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From Jupiter-family to Encke-like orbits

The rôle of non-gravitational forces and resonances

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Abstract. We investigate numerically the transfer routes from Jupiter-family towards Encke-like cometary orbits, including in the model all the planets as well as non-gravitational forces. The numerical integrations are started from orbital elements similar to those of 2P/Encke, changing the perihelion distance q, to obtain starting orbits in the Jupiter family, and the non-gravitational parameter A_2. The results show that some of the model orbits reach the Encke-like stage within a reasonable time, comparable to a typical active cometary lifetime; along the way, at the crossing of mean motion resonances with Jupiter, temporary captures in resonance may occur. Thus, resonances and non-gravitational forces appear to be key factors in the transfer of orbits from the Jupiter family to the Encke region.

Key words. comets: general – comets: individual: 2P/Encke

1. Introduction

Comet 2P/Encke was discovered by P. F. A. Méchain, as a faint naked-eye object, at the Paris Observatory on 17 January 1786. Up to 2004 it passed the perihelion 67 times, 59 of which were observed. Since 1970 the comet has been observed throughout the orbit, and it has the highest number of observed perihelion passages of all periodic comets.

Within the period covered by observations, the orbital elements changed slowly: the argument of perihelion ω from 182° to 186°, the longitude of the ascending node Ω from 337° to 334°, the inclination i from 14° to 12°, while the eccentricity e varied between 0.845 and 0.850 and the perihelion distance q between 0.331 and 0.346 AU. Small perturbations were caused by encounters with Mercury and Jupiter; the closest encounter with Mercury, at a distance of 0.038 AU, took place on 22 November 1848, and the closest one with Jupiter, at a distance of 0.906 AU, on 30 May 1903.

The orbit of the comet is affected by non-gravitational (NG) forces that vary with time; the style II parameter A_1 and A_2 varied, respectively, between −2.17 and +1.14, and between −0.0400 and −0.0016 during the time span covered by observations (Marsden & Williams 1996).

For a long time 2P/Encke was the only periodic comet known with an orbit not allowing deep encounters with Jupiter (Kresák 1979), as also evidenced by the very high value (3.03) of its Tisserand parameter. A significant number of recently discovered small asteroids have orbits similar to that of this comet (Steel et al. 1991); gravitational routes exist that connect the asteroid belt to this type of orbit within a reasonable time span (Valsecchi et al. 1995). On the other hand, the corresponding route from the Jupiter family found by Valsecchi et al. (1995) takes about 10^5 yr, a rather large time compared to the generally accepted physical lifetime of a periodic comet. The computed total mass loss of the comet indicates an active lifetime of between 600 and 1100 revolutions (Whipple & Sekanina 1979), i.e. less than 5000 years of activity; to reconcile this with the timescale for the gravitational transfer from the Jupiter family implies that for most of the time the comet should have been dormant.

A number of studies have been dedicated to the problem of transferring the orbit from the Jupiter family to a completely cis-jovian one. There are two possible sources for the orbital energy needed, namely NG forces and close encounters with the planets.

Regarding the latter, it is clear that, if only encounters with Jupiter take place, then the difficulty would lie in the fact that, after a decelerating close encounter with Jupiter, the orbit of the comet would in general still be coupled to the Jovian orbit, allowing further encounters to take place. Since each of these further encounters could be either decelerating or accelerating, depending on the exact timing of the approach, it would take a very long time before some substantial deceleration of the motion (i.e., reduction of a) would take place, leaving aside the

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question of how to decouple the aphelion of the comet from the Jovian orbit.

As far as encounters with the inner planets are concerned, the main difficulty is constituted by the small masses of these planets, coupled with the rather high encounter velocities. As an example, with the help of Öpik's theory of close encounters we can estimate how close to the Earth 2P/Encke should pass in order to have its aphelion distance raised to 4.4 AU, to become coupled to the orbit of Jupiter. Using the formulae of Valsecchi et al. (2000), we find that the maximum distance from the Earth for a close encounter having such an effect is of about 9 Earth radii; this is also the maximum distance for a close encounter *having the opposite effect*, i.e. lowering the aphelion distance from 4.4 AU down to the present level of about 4.1 AU. Such encounters are very rare, taking place on time scales of millions of years.

Steel & Asher (1996) have investigated the possibility that the cis-Jovian orbit of the 2P/Encke was achieved through the action of NG forces. For this purpose, they integrated the motion of a number of fictitious objects in initial orbits identical to those of 2P/Encke and (6063) Jason, and differing only in the mean anomaly. The goal of the integration was to reach a situation in which the orbit of the fictitious body would become coupled to that of Jupiter, and they used, in the numerical modeling, randomly oriented NG forces whose strength was up to about four times larger than those observed for 2P/Encke. For NG forces of a strength similar to that currently observed, their model orbits were not altered enough to achieve coupling to the orbit of Jupiter.

Two comments on Steel and Asher's work are in order. First, their integrations probed the *opposite* orbital evolution with respect to the one we are discussing, i.e. from Encke-like to Jupiter family orbits. Second, and possibly more important, their modeling of the NG forces consisted of applying, every 170 yr, an instantaneous change to the semimajor axis of the orbit of size

$$\Delta a = \sqrt{2}X \sin\theta, \tag{1}$$

where θ is a random angle in the range 0°–360°, and X is either 0.0037 AU, as observed for 2P/Encke, or 0.0137 AU.

Also Fernández et al. (2002) investigated the problem of the decoupling of cometary aphelia from the orbit of Jupiter, and concluded that it is very difficult to produce what they call "Encke-type" orbits by purely gravitational mechanisms within a reasonable time span. They integrated all the known Jupiter family comets forward in time, over 2 million years, and found only one comet, 1999 WJ_7 P/Korlevic, whose orbital evolution without NG forces leads to such an orbit. They obtained a much higher rate of success in computations in which NG forces also were present.

Their NG force model consists of adding, during the integration, a quantity δv to the modulus of the velocity v, such that

$$\delta v = A \sin\left[\theta_0 + \frac{2\pi(t - t_0)}{T}\right], \tag{2}$$

where δv is the change in velocity in the interval of time $(t, t + \delta t)$, θ_0 is the phase angle angle at the initial time t_0, T is the timescale for the sinusoidal variation of the NG force, and A is the amplitude of variation of this force, given by

$$A = 10^{-6}\frac{(GM_\odot)^2}{va^{3/2}}\delta t. \tag{3}$$

Fernández et al.'s formulation is that of a continuously acting NG force, at variance to Steel and Asher's formulation, and is thus much closer, although presumably still not equivalent, to the standard treatment based on Marsden et al. (1973). On the other hand, for Fernández et al. the region containing Encke-like orbits is rather wide, encompassing orbits with $q < 1.3$ AU and $Q < 4.2$ AU; in the present paper, we use a smaller region, delimited by $q < 0.4$ AU and $Q < 4.2$ AU.

The purpose of this paper is to test the effectiveness of NG forces, in the formulation of Marsden et al. (1973), in allowing the transfer from a Jupiter-family orbit to the abovementioned region of Encke-like orbits ($q < 0.4$ AU and $Q < 4.2$ AU) within a time span of less than 10^5 yr. We do this by means of a number of numerical integrations, exploring a region of initial element space from which the transfer to an Encke-like orbit can be achieved by a simple shortening of a; thus, we start from orbits that have all the elements equal to those of 2P/Encke, apart from the perihelion distance q, for which we choose values large enough to make the orbit well coupled to that of Jupiter.

Within the period covered by observations the NG parameters of 2P/Encke exhibited significant changes (Marsden & Williams 1996). However, in investigations of the long-term behaviour of fictitious orbits, the short-term variations are obviously not known outside the observational period; we have therefore limited ourselves to the use of constant values for them. We could have introduced NG forces varying with time, but the intrinsic arbitrariness of such a procedure would have made it a doubtful improvement over the constant NG parameter strategy.

Table 1. Elements of comet 2P/Encke.

T	1994 Feb. 09.4778
q	0.330915 AU
a	2.20923712 AU
e	0.85021300
P	3.28 yr
ω	186.2703°
Ω	334.7295°
i	11.9405°
Epoch	1994 Feb. 17
A_1	−0.00
A_2	−0.04

2. The numerical model

For our modeling we used the starting elements of 2P/Encke given in Table 1 (Marsden & Williams 1996).

These elements were slightly changed from case to case, increasing q so as to make the orbit belong to the Jupiter family. The numerical integrations of the model orbits were done over

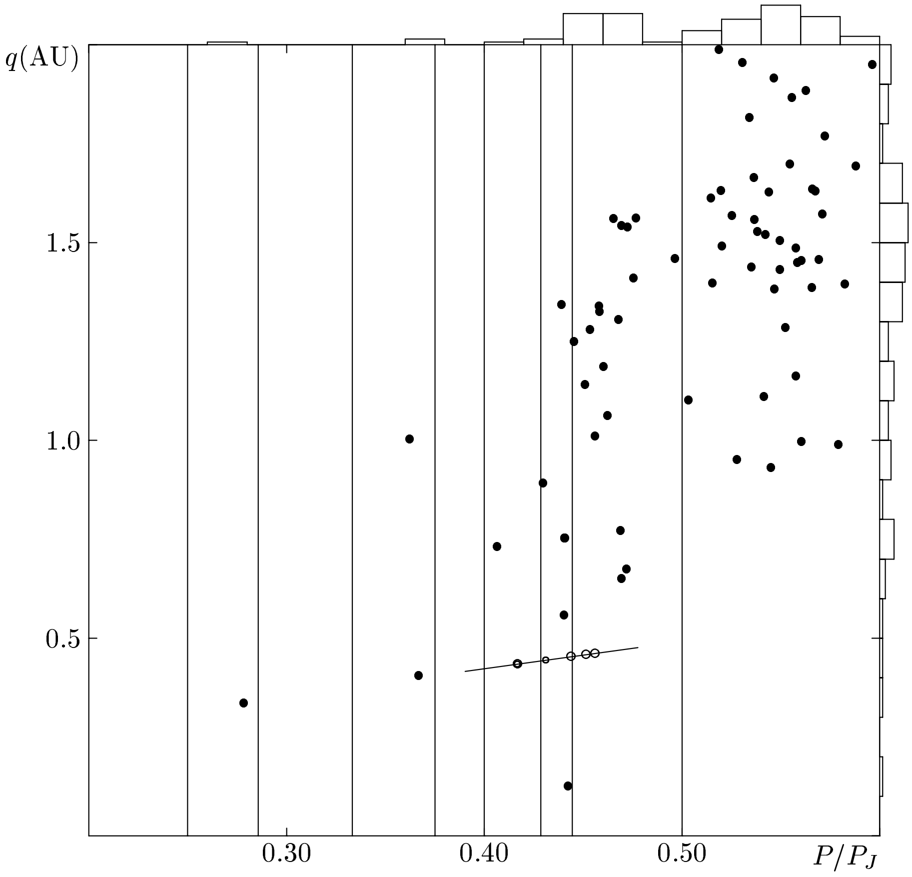

Fig. 1. Plot of the perihelion distances q versus periods (in units of the period of Jupiter) P/P_J for the Jupiter family comets. All the model orbits computed by us lie on the diagonal segment; on the latter, circles denote initial conditions that led to Encke-like orbits: the larger circles denote a transfer time shorter than 20 000 yr, the smaller ones a transfer time shorter than 45 000 yr. On the top and on the right hand side are reported the histograms of the distributions of P/P_J and of q, respectively, for the Jupiter family of comets.

a time span of 16 000 000 d (about 44 000 yr), starting from 1994 Feb. 17.0; osculating elements were output at intervals of 1000 d.

The gravitational model included all the planets; the numerical integration made use of Everhart's RADAU integrator at the 15th order (Everhart 1985), and of the NG force formulation of Marsden et al. (1973).

We tried many different sets of initial conditions, varying the perihelion distance q and the NG parameter A_2; the value of q was changed at the fifth place and A_2 at 2th place after the decimal point. Moreover, the values used for the NG parameter A_2 did not exceed the observed ones. The NG parameter A_1 was in every case put to zero; the reason for this choice is that the secular effect of A_1 on the evolution of the semimajor axis is very small, even for values of this parameter as high as those estimated from observations of 2P/Encke (between −2.17 and 1.14, see Marsden & Williams 1996).

Our initial model orbits cover the region of initial perihelion distance $0.416 < q < 0.476$ AU. This region was selected because it contains a number of known Jupiter-family comets; in it, the value of the Tisserand parameter is about 2.5, a rather typical value for the Jupiter family.

Figure 1 shows the region of our initial conditions, together with the comets of the Jupiter family; it was compiled with data taken from Marsden & Williams (1996). The vertical lines represent, from left to right, the 4/1, 7/2, 3/1, 8/3, 5/2, 7/3, 9/4, and 2/1 mean motion resonances.

Surrounding the region of our initial conditions there is a number of known comets, namely D/1776 G1 Helfenzrieder (q = 0.406, P/P_J = 0.367), 96P/Machholz 1 (0.126, 0.443), 45P/Honda-Mrkos-Pajdušáková (0.559, 0.440), 5D/Brorsen (0.650, 0.470), and D/1770 L1 Lexell (0.674, 0.472). Only two of them have smaller q than those of our model orbits: 96P/Machholz 1 (q = 0.126) and D/1776 G1 Helfenzrieder (q = 0.406). On the other hand, although for higher q the number of observed comets increases substantially, the orbital energy change necessary for the transfer from the Jupiter family to Encke-like orbits grows so much as to render the task of NG forces very hard to accomplish.

3. Results of the integrations

The long-term orbital evolution of our model orbits showed a high sensitivity to small changes of the initial q. Combining the latter with suitable changes of the NG parameter A_2, it is possible to obtain various different evolutions of the model orbits; the evolutions of some of them lead, after some time, to orbits very similar to that of 2P/Encke.

On the basis of our numerical integrations, it is not possible to identify a well defined specific region from which the Encke-like dynamical stage can be reached within a reasonable time. As it is possible to see in Fig. 1, our successful cases are more or less randomly spread within the interval investigated.

The evolution of the three model bodies that gave the best results is shown in Figs. 2–4, while in Fig. 5 is shown a typical

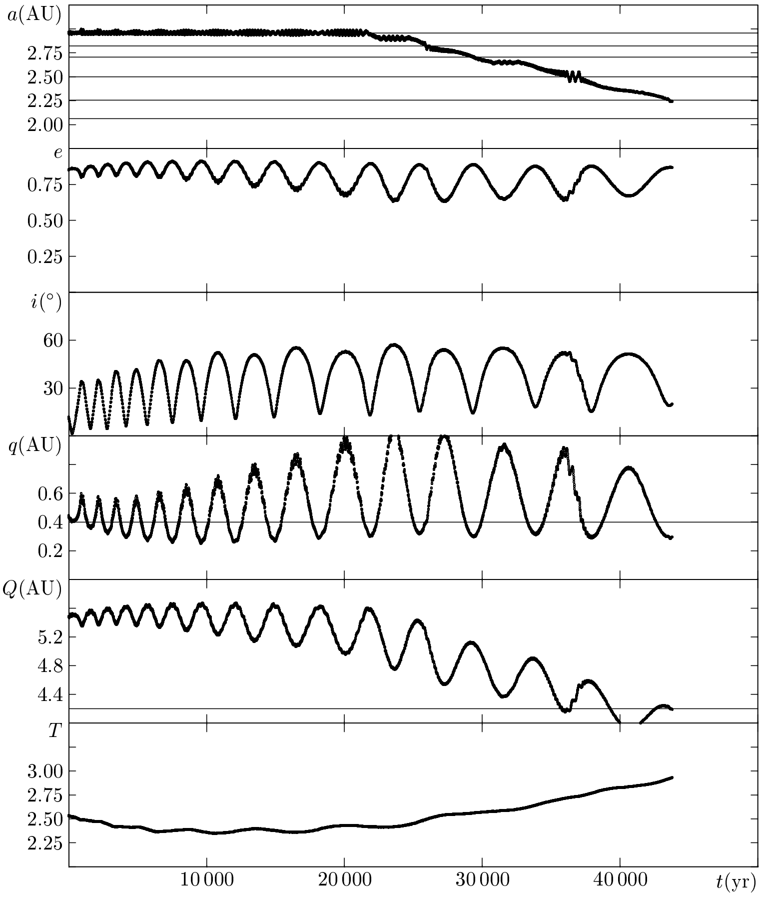

Fig. 2. Time evolution of a (*top*), e, i, q, heliocentric distances at the nodes, and T (*bottom*) for the model orbit with initial orbital elements as in Table 1, but with initial q = 0.444406 AU and $A_2 = -0.04$, over a time span of 16 000 000 d. In the semi-major axis plot, horizontal lines mark (from top to bottom) the 7/3, 5/2, 8/3, 3/1, 7/2, and 4/1 mean motion resonances with Jupiter; in the perihelion and aphelion distances plots, horizontal lines mark the boundary of the region occupied by Encke-like orbits, at q = 0.4 AU, and at Q = 4.2 AU.

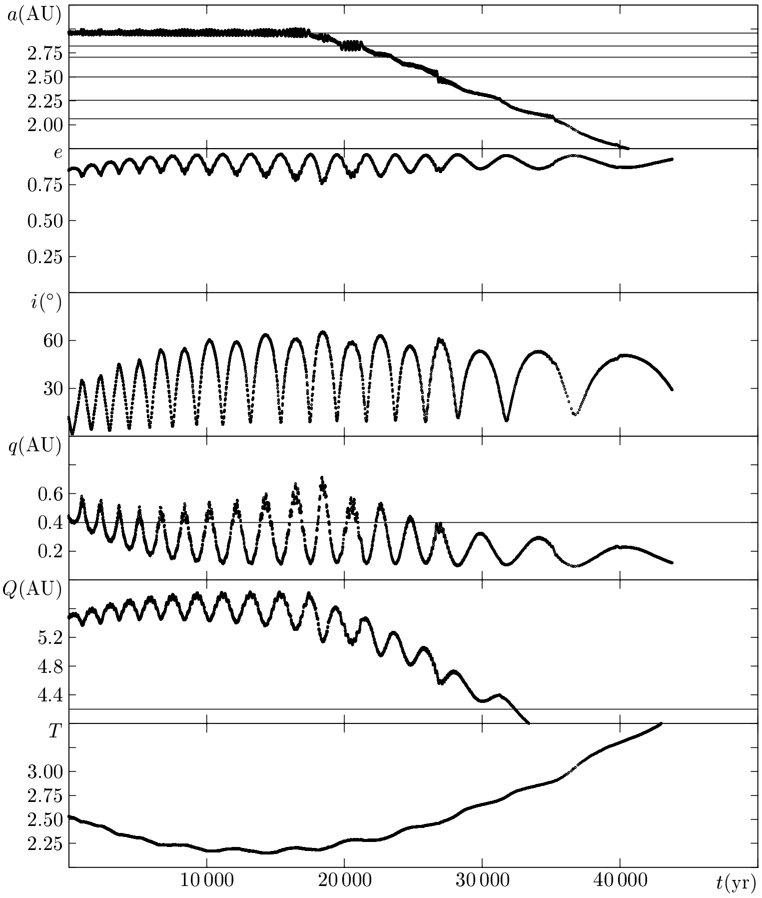

Fig. 3. Same as Fig. 2 for the model orbit with initial q = 0.444386 AU and $A_2 = -0.02$.

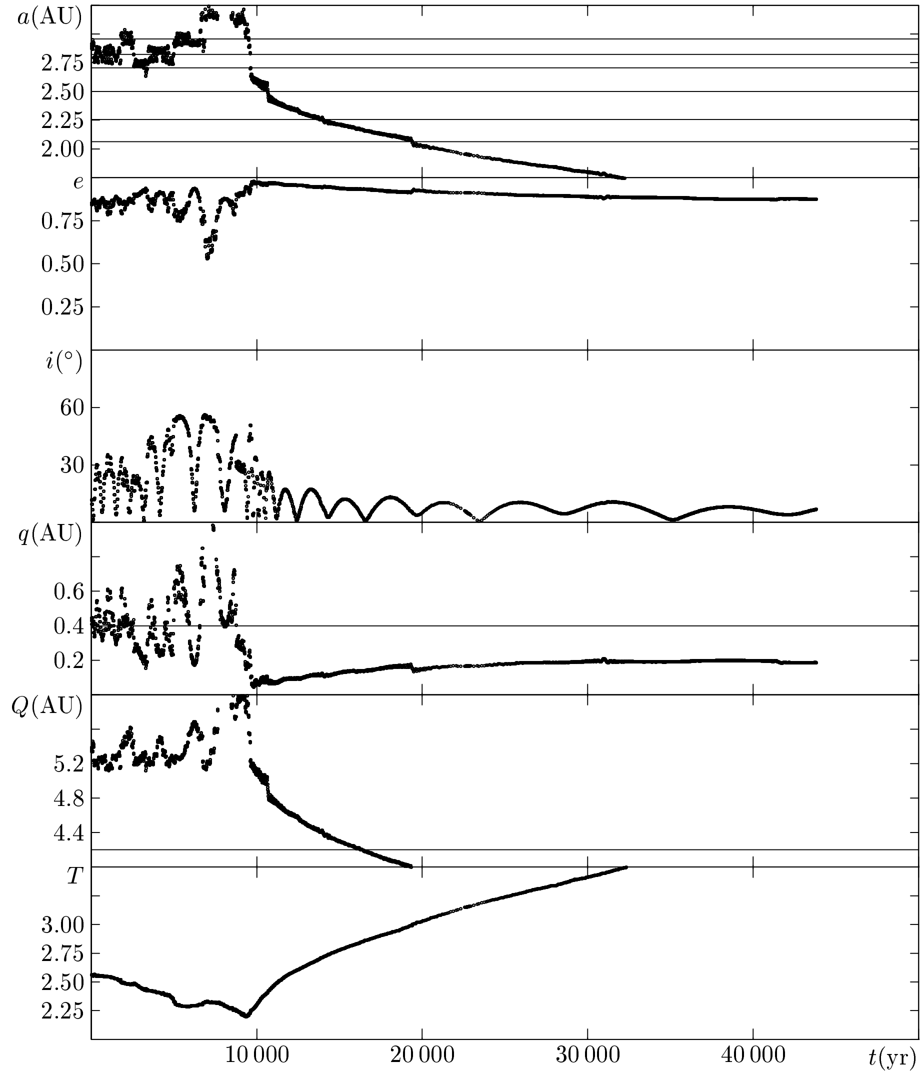

Fig. 4. Same as Fig. 2 for the model orbit with initial $q = 0.434550$ AU and $A_2 = -0.01$.

unsuccessful case. In each figure we show the time evolution of, from top to bottom, the semimajor axis a, eccentricity e, inclination i, perihelion and aphelion distances q and Q, and the Tisserand parameter T.

The object of Fig. 2 had initial $q = 0.444406$ and $A_2 = -0.04$, that of Fig. 3 $q = 0.444386$ and $A_2 = -0.02$, that of Fig. 4 $q = 0.434550$ and $A_2 = -0.01$.

These orbital evolutions are characterized not only by the coupled oscillations of e and i visible in the Figures, induced by planetary secular perturbations, but also by a secular decrease of the semi-major axis a due to the NG forces; it is noteworthy that they become Encke-like within a reasonable time span, compatible with the estimated lifetimes of periodic comets as active objects. Moreover, it is quite noticeable in the plots that the crossing of the resonance conditions, during the secular decrease of the semimajor axis, is sometimes characterized by a temporary permanence in the resonance.

In the case of the model orbit shown in Fig. 2, the Encke-like stage is reached after 45 000 years, while in the case of Fig. 3 it is reached within 32 000 years. In both cases the evolution starts within the 7/3 mean motion resonance, that presumably helps in avoiding the occurrence of close encounters with Jupiter that would disrupt the resonant regime and send the comet elsewhere; during this phase, the increase in the amplitude of the oscillations of e, coupled with the near-constancy of a due to the absence of close encounters, makes the perihelion distance q reach values lower than the initial one, thus making the NG forces more effective. After some time, the comet reaches the lower border (in semimajor axis) of the resonance, and eventually crosses it, starting to evolve more rapidly towards smaller semimajor axis since the effect of NG forces is not any more compensated by resonant perturbations. The rest of the evolution is significantly affected by the crossings of the other mean motion resonances, encountered in sequence.

The model orbit of Fig. 4 reaches the Encke-like stage after only 20 000 years, through a much more irregular evolution. This object starts below the 7/3 resonance, and undergoes some close encounters that lead to a significant increase of the aphelion distance; however, at that point the perihelion distance is so low that the NG forces are able to quickly lower the semimajor axis.

Starting from the same initial conditions of the model orbit of Fig. 2, but setting the NG parameter $A_2 = 0$, we obtain the evolution shown in Fig. 5. The object initially remains in the

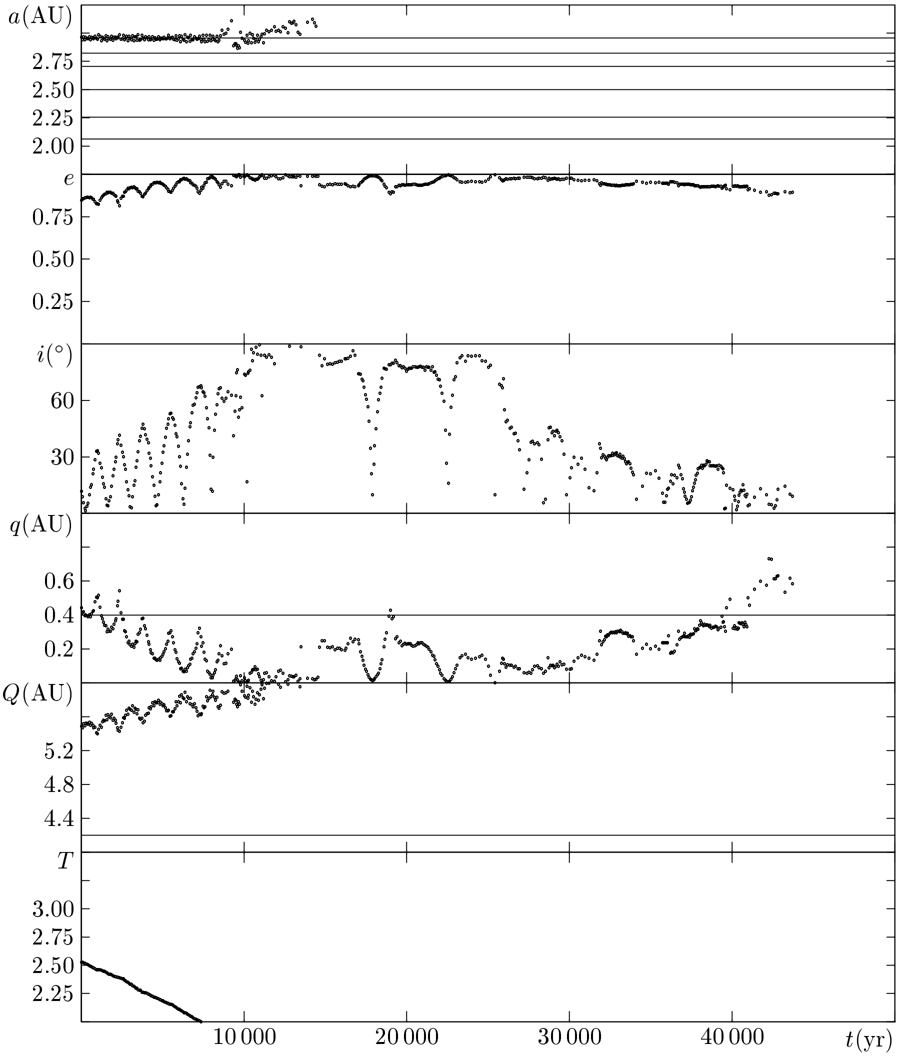

Fig. 5. Same as Fig. 2 for the model orbit initial $q = 0.444406$ AU and $A_2 = 0$.

resonant initial orbit, and its eccentricity grows significantly, allowing q to become very small and Q to go well beyond the orbit of Jupiter; the inclination becomes very high, and the Tisserand parameter becomes smaller than 2, so that the comets does not belong any more to the Jupiter family. Encounters with Jupiter then remove the comet from the inner planetary region.

In other computer experiments we obtained completely different evolutions; in some cases the eccentricity and the inclinations grew dramatically, and led to extreme evolutions without reaching the Encke-like stage; other model bodies did not reach the Encke-like stage within the investigated period and continued to move on relatively stable orbits. We did not find cases in which the Encke-like stage was reached without the action of NG forces; this suggests that these forces play a very important rôle for the evolution of cometary orbits from the Jupiter family into Encke-like orbits.

4. Discussion and conclusions

Our successful cases are characterized by a similar evolution of the orbital elements, in which the semimajor axes and perihelion distances decrease more or less continuously, reaching within a few 10^4 yr the values characterizing the orbit of 2P/Encke. The timescale for this transfer thus appears to be compatible with current estimates of the lifetime of comets as active objects, and our computations, like other studies (Steel & Asher 1996; Fernández et al. 2002), show the necessity of NG forces for the transfer to take place reasonably quickly.

We have analyzed in detail the orbital histories of our model orbits, with the goal of understanding the key factors that affect the transition to Encke-like orbits.

As exemplified by our Figs. 2– 4, the successful evolutions with NG forces are strongly affected by mean motion resonances with Jupiter, in various forms: an object starting its evolution in resonance may be protected by the latter from encounters with Jupiter, thus allowing the lowering of the perihelion distance to the point in which the strength of NG forces significantly affects the evolution; moreover, the crossing of some of the resonances encountered along the way is slowed by temporary captures in resonances, while in other cases the effect of the resonance crossing is rather an acceleration of the time derivative of the semimajor axis.

Figure 6 shows an enlarged view of the evolution of the semimajor axis of the model orbits of Figs. 2 and 3; the rate of variation of the semimajor axis is affected by the presence of mean motion resonances. In particular, we can compare the

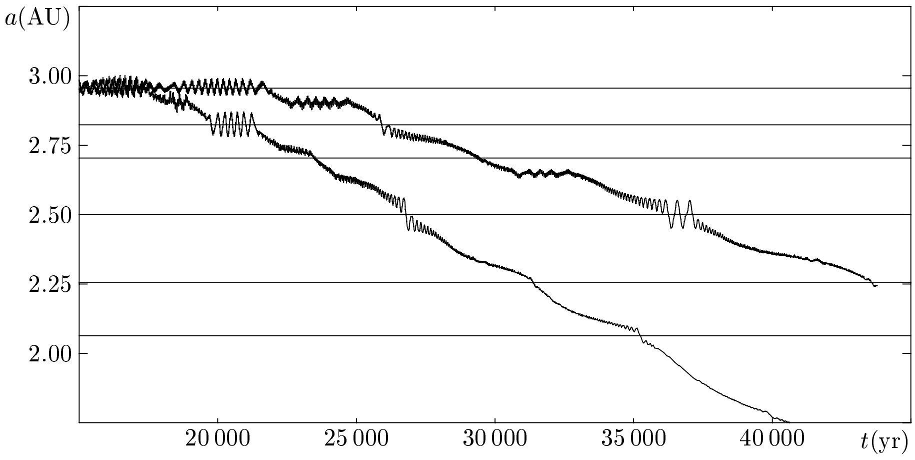

Fig. 6. Enlarged view of the time evolution of the semimajor axis of the model objects of Figs. 2 and 3.

crossing of the 3/1 resonance (at $a \approx 2.5$ AU) of the two objects; for one of them, a temporary capture in resonance occurs, while for the other a quick transition of the semimajor axis from above to below the resonance is clearly recognizable.

Actually, small changes in the NG parameters and/or in the initial conditions of the small body are likely to affect the time spent close to each resonance, and therefore to affect the possibility of a succesful orbital evolution to Encke-like orbits, within a reasonably short time span. The role of resonances appears to be that of temporarily protecting the comet from encounters with Jupiter, allowing the increase in eccentricity that, in turn, leads to stronger NG forces (for constant a, larger e means a decrease of q), thus furthering the inward evolution.

While several previous studies underlined the importance played by NG forces in this problem, the important role of resonances that has just been described seems to have escaped attention so far. We plan to do a systematic study of resonant dynamics in the case of non-negligible NG forces; this study appears to be necessary to be able to assess the probability of transition from the Jupiter family to Encke-like orbits.

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