

# Cepheid distances from infrared long-baseline interferometry

## II. Calibration of the period–radius and period–luminosity relations

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Received 29 October 2003 / Accepted 4 May 2004

**Abstract.** Using our interferometric angular diameter measurements of seven classical Cepheids reported in Kervella et al. (2004, A&A, 416, 941 – Paper I), complemented by previously existing measurements, we derive new calibrations of the Cepheid period–radius (P–R) and period–luminosity (P–L) relations. We obtain a P–R relation of  $\log R = [0.767 \pm 0.009] \log P + [1.091 \pm 0.011]$ , only  $1 \sigma$  away from the relation obtained by Gieren et al. (1998, ApJ, 496, 17). We therefore confirm their P–R relation at a level of  $\Delta(\log R) = \pm 0.02$ . We also derive an original calibration of the P–L relation, assuming the slopes derived by Gieren et al. (1998) from LMC Cepheids,  $\alpha_K = -3.267 \pm 0.042$  and  $\alpha_V = -2.769 \pm 0.073$ . With a P–L relation of the form  $M_\lambda = \alpha_\lambda (\log P - 1) + \beta_\lambda$ , we obtain  $\log P = 1$  reference points of  $\beta_K = -5.904 \pm 0.063$  and  $\beta_V = -4.209 \pm 0.075$ . Our calibration in the  $V$  band is statistically identical to the geometrical result of Lanoix et al. (1999, MNRAS, 308, 969).

**Key words.** stars: variables: Cepheids – cosmology: distance scale – stars: oscillations – techniques: interferometric

### 1. Introduction

The period–luminosity (P–L) relation of the Cepheids is the basis of the extragalactic distance scale, but its calibration is still uncertain at a  $\Delta M = \pm 0.10$  mag level. Moreover, it is not excluded that a significant bias of the same order of magnitude affects our current calibration of this relation. On the other hand, the period–radius relation (P–R) is an important constraint to the Cepheid models (see e.g. Alibert et al. 1999).

Traditionally, there have been two ways to calibrate the P–L relation. For Cepheids in clusters one can use main sequence fitting, assuming that the main sequence is similar to that of the Pleiades. This method has been questioned however, following the release of HIPPARCOS data (e.g., Pinsonneault et al. 1998; but see also Pan et al. 2004; Robichon et al. 1999). Another route to the P–L relation is the Baade–Wesselink (BW) method where one combines photometry and radial velocity data to obtain the distance and radius of a Cepheid. Recent applications of the BW method to individual stars can be found for instance in Taylor et al. (1997) and Taylor & Booth (1998), while the calibration of the P–R and P–L relations using BW distances and radii is demonstrated in Gieren et al. (1998, hereafter GFG98). A requirement of this method is a very accurate measurement of the Cepheid’s effective temperature at all observed phases, in order to determine the angular diameter. Interferometry allows us to bypass this step and its associated uncertainties by measuring *directly* the variation of

angular diameter during the pulsation cycle. As shown by Kervella et al. (2004, hereafter Paper I) and Lane et al. (2002), the latest generation of long baseline visible and infrared interferometers have the potential to provide precise distances to Cepheids up to about 1 kpc, using the interferometric BW method (see Sect. 2).

The main goal of the present paper is to explore the application of this technique to the calibration of the P–R and P–L relations, and to verify that no large bias is present in the previously published calibrations of these important relations. Our sample is currently too limited to allow a robust determination of the P–L relation, defined as  $M_\lambda = \alpha_\lambda (\log P - 1) + \beta_\lambda$ , that would include both the slope  $\alpha_\lambda$  and the  $\log P = 1$  reference point  $\beta_\lambda$ . However, if we suppose that the slope is known a priori from the literature, we can derive a precise calibration of  $\beta_\lambda$ . In Sect. 3, we present our determination of the P–R relation using new angular diameter values from Paper I, as well as previously published interferometric and trigonometric parallax measurements. Section 4 is dedicated to the calibration of the P–L relation reference points  $\beta_\lambda$  in the  $K$  and  $V$  bands. The consequences for the LMC distance are briefly discussed in Sect. 4.5.

### 2. Cepheid distances by interferometry

We have obtained angular diameter measurements for seven Cepheids with the VLT interferometer (Kervella et al. 2004, Paper I). These  $K$ -band measurements were made with the

VINCI instrument (Kervella et al. 2003) fed by two 0.35 m siderostats. Several baselines were used, ranging from 60 m to 140 m. Our measurements, described in detail in Paper I, have a typical precision of 1 to 3%. This is good enough to actually *resolve* the pulsation of several Cepheids; in other words we can follow the change in angular diameter. We have combined these measurements with radial velocity data and derived a radius and distance for four Cepheids of our sample. For the remaining three stars, we were able to derive their mean angular diameters, but the pulsation remained below our detection threshold. This sample was completed by previously published measurements obtained with other instruments.

In the present work, we have retained the limb darkened (LD) angular diameters  $\theta_{LD}$  provided by each author. Marengo et al. (2002, 2003) have shown that the LD properties of Cepheids can be different from those of stable stars, in particular at visible wavelengths. For the measurements obtained using the GI2T (Mourard et al. 1997) and NPOI (Nordgren et al. 2000), the LD correction is relatively large ( $k = \theta_{LD}/\theta_{UD} \approx 1.05$ ), and this could be the source of a bias at a level of a 1 to 2% (Marengo et al. 2004). However, in the infrared, the correction is much smaller ( $k \approx 1.02$ ), and the error on its absolute value is expected to be significantly below 1%. The majority of the Cepheid interferometric measurements was obtained in the *H* and *K* bands (FLUOR/IOTA, PTI, VLTI/VINCI), and we believe that the potential bias introduced on our fits is significantly smaller than their stated error bars. The final answer about the question of the limb darkening of Cepheids will come from direct interferometric observations, that will soon be possible with the AMBER instrument (Petrov et al. 2000) of the VLTI.

The radial velocity data were taken from Bersier (2002). They have been obtained with the CORAVEL spectrograph (Baranne et al. 1979). This instrument performs a cross-correlation of the blue part of a star’s spectrum (3600–5200 Å) with the spectrum of a red giant. A Gaussian function is then fitted to the resulting cross-correlation function, yielding the radial velocity.

In Paper I, we have applied three distinct methods (orders 0, 1 and 2) to derive the distances  $d$  to seven Galactic Cepheids from interferometric angular diameter measurements. Not all three methods can be used to derive the distance for every star, depending on the level of completeness and precision of the available angular diameter measurements:

- **Order 0:** constant diameter model.

This is the most basic method, used when the pulsation of the star is not detected. The average linear diameter  $\bar{D}$  of the star is supposed to be constant and known a priori, e.g. from a previously published P–R relations (such as the relation derived by GFG98). Knowing the linear and angular radii, the only remaining variable to fit is the distance  $d$ .

- **Order 1:** variable diameter model.

We still consider that the average linear diameter of the star is known a priori, but we include in our angular diameter model the radius variation curve derived from the integration of the radial velocity of the star. This method is well suited when the intrinsic accuracy of the angular diameter

measurements is too low to measure precisely the pulsation amplitude. The distance  $d$  is the only free parameter for the fit.

- **Order 2:** interferometric BW method.

The interferometric variant of the BW method (Davis 1979; Sasselov et al. 1994) combines the angular amplitude of the pulsation measured by interferometry and the linear displacement of the stellar photosphere deduced from the integration of the radial velocity curve to retrieve the distance of the star geometrically. This method is also called “parallax of the pulsation”. In the fitting process, the radius curve is matched to the observed angular diameter curve, using both the distance and linear diameter as variables. Apart from direct trigonometric parallax, this method is the most direct way of measuring the distance of a Cepheid. It requires a high precision angular diameter curve and a good phase coverage.

The order 0/1 methods, on one hand, and 2 on the other hand, are fundamentally different in their assumptions, and the distance estimates are affected by different kinds of errors. While the order 2 method errors are due to the interferometric measurement uncertainties (mostly statistical), the order 0/1 distances carry the systematic error bars of the assumed P–R relation. As they are fully correlated for all stars in the sample, they cannot be averaged over the sample. In particular, the order 0/1 diameters cannot be used to calibrate the P–R relation, as they assume this relation to be known a priori.

Due to its stringent requirements in terms of precision, the interferometric BW method (order 2) was applied successfully up to now to five Cepheids only:  $\ell$  Car (Paper I),  $\beta$  Dor (Paper I),  $\eta$  Aql (Paper I; Lane et al. 2002), W Sgr (Paper I) and  $\zeta$  Gem (Lane et al. 2002). However, it is expected that many more stars will be measurable with the required precision in the near future (see Sect. 5).

### 3. Period–radius relation

#### 3.1. Method

The period–radius relation (P–R) of the Cepheids takes the form of the linear expression:

$$\log R = a \log P + b. \quad (1)$$

In order to calibrate this relation, we need to estimate directly the linear radii of a set of Cepheids. We have applied two methods to determine the radii of the Cepheids of our sample: the interferometric BW method, and a combination of the average angular diameter and trigonometric parallax. While the first provides directly the average linear radius and distance, we need to use trigonometric parallaxes to derive the radii of the Cepheids for which the pulsation is not detected. We applied the HIPPARCOS parallaxes (Perryman et al. 1997) to all the order 0/1 measurements, except  $\delta$  Cep, for which we considered the recent measurement by Benedict et al. (2002). Table 1 lists the Cepheid linear radii that we obtain.

**Table 1.** Weighted averages of the interferometric mean angular diameters  $\overline{\theta_{LD}}$  and of the geometric distances  $d$  to nearby Cepheids (bold characters). These values were used to compute the linear radii given in the last two columns. The individual measurements used in the averaging process are also given separately for each star. References: (1) Mourard et al. (1997); (2) Nordgren et al. (2000); (3) Lane et al. (2002); (4) Mozurkewich et al. (1991); (5) Paper I; (6) Benedict et al. (2002); (7) Perryman et al. (1997).

Star	$P$ (d)	$\log P$	Ref. $\theta_{LD}$	$\overline{\theta_{LD}}$ (mas)	Ref. $d$	$d$ (pc)	$R$ ( $R_{\odot}$ )	$\log R$
$\delta$ Cep	5.3663	0.7297		<b><math>1.521 \pm 0.010</math></b>		<b><math>274^{+12}_{-11}</math></b>	<b><math>44.8^{+1.9}_{-1.8}</math></b>	<b><math>1.651^{+0.018}_{-0.018}</math></b>
			(1)	$1.60 \pm 0.12$				
			(2)	$1.52 \pm 0.01$				
					(6)	$273^{+12}_{-11}$		
					(7)	$301^{+64}_{-45}$		
X Sgr	7.0131	0.8459		<b><math>1.471 \pm 0.033</math></b>		<b><math>330^{+148}_{-78}</math></b>	<b><math>52.2^{+23}_{-12}</math></b>	<b><math>1.717^{+0.161}_{-0.118}</math></b>
			(5)	$1.471 \pm 0.033$				
					(7)	$330^{+148}_{-78}$		
$\eta$ Aql	7.1768	0.8559		<b><math>1.791 \pm 0.022</math></b>		<b><math>308^{+27}_{-24}</math></b>	<b><math>59.3^{+5.3}_{-4.6}</math></b>	<b><math>1.773^{+0.037}_{-0.035}</math></b>
			(2)	$1.69 \pm 0.04$				
			(3)	$1.793 \pm 0.070$	(3)	$320^{+32}_{-32}$		
			(5)	$1.839 \pm 0.028$	(5)	$276^{+55}_{-38}$		
					(7)	$360^{+175}_{-89}$		
W Sgr	7.5949	0.8805		<b><math>1.312 \pm 0.029</math></b>		<b><math>400^{+210}_{-114}</math></b>	<b><math>56.4^{+30}_{-16}</math></b>	<b><math>1.751^{+0.184}_{-0.146}</math></b>
			(5)	$1.312 \pm 0.029$	(5)	$379^{+216}_{-130}$		
					(7)	$637^{+926}_{-237}$		
$\beta$ Dor	9.8424	0.9931		<b><math>1.884 \pm 0.024</math></b>		<b><math>323^{+68}_{-42}</math></b>	<b><math>65.4^{+14}_{-8.6}</math></b>	<b><math>1.816^{+0.083}_{-0.061}</math></b>
			(5)	$1.884 \pm 0.024$	(5)	$345^{+175}_{-80}$		
					(7)	$318^{+74}_{-50}$		
$\zeta$ Gem	10.1501	1.0065		<b><math>1.688 \pm 0.022</math></b>		<b><math>362^{+37}_{-34}</math></b>	<b><math>65.6^{+6.7}_{-6.3}</math></b>	<b><math>1.817^{+0.042}_{-0.044}</math></b>
			(2)	$1.55 \pm 0.09$				
			(3)	$1.675 \pm 0.029$	(3)	$362^{+38}_{-38}$		
			(4)	$1.73 \pm 0.05$				
			(5)	$1.747 \pm 0.061$				
		(7)	$358^{+147}_{-81}$					
Y Oph	17.1269	1.2337		<b><math>1.438 \pm 0.051</math></b>		<b><math>877^{+2100}_{-360}</math></b>	<b><math>136^{+325}_{-56}</math></b>	<b><math>2.132^{+0.531}_{-0.231}</math></b>
			(5)	$1.438 \pm 0.051$	(7)	$877^{+2100}_{-360}$		
$\ell$ Car	35.5513	1.5509		<b><math>2.988 \pm 0.012</math></b>		<b><math>597^{+24}_{-19}</math></b>	<b><math>191.2^{+7.6}_{-6.0}</math></b>	<b><math>2.281^{+0.017}_{-0.014}</math></b>
			(5)	$2.988 \pm 0.012$	(5)	$603^{+24}_{-19}$		
					(7)	$463^{+129}_{-83}$		

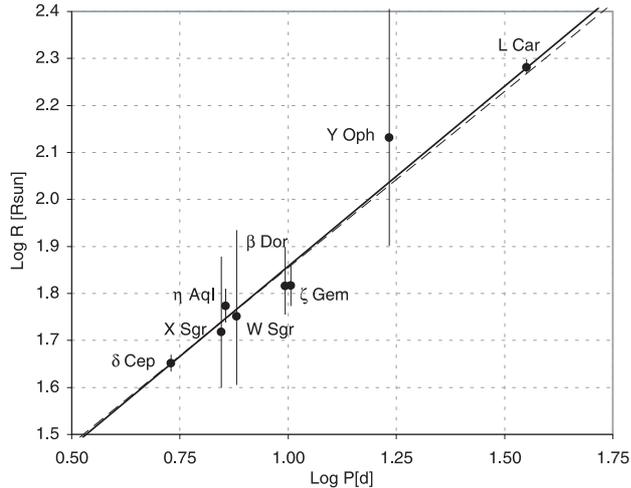
We can use the results from both order 0/1 and 2 methods at the same time, as the obtained linear radii obtained in this way are fully independent on each other. On one hand (BW method), we obtain them considering the *amplitude* of the pulsation and the radial velocity curve, while on the other hand, they are derived from the *average* angular diameter and the trigonometric parallax. As the amplitude of the pulsation and the average diameter values are distinct observables, these two methods can be used simultaneously in the fit.

### 3.2. Calibration results

Figure 1 shows the distribution of the measured diameters on the P–R diagram, based on the values listed in Table 1. When

we choose to consider a constant slope of  $a = 0.750 \pm 0.024$ , as found by GFG98, we derive a zero point of  $b = 1.105 \pm 0.017 \pm 0.023$  (statistical and systematic errors). As a comparison, GFG98 have obtained a value of  $b = 1.075 \pm 0.007$ , only  $-1.6\sigma$  away from our result. The relations found by Turner & Burke (2002) and Laney & Stobie (1995) are very similar to GFG98, and are also compatible with our calibration within their error bars.

Fitting simultaneously both the slope and the zero point to our data set, we obtain  $a = 0.767 \pm 0.009$  and  $b = 1.091 \pm 0.011$ . These values are only  $\Delta a = +0.7\sigma$  and  $\Delta b = +1.2\sigma$  away from the GFG98 calibration. Considering the limited size of our sample, the agreement is very satisfactory. On the other hand, the slopes derived by Ripepi et al. (1997) and



**Fig. 1.** Period–radius diagram deduced from the interferometric observations of Cepheids listed in Table 1. The thin dashed line represents the best-fit P–R relation assuming the slope of GFG98,  $\log R = 0.750 [\pm 0.024] \log P + 1.105 [\pm 0.017 \pm 0.023]$ . The solid line is the best-fit relation allowing both the slope and zero point to vary,  $\log R = 0.767 [\pm 0.009] \log P + 1.091 [\pm 0.011]$ .

**Table 2.** Period–radius relations, assuming an expression of the form  $\log R = a \log P + b$ . For the fitting of  $b$  alone, the slope has been assumed as known a priori from GFG98. In this case, its error bar translates to a systematic uncertainty on the  $b$  value derived from the fit (given in brackets). References: (1) GFG98; (2) Turner & Burke (2002); (3) This work.

Ref.	Fit	$a \pm \sigma_{\text{stat}}$	$b \pm \sigma_{\text{stat}} [\pm \sigma_{\text{syst}}]$
(1)		$0.750 \pm 0.024$	$1.075 \pm 0.007$
(2)		$0.747 \pm 0.028$	$1.071 \pm 0.025$
(3)	$b$ only		$1.105 \pm 0.017 [\pm 0.023]$
(3)	$a, b$	$0.767 \pm 0.009$	$1.091 \pm 0.011$

Krockenberger et al. (1997), both around 0.60, seem to be significantly too shallow.

## 4. Period–luminosity relation

### 4.1. Distance estimates

For the order 0 and 1 methods (Paper I), we used an a priori P–R relation (from GFG98) to predict the true linear diameter of the Cepheids of our sample. This relation relies on the measurement of the photometric flux, effective temperature (classical BW method) and radial velocity. The apparent magnitude also intervenes in the computation of the absolute magnitude, and therefore we cannot use these distance estimates to calibrate the P–L relation without creating a circular reference. For this reason, we have considered only the distances obtained using the interferometric BW method (order 2) for our P–L relation calibration, complemented by the Benedict et al. (2002) trigonometric parallax of  $\delta$  Cep.

**Table 3.** Apparent magnitudes and extinctions in the  $K$  and  $V$  bands for the Cepheid whose distances have been measured directly by interferometry.  $(B - V)_0$  is the mean  $(B - V)$  index as reported in the online database by Fernie et al. (1995). The  $E_{B-V}$  values were taken from Fernie (1990). The extinctions in the  $K$  and  $V$  bands are given respectively in the “ $A_K$ ” and “ $A_V$ ” columns, in magnitudes.

Star	$(B - V)_0$	$E_{B-V}$	$m_K$	$A_K$	$m_V$	$A_V$
$\delta$ Cep	0.66	0.09	2.31	0.03	3.99	0.30
$\eta$ Aql	0.79	0.15	1.97	0.04	3.94	0.49
W Sgr	0.75	0.11	2.82	0.03	4.70	0.36
$\beta$ Dor	0.81	0.04	1.96	0.01	3.73	0.15
$\zeta$ Gem	0.80	0.02	2.11	0.01	3.93	0.06
$\ell$ Car	1.30	0.17	1.09	0.05	3.77	0.58

**Table 4.** Absolute magnitudes of Cepheids measured exclusively using the interferometric Baade-Wesselink method, except for  $\delta$  Cep, whose parallax was taken from Benedict et al. (2002). The same error bars apply to the  $K$  and  $V$  band absolute magnitudes. The Cepheid periods are listed in Table 1. References: (1) Lane et al. (2002); (2) Benedict et al. (2002); (3) Paper I.

Star	Ref.	$d$	$\pm \sigma$	$M_K$	$M_V$	$\pm \sigma$
$\delta$ Cep	(2)	273	$+12$ $-11$	-4.90	-3.49	$+0.09$ $-0.09$
$\eta$ Aql	(1)	320	$+32$ $-32$	-5.60	-4.08	$+0.23$ $-0.21$
$\eta$ Aql	(3)	276	$+55$ $-38$	-5.28	-3.76	$+0.32$ $-0.39$
W Sgr	(3)	379	$+216$ $-130$	-5.10	-3.56	$+0.91$ $-0.98$
$\beta$ Dor	(3)	345	$+175$ $-80$	-5.74	-4.10	$+0.57$ $-0.89$
$\zeta$ Gem	(1)	362	$+38$ $-38$	-5.69	-3.92	$+0.24$ $-0.22$
$\ell$ Car	(3)	603	$+24$ $-19$	-7.86	-5.72	$+0.07$ $-0.08$

### 4.2. Absolute magnitudes

The average apparent magnitudes in  $V$  and  $K$  of  $\delta$  Cep were computed via a Fourier series fit of the data from Moffett & Barnes (1984) and Barnes et al. (1997) for the  $K$  band and Barnes et al. (1997) for the  $V$  band. The sources for the other apparent magnitudes are given in Paper I (Table 1). Following Fouqué et al. (2003), the extinction  $A_\lambda$  has been computed using the relations:

$$A_\lambda = R_\lambda E_{B-V} \quad (2)$$

$$R_V = 3.07 + 0.28 (B - V)_0 + 0.04 E_{B-V} \quad (3)$$

$$R_K = R_V / 11 \simeq 0.279. \quad (4)$$

The resulting extinction values are listed in Table 3, and the final absolute magnitudes  $M_\lambda$  of the Cepheids of our sample are listed in Table 4.

### 4.3. Calibration of the P–L relation

We have considered for our fit the P–L slope measured on LMC Cepheids. This is a reasonable assumption, as it can be measured precisely on the Magellanic Clouds Cepheids, and in addition our sample is currently too limited to derive both the slope and the  $\log P = 1$  reference point simultaneously.

**Table 5.** Period–luminosity relation intercept  $\beta_K$  for a 10 days period Cepheid ( $\log P = 1$ ), in the  $K$  band. We assume an expression of the form  $M_K = \alpha_K (\log P - 1) + \beta_K$ . The slope value is taken from GFG98 ( $\alpha_K = -3.267 \pm 0.042$ ). The systematic error corresponds to the uncertainty on the GFG98 slope.

Ref.	$\beta_K$	$\pm\sigma_{\text{stat}}$	$\pm\sigma_{\text{syst}}$
GFG98	-5.701	$\pm 0.025$	
This work, all stars	<b>-5.904</b>	<b><math>\pm 0.063</math></b>	<b><math>\pm 0.005</math></b>
Without $\delta$ Cep and $\ell$ Car	-5.956	$\pm 0.191$	$\pm 0.006$

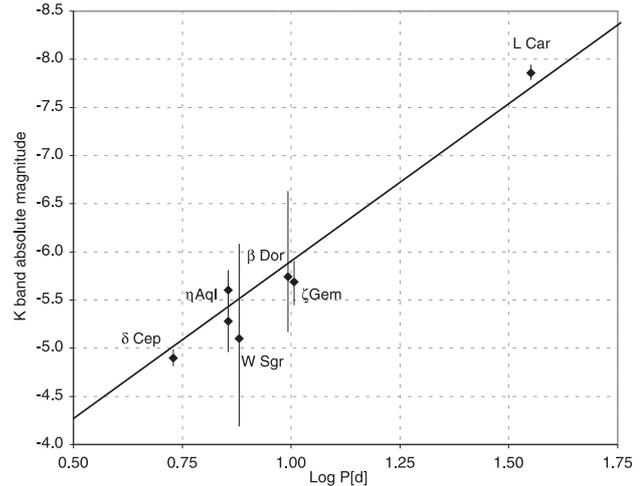
**Table 6.** Period–luminosity relation intercept  $\beta_V$  ( $\log P = 1$ ) in the  $V$  band, derived using the GFG98 slope ( $\alpha_V = -2.769 \pm 0.073$ ).

Ref.	$\beta_V$	$\pm\sigma_{\text{stat}}$	$\pm\sigma_{\text{syst}}$
GFG98	-4.063	$\pm 0.034$	
LPG99	-4.21	$\pm 0.05$	
This work, all stars	<b>-4.209</b>	<b><math>\pm 0.075</math></b>	<b><math>\pm 0.001</math></b>
Without $\delta$ Cep and $\ell$ Car	-4.358	$\pm 0.197$	$\pm 0.010$

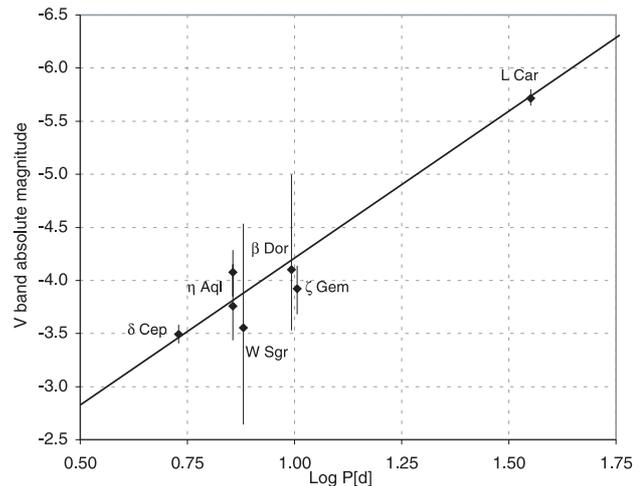
Recently, Fouqué et al. (2003) have revised the P–L slopes derived from the large OGLE2 survey (Udalski et al. 1999), and obtain values of  $\alpha_V = -2.774 \pm 0.042$  and  $\alpha_K = -3.215 \pm 0.037$ . These values are consistent within their error bars with LPG99 ( $\alpha_V = -2.77 \pm 0.08$ ), GFG98 ( $\alpha_V = -2.769 \pm 0.073$ ,  $\alpha_K = -3.267 \pm 0.042$ ) and Sasselov et al. (1997;  $\alpha_V = -2.78 \pm 0.16$ ). Considering this consensus, we have chosen to use the slope from GFG98 to keep the consistence with the P–R relation assumed in Paper I.

Tables 5 and 6 report the results of our calibrations of the P–L relations, and the positions of the Cepheids on the P–L diagram are shown in Figs. 2 and 3. The final  $\log P = 1$  reference points are given in bold characters in Tables 5 and 6. Our calibrations differ from GFG98 by  $\Delta b_K = +0.20$  mag in the  $K$  band, and  $\Delta b_V = +0.14$  mag in  $V$ , corresponding to  $+3.0$  and  $+1.8 \sigma$ , respectively. The sample is dominated by the high precision  $\ell$  Car and  $\delta$  Cep measurements. When these two stars are removed from the fit, the difference with GFG98 is slightly increased, up to  $+0.25$  and  $+0.30$  mag, though the distance in  $\sigma$  units is reduced ( $+1.3$  and  $+1.5$ ). From this agreement,  $\ell$  Car and  $\delta$  Cep do not appear to be systematically different from the other Cepheids of our sample.

It is difficult to conclude firmly to a significant discrepancy between GFG98 and our results, as our sample is currently too limited to exclude a small-statistics bias. However, if we assume an intrinsic dispersion of the P–L relation  $\sigma_{\text{PL}} \approx 0.1$  mag, as suggested by GFG98, then our results point toward a slight underestimation of the absolute magnitudes of Cepheids by these authors. On the other hand, we obtain precisely the same  $\log P = 1$  reference point value in  $V$  as Lanoix et al. (1999, using parallaxes from HIPPARCOS). The excellent agreement between these two fully independent, geometrical calibrations of the P–L relation is remarkable.



**Fig. 2.** Period–luminosity diagram in the  $K$  band using only interferometric BW distances and the  $\delta$  Cep parallax listed in Table 4. The solid line represents the best-fit P–L relation using the slope derived by GFG98 (classical least-squares fit: the individual measurements are weighted by the inverse of their variance).



**Fig. 3.** Period–luminosity diagram in the  $V$  band (slope from GFG98).

#### 4.4. P–L relation slopes in the Galaxy and in the LMC

The question of the difference in slope between the Galactic and LMC Cepheid P–L relations has recently been discussed by Fouqué et al. (2003) and Tammann et al. (2003). These authors conclude that the Galactic slopes are significantly steeper than their LMC counterparts. For example, Tammann et al. (2003) obtain  $\alpha_V[\text{Gal}] = -3.14 \pm 0.10$ , while Fouqué et al. (2003) derive  $\alpha_V[\text{Gal}] = -3.06 \pm 0.11$  and  $\alpha_V[\text{LMC}] = -2.774 \pm 0.042$ .

However, our fit is largely insensitive to the precise value assumed for the P–L relation slope. Considering the steeper Tammann et al. (2003) slope, we obtain a best fit  $\log P = 1$  absolute magnitude of  $\beta_V = -4.211 \pm 0.075 \pm 0.001$ , identical to the calibration obtained using the GFG98 slope. The small systematic error bar that we obtain on  $\beta_V$  (corresponding to the  $\pm 0.10$  error on  $\alpha_V$ ) shows the weakness of the correlation between  $\alpha$  and  $\beta$  in our fit. However, the reduced  $\chi^2$  of the fit

is significantly larger with this steeper slope ( $\chi_{\text{red}}^2 = 1.25$ ) than with the LMC slope from GFG98 ( $\chi_{\text{red}}^2 = 0.53$ ).

#### 4.5. The distance to the LMC

The apparent magnitudes in  $V$  and  $K$  of a 10 day period Cepheid in the Large Magellanic Cloud (LMC) derived by Fouqué et al. (2003) from the OGLE Cepheids are  $ZP_K = 12.806 \pm 0.026$  and  $ZP_V = 14.453 \pm 0.029$ . These authors assumed in their computation a constant reddening of  $E(B - V) = 0.10$  for all the LMC Cepheids they have used (more than 600). Our calibrations of the Galactic Cepheids P–L relations in  $K$  and  $V$  thus implies LMC distance moduli of  $\mu_K = 18.71 \pm 0.07$  and  $\mu_V = 18.66 \pm 0.08$ , respectively.

From a large number of photometric measurements of LMC and SMC Cepheids obtained in the framework of the EROS programme, Sasselov et al. (1997) have shown that a  $\delta\mu$  correction has to be applied to the LMC distance modulus to account for the difference in metallicity between the LMC and the Galactic Cepheids. They have determined empirically a value of:

$$\delta\mu = \mu_{\text{true}} - \mu_{\text{observed}} = -0.14 \pm 0.06 \quad (5)$$

this correction has been questioned by Udalski et al. (2001), based on Cepheid observations in a low metallicity galaxy (IC 1613), and its amplitude is still under discussion (Fouqué et al. 2003).

Averaging our  $K$  and  $V$  band zero point values (without reducing the uncertainty, that is systematic in nature), we obtain a final LMC distance modulus of  $\mu_0 = 18.55 \pm 0.10$ . This value is only  $+0.8 \sigma$  away from the  $\mu_0 = 18.46 \pm 0.06$  value obtained by GFG98, and  $-1 \sigma$  from the  $\mu_0 = 18.70 \pm 0.10$  value derived of Feast & Catchpole (1997). It is statistically identical to the LMC distance used by Freedman et al. (2001) for the *HST Key Project*,  $\mu_0 = 18.50 \pm 0.10$ . Alternatively, if we consider the smaller metallicity correction of  $\delta\mu = 0.06 \pm 0.10$ .

## 5. Conclusion and perspectives

We have confirmed in this paper the P–R relation of GFG98 and Turner & Burke (2002), to a precision of  $\Delta(\log R) = \pm 0.02$ . We also derived an original calibration of the P–L relations in  $K$  and  $V$ , assuming the slopes from GFG98 that were established using LMC Cepheids. Our P–L relation calibration yields a distance modulus of  $\mu_0 = 18.55 \pm 0.10$  for the LMC, that is statistically identical to the value used by Freedman et al. (2001) for the *HST Key Project*. We would like to emphasize that this result, though encouraging, is based on six stars only (seven measurements, dominated by two stars), and our sample needs to be extended in order to exclude a small-number statistical bias. In this sense, the P–L calibration presented here should be considered as an intermediate step toward a final and robust determination of this important relation by interferometry.

While our results are very encouraging, the calibration of the PR and PL relations as described here may still be affected by small systematic errors. In particular the method relies on the fact that the displacements measured through interferometry and through spectroscopy (integration of the radial velocity

curve) are in different units (milli-arcseconds and kilometers respectively) but are the same physical quantity. This may not be the case. The regions of a Cepheid’s atmosphere where the lines are formed do not necessarily move homologously with the region where the  $K$ -band continuum is formed. This means that the two diameter curves may not have exactly the same amplitude; there could even be a phase shift between them. As discussed in Sect. 2, the limb darkening could also play a role at a level of  $\approx 1\%$ . A full exploration of these effects is far beyond the scope of this paper. We can nevertheless put an upper bound on the systematic error that could result from this mismatch. Our PL relation can be compared to that derived from Cepheids in open clusters, whose distances are obtained via main sequence fitting. The two distance scales are in excellent agreement (Gieren & Fouqué 1993; Turner & Burke 2002). These distances are consistent with a Pleiades distance modulus of 5.56; if anything they are slightly larger.

The availability of 1.8 m Auxiliary Telescopes (Koehler et al. 2002) on the VLTI platform in 2004, to replace the current 0.35 m Test Siderostats, will allow to observe many Cepheids with a precision at least as good as the observations of  $\ell$  Car reported in Paper I (angular diameters accurate to 1%). In addition, the AMBER instrument (Petrov et al. 2000) will extend the VLTI capabilities toward shorter wavelengths ( $J$  and  $H$  bands), thus providing higher spatial resolution than VINCI ( $K$  band). The combination of these two improvements will extend significantly the accessible sample of Cepheids, and we expect that the distances to more than 30 Cepheids will be measurable with a precision better than  $\pm 5\%$ . This will provide a high precision calibration of both the  $\log P = 1$  reference point (down to  $\pm 0.01$  mag) and the slope of the Galactic Cepheid P–L. As the galaxies hosting the Cepheids used in the *Key Project* are close to solar metallicity on average (Feast 2001), this Galactic calibration will allow us to bypass the LMC step in the extragalactic distance scale. Its attached uncertainty of  $\pm 0.06$  due to the metallicity correction of the LMC Cepheids will therefore become irrelevant for the measurement of  $H_0$ .

*Acknowledgements.* D.B. acknowledges partial support from NSF grant AST-9979812. P.K. acknowledges support from the European Southern Observatory through a post-doctoral fellowship. Based on observations collected at the VLT Interferometer, Cerro Paranal, Chile, in the framework of the ESO shared-risk programme 071.D-0425 and an unreferenced programme in P70. The VINCI/VLTI public commissioning data reported in this paper have been retrieved from the ESO/ST-ECF Archive (Garching, Germany). This research has made use of the SIMBAD database at CDS, Strasbourg (France). We are grateful to the ESO VLTI team, without whose efforts no observation would have been possible.

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