathrm{M} = 0.25$ in Fig. 1. From the figure we see that the cross term is negligible. In the ΛCDM model it is hence well justified to use the approximation $|\Delta^\mathrm{SW} + \Delta^\mathrm{ISW}|^2 \approx (\Delta^\mathrm{SW})^2 + (\Delta^\mathrm{ISW})^2$ when calculating the ISW effect. Generally this does not have to be true, however, and one can wonder whether the ISW can be responsible for the observed low quadrupole. We will in the following sections demonstrate that this is possible in principle, but that in order to do so unusual expansion histories which are hard to motivate physically are required.

3. Suppressing the large scale power with non-standard evolution

To study how one can reduce the large scale CMB power with the ISW effect we consider a number of different models.

3.1. Jump in f(t)

As a simple starting point for a modified model of the universe, we consider an evolution history such that f undergoes a jump at some redshift:

$$f(\eta) = f_1 + f_0\theta(\eta - \eta_c). \tag{17}$$

Depending on the choice of f_1, we have two possible scenarios: if evolution is standard, $a \sim t^{2/3}$, at early times, $\eta < \eta_c$, then $f_1 = 3/5$. If, on the other hand, we wish to have standard expansion at late times, then $f_1 + f_0 = 3/5$.

Before and after the jump, f is constant and the derivative vanishes. Inserting *Ansatz* (17) into Eq. (16) we find that

$$\widetilde{K}_l^2 = \frac{1}{2l(l+1)}\left(\frac{1}{100} + f_0^2\right) + \frac{1}{5}f_0 I_l(1 - \eta_c/\eta_0). \tag{18}$$

Solving for $\widetilde{K}_l^2 = 0$, we note that physical solutions are only found if the condition

$$l(l+1)I_l(1 - \eta_c/\eta_0) > \frac{1}{2} \tag{19}$$

holds. However, from the definition of I_l it is evident that $I_l(\xi) < 1/2l(l+1)$, $\xi < 1$ and $l(l+1)I_l(1) = 1/2$. Hence, $\widetilde{K}_l^2$ can only be set to zero if $\eta_c = 0$, in which case $I_l(1) = 1/2(l(l+1))$ and $f_0 = -1/10$. But this choice leads to vanishing power at all of the multipoles considered as is obvious from Eq. (9).

Similarly, it is straightforward to see that given a jump at η_c, one can reduce power at a given multipole, l, to a minimum value of

$$^{\min}\widetilde{K}_l^2 = \frac{1}{200l(l+1)} - \frac{1}{50}l(l+1)I_l^2(1 - \eta_c/\eta_0). \tag{20}$$

However, we are interested in suppressing power at a given multipole *relative* to other multipoles. In particular we wish to suppress C_2 with respect to a normalization scale which we choose to be C_{10}. Minimizing K_2^2/K_{10}^2 we find that $f_0 = -1/10$ (this result does not depend on which two multipoles we are considering). Note that choosing $f_0 = -1/10$ implies vanishing power at all multipoles only if $\eta_c = 0$. The expansion rate now depends on whether we are considering early or late time standard expansion. If we have standard matter dominated evolution at early times, $f_1 = 3/5$ and hence at late times $f = 0.5$. This corresponds to $a \sim t$ as can be seen from Eq. (17). If evolution is standard at late times, $f_1 + f_0 = 3/5$ and therefore at early times $f = f_1 = 0.7$, corresponding to $a \sim t^{0.4}$.

The effect of varying η_c is shown in Fig. 2. We see that the earlier the transition happens, the stronger the suppression. In the same figure we have plotted the normalization factor as a function of transition time.

From Fig. 2 it seems that one may be able to suppress the quadrupole by slightly changing the power law expansion from the matter dominated one. However, since we now have evidence of a non-zero cosmological constant, which acts in a different direction, one must include the effects ΛCDM universe.

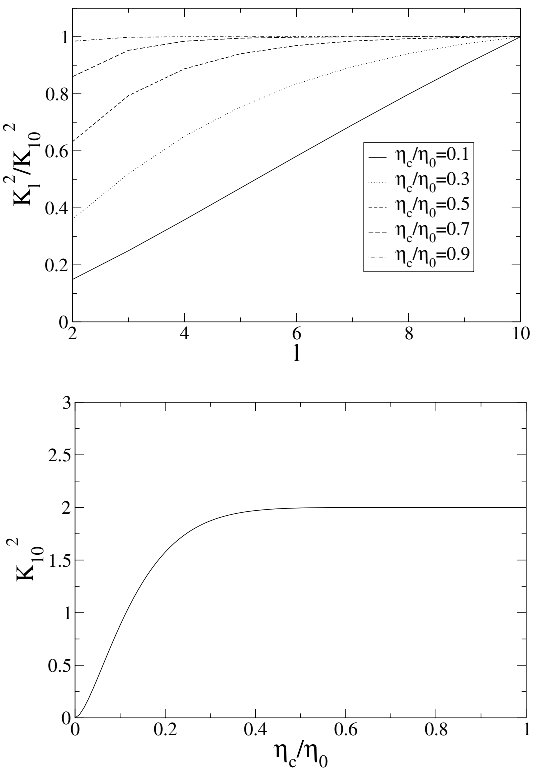

Fig. 2. K_l^2 (*top*) and K_{10}^2 (*bottom*) for different values of η_c/η_0, $f_0 = -0.1$.

3.2. Jump in f(t) in a ΛCDM universe

On might be inclined to believe that a jump in f at a very high redshift is decoupled from the effects due to acceleration at low redshifts in a standard ΛCDM cosmology. However, they are not decoupled but a cross term is present again and hence one cannot extend the conclusions of the previous section to the ΛCDM model.

Let us therefore consider a modification of the ΛCDM cosmology by adding a jump to f at some redshift:

$$f(\eta) = f_\Lambda(\eta) + f_0\theta(\eta - \eta_c), \tag{21}$$

where f_Λ is calculated in a ΛCDM universe with $\Omega_M = 0.25$. Inserting this into Eq. (16), we get

$$\widetilde{K}_l^2 = \widetilde{K}_l^{2,\Lambda} + \frac{f_0^2}{2l(l+1)} + \frac{1}{5}f_0 I_l\left(1 - \frac{\eta}{\eta_c}\right) + 2f_0\int_0^{\eta_0} \mathrm{d}\eta\, \frac{\mathrm{d}f_\Lambda}{\mathrm{d}\eta} I_l\left(\frac{\eta_0 - \eta}{\eta_0 - \eta_c}\right), \tag{22}$$

where $\widetilde{K}_l^{2,\Lambda}$ is the ΛCDM contribution.

We have plotted the quadrupole power relative to the normalization scale in Fig. 3 for different values of f_0 and η_c. From the figure we see that overall it is difficult to suppress the relative quadrupole power by a large amount. The maximum suppression can be found numerically,

$$\frac{K_2^2}{K_{10}^2} \approx 0.64,\ f_0 \approx -0.15,\ \frac{\eta_c}{\eta_0} \approx 0.67. \tag{23}$$

Hence, adding the effects of the ΛCDM universe, makes the suppression more difficult compared to a EdS model

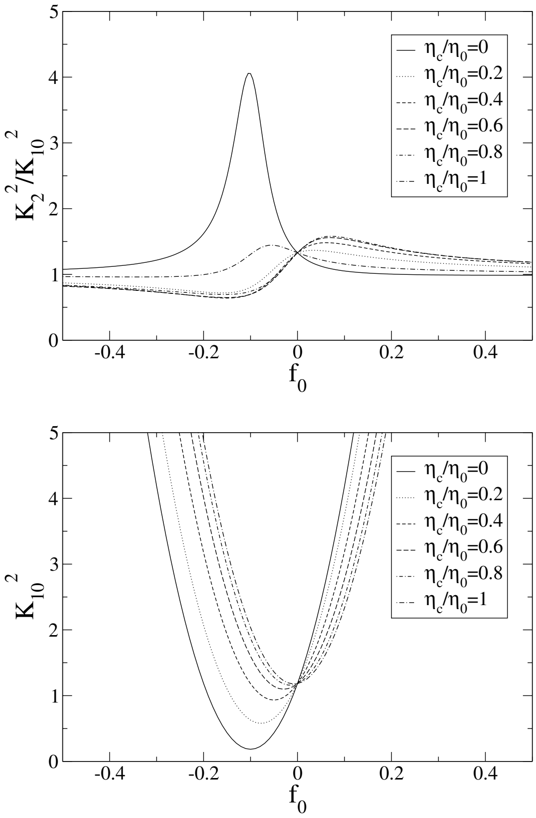

Fig. 3. The relative quadrupole power for different η_c (*top*) and the normalization factor (*bottom*) as a function on f_0.

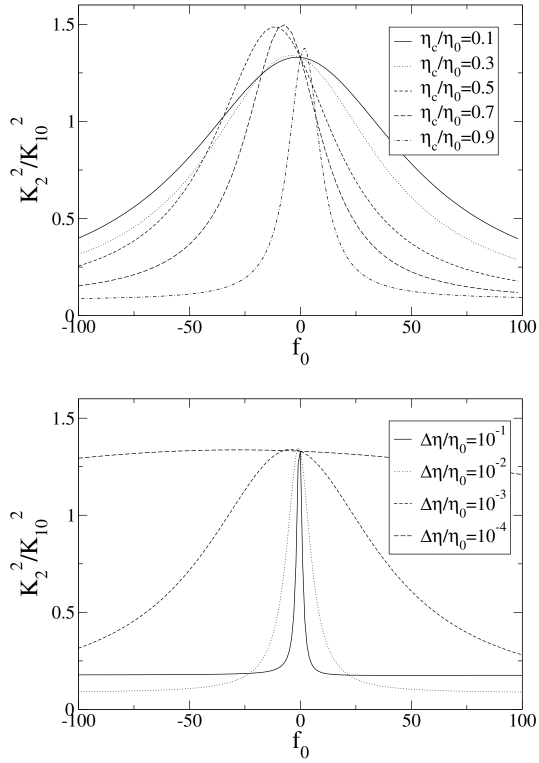

Fig. 4. The relative quadrupole power as a function of f_0 for different values of η_c/η_0 ($\Delta\eta/\eta_0 = 10^{-3}$) (*top*) and $\Delta\eta/\eta_0$ ($\eta_c/\eta_0 = 0.3$) (*bottom*).

with a non-standard period discussed in the previous section. Furthermore, the length of non-standard evolution required is long and the corresponding expansion, $a \sim t^{1.2}$, is significantly different from standard matter dominated behavior.

3.3. ΛCDM with a crunch

As a possibly more realistic model, we study a ΛCDM model where the universe undergoes a period of non-standard growth sometime, but not immediately, after recombination. Such a scenario can be straightforwardly modelled by

$$f(\eta) = f_\Lambda(\eta) + f_0\theta\left(\frac{1}{2}\Delta\eta - |\eta-\eta_c|\right), \tag{24}$$

where $\Delta\eta$ is the duration of the non-standard phase. If the required f_0 is small compared to f_Λ, one can consider physical models that give small modifications to the expansion history by decaying dark matter models etc.

Inserting the *Ansatz* (24) into Eq. (16), it is easy to see that

$$\begin{aligned}\widetilde{K}_l^2 \;=\; & \widetilde{K}_l^{2,\Lambda} + f_0^2\left(\frac{1}{2l(l+1)} - 2I_l\left(\frac{\eta_0-\eta_1}{\eta_0-\eta_2}\right)\right)\\ & +\frac{1}{5}f_0\left(I_l\left(1-\frac{\eta_1}{\eta_c}\right) - I_l\left(1-\frac{\eta_2}{\eta_c}\right)\right)\\ & +2f_0\int_0^{\eta_0}\mathrm{d}\eta\,\frac{\mathrm{d}f_\Lambda}{\mathrm{d}\eta}\left(I_l\left(\frac{\eta_0-\eta_1}{\eta_0-\eta}\right) - I_l\left(\frac{\eta_0-\eta_2}{\eta_0-\eta}\right)\right),\end{aligned} \tag{25}$$

where $\eta_1 \equiv \eta_c - \Delta\eta/2,\ \eta_2 \equiv \eta_c + \Delta\eta/2$.

Now we have three free parameters in the model, f_0 and the time and duration of the period of non-standard growth, η_c and $\Delta\eta$. We have explored the parameter space numerically and the result are shown in Figs. 4 and 5. From Fig. 4 we see that the relative quadrupole power as a function of f_0 has a peaked shape. The position of the peak is determined by η_c whereas $\Delta\eta$ determines the width. Note that these figures indicate that one can reduce the relative quadrupole power quite significantly. Looking at the overall normalization, shown in Fig. 5, we see that as $|f_0|$ is increased the overall power increases rapidly. As a general feature we see that in order to decrease the relative quadrupole power, we need to increase f_0 which in turn increases K_{10}^2 and hence changes the overall CMB normalization.

Given K_{10}^2, one can consider how much the quadrupole can be suppressed in this model. Scanning the parameters we have found that in order to reduce the quadrupole power by half, we typically need to change the overall normalization by a factor of 3−4.

In order to explain the lack of large scale power while not changing the history of the universe too radically, we would like to have a very short period of non-standard growth occurring at a high enough redshift. Choose say, $\Delta\eta/\eta_0 = 0.001$, and approximate the evolution of the universe by ΛCDM for purposes of calculating redshifts. The value of f_0 is set by requiring that K_{10}^2 is not bigger than ∼2. The multipole power for such a model is shown in Fig. 6 for $z_c \sim 10,\ 30$. We see that one can suppress the quadrupole effectively by a short period of non-standard growth. It is also evident that the later this occurs, the larger the effect.

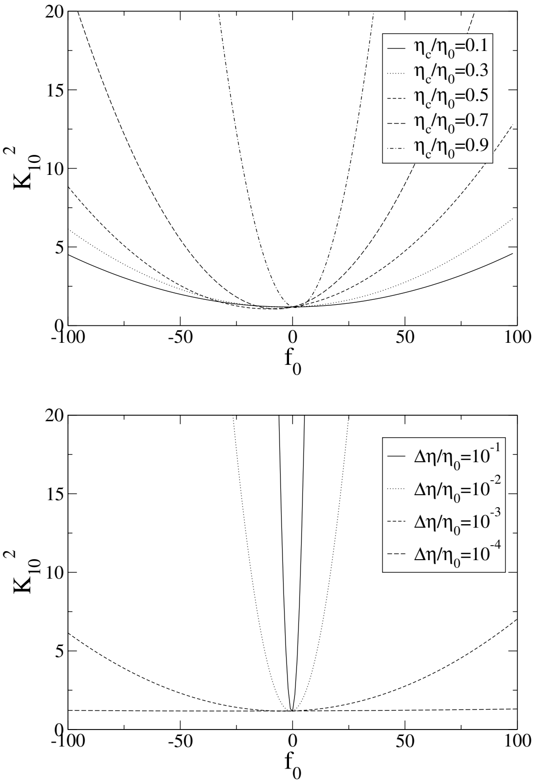

Fig. 5. Normalization as a function of f_0 for different values of η_c/η_0 ($\Delta\eta/\eta_0 = 10^{-3}$) (*top*) and $\Delta\eta/\eta_0$ ($\eta_c/\eta_0 = 0.3$) (*bottom*).

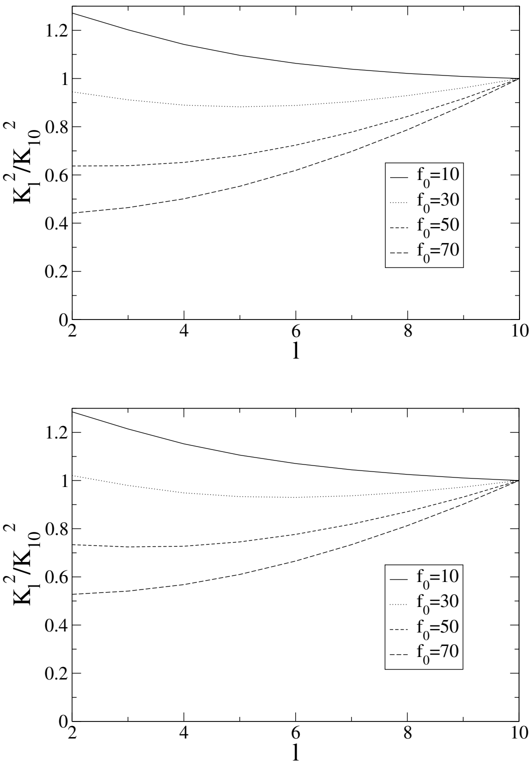

Fig. 6. Relative power at large scales with $z_c \sim 10$ (*top*) and $z_c \sim 30$ (*bottom*).

However, the physical model corresponding to such a universe is very peculiar. From Eq. (11) we see that the required value of f_0 corresponds to a universe which evolves as $a \sim 1/t$ i.e. the universe shrinks! In terms a cosmic fluid this implies that the universe is briefly dominated by a phantom fluid with an equation of state of $p = -\frac{5}{3}\rho$. Such a model seems very contrived and indicates that one needs to modify the cosmological standard model radically in order to explain the low quadrupole by the ISW effect.

3.4. Modified Friedmann equations

As a final example on the possibility of suppressing the relative quadrupole power, we consider models where the Friedmann equation is modified from the ordinary one. Such models can be interesting from the point of view of the ISW effect since linear fluctuations grow differently from in the standard scenario and one can possibly use the ISW effect to constrain such models (Corasaniti et al. 2003). As a particular example, consider the Modified Polytropic Cardassian (MPC) model (Wang et al. 2003), in which the Friedmann equation is modified in such a way that SNIa observations are fit by having universe filled only with matter,

$$H^2 = \frac{8\pi G}{3}\rho_M\left(1+\left(\frac{\rho_M}{\rho_c}\right)^{q(n-1)}\right)^{1/q}, \tag{26}$$

where ρ_M is the energy density of matter, ρ_c is the critical density at which the non-standard terms begin to dominate and $q > 0$, $n < 2/3$ are parameters. The growth of linear perturbations can be radically different from the ΛCDM model in such models (Multamäki et al. 2003). With large q there is a period where linear growth is enhanced compared to the ΛCDM model and hence the MPC-models are potentially interesting for suppressing the quadrupole.

We have calculated the ISW effect for different sets of parameters. Even though at large q the linear growth of fluctuations is larger than in the ΛCDM model, we find that the quadrupole is enhanced for large q. In fact, the quadrupole can only be suppressed at small q. This is demonstrated in Fig. 7, where we have plotted the normalized large scale power for $q = 1,\ 5$, $n = -0.6,\ -0.3,\ 0,\ 0.3,\ 0.6$. The figure is plotted for $\Omega_M = 0.25$. From the figure we can see that for small q, the quadrupole can be suppressed for certain values of n but at large q the quadrupole is more likely to be enhanced.

4. ISW and linear growth

Cosmic variance is obviously a significant hindrance when considering the low order multipoles. Hence, it can be difficult to make observationally significant predictions on the shape of the power spectrum at low l. However, non-standard cosmological evolution does not only have an effect on the shape but also on the overall normalization of the power spectrum due to the ISW effect, which can be a useful tool in differentiating between different models. As we have seen in the previous section, non-standard evolution can, in addition to suppressing the low multipoles, also have a significant effect on the normalization.

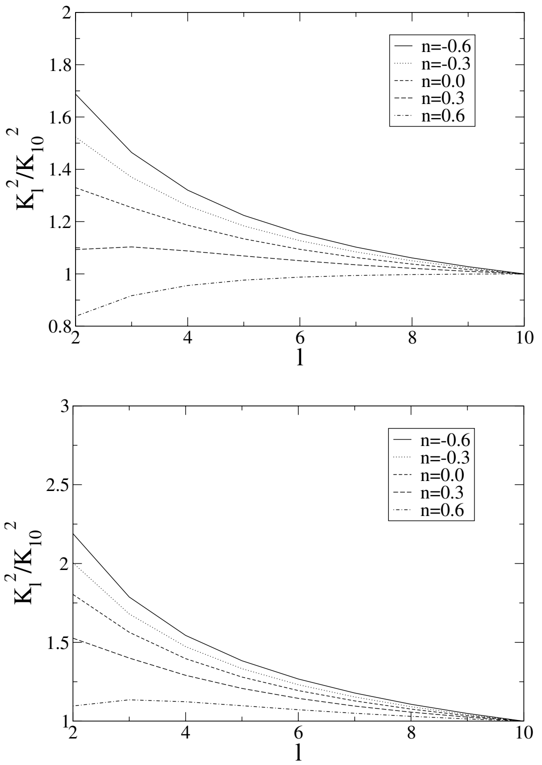

Fig. 7. K_l^2/K_{10}^2 in the MPC model, $q = 1$ (*top*), $q = 5$ (*bottom*).

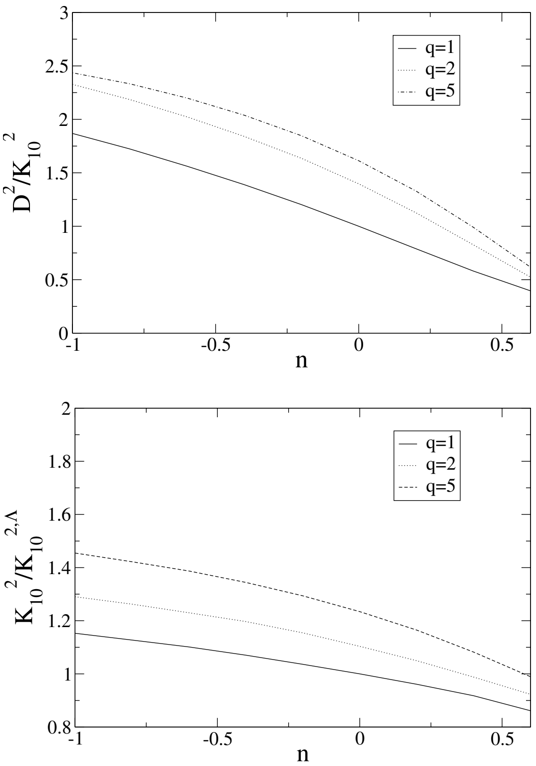

Fig. 8. D/K_l^2 (*top*) and K_{10}^2 (*bottom*) relative to ΛCDM in the MPC model for different values of q as a function of n.

From Eq. (8) it is clear that the power spectrum has two contributions: the amplitude of the primordial fluctuations A and the ISW effect. The CMB observations measure this product and not the amplitude of the primordial fluctuations directly. On the other hand, the amplitude of the primordial fluctuations is also probed by matter fluctuations. A common measure of the matter fluctuations is the current value of rms mass fluctuations in a sphere of radius 8 Mpc/h, σ_8. Given the amplitude of primordial fluctuations and a particular cosmological model, the value of σ_8 can be calculated. WMAP (Spergel et al. 2003) results indicate that $\sigma_8 = 0.84 \pm 0.04$ (best fit model with a running spectral index).

Both probes of primordial fluctuations depend on A in a simple way, $\sigma_8 \propto AD$, $C_l \propto A^2 K_l^2$, respectively where D is the current value of the linear growth factor in a particular cosmological model. The observational quantitities, σ_8 and C_l, are therefore related to the theoretical quantities, D and K_l^2 by

$$\frac{\sigma_8}{l(l+1)C_l} \propto \frac{D^2}{K_l^2}. \tag{27}$$

The evolution of the linear growth factor D in a cosmological model with a general Friedmann equation is determined by (Multamäki et al. 2003)

$$\frac{\mathrm{d}^2 D}{\mathrm{d}\tau^2} + \left(2 + \frac{\dot{\overline{H}}}{\overline{H}^2}\right)\frac{\mathrm{d}D}{\mathrm{d}\tau} + 3c_1 D = 0, \tag{28}$$

where $\tau = \ln(a)$, $\overline{H}$ is the unperturbed Hubble rate and c_1 is determined by the expansion

$$3\frac{1+\delta}{\overline{H}^2}\Big((\dot{H} + H^2) - (\dot{\overline{H}} + \overline{H}^2)\Big) \equiv 3(1+\delta)\sum_{n=1} c_n \delta^n. \tag{29}$$

Here $\delta = \rho/\bar{\rho} - 1$ is the local density contrast and $H = H(\rho)$ is the perturbed Hubble rate.

Equation (28) is straightforwardly solvable numerically for each cosmological model, along with the ISW effect. To illustrate, we have plotted D^2/K_l^2 at $z = 0$ as well as $K_{10}^2/K_{10}^{2,\Lambda}$ for the MPC model for different values of q and n in Fig. 8. Both figures are normalized to ΛCDM values. From the figures it is evident that the additional information from σ_8 helps to discriminate between models.

In order to discriminate between dark energy models, it is hence important to combine CMB observations with the measurements of σ_8 from matter surveys, as pointed out in (Kunz et al. 2003). Measuring the CMB alone cannot tell us what is the actual amplitude of the initial perturbations, but we must combine it with σ_8 to sidestep the issue. Note that since the ISW effect can enhance the initial fluctuations, one can in principle have less initial power from inflation. The ISW effect can therefore be also significant from the point of view of inflationary model building.

As we have seen, the typical situation is where the ISW tends to increase the large-scale CMB power leading to a lower normalization and a lower value for σ_8. If the dark energy clusters on large scales, there is an extra modification to the matter power spectrum for comoving wavenumbers less than

$$k_Q \sim 10^{-3}\sqrt{(1-w)(2+2w-w\Omega_\mathrm{m})}\, h\,\mathrm{Mpc}^{-1}, \tag{30}$$

where w is the effective equation of state parameter (Ma et al. 1999). The main effect is again to lower σ_8 compared to standard ΛCDM. This suggests that it should be possible to use σ_8

to discriminate between dark energy models. Note, however, that several other effects have a similar impact on σ_8. For example, massive neutrinos reduce power on comoving wavenumbers greater than

$$k_{\rm nr} \approx 0.02 \left(\frac{m_\nu}{1\ \rm eV}\right)^{1/2} \Omega_{\rm m}^{1/2}\ h\,{\rm Mpc}^{-1}, \tag{31}$$

where m_ν is the common neutrino mass (i.e. we have assumed three equal-mass neutrinos). As long as a neutrino mass as large as $m_\nu \sim 0.1$ eV cannot be excluded, it is hard to use the clustering amplitude to distinguish between models of dark energy. As an illustration, standard ΛCDM (flat universe with $\Omega_{\rm m} = 0.3$, scale-invariant adiabatic primordial fluctuations) gives $\sigma_8 = 0.93$ after normalizing to COBE. With a constant equation of state $w = -0.7$ for the dark energy component, and the remaining parameters fixed, one gets $\sigma_8 = 0.79$, whereas ΛCDM with a contribution $\Omega_\nu h^2 = 0.005$ from massive neutrionos to the dark matter density (corresponding to $m_\nu = 0.15$ eV) gives $\sigma_8 = 0.82$.

5. Discussion and conclusions

In this work we have studied the importance of the ISW effect in non-standard cosmologies. A possible signature is the observed lack of large scale power in the cosmic microwave background radiation. Our discussion is relevant to flat universes where the dark energy component does not fluctuate, like the ΛCDM model. Extending the discussion to the case where dark energy also fluctuates is left for future work.

In the ΛCDM model, the ISW effect acts to enhance the fluctuations on large scales in such a way that effectively, $|\Delta^{\rm SW} + \Delta^{\rm ISW}|^2 \approx |\Delta^{\rm SW}|^2 + |\Delta^{\rm ISW}|^2$. As we have argued, in the general case this does not have to hold, and the cross term can be significant. To illustrate the point, we have studied different examples of models where the ISW effect can reduce the large scale power. For example, we have seen that in an EdS-universe which undergoes a period of non-standard growth, one can easily suppress the quadrupole as is shown in Fig. 2. Unfortunately, in addition to the fact that the EdS-model is not compatible with the cosmological concordance model, strong suppression also requires that the universe evolves in a non-standard way throughout the most of its history since recombination. If one further assumes that we should recover the ΛCDM behaviour at low redshifts, the situation becomes worse as one cannot then suppress the quadrupole as much.

Considering a more realistic model where there is a brief period of non-standard growth within the ΛCDM model, we have seen that one can reduce the large scale power by a short phase on non-standard growth. However, during such a phase the expansion is very different from a matter dominated universe and in fact the universe needs to contract briefly, $a \sim 1/t$. Physically the universe would have to go through a short phase dominated by "phantom energy" with $p = -\frac{5}{3}\rho$, before going back to ΛCDM expansion. It is hard to think of any realistic physical mechanism for such a behaviour, and the lesson to take away from this exercise is that the explanation for the apparent lack of large-scale power in the CMB is unlikely to be found in an unusual ISW effect.

It is not obvious that one cannot device a model where we can produce the observed multipoles exactly, i.e. one can consider the inverse problem of going from the power spectrum to the evolution of the scale factor. This is most likely an academic question, unless one finds that the evolution is not modified too radically from the standard picture. Again, we leave these questions for further studies.

In addition to possibly alleviating the problems associated with the lack of large scale power, one can potentially also use the ISW to differentiate between different dark energy models in a way that is independent of the amplitude of primordial fluctuations. Combined with CMB independent observations of σ_8, the ISW effect gives constraints on dark energy models. Interestingly, since we only observe the CMB filtered through the ISW effect, one can speculate whether the amplitude primordial fluctuations can be in fact lower than what is typically assumed.

The integrated Sachs-Wolfe effect is a useful tool for cosmology. It probes the whole history of cosmological evolution from recombination until the present time. It can act to enhance, but also reduce, power on large scales. We have shown that the latter case, reduction in large scale power, is possible in principle, but such a scenario is physically difficult to realize and is unlikely to occur for realistic expansion histories.

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