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Letter to the Editor

Neutron star oceans: Instability, mixing, and heat transport

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Abstract. Stability of the ocean of magnetic neutron stars is considered. We argued that the ocean is unstable if the temperature varies along the surface. The instability grows on a time scale ~0.1−100 s depending on the lengthscale of perturbations and generates a weak turbulence. Turbulence can be responsible for mixing between the surface and deep ocean layers and can enhance heat transport in the surface region.

Key words. MHD – convection – stars: neutron – instabilities

1. Introduction

The structure of the surface layer in neutron stars (NSs) is of general interest to studies of thermal radiation and NS cooling. The matter is likely in a melted state in the surface layer and forms a liquid ocean. The depth of this ocean depends crucially on the temperature and chemical composition and varies within a wide range. Crystallization of liquid occurs when the ion coupling parameter $\Gamma = Z^2e^2/(ak_BT)$ reaches the critical value $\Gamma = \Gamma_m \approx 170$ (Slattery et al. 1980); $a = (3/4\pi n_i)^{1/3}$, n_i and Z are the number density and charge of ions; T is the temperature, and k_B is the Boltzmann constant. Then, the crystallization temperature is

$$T_m = \frac{Z^2e^2}{ak_B\Gamma_m} \approx 1.3 \times 10^5 Z^{5/3} x^{1/3} \left(\frac{170}{\Gamma_m}\right) \text{ K}, \quad (1)$$

where $x = Z\rho/A \times 10^6$ g/cm^3, A is the atomic number of ions. In NSs with the surface temperature $\geq 10^6$ that are of interest to studies of thermal radiation, crystallization occurs at the density $\rho \geq 10^9$ g/cm^3 if $Z \leq 26$. The depth from the surface, h, is related to ρ by $x^{2/3} \approx h(h + 2H)/H^2$ (Urpin & Yakovlev 1979) where $H = Zm_ec^2/Agm_p$ and g is the gravity. For the "standard" NS, $H \sim 10$ m, and the density 10^9 g/cm^3 corresponds to the depth ~100 m.

Hydrodynamic phenomena in the ocean play an important role in the magnetic and thermal evolution, generation of quasi-periodic oscillations (Bildsten & Cutler 1995; Bildsten et al. 1996) and r-modes (Yoshida & Lee 2001, 2002), and in mixing. The spectral modelling is usually based on the assumption that NS atmospheres are hydrogen (see, e.g., Zavlin & Pavlov 2002). The ocean, however, cannot be entirely hydrogen. Hydrodynamic motions can mix the ocean and transport heavy elements to the surface. Recently, Sanwal et al. (2002) reported the first detection of absorbtion features in an isolated NS, 1E 1207.4-5209. Their analysis demonstrates that the atmosphere cannot be hydrogen. Applying the atomic model for magnetized NS atmospheres, Hailey & Mori (2002) found that the features can be associated to He-like oxygen or neon. The presence of heavy elements in the atmosphere can naturally be explained by mixing with deep ocean layers. Despite the lines of heavy elements have not been observed yet in most isolated NSs these features can be a general phenomenon if mixing is efficient.

Mixing is oftenly atributed to hydrodynamic instabilities, e.g., convection. The standard convection arises if ∇T is superadiabatic and occurs only in the atmosphere of hot NSs with a weak magnetic field (Miralles et al. 1997). Likely, convection is not the only instability that occurs in NSs. Stability properties of the ocean can be complicated because of a strong magnetic field, $B \geq 10^{12}$ G, and the temperature gradient that is not parallel to $\boldsymbol{g}$. Generally, anisotropy caused by the magnetic field can be the reason of diffusive instabilities (see, e.g., Balbus 2000). Note that the magnetic field in NSs can have a spot-like structure with a small scale field being stronger than the mean field (Urpin & Gil 2003) that provides an additional complexity. In this paper, we consider stability of the NS ocean and argue that non-parallel ∇T and $\boldsymbol{g}$ should be the reason of instability that can mix the surface layers.

2. The dispersion equation

Consider a linear stability of a magnetic NS ocean assuming that the temperature departs from the spherical symmetry in the unperturbed state. We do not specify the mechanism responsible for these departures but it can be, for example, anisotropic heat transport in the magnetic field (see, e.g., Schaaf 1990; Potekhin & Yakovlev 2001) or heating due to relativistic

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particles created in the pulsar's acceleration zones and bombarding the NS surface (Ruderman & Sutherland 1975). Since the gravity, $\boldsymbol{g}$, is approximately radial in the ocean, ∇T can generally be non-parallel to $\boldsymbol{g}$. In this *Letter*, we neglect the influence of a compositional gradient on stability and use the Boussinesq approximation since the growth time of instability is longer than the period of sound waves (see, e.g., Landau & Lifshitz 1959). Then, the equations governing the velocity, magnetic field and thermal balance are

$$\dot{\boldsymbol{v}} + (\boldsymbol{v} \cdot \nabla)\boldsymbol{v} = -\frac{\nabla p}{\rho} + \boldsymbol{g} + \frac{1}{4\pi\rho}(\nabla \times \boldsymbol{B}) \times \boldsymbol{B}, \quad (2)$$

$$\nabla \cdot (\rho \boldsymbol{v}) = 0, \quad (3)$$

$$\dot{\boldsymbol{B}} - \nabla \times (\boldsymbol{v} \times \boldsymbol{B}) = -\nabla \times [\hat{\eta} \cdot (\nabla \times \boldsymbol{B})], \quad (4)$$

$$\nabla \cdot \boldsymbol{B} = 0, \quad (5)$$

$$\dot{T} + \boldsymbol{v} \cdot (\Delta \nabla T) = \nabla \cdot (\hat{\chi} \cdot \nabla T), \quad (6)$$

where $\Delta \nabla T = \nabla T - \nabla_{\rm ad} T$ is a difference of the real and adiabatic temperature gradients; $\hat{\chi} = \hat{\kappa}/\rho c_{\rm p}$, $\hat{\eta} = c^2 \hat{R}/4\pi$; $\hat{\kappa}$ and $\hat{R}$ are tensors of the thermal conductivity and electrical resistivity, and $c_{\rm p}$ is the specific heat at p = const. Following tensor operations on Eqs. (4) and (6), we have

$$\hat{\chi} \cdot \nabla T = \chi_{\parallel} \nabla_{\parallel} T + \chi_{\perp} \nabla_{\perp} T + \chi_{\wedge} \boldsymbol{b} \times \nabla T,$$

where $\chi_{\parallel,\perp}$ are the tensor components along and across $\boldsymbol{B}$, $\chi_{\wedge}$ is the Hall component; $\boldsymbol{b} = \boldsymbol{B}/B$. An analogous expression can be written for $\hat{\eta}$. Note that $\eta_{\parallel} = \eta_{\perp} = \eta_{\wedge}/\alpha$ where α is the Hall parameter that generally can be large,

$$\alpha = \Omega_{\rm Be} \tau \approx \frac{9.9 \times 10^3 B_{13}}{Z\Lambda(1 + x^{2/3})}, \quad (7)$$

$\Omega_{\rm Be}$ and τ are the electron gyrofrequency and relaxation time; $B_{13} = B/10^{13}$G, $\Lambda \approx 2$ is the Coulomb logarithm.

We neglect viscosity in the momentum Eq. (2). Generally, both electrons and ions contribute to the shear viscosity. The ion viscosity is dominating in a low density region but is smaller than η and χ. The electron viscosity is important in deep layers (Itoh et al. 1987). The ratio of the electron viscosity and magnetic diffusivity is

$$\frac{\nu_{\rm e}}{\eta_{\parallel}} = \frac{2.2 \times 10^2}{AZ\Lambda} \frac{x^{5/3}}{(1 + x^{2/3})^2}. \quad (8)$$

Viscosity becomes comparable to $\eta_{\parallel}$ only if $\rho \geq 10^8$–10^9 g/cm^3, and the ocean consists of light elements. Therefore, neglecting viscosity in Eq. (2) is qualitatively justified.

The unperturbed ocean is in hydrostatic equilibrium,

$$\frac{\nabla p}{\rho} = \boldsymbol{G} = \boldsymbol{g} + \frac{1}{4\pi\rho}(\nabla \times \boldsymbol{B}) \times \boldsymbol{B}. \quad (9)$$

Taking the curl of this equation, we have

$$\nabla \rho \times \boldsymbol{G} = -\frac{1}{4\pi} \nabla \times [(\nabla \times \boldsymbol{B}) \times \boldsymbol{B}]\,. \quad (10)$$

Hence, $\nabla \rho$ and ∇T have components perpendicular to $\boldsymbol{G}$ only if $\boldsymbol{B}$ is not force-free in the ocean.

Equations governing small perturbations can be obtained by linearization of Eqs. (2)–(6). The linearized equations should be complemented by the equation of state linking perturbations of the pressure, density, and temperature. Since in the Boussinesq approximation perturbations of the pressure are small, the density perturbation, ρ_1, can be expressed in terms of the temperature perturbation, T_1, by $\rho_1 \approx -\rho\beta T_1/T$ where $\beta = -(\partial \ln \rho / \partial \ln T)_p$ is the coefficient of thermal expansion.

We consider short-wavelength perturbations with spatial and temporal dependence $\exp(\gamma t - i\boldsymbol{k} \cdot \boldsymbol{r})$ where $\boldsymbol{k}$ is the wavevector. Then, the dispersion equation corresponding to the linearized Eqs. (2)–(6) reads

$$\gamma^5 + a_4 \gamma^4 + a_3 \gamma^3 + a_2 \gamma^2 + a_1 \gamma + a_0 = 0\,, \quad (11)$$

where

$$a_4 = \omega_\chi + 2\omega_\eta,$$
$$a_3 = \omega_\eta^2 + \omega_\wedge^2 + 2\omega_\eta \omega_\chi + 2\omega_{\rm A}^2 - \omega_{\rm g}^2,$$
$$a_2 = \omega_\chi \left(\omega_\eta^2 + \omega_\wedge^2 + 2\omega_{\rm A}^2\right) + 2\omega_\eta \left(\omega_{\rm A}^2 - \omega_{\rm g}^2\right),$$
$$a_1 = \omega_{\rm A}^4 + 2\omega_\eta \omega_\chi \omega_{\rm A}^2 - \omega_{\rm g}^2 \left(\omega_\eta^2 + \omega_\wedge^2 + \omega_{\rm A}^2\right),$$
$$a_0 = \omega_{\rm A}^2 \left(\omega_\chi \omega_{\rm A}^2 - \omega_\eta \omega_{\rm g}^2 - \omega_\eta \omega_{\rm gH}^2\right),$$

and characteristic frequencies are given by

$$\omega_\eta = \eta_{\parallel} k^2\,, \quad \omega_\chi = \chi_{\perp} k^2 + (\chi_{\parallel} - \chi_{\perp})(\boldsymbol{k} \cdot \boldsymbol{b})^2\,,$$
$$\omega_\wedge = \eta_\wedge k(\boldsymbol{k} \cdot \boldsymbol{b})\,, \quad \omega_{\rm A} = (\boldsymbol{k} \cdot \boldsymbol{B})/\sqrt{4\pi\rho}\,,$$
$$\omega_{\rm g}^2 = \frac{\beta}{T} \boldsymbol{D} \cdot \Delta \nabla T\,, \quad \omega_{\rm gH}^2 = \frac{\alpha\beta(\boldsymbol{k} \cdot \boldsymbol{b})}{k^2 T} \Delta \nabla T \cdot (\boldsymbol{k} \times \boldsymbol{D}),$$

where $\boldsymbol{D} = \boldsymbol{G} - \boldsymbol{k}(\boldsymbol{k} \cdot \boldsymbol{G})/k^2$ is the component of $\boldsymbol{G}$ perpendicular to $\boldsymbol{k}$.

3. Criterion and growth rate of instability

Equation (11) has roots with Re$\gamma > 0$ (unstable modes) if one of the following inequalities is fulfilled

$$a_4 < 0\,, \quad a_0 < 0\,, \quad A_1 \equiv a_4 a_3 - a_2 < 0\,,$$
$$A_2 \equiv a_2(a_4 a_3 - a_2) - a_4(a_4 a_1 - a_0) < 0\,,$$
$$A_3 \equiv (a_4 a_1 - a_0)\left[a_2 (a_4 a_3 - a_2) - a_4 (a_4 a_1 - a_0)\right]$$
$$-a_0 (a_4 a_3 - a_2)^2 < 0\,, \quad (12)$$

(e.g., Aleksandrov et al. 1985). Note that despite Eq. (11) has five roots and there are five Routh-Hurwitz criteria of instability (12), there is no one-to-one correspondence between the roots and criteria. It is often the case that when one criterion is satisfied, some others are as well, and in reality only one (or few) criterion is (are) involved.

We consider only the criterion $A_3 < 0$ that is most relevant to the NS oceans. For a very short wavelength, $\lambda = 2\pi/k$, dissipative frequences ω_η, $\omega_\wedge$, and ω_χ become larger than dynamical frequences $\omega_{\rm A}$, $\omega_{\rm g}$, and $\omega_{\rm gH}$, and neither of the conditions (12) is satisfied. Therefore, we consider the most interesting case $\min(\omega_{\rm A}, \omega_{\rm g}, \omega_{\rm gH}) > \max(\omega_\eta, \omega_\wedge, \omega_\chi)$. This inequality is usually equivalent to $\omega_{\rm gH} > \omega_\chi$ since $\omega_{\rm gH}$ is the smallest dynamical frequency and ω_χ is the largest dissipative frequency. We can rewrite the condition $\omega_{\rm gH} > \omega_\chi$ as

$$\lambda > \lambda_{\rm c} = \frac{2\pi\chi_\parallel^{1/2}}{(\alpha\beta\delta g)^{1/4}} \left(\frac{\Delta\nabla T}{T}\right)_{\rm t}^{-1/4} \sim \frac{0.1Z^{1/2}T_7^{1/2}x^{1/6}}{\left[\delta(1+x^{2/3})\right]^{1/4}} \ {\rm cm}, \quad (13)$$

where $T_7 = T/10^7$ K and $\delta = k_t/k$, the subscript "t" denotes the component perpendicular to $\boldsymbol{G}$; $(\Delta\nabla T/T)_t \equiv 1/L$ with L being the temperature lengthscale along the surface, $L = 10^6$ cm in Eq. (13); we assume that $\rho < 10^{10}$ g/cm^3 in the ocean and use the electron thermal conductivity by Urpin & Yakovlev (1980).

Under Eq. (13), the criterion $A_3 < 0$ is equivalent

$$(\omega_{\rm gH}^2 - \omega_{\rm g}^2)\left[\omega_{\rm gH}^2 + \omega_{\rm g}^2(1-\zeta)\right] > 0, \quad (14)$$

where $\zeta = \omega_\chi\omega_{\rm g}^2/\omega_\eta\omega_{\rm A}^2$. If $\omega_{\rm gH}^2 = 0$ then Eq. (14) reduces to the well known criterion of oscillatory convection

$$\omega_\chi\omega_{\rm g}^2 > \omega_\eta\omega_{\rm A}^2 \quad (15)$$

(see, e.g., Chandrasekhar 1961). However, usially $\zeta < 1$ in the surface layer of NSs, and Eq. (15) is not fulfilled except very hot stars with a low magnetic field (Miralles et al. 1997). If $|\zeta| \ll 1$, then Eq. (14) simplifies,

$$|\ \omega_{\rm gH}^2\ | > |\ \omega_{\rm g}^2\ |\ . \quad (16)$$

This criterion as well as the condition (14) can always be satisfied by a corresponding choice of $\boldsymbol{k}$ once $(\Delta\nabla T)_{\rm t} \neq 0$.

Introducing local coordinates with the z-axis antiparallel to $\boldsymbol{G}$ and the x-axis aligned with $\boldsymbol{k}_{\rm t}$, we have $\boldsymbol{G} = -G\boldsymbol{e}_z$, $\boldsymbol{k} = k_x\boldsymbol{e}_x + k_z\boldsymbol{e}_z$; $\boldsymbol{e}_x, \boldsymbol{e}_y$, and $\boldsymbol{e}_z$ are the unit vectors. Then, Eq. (16) reads

$$\alpha\ |\ \boldsymbol{k}\cdot\boldsymbol{b}\ |\ |\ \Delta\nabla T_y\ | > |\ k_x\Delta\nabla T_z - k_z\Delta\nabla T_x\ |, \quad (17)$$

where $\Delta\nabla T_{x,y,z}$ are the corresponding components of $\Delta\nabla T$. It is seen that instability occurs only if $\boldsymbol{k}$ has a non-zero component perpendicular to the plane $(\boldsymbol{G}, \Delta\nabla T)$. Perturbations are suppressed if $\boldsymbol{k}$ is parallel to this plane. Usually, the component of $\Delta\nabla T$ parallel to the NS surface is relatively small compared to the radial one. Then, Eq. (17) is fulfilled if

$$\left|\frac{k_x}{k_z}\right| < \alpha\frac{|b_z\Delta\nabla T_y|}{|\Delta\nabla T_z|}, \quad (18)$$

and only perturbations with $|k_z| \gg |k_x|$ can be unstable. Since $\boldsymbol{kv} = 0$, this implies that the velocity of unstable perturbations has a small vertical component and a large component parallel to the surface.

The dispersion Eq. (11) can be solved by making use of the perturbation method if $\min(\omega_{\rm A}, \omega_{\rm g}, \omega_{\rm gH}) > \max(\omega_\eta, \omega_\wedge, \omega_\chi)$ and Eq. (13) is satisfied. We can expand the growth rate as $\gamma = \gamma^{(0)} + \gamma^{(1)} + ...$ where $\gamma^{(0)}$ and $\gamma^{(1)}$ are terms of the zeroth and first order in dissipative frequencies, respectively. The corresponding expansion should be made for the coefficients of Eq. (11) as well. Generally, we can restrict ourselves in the linear terms in dissipative frequencies. In the zeroth order when dissipation is neglected, Eq. (11) reduces to a quadratic equation,

$$\gamma^{(0)\,4} + \gamma^{(0)\,2}(2\omega_{\rm A}^2 - \omega_{\rm g}^2) + \omega_{\rm A}^2\left(\omega_{\rm A}^2 - \omega_{\rm g}^2\right) = 0. \quad (19)$$

This equation describes four modes with the frequencies

$$\gamma_{1,2}^{(0)\,2} = -\omega_{\rm A}^2, \quad \gamma_{3,4}^{(0)\,2} = -\omega_{\rm A}^2 + \omega_{\rm g}^2. \quad (20)$$

The fifth root of Eq. (11) vanishes in the zeroth order, $\gamma_5^{(0)} = 0$. Usually, $\gamma^{(0)\,2} < 0$ in NSs because the inequality $\omega_{\rm g}^2 > \omega_{\rm A}^2$ requires large ∇T that does not exist in NSs. Therefore, instability does not occur in the zeroth order.

The real parts of $\gamma^{(1)}$ that determine instability are

$${\rm Re}\ \gamma_{1,2}^{(1)} = \frac{\omega_\eta}{2\omega_{\rm g}^2}\left(\omega_{\rm gH}^2 - \omega_{\rm g}^2\right), \quad (21)$$

$${\rm Re}\ \gamma_{3,4}^{(1)} = -\frac{\omega_\eta\omega_{\rm A}^2}{2\omega_{\rm g}^2\left(\omega_{\rm A}^2 - \omega_{\rm g}^2\right)}\left[\omega_{\rm gH}^2 + \omega_{\rm g}^2(1-\zeta)\right], \quad (22)$$

$$\gamma_5^{(1)} = \frac{\omega_\eta\left(\omega_{\rm g}^2 + \omega_{\rm gH}^2\right) - \omega_\chi\omega_{\rm A}^2}{\omega_{\rm A}^2 - \omega_{\rm g}^2}. \quad (23)$$

The modes 1–4 are oscillatory and can be unstable if the condition (14) is satisfied. If

$$\omega_{\rm gH}^2 > \omega_{\rm g}^2 \ \text{ and } \ \omega_{\rm gH}^2 > \omega_{\rm g}^2(\zeta - 1), \quad (24)$$

then the modes 1, 2 are unstable but the modes 3, 4 are stable if $\omega_{\rm g}^2 > 0$, and the modes 1, 2 are stable but the modes 3, 4 are unstable if $\omega_{\rm g}^2 < 0$. On the contrary, if

$$\omega_{\rm gH}^2 < \omega_{\rm g}^2 \ \text{ and } \ \omega_{\rm gH}^2 < \omega_{\rm g}^2(\zeta - 1), \quad (25)$$

then the modes 1, 2 are stable whereas the modes 3 and 4 are unstable if $\omega_{\rm g}^2 > 0$, and the modes 1, 2 are unstable but the modes 3, 4 are unstable if $\omega_{\rm g}^2 < 0$.

The fifth root corresponds to a non-oscillatory mode. The condition of instability of this mode,

$$\omega_\eta(\omega_{\rm g}^2 + \omega_{\rm gH}^2) - \omega_\chi\omega_{\rm A}^2 > 0, \quad (26)$$

generalizes the Chandrasekhar criterion of convection for non-parallel $\boldsymbol{g}$ and ∇T and is equivalent to $a_0 < 0$.

4. Discussion

The NS oceans are always hydrodynamically unstable if T varies over the surface. The growth rate of instability depends on ∇T and λ and can vary within a wide range. If $\omega_{\rm A}^2 \gg \omega_{\rm g}^2$ and $\zeta < 1$, then the growth rate is

$${\rm Re}\ \gamma \approx \pm\frac{\omega_\eta}{2\omega_{\rm g}^2}\left(\omega_{\rm gH}^2 \pm \omega_{\rm g}^2\right). \quad (27)$$

The growth rate is maximal if $\boldsymbol{k}$ lies in the plane perpendicular to the plane $(\boldsymbol{G}, \Delta\nabla T)$. Then $\Delta\nabla T_x = 0$ and, assuming that Eq. (18) is fulfilled, we have from Eq. (27)

$${\rm Re}\,\gamma \sim \frac{\eta_\parallel k^2}{2}\frac{\alpha b_z|\Delta\nabla T_y|}{|\Delta\nabla T_z|}\frac{k_z}{k_x} \sim 40k^2\ \frac{B_{13}}{x}\ \frac{H}{L}\ \left|\frac{k_z}{k_x}\right|\ {\rm s}^{-1}, \quad (28)$$

where $H = T/|\Delta\nabla T_z| \approx 10^3$ cm is the vertical lengthscale of the temperature. The ratio k_z/k_x in Eq. (28) is maximal if the wavelength of perturbations along the surface is large (say, $\sim 0.01L$) but the vertical wavelength is minimal. For such perturbations, $|Hk_z/Lk_x| \sim 0.01H/\lambda_z$ where λ_z is the vertical wavelength and, hence,

$$\mathrm{Re}\,\gamma \sim 10^{-2}\ B_{13}x^{-1}\ \lambda_{z2}^{-3}\ \ \mathrm{s}^{-1}, \qquad (29)$$

where $\lambda_{z2} = \lambda_z/10^2$ cm. The growth rate increases with decreasing λ_z and reaches its maximum at $\lambda_z \sim \lambda_c$. For example, if $\lambda_z \sim 10$ cm and $B \sim 10^{13}$ G then the growth time is ~0.1 s in the layer where $\rho \sim 10^6$ g/cm^3.

The main driving forces of instability are the Hall effect and horizontal advection of heat and, as a result, the growth rate (28) is proportional to the Hall conductivity, $\eta_\wedge = \eta_\| \alpha$, and ∇T_y. This point can be clarified by a simple qualitative consideration. The amplitude of magnetic perturbations is changed mainly by the Hall effect. For example, the amplitude B_{1y} increases after Δt by

$$\frac{\Delta B_{1y}}{\Delta t} \sim \eta_\wedge(\boldsymbol{k b})(\boldsymbol{k} \times \boldsymbol{B}_1)_y \sim \eta_\wedge(\boldsymbol{k b})\frac{k^2}{k_x}B_{1z} \qquad (30)$$

(we use the divergence-free condition (5) for $\boldsymbol{B}_1$); small perturbations are marked by the subscript "1". Since unstable perturbations are approximately Alfvenic in the ocean, the components of velocity and magnetic field are related by $B_{1z} \sim B_{1y}(v_{1z}/v_{1y})$. For the considered instability, perturbations of the temperature are small, $T_1 \propto (\boldsymbol{v}_1 \cdot \Delta\nabla T) \approx 0$, then $v_{1z}/v_{1y} \sim \Delta\nabla T_y/\Delta\nabla T_z$. Substituting these expressions into Eq. (30), we obtain the growth rate (28).

The growth time of instability is relatively short and, likely, it operates in a non-linear regime. We estimate the saturated velocity using the mixing-length model (e.g., Schwarzschild 1958) that assumes that the turn-over time of turbulence is of the order of the growth time of instability. Then, the vertical turbulent velocity is approximately given by $v_{\mathrm{T}z}(\lambda_z) \sim \lambda_z\ \mathrm{Re}\,\gamma(\lambda_z)$ or

$$v_{\mathrm{T}z} \sim B_{13}x^{-1}\lambda_{z2}^{-2}\ \mathrm{cm/s}. \qquad (31)$$

Since the instability is anisotropic, the saturated turbulent velocity should be anisotropic as well. From Eqs. (3) and (30), we can estimate the turbulent velocity along the surface as $v_{\mathrm{T}x} \sim v_{\mathrm{T}z}(\lambda_x/\lambda_z)$.

Turbulent motions can enhance transport in the ocean. The coefficient of turbulent diffusion in the vertical direction is relatively large,

$$\nu_\mathrm{T} \sim v_{\mathrm{T}z}\lambda_z \sim 10^2 B_{13}x^{-1}\ \lambda_{z2}^{-1}\ \mathrm{cm^2/s}. \qquad (32)$$

For example, $\nu_\mathrm{T} \sim 10^2$ cm^2/s if the characteristic vertical wavelength is ~1 m and $B_{13} = x = 1$. Such diffusion is sufficient to mix the layer with the depth ~10 m on a timescale ~hours. Turbulent diffusion can be compared to the gravitational sedimentation in the ocean. The coefficient of interspecies diffusion (Brown et al. 2002) reads in our notations

$$\mathcal{D} \sim \frac{4 \times 10^{-3}}{A^{0.5}Z^{0.7}Z_2^{0.3}}\ \frac{T_7^{1.2}}{x^{0.6}}\ \mathrm{cm^2\ s^{-1}}, \qquad (33)$$

where Z_2 is the charge of a trace component. This quantity is usually much smaller than ν_T. The turbulent diffusivity is also few orders of magnitude larger that the electron and ion shear viscosities calculated by Itoh et al. (1987) except layers with $\rho \geq 10^9$ g/cm^3.

Apart from transport of heavy elements to the surface, turbulence can also enhance heat transport. The turbulent thermal diffusivity is $\sim\nu_\mathrm{T}$, and the ratio of turbulent and electron thermal diffusivities is

$$\frac{\nu_\mathrm{T}}{\chi_\|} \sim B_{13}T_7^{-1}\left(1 + x^{2/3}\right)x^{-1}\lambda_{z2}^{-1}. \qquad (34)$$

Generally, these quantities can be comparable. Since ∇T is subadiabatic, turbulent motions increase the difference between the surface and internal temperature. Turbulent transport along the surface and vertically should be different for the considered instability with more efficient diffusion along the surface. This can reduce the surface temperature gradient and decrease the contrast between the polar and equatorial temperature.

Note that the chemical stratification can influence the stability properties of the ocean as well. This influence is two-fold. If stratification is spherical then it provides the stabilizing effect increasing ω_g^2 (the Ledoux effect). However, if stratification is non-spherical (for example, in accreting NSs) then it can cause instability of the same nature as considered above but driven by the surface chemical gradient.

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References

Aleksandrov, A., Kolmogorov, A., & Laurentiev, M. 1985, Mathematics: Its Content, Methods, and Meaning (Cambridge: MIT)
Balbus, S. 2000, ApJ, 534, 420
Bildsten, L., & Cutler, C. 1995, ApJ, 449, 800
Bildsten, L., Ushomirski, G., & Cutler, C. 1996, ApJ, 460, 827
Brown, E., Bildsten, L., & Chang, P. 2002, ApJ, 574, 920
Chandrasekhar, S. 1961, Hydrodynamic and hydromagnetic stability (Oxford: Clarendon)
Hailey, C., & Mori, K. 2002, ApJ, 578, L133
Itoh, N., Kohyama, Y., & Takeuchi, H. 1987, ApJ, 317, 733
Landau, L., & Lifshitz, E. 1971, Fluid mechanics (London: Pergamon Press)
Miralles, J., Urpin, V., & van Riper, K. 1997, ApJ, 480, 358
Potekhin, A., & Yakovlev, D. 2001, A&A, 374, 213
Ruderman, M., & Sutherland, P. 1975, ApJ, 196, 51
Sanwal, D., Pavlov, G., Zavlin, V., & Teter, M. 2002, ApJ, 574, L61
Schaaf, M. 1990, A&A, 227, 61
Schwarzschild, M. 1958, Structure and evolution of the stars (Princeton: Princeton Univ. Press)
Slattery, V., Doolen, G., & De Witt, H. 1980, Phys. Rev. A, 21, 2087
Urpin, V., & Gil, J. 2004, A&A, 415, 305
Urpin, V., & Yakovlev, D. 1979, Astrofizika, 15, 647
Urpin, V., & Yakovlev, D. 1980, SvA, 24, 425
Yoshida, S., & Lee, U. 2001, ApJ, 546, 1121
Yoshida, S., & Lee, U. 2002, A&A, 395, 201
Zavlin, V., & Pavlov, G. 2002, Proc. of the 270th Heraeus seminar on neutron stars, pulsars and supernova remnants, ed. W. Becker, H. Lesch, & J. Truemper, MPE Reports, 278, 273