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Magellanic Cloud Cepheids: Pulsational and evolutionary modelling vs. observations

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Abstract. The pulsational properties of the Cepheid models along the evolutionary tracks from the Padova group (Girardi et al.), as calculated with our turbulent convective pulsation code, are in good agreement with the resonance constraints imposed by the observational OGLE-2 data of the Small and Large Magellanic Clouds. Our study suggests that the $P_4/P_1 = 1/2$ resonance for the overtone Cepheids occurs for periods clustering around 4.2 d, in disagreement with the suggestion of Antonello & Poretti based on the observations of light curves, but in agreement with Kienzle et al. and Feuchtinger et al. For the fundamental Cepheids the lowest order Fourier decomposition coefficients from the light curves, viz. R_{21} and ϕ_{21} can be used to locate the resonance region, but not so for the first overtone Cepheids. Here, the radial velocity curves can be used to locate the overtone resonance region, or in their absence, one needs to resort to numerical hydrodynamic modelling.

Key words. stars: oscillations – stars: variables: Cepheids – galaxies: Magellanic Clouds – stars: distances – stars: evolution

1. Introduction

Cepheids are perhaps the best observed variable stars. They are very interesting heat engines from a physical point of view, and it is hardly necessary to stress their importance as cosmological distance indicators. Obviously, a good theoretical understanding of these important stars is essential, and they have received the most theoretical attention among the variable stars, going as far back as a half-century.

It was first noted by Hertzsprung (1926) that the shape of the light curves (LC) of the Galactic fundamental (F) mode Cepheids exhibit a progression with pulsation period in the vicinity of 10 days. Subsequently Payne-Gaposhkin (1947) found concomitant sharp variations in the Fourier decomposition coefficients of the LCs. Later, Simon & Schmidt (1976) noted a correlation of the location of the bump on the LC with the location of a resonance between the excited fundamental pulsation mode and the second overtone ($P_2/P_0 = 1/2$).

Intrigued by these features Buchler & Goupil (1984) developed the mathematical "amplitude equation" formalism that is necessary to understand how the occurrence of an internal resonance can produce, both qualitatively and quantitatively, a progression of the Fourier coefficients of the LCs and of the radial velocity (V_r) curves (see also Buchler 1993; Buchler & Kovács 1986). The amplitude equations for a given star yield the modal amplitudes as a function of the stellar parameters, such as the period or $T_{\rm eff}$ for example, and these in turn are related to the Fourier coefficients of the LC.

Klapp et al. (1985) and Kovács & Buchler (1989) applied this formalism to the specific case of the F Cepheid progression. A comparison of a number of sequences of full amplitude hydrodynamic Cepheid models in Figs. 3, 5 and 7 of Buchler et al. (1990) shows very clearly that the structure of the (magnitude) Fourier coefficients $\phi^{\rm m}_{21} = \phi^{\rm m}_2 - 2\phi^{\rm m}_1$ and $R^{\rm m}_{12} = A_2/A_1$ correlates very well with the period ratio $P_{\rm r} = P_2/P_0$, which is indicative of the resonance, rather that with the period P_0 itself. (We note that it is because of the finite width of the IS that stars with different masses and luminosities, and thus different periods can have the same $P_{\rm r}$ as Fig. 1 below will show. A corollary is that there is no real Cepheid resonance center, but rather a resonance region.) For the observed Cepheids we know the periods, but we do not have any direct information on the period ratios $P_{\rm r}$. We therefore try to take advantage of the behavior of the Fourier coefficients to localize the resonance region.

The Fourier coefficients $\phi^{\rm v}_{21}$ and $R^{\rm v}_{21}$ of the observed $V_{\rm r}$ curves of Galactic F Cepheids also show structure in the vicinity of a 10 d period (e.g., Kovács et al. 1990), but the structure is different from that of the LCs. This is to be expected because the luminosity $L \sim R^2T^4$ involves not just the radial displacement eigenvectors but also the temperature

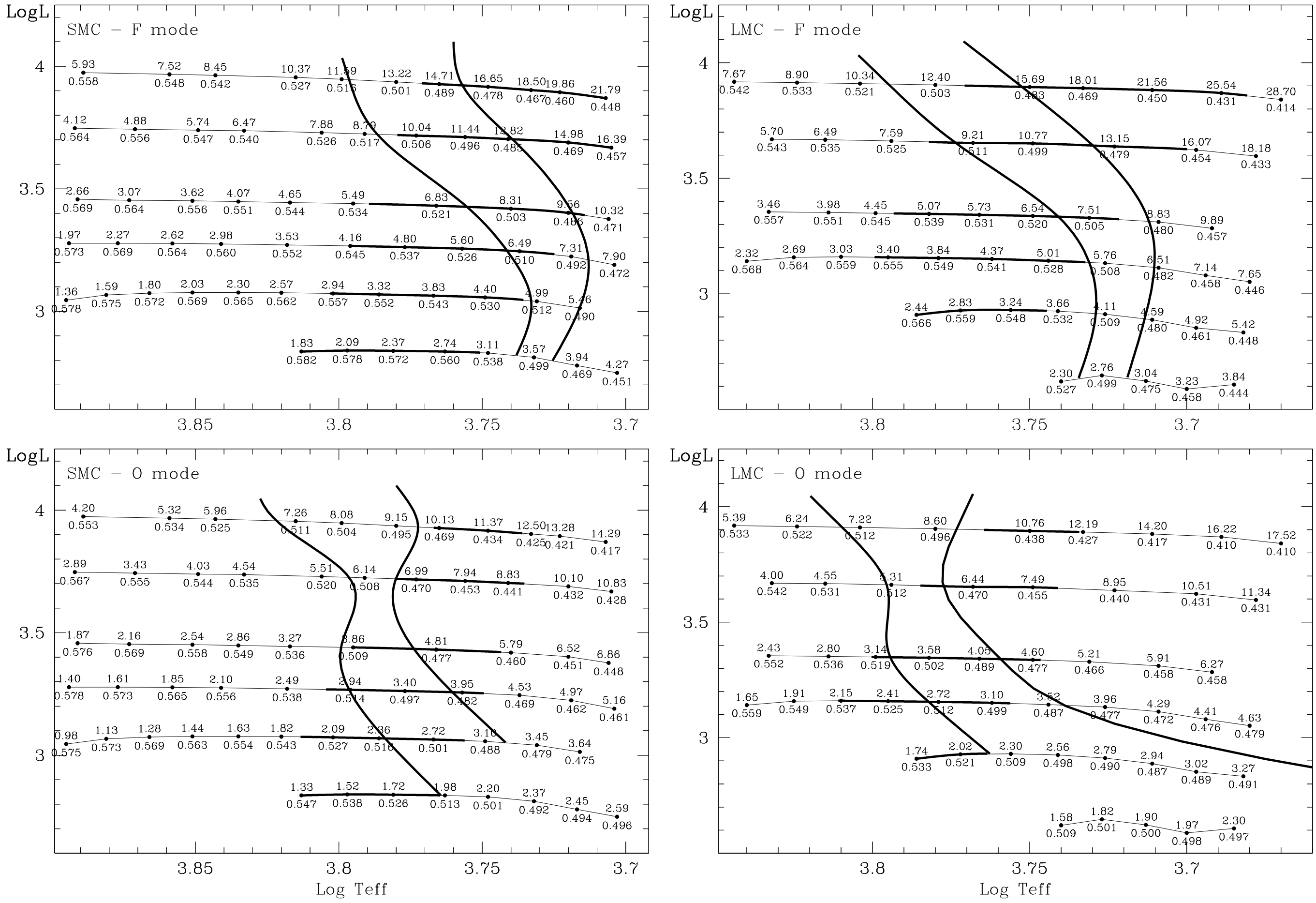

Fig. 1. F and O1 Cepheid models in SMC and LMC: sections of the 3.5, 4.0, 4.5, 5.0, 6.0 and 7.0 $M_\odot$ Padova (Z = 0.004 and 0.008) tracks corresponding to the third crossing of the IS; the periods [d] are noted above the tracks, the period ratios P_r below; the *thickened sections* of the tracks represent the linear IS (linearly unstable models); the pairs of thick *vertical curves* delimit the resonance regions, $P_\mathrm{r} = 0.5 \pm 0.015$.

eigenvectors. The coefficients in the transformation from the modal amplitudes to LCs and to V_r curves are therefore different as explained in Kovács & Buchler (1989).

Thus, for example, in the F Cepheids the sharp feature in the magnitude phases ϕ_{21}^m occurs close to 10 d, and R_{21}^m has a minimum there (cf. e.g., Fig. 6 of Moskalik et al. 1992). The corresponding V_r phase ϕ_{21}^v has a rapid, but not sharp variation from 5 to 15 d, while the minimum for R_{21}^v is less pronounced and appears at a somewhat larger period of ~12 d. Either one of the two coefficients, ϕ_{21}^m or R_{21}^m, can therefore serve as a criterion for locating the resonance, but ϕ_{21}^v and R_{21}^v are less useful discriminants.

We now turn to the O1 Cepheids. Antonello & Poretti (1986) suggested evidence for an internal resonance of the excited first overtone with the fourth overtone $P_\mathrm{r} = P_4/P_1 \sim 1/2$ in the vicinity of $P_1 \sim 3.2$ d, on the basis of the sharp variations of ϕ_{21}^m of the Galactic LC Fourier coefficients. Linear models of O1 Cepheids indeed display such a resonance in that general vicinity (Antonello 1994; Buchler et al. 1996), the exact location depending however on the $M-L$ relation that is used. The first results of nonlinear radiative model did not correctly reproduce the variation of the Fourier coefficients (Antonello & Aikawa 1995). In contrast, on the basis of the V_r Fourier coefficients Kienzle, Moskalik et al. (1999), instead, put the resonance near 4.6 d. The reason for this discrepant conclusion is that the Fourier coefficients, especially ϕ_{21}^m have a substantially different behavior for the LCs and the V_r curves. See e.g., Feuchtinger et al. (2000, hereafter FBK) for a juxtaposition.

In order to resolve this discrepancy it was necessary to make a detailed numerical hydrodynamical survey of full amplitude pulsations (FBK). The code contains a time-dependent mixing length model for convection because purely radiative models had failed to reproduce the observed features of the Fourier coefficients, in particular near 3.2 d. This survey clearly put the resonance near 4.2 d. But it also showed that only the broad maximum of ϕ_{21}^v correlates with the resonance.

The conclusion is that while it is true that sharp features in the Fourier coefficients generally indicate the presence of internal resonances, there is *no simple* a priori criterion for locating precisely the resonance region from either the LC or from the V_r data. Numerical hydrodynamic modelling of full amplitude pulsations is needed to determine the criterion to be used and the location of the resonance.

The Magellanic Cloud F Cepheid LCs appear very similar to those of the Galactic ones in terms of their Fourier coefficients (e.g., Beaulieu et al. 1995; Beaulieu & Sasselov 1997). What matters in particular for this paper is that the resonant features are very similar. The observed LC features of the LMC and SMC F Cepheids occur in the same place as for the Galactic ones. A systematic and comparative hydrodynamic full amplitude survey of Galactic and MC F Cepheids is still lacking, and in its absence we make the reasonable assumption that the Fourier structure as a function of P_r is the same.

There are some small differences though for the overtones. Despite a fair amount of scatter, it would appear that the prominent sharp features, namely the minimum in R_{12}^{m} occurs near 2.7 and 2.4 d for the LMC and SMC O1 Cepheids respectively (Beaulieu et al. 1995; Beaulieu & Sasselov 1997; and the larger OGLE sample by Udalski et al. 1999a,b), whereas they occur closer to 3.2 d for the Galactic O1 Cepheids (Antonello & Poretti 1986). There is a similar small shift in the sharp drop in ϕ_{21}^{m} to lower periods. In the absence of a numerical hydrodynamic survey similar to that of FBK, we will therefore assume on the basis of these shifts in R_{21}^{m} that there are probably corresponding shifts of −0.5 and −0.7 d of the resonance period for the LMC and SMC O1 Cepheids.

In a previous attempt to use the resonances to obtain constraints on the M and L of the Cepheid models Buchler et al. (1996; see also Simon & Kanbur 1994; Aikawa & Antonello 2000) made the assumption that the resonance boundaries, for example for F Cepheids, were defined for the resonance center, viz. $P_2/P_0 = 1/2$, at $P_0 = 11$ d at the blue edge, and 9 d at the red edge of the IS. This led to their suggestion that the derived masses for the SMC and LMC were too small. The current paper reexamines this assumption. Figure 1 and the discussion below show that instead one should define the resonance boundaries to be $P_r = P_2/P_0 = 0.5 + \Delta$ at the blue edge and $P_r = 0.5 - \Delta$ at the red edge, with $\Delta = 0.015$ for example, both for $P_0 = 10$ d and $P_1 = 4.2$ d for O1 Cepheids). If one repeats their procedure with these new conditions one finds that the sensitivity to the exact values of the resonance boundaries (P_r) is too great to constrain M and L.

Hence we are led here to an alternate comparison, still involving the resonances, that is based on a combination of stellar evolution tracks and pulsation properties. Other studies (Baraffe et al. 1998; Alibert et al. 1999; Bono et al. 2000a,b, 2001) usually concentrate on the comparison of the models with the observations in the period – luminosity planes in different bands and ignore the strong constraints on dynamics coming from the resonances. The main purpose of this paper is to check how well the models satisfy these resonance constraints when confronted with the observational OGLE-2 Magellanic Cloud (MC) data.

1.1. Models on the tracks of Girardi et al.

In Fig. 1 we display sections of the evolutionary tracks of Girardi et al. (2000) for $Z = 0.004$ and for $Z = 0.008$ corresponding to the third crossing of the instability strip (IS). They are meant to be approximately representative of the average properties of the SMC and LMC Cepheids. We do not show the second crossings of the IS here because they will give similar results.

We note the well known fact that the blueward extents of the low mass evolutionary tracks, e.g., for 3.5 $M_\odot$ are too short, and for the LMC the 3.0 $M_\odot$ track does not even penetrate the IS. This problem is not specific to the Padova tracks, but is encountered by all recent evolution calculations. See Cordier et al. (2003) for a recent discussion of this problem with evolutionary tracks and a possible explanation. However the problem occurs outside both the F and the O1 Cepheid resonance regions that we are concerned with here.

As far as pulsation modelling is concerned the Padova evolutionary models provide us with a mass M, a luminosity L, and effective temperature $T_{\rm eff}$. For a given composition (specified by X and Z) these three quantities *uniquely specify the stellar envelope* i.e., without a need to know the properties of the stellar interior. (One could equivalently, but much less conveniently characterize the Cepheid envelope instead by the luminosity, temperature and pressure at the core radius, for example). We note that the envelope composition is uniform having recently undergone a fully convective stage. The problems associated with core convection and overshooting (e.g., Cordier et al. 2003) thus need not concern us here because pulsation is limited to the part of the star (the envelope) that is located above the burning shells (i.e., $T <$ a few million K).

The equilibrium Cepheid models and their *linear* vibrational stability properties are computed with our turbulent convective, TC code (e.g., Yecko et al. 1998; Kolláth et al. 2002) for the model sequence along the Padova tracks.

One might think that there is a remaining inconsistency because Girardi et al. (2000) use standard (time-independent) mixing length in the envelope, whereas we have additional (α) parameters associated with our time-dependent mixing length model, which also contains a turbulent flux and a turbulent pressure. The structure of the envelope turns out to be relatively insensitive to these differences, as are the pulsation periods of the lowest modes. In contrast, as one may expect, the modal stability is more sensitive to the α's. We can therefore safely adopt the M, L and $T_{\rm eff}$ given by Girardi et al. and expect our envelopes to be very close to their evolutionary ones.

The top plots in Fig. 1 show the F Cepheid models and the bottom plots the O1 Cepheid models. The numbers above the tracks represent the linear pulsation periods, P_0 and P_1 [d], respectively. (We recall here that the nonlinear and linear pulsation periods are known to differ only by a few tenths of a percent.)

The numbers below the tracks refer to the (linear) period ratios $P_r = P_2/P_0$ and $P_r = P_4/P_1$, respectively, in days. The pairs of thick vertical curves delineate the boundaries of the resonance regions which we define as $0.485 < P_r < 0.515$.

The thickened portions of the tracks in Fig. 1 represent the linear IS, i.e., the region where the F or O1 Cepheid model, resp., are linearly unstable. The actual IS is a little narrower because nonlinear effects are known to shift the F blue edge a little lower in temperature, 0−200 K, depending on M, and the O1 red edge a few hundred K to higher temperatures (e.g., Fig. 2 of Buchler 2000). In addition, it is well known that there is a region where either F or O1 pulsation is possible depending on the direction of the evolutionary track (e.g., Kolláth et al. 2002).

In Fig. 1 the resonant F Cepheids therefore should fall in the *resonance regions* defined by the resonance boundaries,

$P_2/P_0 = 0.485$ and $P_2/P_0 = 0.515$ (vertical curves) and the boundaries of the IS. (The pulsationally unstable models marked by the thickened sections of the tracks.) These are the approximate values for which the (cos) Fourier coefficients $R^{\rm m}_{21}$ and $\phi^{\rm m}_{21}$ have an egregious behavior, viz. the $R^{\rm m}_{21}$ have a dip, and the phases $\phi^{\rm m}_{21} \gtrsim 5.5$ or $\phi^{\rm m}_{21} \lesssim 2.0$.

We see that for the resonant Cepheids, independently of Z and of the pulsation mode, the "NW" corner on the *blue edge* of the IS, with the highest L and M has the highest $P_{\rm r}$. For example for the SMC F Cepheids one finds $M \sim 7.3\ M_\odot$, with a period ($P_0 \sim 16.5$ d). Conversely, the "SE" corner of the *red edge* of the resonant IS, with the lowest L and M has the lowest $P_{\rm r}$. For the SMC F Cepheids $M \sim 4\ M_\odot$ with $P_0 \sim 5$. The resonant F Cepheid models along the $Z = 0.004$ Padova tracks therefore have periods ranging from ~5 to 16.5 d. A similar topography is seen to obtain for the LMC Cepheids.

From the periods that are indicated in Fig. 1 one can readily infer that constant period curves run NW to SE, albeit with a slope that is shallower than that of the constant $P_{\rm r}$ curves. If instead the resonance regions were defined by the edges of the IS and the edges of the constant period curves, they would be reasonably close to the resonance regions that we have just defined on the basis of $P_{\rm r}$, as one would expect from the correlation of $R^{\rm m}_{21}$ with the period in the OGLE data, for example.

A similar situation occurs for the overtone Cepheids where the resonant O1 Cepheid models along the $Z = 0.004$ Padova tracks have periods ranging from ~2.0 to 6.5 d.

The constant $P_{\rm r}$ curves that demarcate the resonant O1 models show a pinch near 6 $M_\odot$. Interestingly, this is not a numerical artifact, but can be traced to the nonmonotone behavior of the the period P_4 of the fourth overtone, because of the occurrence of a strange mode (Buchler & Kolláth 2001, e.g., Fig. 2; Buchler et al. 1997).

For the $Z = 0.008$ Padova tracks (third crossing of the IS) that are approximately representative of the LMC Cepheids, we obtain a very similar overall picture. In particular, we find approximately the same period ranges for the resonant F Cepheid models ($P_0 \sim 5.5$ to 16 d) and for the resonant O1 Cepheid models ($P_1 \sim 2.2$ to 7.2 d). Note that a similar pinch in the resonance curves also occurs for the $Z = 0.008$ tracks.

1.2. Comparison with the OGLE-2 Cepheids

1.2.1. Instability strip: Model stability

Beaulieu et al. (2001, hereafter BBK) analyzed the SMC and LMC OGLE Cepheids (OGLE-2 web database) with the intent of extracting $M - L$ relations from the observational data. They first converted the observed magnitudes and colors into L and $T_{\rm eff}$ with assumptions about reddening and distance moduli. We shall refer to these observationally derived L, $T_{\rm eff}$ and periods as defining the *OGLE stars*. From these OGLE star data BBK then derived stellar masses M as well as linear stability properties with the help of a linear pulsation code. In the following we shall refer to these as the *OGLE models* when there is a need to distinguish between them.

We recall that BBK had concluded that observational luminosity and reddening uncertainties cause a small spread in $T_{\rm eff}$ that is difficult to disentangle from the spread in $T_{\rm eff}$ due to the width of the IS. The derived M, L and $T_{\rm eff}$ values for the OGLE models thus contain small errors. The two consequences are that (1) in an $M-L$ diagram one obtains a swarm rather than a narrow strip as one would expect from a relatively homogeneous group of Cepheids, and that (2) in an $T_{\rm eff} - L$ plot some stars necessarily fall outside the actual IS.

Figures 2 and 3 show theoretical HR diagrams (Log L vs. Log $T_{\rm eff}$) in which we redisplay the OGLE F Cepheid stars for the BBK's (preferred) choice B of distance modulus, namely 18.55 ± 0.10 for the LMC and 19.97 ± 0.15 for the SMC, and mean reddenings of $E(B-V) = 0.1$ and 0.08, respectively. These figures are identical except for the resonance criteria to be addressed below. In these figures we now represent the linearly stable/unstable OGLE models with open circles/dots. Despite the above mentioned observational uncertainties in the derived M, L and $T_{\rm eff}$ we see that the majority of the OGLE Cepheid models are unstable, as they should be. There appears to be some small systematic discrepancy, however, for the low luminosity F Cepheids on the red side, both in the LMC and SMC that goes beyond the uncertainties in the model parameters. Considering that the linear growth rates are the least certain of the calculated pulsation quantities, because they depend on the α parameters that are used in the time-dependent mixing length equations we are not too concerned because a small adjustment of the α parameters in the convective terms could fix this problem. We stress again that, in contrast, the periods are largely independent of the α s.

In Figs. 4 and 5 we reproduce the OGLE O1 Cepheids. One notes that the vast majority of the OGLE models are linearly unstable, and that the region occupied by the unstable evolutionary models coincides well with that of the unstable OGLE models.

Overall, the figures thus indicate good agreement between the tracks, the pulsation calculations and the observations as far as the stability of the models is concerned.

1.2.2. Resonance conditions

We now go on to examine how well the resonance information along the evolutionary tracks agrees with that of the Fourier coefficients of the OGLE LCs. We start with the F Cepheids, both in SMC and LMC.

Fundamental mode Cepheids:

As we have already discussed in the Introduction, for the observed F Cepheids we know the periods, but we do not have any direct information on the *period ratios* $P_{\rm r}$. However, near $P_{\rm r} \sim 0.50$ the period ratio correlates with the dip of the LC's Fourier amplitude ratio $R^{\rm m}_{21}$. It also correlates well with the egregious values of the phases $\phi^{\rm m}_{21}$ in the resonance region, viz. $\lesssim$2.0 and $\phi^{\rm m}_{21} \gtrsim 5.3$. The phases are normalized to $[0, 2\pi]$, mod 2π.

In Figs. 2 and 3 we display $T_{\rm eff} - L$ plots for the F Cepheids. The OGLE F Cepheids are represented by dots for the linearly unstable models, and by open circles for the stable ones. Superposed are the Padova tracks where again the thickened sections represent the linearly unstable models (as in Fig. 1),

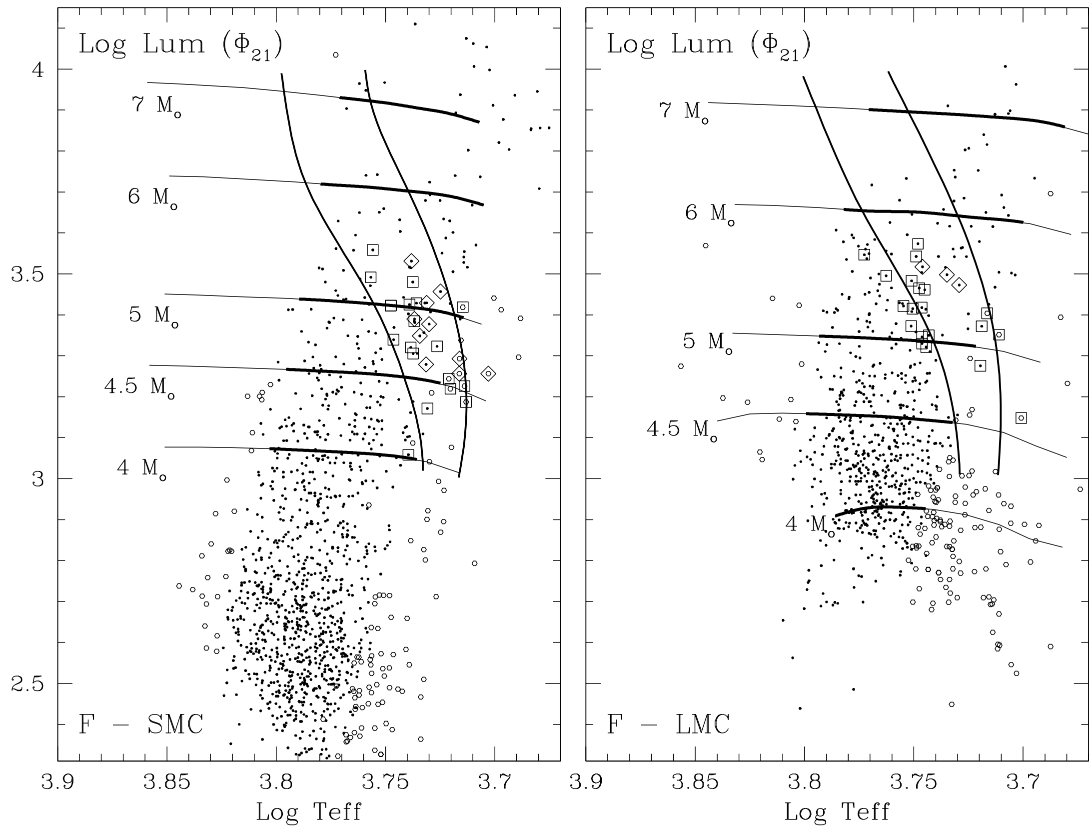

Fig. 2. MC F Cepheids – ϕ_{21}^{m} resonance criterion: *dots (open circles)* represent linearly stable (unstable) OGLE models; *squares*: $\phi_{21}^{m} > 5.45$ (and $P_0 < 20$), *tilted squares*: $0 < \phi_{21}^{m} < 2$. The resonant Cepheid models lie between the vertical curves and the envelopes of the thickened tracks (instability strip).

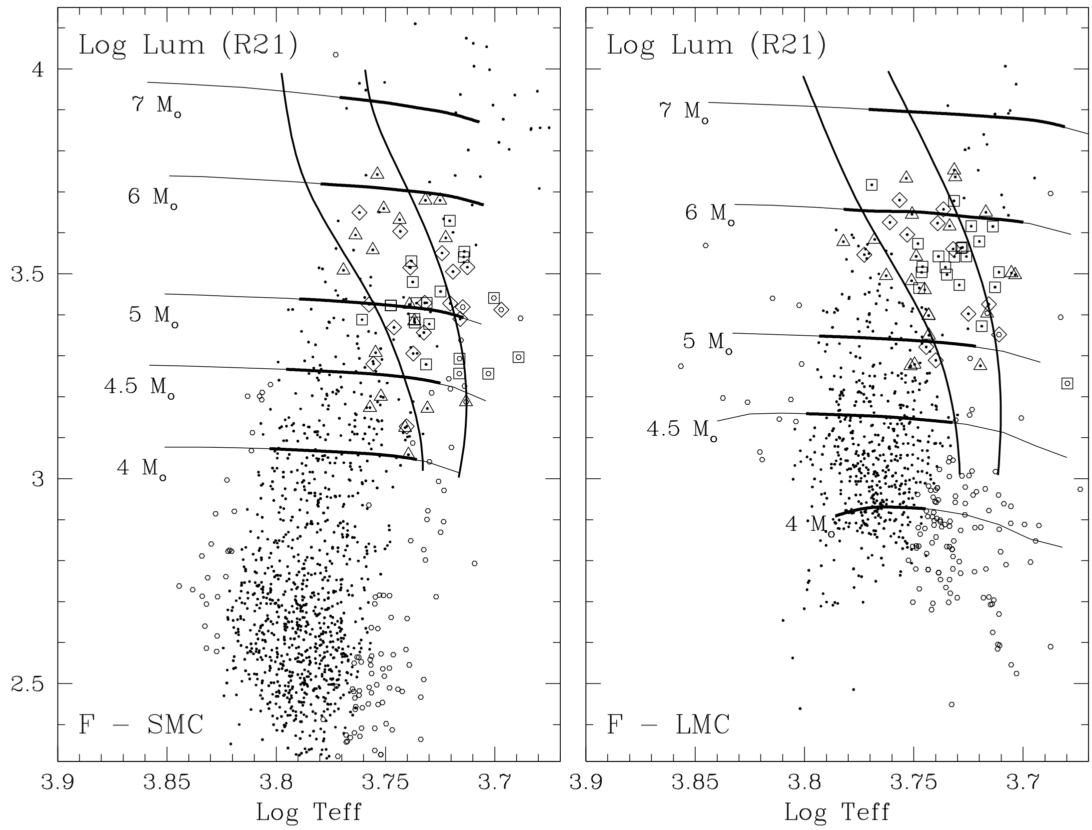

Fig. 3. MC F Cepheids – R_{21}^{m} resonance criterion: *dots (open circles)* represent linearly stable (unstable) OGLE models; *squares*: $R_{21}^{m} < 0.15$ (and $P_0 < 16$), *tilted squares*: $0.15 < R_{21}^{m} < 0.2$ (and $P_0 < 16$), *triangles*: $0.2 < R_{21}^{m} < 0.25$ (and $6 < P_0 < 16$). The resonant Cepheid models lie between the vertical curves and the envelopes of the thickened tracks (instability strip).

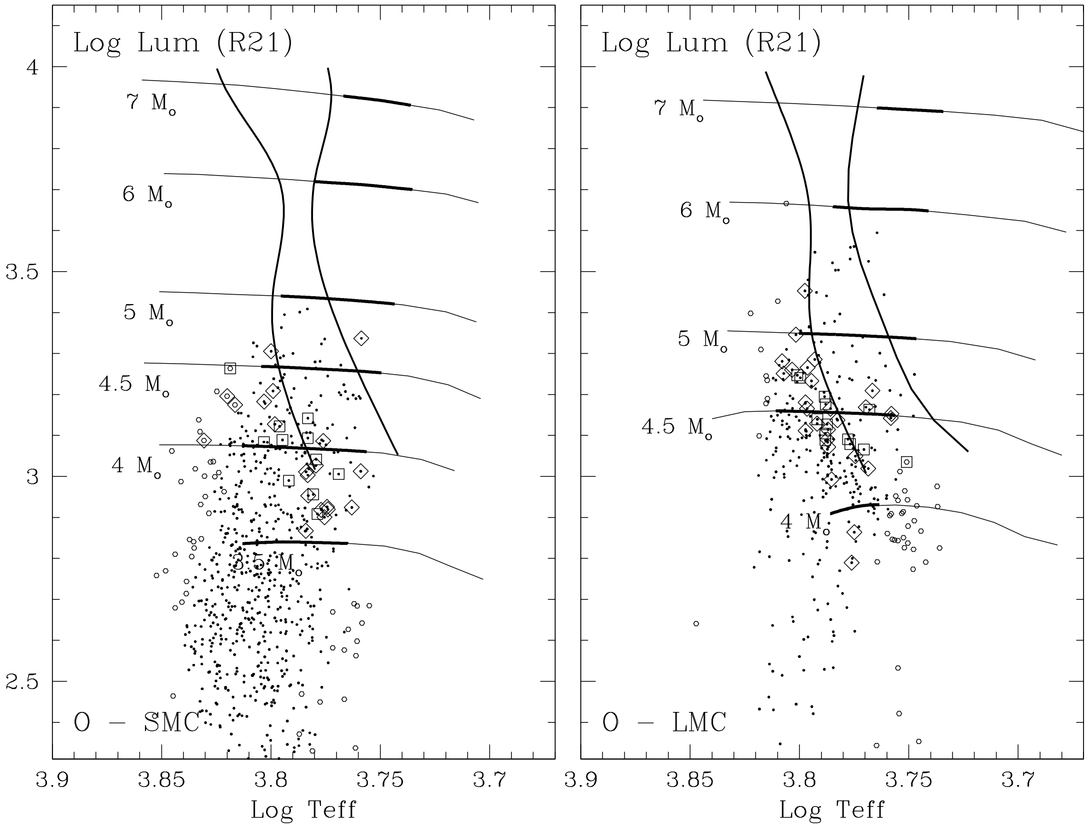

Fig. 4. MC O1 Cepheids – R_{21}^{m} resonance criterion: *squares*: $R_{21}^{\mathrm{m}} < 0.05$ (and $P_1 < 4.5$), *tilted squares*: $0.05 < R_{21}^{\mathrm{m}} < 0.07$ (and $P_1 < 4.5$). The resonant Cepheid models lie between the vertical curves and the envelopes of the thickened tracks (instability strip). The apparent lack of agreement is discussed in the text.

and the vertical curves denote the edges of the resonance regions, defined by $0.485 < P_{\mathrm{r}} < 0.515$.

In Fig. 2 we use the Fourier phase (ϕ_{21}^{m}) resonance criterion. We have surrounded by *squares* the stars for which $\phi_{21}^{\mathrm{m}} > 5.45$ (and $P_0 < 20$), and by *tilted squares* those for which $\phi_{21}^{\mathrm{m}} < 2$. The position of these resonant OGLE stars is seen to agree rather well with the resonance region that we have defined with the help of the models along the Padova tracks, despite the large scatter in the OGLE Fourier coefficients that manifests itself in our figures.

In Fig. 3 we use instead the resonance criterion based on the R_{21}^{m} coefficients. We surround the stars for which $R_{21}^{\mathrm{m}} < 0.15$ and $P_0 < 16$ by square boxes, those for which $0.15 < R_{21}^{\mathrm{m}} < 0.20$ and $P_0 < 16$ by tilted squares, and those for which $0.20 < R_{21}^{\mathrm{m}} < 0.25$ and $6 < P_0 < 16$ by triangles. We have chosen these numerical values on the basis of the plots of R_{21}^{m} vs. period of the OGLE Cepheids (Udalski et al. 1999a,b).

One notes that this second criterion gives results that are very similar to those derived with the ϕ_{21}^{m} criterion. Again there is excellent agreement between the F Cepheid models along the Padova tracks and the OGLE F Cepheids.

First overtone Cepheids:

We have already noted in the Introduction that for the O1 Cepheids neither the phase ϕ_{21}^{m} nor the amplitude ratio R_{21}^{m} are useful discriminants for the location of the resonance (FBK). Figure 4 shows that if we persist in using R_{21}^{m} as a resonance criterion, where the squares are stars with $R_{21}^{\mathrm{m}} < 0.05$ and $P_1 < 4.5$, and the tilted squares those with $0.05 < R_{21}^{\mathrm{m}} < 0.07$ and $P_1 < 4.5$, we see that these "resonant" stars fall substantially below the resonance region defined by the evolutionary tracks and pulsation theory. The reason, as we have already pointed out, is that the behavior of R_{21}^{m} through the resonance is different (as a function of P_{r}) for the O1 Cepheids and for the F Cepheids. This point was argued in FBK where it was shown that nonlinear Galactic overtone Cepheid models give excellent agreement with the observational Fourier decomposition data, but that the resonance occurs around 4.2 d, rather than 3.2 d. Kienzle et al. (1999) had also suggested a resonance at 4.6 d on the basis of the Fourier decomposition of the V_{r} curves of the Galactic overtone Cepheids.

Since neither the ϕ_{21}^{m} nor the R_{21}^{m} resonance criteria are very simple or useful in the case of O1 Cepheids, we turn to the period P_1 as an alternate resonance criterion in Fig. 5, even though it is less restrictive as pointed out in the introduction. The different period ranges are indicated with different symbols, viz. circles: $4.45 < P_1 < 4.95$, tilted squares: $3.95 < P_1 < 4.45$, squares: $3.45 < P_1 < 3.95$, hexagons: $2.95 < P_1 < 3.45$, triangles: $2.45 < P_1 < 2.95$.

In the Introduction we suggested on the basis of the comparative location of the minima of R_{21}^{m} that in going from the Galaxy to the LMC and SMC one might expect similar shifts of -0.5 and -0.7 d in the locations of the resonance regions. The latter would thus be centered on $P_1 \sim 3.7$ d

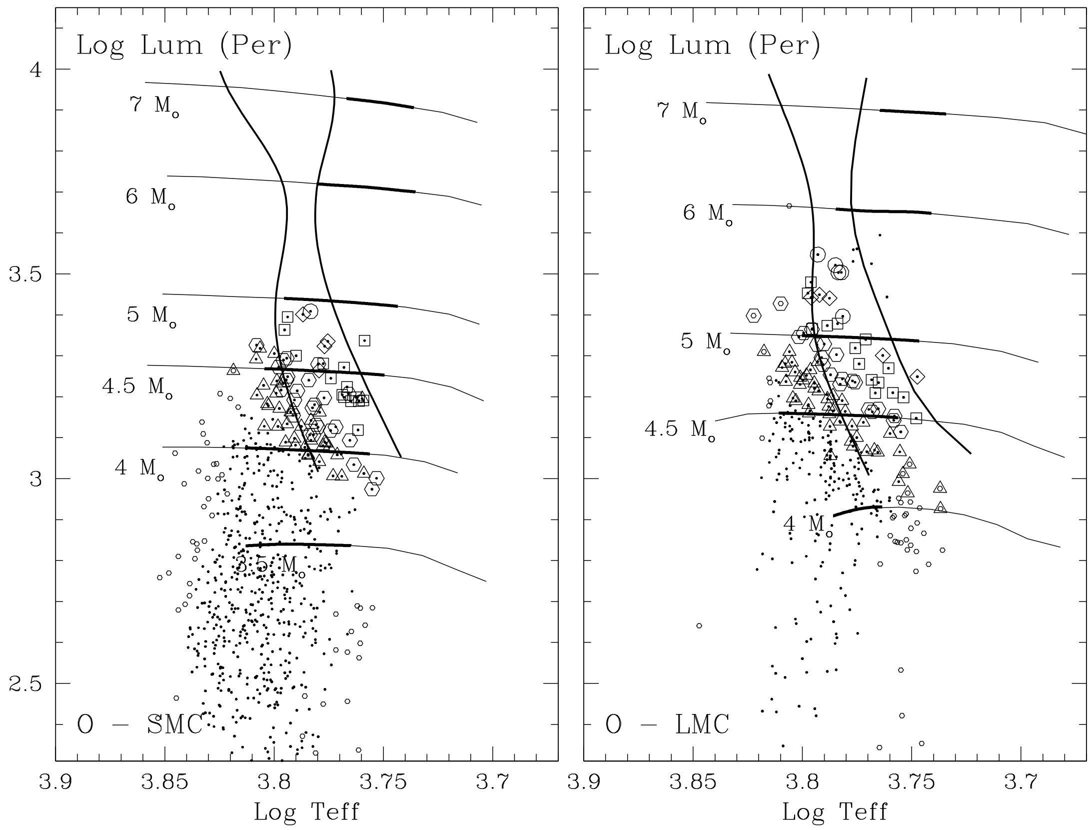

Fig. 5. SMC O1 Cepheids – period "resonance" criterion: *dots (open circles)* represent linearly stable (unstable) OGLE models; *circles*: $4.45 < P_1 < 4.95$, *tilted squares*: $3.95 < P_1 < 4.45$, *squares*: $3.45 < P_1 < 3.95$, *hexagons*: $2.95 < P_1 < 3.45$, *triangles*: $2.45 < P_1 < 2.95$. The resonant Cepheid models lie between the vertical curves and the envelopes of the thickened tracks (instability strip).

for the LMC (in the region of the squares in Fig. 5) and on ~3.5 d for the SMC (in the region straddling the squares and the hexagons). Figure 5 is certainly compatible with these conclusions.

In this paper we have shown the properties of the OGLE Cepheids obtained with BBK's choice B of distance modulus and reddening. For completeness we have repeated the same calculations with their choice A. It is noteworthy that choice B, which BBK labelled as preferred on other grounds, also gives better agreement between the resonance properties of the observations and the models than choice A.

2. Conclusions

We have used the properties of the internal 2:1 resonance between fundamental and second overtone modes for the F Cepheids (originally known as the Hertzsprung progression), as well as the 2:1 resonance between the first and fourth overtone modes for the O1 Cepheids to compare the observational, stellar evolutionary and pulsational properties. We find a very good agreement between the evolutionary tracks of Girardi et al. (2000), our turbulent convective pulsation code and the OGLE observational constraints.

The only major, and well known disagreement is the shortness of the blueward evolutionary tracks for low masses where they do not even penetrate into the instability strip, but this occurs outside the resonance regions.

Another, but smaller disagreement occurs in the precise location of the ISs and their widths. It has its origin partially in the errors in the L and $T_{\rm eff}$ which themselves come from reddening and magnitude uncertainties in the observational OGLE data. From the modelling side uncertainties arise from the values of the α parameters that enter the pulsation code through time-dependent mixing length theory. In addition, nonlinear effects that we have not considered here shift the F blue edge to lower $T_{\rm eff}$ and the O1 blue edge to higher $T_{\rm eff}$.

In the past the sharp features in the Fourier coefficients, that are indicative of the presence of internal resonances, have been used to localize internal resonances. This has worked reasonably well for the Galactic F Cepheids, but as we have discussed here, in general, there exists *no simple* a priori criterion for localizing precisely the resonance region from either the LC or from the $V_{\rm r}$ data. One needs to resort to full amplitude numerical hydrodynamic modelling to determine the criterion to be used to localize the resonance center. We find that we get much better agreement between the resonant models and the OGLE data if we put the O1 Cepheid resonance center ($P_4/P_1 = 1/2$) at ~3.7 d for the LMC and at ~3.5 d for the SMC, rather than where the ϕ_{21} has its sharp drop. The same suggestion was reached already by FBK in their detailed study of Galactic Cepheids where this resonance appear ~4.2 d.

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