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The polarimetric properties of cometary dust and a possible effect of dust aging by the Sun

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Abstract. After the last apparition of comet Halley in 1985–86, a number of other comets were observed in polarimetry with IHW-continuum filters. From the in situ dust measurements of Halley, dust size distribution functions were obtained (Lamy et al. 1987, A&A, 187, 767; Mazets et al. 1986, Nature, 321, 276), which were later used by several authors to interpret polarisation data of Halley. In the present work, polarimetric data of various comets have been analyzed, using Mie theory and assuming that the composition of dust particles does not differ from comet to comet (Delsemme 1987, ESA SP-278, 19). The individual grain size distribution functions so obtained for various comets suggest different values for the relative abundance of coarser grains. Introducing a "relative abundance of coarser grains" index g, we study any possible dependence of g on the dynamical age of a comet, where the dynamical age can be defined in terms of some of the orbital parameters of the comet. For the four non-periodic comets available for our analysis, we find a clear empirical relation $g = -2.5.q^{2/3}$ emerging from our work. This relation strengthens the concept that comets whose grains are processed more by the solar radiation (these comets may be dynamically older) contain a relatively smaller number of finer grains. The case for other periodic comets is also discussed here.
Since the work has been carried out using Mie scattering theory meant for perfect compact spheres, it is also suggested to repeat the calculations with more realistic porous grains in a follow-up paper.

Key words. solar system: general – comets: general – techniques: polarimetric

1. Introduction

The measurement of polarisation of the scattered radiation from comets, over various phase angles and wavelengths, provides an excellent tool to study cometary dust properties. The polarisation is caused mainly by scattering of solar radiation by cometary dust grains. At certain wavelengths, however, the polarisation features, are contaminated due to the polarisation present in the cometary molecular line emissions. Since the last apparition of comet Halley (1985–86), observers have been using a set of filters, centered at $\lambda = 0.365, 0.484, 0.684\,\mu$m, to avoid contamination by line emission. Such filters, commonly known as IHW (International Halley Watch) filters, have made comparison of polarisation data of various comets easy.

Analysis of these polarisation data reveals the physical properties of the cometary grains, which include size distribution, shape and complex refractive index. The in situ spacecraft measurement of Halley gave us the first direct evidence of grain mass distribution (Mazets et al. 1986). Lamy et al. (1987) compared the data from various space-crafts like Vega I, Vega II and Giotto, and arrived at the grain size distributions for Halley for various bulk densities. Based on (i)the power law dust distribution (Mazets et al. 1986); and (ii) Mie Scattering formulation, Mukai et al. (1987) and Sen et al. (1991a) analysed the polarisation data of comet Halley and derived a set of values for complex refractive indices of cometary grains. These values can be used to characterise the composition of cometary grains.

However, there is a modified form of power law dust size distribution differing substantially in the abundance of larger particles, which has been successfully used to explain the observed Spectral Energy Distribution (SED) in the thermal (IR) emission from cometary particles (Harker et al. 2002; Hanner & Hayward 2003).

Since the last apparition of Halley's comet, many other comets were observed in polarimetry and the analysis of these data clearly shows that the dependence of polarisation on phase angle and wavelength varies widely from comet to comet (Chernova et al. 1993; Levasseur-Regourd et al. 1996; Hadamcik & Levasseur-Regourd 2003a, etc.).

Comet Austin was observed polarimetrically by Sen et al. (1991b) and the authors compared the data with those for Halley. The two comets exhibited different types of phase angle dependence at the same wavelength. Using Mie Theory the authors argued that the observed differences can be explained if at least one of the two grain properties viz. size distribution and composition differs from one comet to other.

Following the suggestion of Delsemme (1987) that grain composition is less likely to differ between any two comets, Sen et al. (1991b) also showed that a better fit of the data to the predictions of Mie theory indeed results if variations in size distribution alone are considered to be present. The sizes are expected to increase with the dynamical age of comets, due to sintering (among other processes) by solar radiation (Delsemme 1987). Halley being a dynamically older comet than Austin one may find it reasonable to expect the grains of Austin to be finer than those of Halley.

The increase in size with age can have reasons other than sintering. The smaller grains are preferentially pushed away by solar radiation pressure, leaving the larger ones in orbit around the nucleus of the comet. It has also been observed that the composition of the nucleus does not seem to differ from one comet to another (A'Hearn 1999). Now since the nucleus is the sole source of grains in comets, we may expect that the composition of grains does not differ from one comet to other. Therefore, if required we may vary the size distribution to fit the observed data to model.

Harker et al. (2002) suggested a mechanism in which the action of solar radiation increases the size of the nuclear pore, through which grains are released. A larger pore size (caused due to nearness to the Sun) allows larger grains to be released from the nucleus. Thus the action of solar radiation on the surface of the nucleus alters the grain size distribution towards larger sizes. Also a good model fit of the observed IR data of Comets Hale-Bopp and Mueller(C/1993 A1) was obtained by the authors with a change in size distribution alone, rather than a change in composition (mineralogy). Harker et al. (2002) had also suggested that different grain compositions between comets are less likely, but a different grain size distribution could play an important role to explain differences in IR emission from different comets.

With this background, in the present work we try to understand whether the observed differences in polarisation behaviour of different comets can be understood in terms of the variation in grain size distribution.

Levasseur-Regourd et al. (1996), studied a polarimetric data base of 22 comets and from the nature of the phase angle dependence concluded that there is a clear evidence for two classes of comets. More recently Hadamcik & Levasseur-Regourd (2003a) compared the imaging polarimetry of seven different comets and suggested that Hale-Bopp itself represents a third class, marked by unusually high polarisation. The behaviour of polarisation and polarimetric colours of different regions of several comets were discussed taking into account different grain properties.

In the present work, we use the post-Halley polarimetric observations of various comets and analyze their behaviour with the following objectives:

(i) we extend the assumption made by Sen et al. (1991b) and the idea put forth by Delsemme (1987) to all other comets, so that one can characterize each comet by an individual grain size distribution, with fixed complex refractive index for all comets;
(ii) we estimate the relative abundance of coarser grains in a comet (as derived from the grain size distribution) and explore if such relative abundances are in anyway related to its dynamical age.

2. On the in situ dust measurements of Halley

During the last apparition of Halley's comet the various space probes on board Vega I, Vega II and Giotto carried out measurements to determine the number density of particles of given masses. However, the exact determination of the particle size distribution function from the above data needs a number of assumptions to be made. Hence, it is crucial to analyze the ground based observations (related to dust properties), with reference to the in situ observations, to check the consistency of the two sets of results. In this context, amongst various other types of measurements, the polarimetry of comets in the continuum plays an important role in the study of cometary dust properties.

Based on the SP-2 experiment on-board Vega space-craft, Mazets et al. (1986) had suggested a set of power laws (with separate indices for different mass ranges) for the particle mass distribution over the range 10^{-16} g to 10^{-7} g. Subsequently, Mukai et al. (1987) used these distribution functions to explain their optical polarimetric observations of Halley. Assuming the grain bulk density to be 1 g cm^{-3}, they arrived at the following size distribution functions:

$$N(s) \sim s^{-2}, \qquad s < 0.62\,\mu\mathrm{m} \tag{1}$$

$$N(s) \sim s^{-2.75}, \qquad 0.62\,\mu\mathrm{m} < s < 6.2\,\mu\mathrm{m} \tag{2}$$

$$N(s) \sim s^{-3.4}, \qquad s > 6.2\,\mu\mathrm{m}. \tag{3}$$

Sen et al. (1991a) followed the same approach in their analysis of the polarimetric data of Halley. Lamy et al. (1987) combined the in situ dust measurements from Vega-I, Vega-II and Giotto and modelled the dust mass distribution as a polynomial of the form:

$$\log N_{\mathrm{c}}(m) = \sum_{i=0}^{3} a_i (\log\ m)^i \tag{4}$$

where N_{c}(m) is the cumulative number density of dust particles with mass $> m$ and the coefficients (a_i) are determined by the least-squares method.

Lamy et al. (1987) further derived the differential spatial density $N(s)$ as a function of grain radius (s), given by

$$N(s) = -\frac{3N_{\mathrm{c}}}{s} \sum_{i=1}^{3} i a_i (\log\ m)^{i-1} \tag{5}$$

The size distribution function derived by Mukai et al. (1987) on the basis of the work reported by Mazets et al. (1986) has three discrete size ranges, and the size distribution function changes its value abruptly over the three ranges due to the presence of three distinct values of power law index. In contrast the size

Table 1. The log of grain radius (s) and log of differential spatial density ($N(s)$) as derived from Lamy et al. (1987).

log (s) (s in cm)	log $N(s)$ ($N(s)$ in cm^{-4})
–7.0	3.50
–6.5	2.67
–6.0	1.90
–5.5	1.18
–5.0	0.44
–4.5	–0.39
–4.0	–1.37
–3.5	–2.53
–3.0	–3.91
–2.5	–5.57
–2.0	–7.54

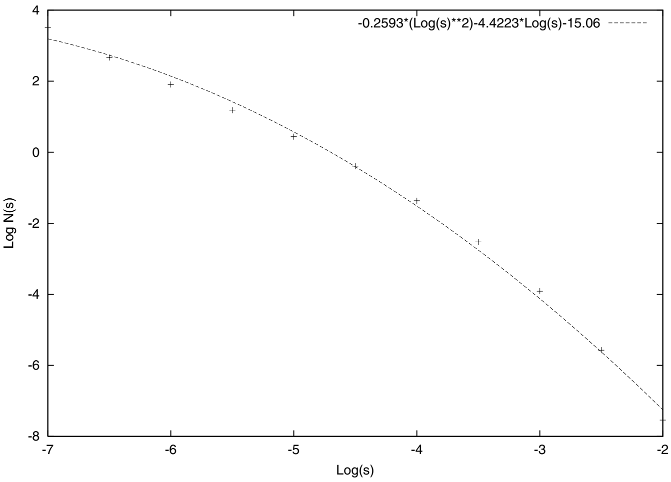

Fig. 1. Log of grain radius (s in cm) against the log of differential spatial density ($N(s)$ in cm^{-4}) as obtained from Lamy et al. (1987) for comet Halley derived through space-craft experiments (Table 1). The dotted curve represents the best-fit polynomial equation as derived in the present work.

distribution function as in Eq. (5) (from Lamy et al. 1987) has a smooth behaviour.

The dust distribution function derived by Mazets et al. (1986) is actually based on only Vega II results, while the work of Lamy et al. (1987) is based on the results of three spacecraft. Since in this work we plan to analyze polarimetric data of various comets, we proceed with the dust distribution function suggested by Lamy et al. (1987). These authors have listed the values of bulk density and (n, k) for different grain materials and have justified that the value 2.2 for bulk density in most cases corresponds to materials like chondrite, magnetite or silicates. We use this value for the bulk density to construct Table 1, which gives values of log (s) and corresponding log $N(s)$.

The plot of the data and the best-fit second degree polynomial curve are shown in Fig. 1. The second degree polynomial used has the form,

$$\log N(s) = a(\log s)^2 + b(\log\ s) + c \quad (6)$$

where
$a = -0.2593$
$b = -4.422$
$c = -15.06$.
The polynomial seems to fit the data of Lamy et al. (1987) quite well, as can be seen in Fig. 1. We use this grain model in our subsequent discussions.

However, we note that the grain size distributions used by Mukai et al. (1987) (Eqs. (1)–(3)), or the one derived from Lamy et al. (1987) (Eq. (6)) are basically the ones obtained after the last apparition of comet Halley in 1985–86 and were invoked in explaining the polarisation properties of comets. In the post-Halley era, infrared observations of different comets have given much new information to be used to diagnostics to understand grain properties. In a recent imaging polarimetric study of comet Hale-Bopp, a very useful discussion has been given by Hadamcik & Levasseur-Regourd (2003b) on the polarimetric results with reference to the results obtained by other diagnostics viz., albedo derived from NIR observations, 10 μm silicate emission feature, emission in the sub-millimeter domain, colour temperature and brightness structure of grains. In Sect. 1 we have already mentioned that a different grain size distribution has been used in the interpretation of NIR emission (Harker et al. 2002; Hanner & Hayward 2003).

A discussion of the grain model of Lamy et al. (1987) (modified here as Eq. (6)) in the context of recent results obtained from other diagnostics is expected to lead to a refinement in the grain size distribution used in the study of comets. However, it requires a very detailed analysis. So far no unified grain model has been suggested that takes care of both types of observations. Apparently, failure in fixing a unified grain model for both types of observations may be due to the fact that grains that are powerful polarizers may not be good emitters in IR. For example, Harker et al. (2002) have commented that by fitting the thermal grain model to NIR spectra of comet Hale-Bopp without including a scattered light component, Hayward et al. (2000) derived a smaller peak grain size much outside 1σ uncertainty.

Therefore, without going further into this analysis in an attempt to find a unified grain model, we here choose the grain model which has been successfully used earlier to explain cometary polarisation (Mukai et al. 1987; Krishnaswamy & Shah 1988; Sen et al. 1991a). This amounts to our selection of Eq. (6) as a slightly modified grain model for the present work.

The detectors on-board the Vega and Giotto spacecraft had sensitivities as low as 10^{-16} g, and it was observed that the particle number density continued to increase till the detection limit was reached (Mazet et al. 1987). Assuming spherical particles of density 1 or 2.2 g per cc, one derives a lower limit for the particle radius of 0.01 μm. However, as the 0.001–20.0 μm size range has been already used by Sen et al. (1991a) and Krishnaswamy & Shah (1988) for the analysis of polarimetry results, we continue to use the same size range, so that very small particles are not left out. This has also been done with a view to comparing the present studies with previous similar

Table 2. The (n, k) values obtained by previous authors and in our present work, for comet Halley at different wavelengths.

λ	n	k	Authors
0.365 μm	1.392	0.024	Mukai et al. (1987)
	1.387	0.032	Sen et al. (1991a)
	1.403	0.024	Present work
0.484 μm	1.387	0.031	Mukai et al. (1987)
	1.375	0.040	Sen et al. (1991a)
	1.390	0.026	Present work
0.620 μm	1.385	0.035	Mukai et al. (1987)
0.684 μm	1.374	0.052	Sen et al. (1991a)
	1.386	0.038	Present work

polarimetric studies. The selection of 0.001 μm or 0.01 μm as the lower limit of the size range changes the calculated value of percentage polarisation only at the fourth place after the decimal. The selection of a lower limit as 0.001 μm or 0.01 μm in no way changes the conclusions arrived at in this work. Also we note we have explicitly fixed the lower limit of the grain size distribution at 0.001 μm, but we may as also assume the lower limit to be 0.01 μm, if we want to make a comparison with other similar work.

2.1. Polarimetric data of Halley and grain characteristics

During the last apparition of Halley's comet, IHW was coordinating the ground based observations and suggested a set of eight narrow band interference filters for polarimetry and photometry, out of which three correspond to the continuum.

Based on the grain model of Mazet et al. (1987) and Mie Theory, Mukai et al. (1987) found a set of three complex refractive indices (n, k) at three IHW continuum wavelengths which best match their observations.

Again Sen et al. (1991a) combined their polarimetric observations with those of other investigators and minimized the sum of squares of differences between observed polarisation and calculated polarisation values to estimate (n, k) values and found refractive indices only slightly different from those of Mukai et al. (1987).

In the present study, we consider Eq. (6) for the grain distribution rather than Eqs. (1)–(3) and use a value of 2.2 g cm^{-3} for the bulk density of grains as justified by Lamy et al. (1987). We also choose a grain size range 0.001 μ to 20 μm, as discussed earlier. Using the Mie theory, we determine the best fit values of (n, k) at which the sum of squares of differences between the calculated and observed values of polarisation becomes minimum. These values are listed in Table 2.

In Figs. 2–4, we show the curves that give the calculated values of polarisation against the observed polarisation values reported by various authors, at wavelengths λ = 0.365, 0.484, 0.684 μm respectively.

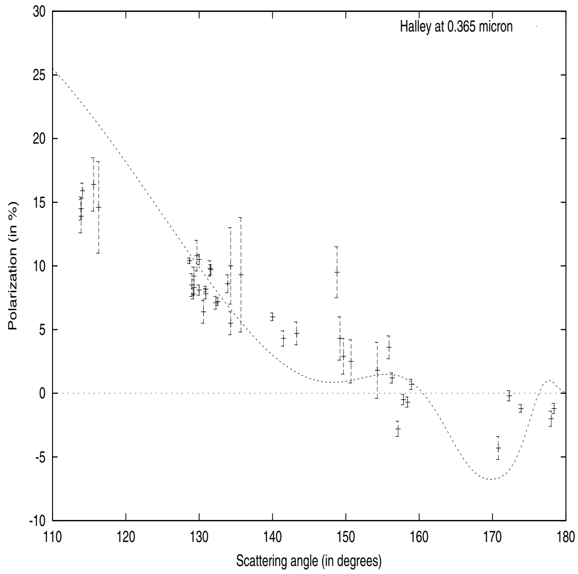

Fig. 2. The observed polarisation values of comet P/Halley at λ = 0.365 μm. The dotted curve represents the calculated values for Mie type scattering with $(n, k) = (1.403, 0.024)$.

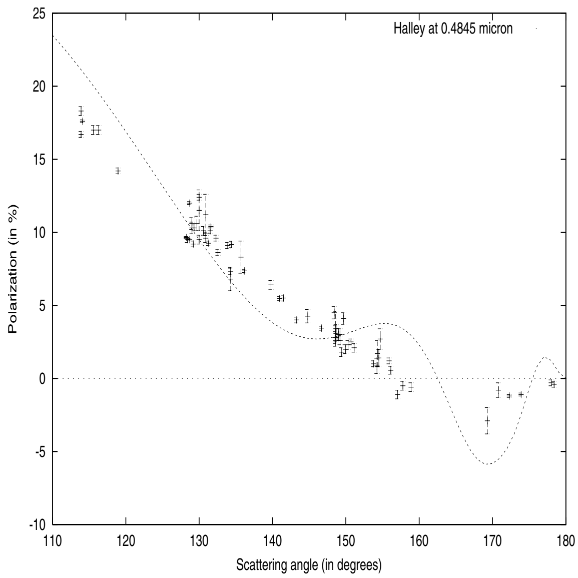

Fig. 3. The observed polarisation values of comet P/Halley at λ = 0.484 μm. The dotted curve represents the calculated values for Mie type scattering with $(n, k) = (1.390, 0.026)$.

2.2. Polarimetric properties of other comets

Polarimetry has always been considered a powerful tool in the study of cometary dust properties (Sen 2001). In the present work, we compile data on the polarisation observations that were made through IHW continuum filters and published in various journals. No claim to completeness is made here, but whatever data was available has been included. When including data, we imposed the selection criterion that the number of data points should be at least five, since the number of fitting parameters is of the same order. Table 3 lists the names of the comets that were considered in this work, and the corresponding reference for the source of data. In the same

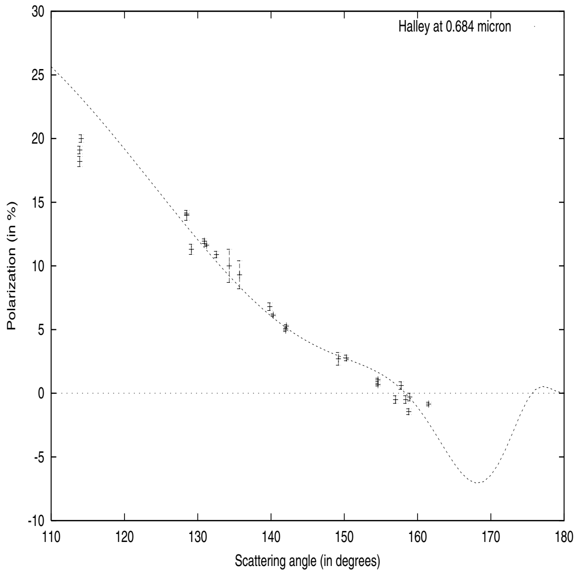

Fig. 4. The observed polarisation values of comet P/Halley at $\lambda = 0.684\ \mu$m. The dotted curve represents the calculated values for Mie type scattering with $(n, k) = (1.386, 0.038)$.

table, we also note the two orbital elements q (perihelion distance in AU) and T (time period in years), which we use in the subsequent section.

The polarisation data used here are reported at various phase angles, and if we assume Mie theory, we can fit the observed data to the expected curve, with (n, k) and the coefficient a, b, c (of Eq. (6)) as the free parameters. From Eq. (6) one can show the calculated polarisation value will not depend upon c, as it cannot influence the relative abundances of different sizes. Therefore, as has already been discussed in Sect. 1, we keep the composition (n, k) fixed and vary the size distribution (a, b) alone.

Thus, if we narrow down our search procedure by fixing the (n, k) values to those of Halley and try fitting the parameters a, b of Eq. (6), we can obtain individual grain size distribution functions for different comets by specifying a, b. It is clearly seen from Eq. (6) that the value of $\frac{\mathrm{d}\log(N(s))}{\mathrm{d}\log s}$ is proportional to the relative abundances of coarser grains. Equation (6) further suggests that

$$\frac{\mathrm{d}\log(N(s))}{\mathrm{d}\log\ s} = 2a\log\ s + b, \tag{7}$$

which can be fixed at a definite value of s (say $s = 10^{-7}$ cm or $0.001\ \mu$m) for purposes of comparison between various comets. This can be done by adjusting the value of c among the various comets – a change in c will not change the calculated value of the polarisation.

From Eq. (6),

$$N(s) = 10^{a(\log\ s)^2 + b(\log\ s) + c}. \tag{8}$$

We may write:

$$\frac{\mathrm{d}N(s)}{\mathrm{d}s} = \frac{\mathrm{d}N(s)}{\mathrm{d}\log\ s}\frac{\mathrm{d}\log\ s}{\mathrm{d}s} = N(s)(2a\log\ s + b)\frac{1}{s}\log_e 10.$$

Therefore,

$$\frac{\mathrm{d}N(s)}{\mathrm{d}s} = \frac{N(s)}{s}\left(\frac{\mathrm{d}\log N(s)}{\mathrm{d}\log s}\right)\log_e 10.$$

Now if we normalise the $N(s)$ values of different comets at a fixed value of s , say at $s = 10^{-7}$ cm, we may write

$$\frac{\mathrm{d}\log(N(s))}{\mathrm{d}\log\ s} = \text{const.} * \frac{\mathrm{d}(N(s))}{\mathrm{d}s}, \tag{9}$$

This value of $\frac{\mathrm{d}\log(N(s))}{\mathrm{d}\log\ s}$ as expressed by Eq. (7) can be considered as a "relative abundance of coarser grain" index which we denote by g. Thus, $g = -14a + b$. Note that g can be considered as the gradient of the tangent drawn to the grain distribution curve $N(s)$ at the point $s = 10^{-7}$ cm. We determine the best fit values of a and b required to minimise the sum of the squares of the differences between the observed and calculated values of polarisation. The values of g so calculated for different comets are listed in Table 3. We see from Table 3, that $\frac{\mathrm{d}\log(N(s))}{\mathrm{d}\log\ s}$ or the g value for Halley is –0.79, and that of Austin is –1.28, at $s = 0.001\ \mu$m. This suggests that comet Halley contains a relatively larger number of coarser grains than Austin.

The infrared spectrum of comet Austin, however, suggests the presence of larger particles. But for reasons already discussed in Sect. 2, we do not make any further attempt to determine a unified grain model to include these two sets of results.

3. A model to relate grain size distribution with grain aging

It is apparent from the work of Levasseur-Regourd (1996) and Hadamcik & Levasseur-Regourd (2003a) that comets exhibit different kinds of phase angle dependence on polarisation. As already explained in Sect. 2, we try to explain these polarimetric differences in terms of differences in grain size distribution.

In our present work, we have introduced and estimated a parameter g which represents the relative abundance of coarser grains in different comets.

The sample of comets included in our calculations have widely different values of perihelion distances (q); four of them are non-periodic and the rest are periodic (see Table 3). If we define "dynamical age" in terms of a meaningful combination of some orbital parameters, naturally q will be an important parameter. There are many ways of defining the dynamical age of a comet. However, in the present case of non-periodic comets, we choose q as the only important parameter and suggest an empirical relation of the type

$$g = Dq^n \tag{10}$$

to find any possible relation between g and q. Here the constants D and n can be determined by first linearising the equation and making a least square fit to our data for the four non-periodic comets viz. Austin, Bradfield, Hyakutake and Levy. In Fig. 5, we plot $\log(-g)$ against $\log(q)$. It is clear that a straight line of the form $\log(-g) = \log(2.5) + \frac{2}{3}\log(q)$ fits the linearised data very well. Thus we can write the following mathematical relations

$$g = -2.5q^{2/3} \tag{11}$$

Table 3. The "relative abundance of coarser grains" (g) for different comets along with their orbital parameters.

Comet	Scatt. angle range (°)	No. of data points	q	T	Reference	Estimated values of a, b, g	Source of pol. data
Austin (1990 V)	72–165 (λ = 485 nm)	6	0.350	∞	IAUC 4972/ MPC 16001	–0.283, –5.24, –1.28	Chernova et al. (1993) Sen et al. (1991b)
	71–117 (λ = 684 nm)	4					
Bradfield (1987 XIII)	124–147 (λ = 485 nm)	7	0.871	∞	IAUC 4442	–0.169, –4.57, –2.20	Kikuchi et al. (1989) Chernova et al. (1993)
Faye (1991n)	154–157 (λ = 485 nm)	4	1.59	7.34	MPC 27081	–0.184, –4.35, –1.77	Chernova et al. (1993)
Hale-Bopp (C/1995 O1)	133–163 (λ = 485 nm)	29	0.914	4000	†	–0.248, –4.82, –1.35	Ganesh et al. (1998) Manset & Bastien (2000)
	133–177 (λ = 684 nm)	57					
Halley (1986 III)	114–178 (λ = 365 nm)	43	0.587	76.1	†	–0.259, –4.42, –0.79	Bastien et al. (1986) Kikuchi et al. (1987)
	114–178 (λ = 485 nm)	71					Le Borgne et al. (1987) Sen et al. (1991a)
	114–162 (λ = 684 nm)	25					Chernova et al. (1993)
Hyakutake	69–143 (λ = 485 nm)	11	0.230	∞	IAUC 6329	–0.257, –4.50, –0.91	Joshi et al. (1997) Kiselev & Velichko (1998)
(1996 B2)	69–143 (λ = 684 nm)	13					
Kopff (1983 XIII)	143–162 (λ = 485 nm)	6	1.59	6.46	MPC 34423	+0.174, –1.23, –3.67	Chernova et al. (1993)
Levy (1990 XX)	122–161 (λ = 485 nm)	16	0.94	∞	†	–0.049, –3.17, –2.48	Chernova et al. (1993)

† Marsden & Williams (1995).

or

$$\frac{\mathrm{d}\log(N(s))}{\mathrm{d}\log s} = -2.5q^{2/3}. \tag{12}$$

This is the simple model which is suggested by our analysis of the data for the four non-periodic comets. However, if one wants to include periodic comets, one can again assume a simple model, where the grain aging is multiplied by the number of times the comet has revolved around the Sun. This can be done by modifying the Eq. (11) to

$$g = -2.5q^{2/3}\frac{1}{(1+(k/T)^m)} \tag{13}$$

where k is a constant having the dimension of time (in years), T is the period of the comet in years and m is some unknown index. Clearly, for non-periodic comets, Eq. (13) reduces to Eq. (11).

At this stage, one may want to determine k and m for periodic comets. Here, we have four comets, viz. Halley, Hale-Bopp, Faye and Kopff. Unfortunately, for comets Faye and Kopff we have only 4 and 6 polarimetric data points respectively, from which to calculate g. Therefore, there is no strong case for including these two comets in our analysis, (see Sect. 2.2). As a result in Fig. 5 we plot the $\log(-g)$ vs. $\log(q)$ values for Halley and Hale-Bopp only.

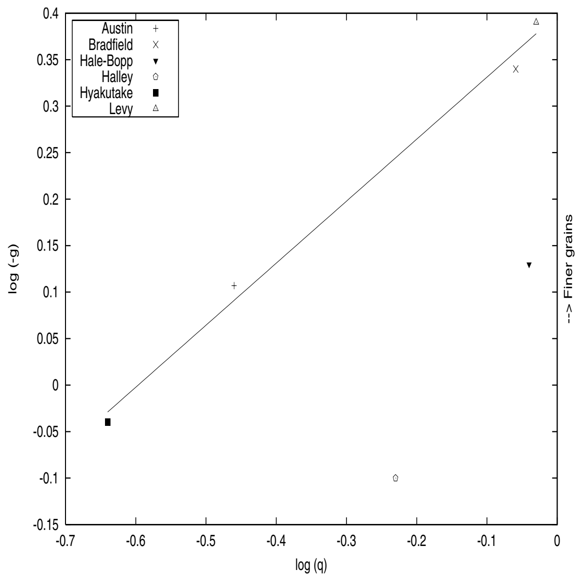

Fig. 5. Log of perihelion distance against log $(-g)$, where g is the relative abundance of coarser grains, as discussed in the paper. The straight line represents the equation $\log(-g) = \log(2.5) + \frac{2}{3}\log(q)$.

However, Halley and Hale-Bopp are two well-studied comets and taking their g values into account we find the values of the two unknowns m and k to be 0.12 and 375 years, respectively. This allows us to write the following equation for periodic comets:

$$g = -2.5q^{2/3} \frac{1}{(1 + (375/T)^{0.12})}. \quad (14)$$

However, the introduction of Eq. (14) at this stage is only exploratory. To suggest a model for periodic comets, one should determine the values of the unknowns m and k from a larger sample of periodic comets.

The meaning of Fig. 5 is as follows: for non-periodic comets we see that the relative abundance of coarser grains g and nearness to the Sun (or perihelion distance q) are related by the equation $g = -2.5q^{2/3}$. Thus, all the non-periodic comets lie along the straight line $\log(-g) = \log(2.5) + \frac{2}{3}\log(q)$. The fit appears to be very good. The effect of the Sun (measured by nearness to the Sun or perihelion distance) clearly causes the relative abundance of coarser grains to increase.

The periodic comets Faye, Kopff, Halley and Hale-Bopp are not expected to fall on this straight line. This is so because here the effect of the Sun is not measured by perihelion distance alone, but also by how many times the comet has revolved around the Sun. For this case another model (Eq. (14)) has been suggested. Thus, short-period comets are expected to deviate more from this straight line than long-period comets. This is exactly what we see in Fig. 5. Comet Hale-Bopp (period 4000 years) seems to be placed closer to the straight line than Halley (period 76 years). The position of the comets Faye and Kopff are not to be taken seriously as there are very few data points corresponding to them. Also these two, being periodic comets, are not expected to fall on the straight line.

4. Discussion

In the present work Mie Scattering theory has been used to match the observed cometary polarisation data. With this theory one can generate polarisation values for light scattered by compact spheres. However, we now know definitely that the cometary particles are "fluffy aggregates" or porous, with irregular shapes. Because of the difficulties involved in the calculations of scattering from porous particles, Mie calculations on spherical particles are widely used as an approximation to the true situation. There have been many recent developments, however, in the field of scattering by porous grains which need to be mentioned here.

Greenberg & Hage (1990) originally proposed the existence of large numbers of porous grains in the coma of comets to explain the spectral emission at 3.4 and 9.7 μm. Model calculations have been done by Hage & Greenberg (1990) for particles with various porosities, sizes and compositions to generate different scattering properties. The typical properties of such porous particles are enhanced absorption and emission features, lower albedo, etc., compared to those of Mie spheres. Dollfus (1989) discussed the results of laboratory experiments by microwave simulation and laser scattering on various complex shapes with different porosities. These results were later compared to the observed polarimetric data on Halley's comet. It was also pointed out that the observed circular polarisation in the coma of various comets could be a good indicator of aligned elongated grains. This view was further strengthened by Rosenbush et al. (1997) when they observed circular polarisation in comet Hale-Bopp. Xing & Hanner (1997) have carried out elaborate calculations with porous grains of various shapes and sizes using the Discrete Dipole Approximation (DDA) method. The polarisation values thus obtained were compared with the observed polarisation data for various comets. The "aggregate structure" considered by them to represent porosity, was found to suppress large amplitude fluctuations in polarisation as observed for single spheres. This work also explained negative polarisation in a more satisfactory manner. Further, it was concluded that the "equivalent volume spheres" is a poor approximation to the polarisations caused by aggregates.

In one recent work Kerola & Larson (2001) used T-matrix formulation to calculate polarisation properties of non-spherical particles and applied the results to the polarimetric measurements of comet Hale-Bopp.

These new approaches with porous aggregates and different shapes in general produced a better fit of the observed polarisation data of various comets. Thus the favoured grain model is now that of "fluffy" grains with irregular shapes, rather than Mie's compact spheres. The fluffiness may change as a function of size, the smaller ones being almost spherical, but the larger ones having a more fluffy structure.

Ideally the calculations presented in this paper should be repeated with such a grain model. But the calculations are very complicated and demand a lot of computer time. Owing to this we leave our present work as it is and admit that the Mie Theory has been used as an approximation only as grains are probably "fluffy". However, we plan a follow-up paper, where calculations will be made assuming porous "fluffy" grains, rather than compact Mie spheres as has been presently done.

We also note that any grain model which is suggested to explain cometary polarisation should also be able to explain the Spectral Energy Distribution (SED) in the Near Infra Red (NIR) part of the spectrum. The cometary grain size distribution function as discussed in the present work (with a possible dependence on the dynamical age of the comet, in terms of grain aging) should also have some implications for the observed SED in the NIR region. Recent work by Hanner & Hayward (2003) discusses clearly the role of dust size distributions on the NIR flux. The different slopes in the grain size distributions as considered by the authors can be related to our g parameter which expresses the richness of coarser grains. As discussed by Hanner & Hayward (2003), small grains are hotter and they contribute more to the total emission. According to our present work, the dynamically newer comets are richer in fine grains and thus one should now be able to distinguish them in terms of their NIR flux. A systematic approach in this direction is beyond the scope of the present work. We can only mention the possible areas where this problem can be further addressed.

5. Conclusions

Based on the in situ spacecraft dust measurements and ground based polarimetric observations of Halley and also on the ground-based observations of other comets in the post-Halley era, we have arrived at the following conclusions from our present work:

1. The dust size distribution function $N(s)$ (with a bulk density of dust = 2.2 g cm^{-3}) for Halley has been derived in this work following Lamy et al. (1987), which is

   $$\log N(s) = -0.2593(\log s)^2 - 4.422(\log s) - 15.06.$$

2. The complex refractive indices of Halley's grains as derived from present work are: (1.403, 0.024), (1.390, 0.026) and (1.386, 0.038) at wavelengths 0.365, 0.485. 0.684 μm, respectively.
3. From the present work we derive individual grain size distributions of several observed comets. Assuming all the comets to have the same composition of grains, individual grain size distribution functions can be set up for them. Equation (8) represents the grain size distribution of an individual comet, where values of the constants a and b are tabulated in Table 3.
4. We introduce and estimate a parameter $g(= -14a + b)$ which signifies the relative abundance of coarser grains in a comet, and based on the data for four non-periodic comets, we establish that

   $$g = -2.5q^{2/3}$$

   where "q" is the perihelion distance. This mathematical relation quantifies the effect of solar radiation (in terms of orbital parameter or dynamical age) on the grain size distribution.
5. For periodic comets one can also derive an expression for g in terms of the orbital parameters as in Eq. (14). However, data on more periodic comets are needed to determine any such model in a more definite way.
6. The above model is suggested based on calculations which use Mie scattering theory, meaningful only for ideal compact spheres. However, as cometary grains are probably "fluffy", we plan a follow-up paper where calculations will be repeated with more realistic "fluffy" grains.

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