

Apsidal motion in eccentric eclipsing binaries: V871 Aql, V345 Lac, V401 Lac and CR Sct[★]

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Abstract. About thirty new times of minimum light recorded with photoelectric or CCD photometers were obtained for several little studied eccentric-orbit eclipsing binaries V871 Aql ($P = 3^d0$, $e = 0.13$), V345 Lac (7^d5 , 0.46), V401 Lac (2^d0 , 0.16) and CR Sct (4^d2 , 0.09). Their O–C diagrams were analyzed using all reliable timings found in the literature and in all cases an apsidal motion was either discovered or confirmed. The new values for the elements of the apsidal motion were derived. We find periods of apsidal motion of about 255, 7000, 79.4 and 4400 years for V871 Aql, V345 Lac, V401 Lac and CR Sct, respectively.

Key words. stars: binaries: eclipsing – stars: general – stars: fundamental parameters

1. Introduction

The study of apsidal motion in detached eclipsing binary systems with eccentric orbits (EEB) is a rewarding area of research, which requires only moderate or small telescopes equipped with a photoelectric photometer or a CCD camera. A detailed analysis of the period variations can be performed using times of minimum light observed throughout the apsidal motion cycle, and from this both the orbital eccentricity and the period of rotation of the periastron can be obtained with a high accuracy. Moreover, this provides independent information for the analysis of the light curves (Giménez 1994). Similar photometric studies of apsidal motion in EEB's were published regularly during the seventies by Helmut Busch, Hartha observatory, and later e.g. by Kh. F. Khaliullin, Moscow University, or J. V. Clausen, Copenhagen University. A catalogue of eclipsing binaries that are suitable for photometric monitoring was provided by Hegedüs (2000) while a catalogue of known binaries with apsidal motion was published by Petrova & Orlov (1999).

In this paper, we report new results of our observational project initiated more than ten years ago with the main purpose of monitoring the eclipsing binaries with eccentric orbits. In particular, the four northern-hemisphere summer

objects V871 Aql, V345 Lac, V401 Lac and CR Sct analyzed here are relatively little studied and rather faint eclipsing binaries, although the O–C diagrams for V345 Lac and CR Sct were published by Kreiner et al. (2001). With the exception of V401 Lac, the apsidal motion is announced here for the first time. To the best of our knowledge, no spectroscopic observations have been published for these systems.

2. Observations of minimum light

In order to enlarge the number of times of minimum light, new observations for all systems were carried out. New photoelectric photometry was obtained at five different observatories with the aim to secure several new, well-covered primary and secondary minima for all variables:

- Ondřejov Observatory, Czech Republic: the 0.65-m reflecting telescope with the CCD camera SBIG ST-8 or Apogee AP-7 and *R* or *V* filters;
- Hvar Observatory, Croatia: the 0.65-m reflecting telescope with the classical photoelectric photometer and Johnsons *UBV* filters;
- Tien-Shan High Altitude Station of the Sternberg Astronomical Institute, Kazakhstan: 0.70-m telescope with the photoelectric photometer and *WBVR* filters;

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[★] Partly based on photoelectric observations secured at the Hvar Observatory, Zagreb University, Croatia.

Table 1. New times of minimum light.

System	JD Hel.- 2 400 000	Error [day]	Epoch	Method filter	Reference observatory
V871 Aql	49 925.300	0.001	3823.0	CCD, <i>R</i>	Ondřejov, this paper
	50 242.5359	0.0005	3930.5	CCD, <i>R</i>	Ondřejov, this paper
	51 680.4929	0.0005	4417.5	CCD, <i>R</i>	Ondřejov, this paper
	52 467.5362	0.0001	4684.0	CCD, <i>R</i>	Ondřejov, this paper
	52 492.4813	0.0001	4692.5	CCD, <i>R</i>	Ondřejov, this paper
V345 Lac	50 201.495	0.003	2517.0	CCD, <i>R</i>	Ondřejov, this paper
	50 286.53	0.02	2528.5	CCD, <i>R</i>	Diethelm (1996)
	50 673.4854	0.0002	2580.0	CCD, <i>R</i>	Ondřejov, this paper
	50 713.5572	0.0005	2585.5	CCD, <i>R</i>	Ondřejov, this paper
	51 717.47874	0.0001	2719.5	CCD, <i>V</i>	Ondřejov, this paper
	51 777.4102	0.0004	2727.5	CCD, <i>R</i>	Ondřejov, this paper
	52 239.29095	0.0001	2789.0	CCD, <i>V</i>	Ondřejov, this paper
	52 651.3427	0.0003	2844.0	CCD, <i>R</i>	Lelekovice, this paper
V401 Lac	48 170.268	0.001	-170.0	pe, <i>H_p</i>	Hipparcos, this paper
	48 501.7900	–	0.0	pe, <i>H_p</i>	Hipparcos, ESA (1997)
	48 514.66	0.01	6.5	pe, <i>H_p</i>	Hipparcos, this paper
	48 781.82	0.01	143.5	pe, <i>H_p</i>	Hipparcos, this paper
	51 842.3465	0.0002	1713.0	CCD, –	Hegedüs et al. (2003)
	51 843.418	–	1713.5	computed	this paper, see text
	52 148.5116	0.0001	1870.0	CCD, –	Ostrava, this paper
	52 464.4306	0.0001	2032.0	CCD, <i>R</i>	Ostrava, this paper
	52 891.5075	0.0003	2251.0	pe, <i>UBV</i>	Ak & Filiz (2003)
	52 898.4022	0.0007	2254.5	pe, <i>UBV</i>	Ak & Filiz (2003)
	52 940.2601	0.0003	2276.0	pe, <i>UBV</i>	Hvar, this paper
CR Sct	45 172.388	0.005	4079.5	pe, <i>V</i>	Tien-Shan, this paper
	45 564.3076	0.0005	4173.0	pe, <i>V</i>	Tien-Shan, this paper
	50 232.558	0.001	5286.5	CCD, <i>R</i>	Ondřejov, this paper
	50 318.4346	0.0002	5307.0	CCD, <i>R</i>	Ondřejov, this paper
	51 364.4907	0.0002	5556.5	CCD, <i>R</i>	Ondřejov, this paper
	51 777.3707	0.0002	5655.0	CCD, <i>R</i>	Ondřejov, this paper
	51 798.33117	0.0001	5660.0	CCD, <i>R</i>	Ondřejov, this paper
	52 454.49927	0.0001	5816.5	CCD, <i>R</i>	Ondřejov, this paper
	52 802.4640	0.0004	5899.5	CCD, <i>R</i>	Ondřejov, this paper
52 846.41902	0.0001	5910.0	CCD, <i>R</i>	Ondřejov, this paper	

- Private observatory of K.H. at Lelekovice, Czech Republic: the 0.35-m Newtonian telescope with the CCD camera SBIG ST-6V and *R* filter;
- Johann Palisa Observatory and Planetarium of Technical University, Ostrava, Czech Republic: the 0.15-m or 0.09-m telescopes with CCD camera Meade Pictor XTE 416 or SBIG ST-7.

The CCD measurements in Ondřejov, Lelekovice and Ostrava observatories were flat-fielded via sky exposures taken at dusk or dawn. Several comparison stars were chosen on the same frame as the variables. No variations in the brightness of these stars exceeding the expected error of measurements were detected (typically $\sigma \approx 0.005$ mag in Ondřejov). No correction

for differential extinction was applied, due to the proximity of the comparison stars to the variable and the resulting negligible differences in air mass.

The photoelectric measurements of CR Sct at Tien-Shan observatory were done using the standard photoelectric photometer and *V* filter of the *WBVR*-photometric system. Concerning this equipment see Khaliullin et al. (1985) for more details. The new times of primary and secondary minima and their errors were determined by fitting polynomials to the data by the least squares and the bisecting chord method and are given in Table 1. Experience shows that this method gives smaller errors than the often used Kwee & Van Woerden (1956) method. (For instance, the latter method gives HJD 2 452 239.2898 \pm 0.0056 for a well-covered

Table 2. Apical motion elements for V871 Aql, V345 Lac, V401 Lac and CR Sct.

Parameter	Unit	V871 Aql	V345 Lac	V401 Lac	CR Sct
T_0	HJD	2 438 637.1489 (9)	2 431 343.5977 (7)	2 448 501.8816 (12)	2 428 069.6810 (7)
P_s	days	2.9526698 (12)	7.4919207 (8)	1.9500920 (12)	4.1923467 (11)
P_a	days	2.9527633 (12)	7.4919426 (8)	1.9502231 (12)	4.1923577 (11)
e		0.1295 (12)	0.456 (8)	0.161 (7)	0.089(4)
$\dot{\omega}$	deg cycle ⁻¹	0.0114 (12)	0.00106 (10)	0.0242 (25)	0.00094 (9)
$\dot{\omega}$	deg yr ⁻¹	1.41 (14)	0.052 (5)	4.53 (45)	0.0082 (8)
ω_0	deg	178.2 (1.2)	238.4 (1.0)	16.4 (1.1)	68.9 (1.3)
U	years	255 (26)	7000 (700)	79.4 (8)	4400 (450)

minimum of V345 Lac. Note that the value agrees with the value adopted here within the quoted errors – see Table 1). Only the bottom parts of the eclipses were used. All epochs in Table 1 are calculated from the light elements given in Table 2, the other columns are self-explanatory.

3. Apical motion analysis

The apical motion in all systems was studied by means of an O–C diagram analysis. For a more accurate calculation of the apical motion rate, the method described by Giménez & García-Pelayo (1983) and revised by Giménez & Bastero (1995), was used. This is a weighted least squares iterative procedure, including terms in the eccentricity up to the fifth order. Due to the relatively large values of the orbital eccentricity of some studied systems, we used all terms in our calculation. The relation for the prediction of the times of minimum, used for the minimization by the least-squares method, is also given in Wolf & Šarounová (1995). There are five independent variables (T_0 , P_s , e , $\dot{\omega}$, ω_0) to be determined in this procedure. The relation between the sidereal and the anomalistic period, P_s and P_a , is given by

$$P_s = P_a (1 - \dot{\omega}/360^\circ)$$

and the period of apical motion by

$$U = 360^\circ P_a / \dot{\omega}.$$

We have collected all reliable times of minimum light available in the literature. All new precise photoelectric and CCD times of minimum were used with a weight of 10 or 20 in our computation. The current photographic as well as some of our less precise measurements were weighted with a factor of 5, while the earlier visual and photographic times (esp. the times of the mid-exposure of a photographic plate) were given a weight of 1 or 0 because of the large scatter in these data. We tested the stability of the results with respect to our – somewhat arbitrarily chosen – weighting scheme. It turned out that the results show some dependence on the weighting but this is mainly related to the less ideal distribution of available observations. For this reason, the results must be considered preliminary.

3.1. V871 Aql

The detached eclipsing binary V871 Aquilae (=SVS 1215 = FL 2526; $\alpha_{2000} = 18^h 42^m 32^s$, $\delta_{2000} = -3^\circ 10.0'$, $V_{\max} = 13.6$ mag; Sp. G2) is a neglected, rather faint eclipsing binary with eccentric orbit ($e = 0.13$) and an orbital period of about 3 days. It was discovered to be a variable star in the field of SA 110 by Kurochkin (1956), who derived the first light elements

$$\text{Pri. Min.} = \text{HJD } 2\,432\,764.365 + 2^d 95300 \cdot E.$$

Later, Kurochkin (1979) improved the light elements with a shorter orbital period

$$\text{Pri. Min.} = \text{HJD } 2\,438\,637.28 + 2^d 952682 \cdot E.$$

Since its discovery, no photometric, spectroscopic or period study of this star was published as far as we know. Ten new epochs were derived from the photographic measurements given in Kurochkin (1979). A total of 15 times of minimum light were used in our analysis, with 8 secondary eclipses among them.

The computed apical motion elements and their internal errors of the least squares fit (in brackets) are given in Table 2. In this table P_s denotes the sidereal period, P_a the anomalistic period, e represents the eccentricity and $\dot{\omega}$ is the rate of periastron advance (in degrees per cycle or in degrees per year). The zero epoch is given by T_0 and the corresponding position of the periastron is represented by ω_0 . The O–C residuals for all times of minimum with respect to the linear part of the apical motion equation are shown in Fig. 1. The non-linear predictions, corresponding to the fitted parameters, are plotted as continuous and dashed curves for primary and secondary eclipses, respectively.

3.2. V345 Lac

The detached eclipsing binary V345 Lacertae (=GSC 3986.2900 = VV 464 = FL 3354; $\alpha_{2000} = 22^h 18^m 43.4^s$, $\delta_{2000} = +54^\circ 40' 33.5''$, $V_{\max} = 11.3$ mag; Sp. B8) is the next relatively little known, early-type eccentric binary candidate for the detection of the apical motion. It has a high orbital eccentricity of $e \approx 0.5$ and a longer period of about 7.5 days. It was discovered to be a variable star photographically by

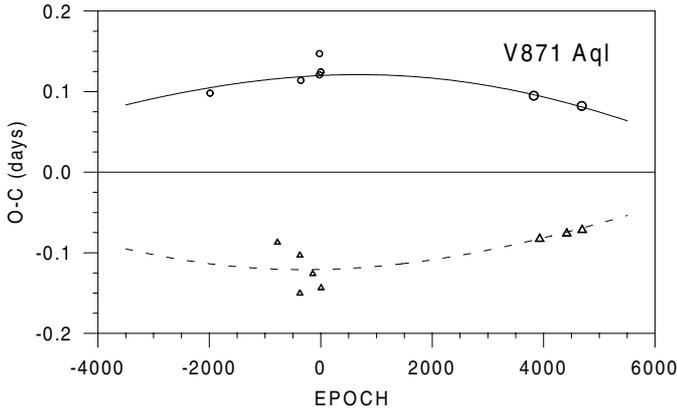


Fig. 1. The O–C diagram for the times of minimum of V871 Aql. The continuous and dashed curves represent predictions for the primary and secondary eclipses, respectively. The individual primary and secondary minima are denoted by circles and triangles, respectively. Larger symbols correspond to the photoelectric or CCD measurements which were given higher weights in the calculations.

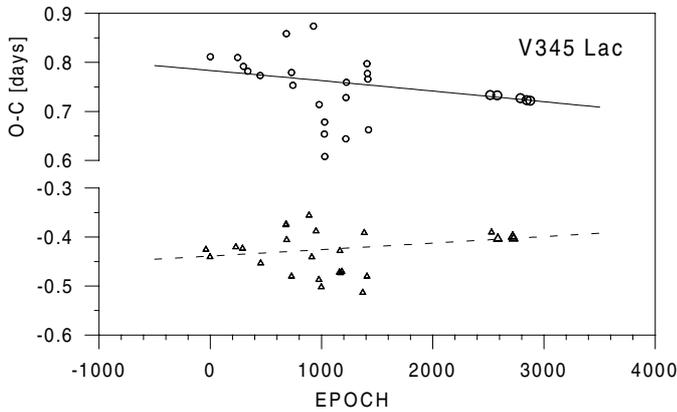


Fig. 2. O–C graph for V345 Lac. See legend for Fig. 1.

Miller & Wachmann (1973), who obtained first different light elements for primary and secondary minimum

$$\begin{aligned} \text{Pri. Min.} &= \text{HJD } 2\,433\,569.493 + 7^{\text{d}}491746 \cdot E, \\ \text{Sec. Min.} &= \text{HJD } 2\,433\,564.526 + 7^{\text{d}}491761 \cdot E, \end{aligned}$$

and recommended this eclipsing variable for future apsidal motion study. The photographic light curve and new times of minimum light were later found by Busch (1978) from the plates of the Sonnenberg and Hartha Observatories. He derived the light elements with rather longer orbital period:

$$\begin{aligned} \text{Pri. Min.} &= \text{HJD } 2\,431\,344.407 + 7^{\text{d}}491862 \cdot E, \\ \text{Sec. Min.} &= \text{HJD } 2\,431\,346.954 + 7^{\text{d}}491862 \cdot E. \end{aligned}$$

He also announced the possibility of an apsidal motion in this system and found a relatively large value of the orbital eccentricity $e = 0.50$ and $\omega = 240$ deg. Our new CCD times are given in Table 1. A total of 50 times of minimum light were used in our analysis, with 25 secondary eclipses among them. The resulting apsidal motion parameters are again given in Table 2. The O–C residuals for all times of minimum with respect to the linear part of the apsidal motion equation are shown in Fig. 2 as explained above.

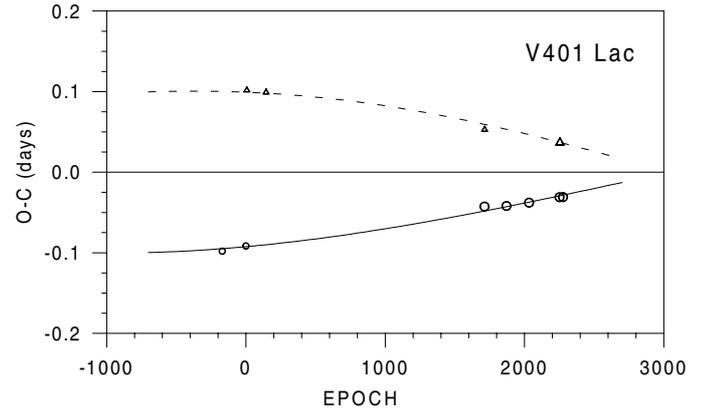


Fig. 3. The O–C diagram for V401 Lac. See legend for Fig. 1.

3.3. V401 Lac

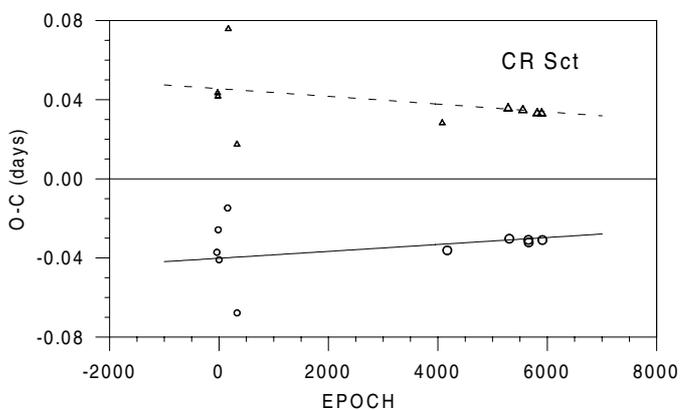
The detached eclipsing binary V401 Lacertae (=HD 210308 = HIP 109283 = BD +48 3621 = ADS 15 661 A; $\alpha_{2000} = 22^{\text{h}}8^{\text{m}}21.3^{\text{s}}$, $\delta_{2000} = +49^{\circ}13'15.6''$, $V_{\text{max}} = 7.9$ mag; Sp. A0) is a newly discovered and relatively bright early-type binary with an orbital eccentricity of $e = 0.16$ and a short period of about 2 days. It was discovered to be a variable star by the ESA satellite Hipparcos – see Perryman et al. (1997) who also give the first light elements

$$\text{Pri. Min.} = \text{HJD } 2\,448\,501.7900 + 1^{\text{d}}95010 \cdot E.$$

Bulut & Demircan (2001) obtained a new *UBV* light curve at Tübitak Observatory during 2000 and 2001. They also announced the possibility of a rapid apsidal motion in this system and predicted an apsidal-motion period of about 150 years. Our new CCD times of minima obtained at Ostrava in July and August 2001 are given in Table 1. Another primary minimum was obtained photoelectrically at Hvar Observatory in October 2003. The nearby comparison star HD 210119 = BD+48°3613 ($V = 8^{\text{m}}35$, $B-V = 0^{\text{m}}10$, $U-B = 0^{\text{m}}08$) was used during these *UBV* observations which consisted of 10-second integrations in each filter. They were carefully reduced to the standard *UBV* system and corrected for differential extinction using the reduction program HEC 22 rel. 14 (Harmanec & Horn 1998). The standard errors of these measurements were about 0.011, 0.008 and 0.006 mag in *U*, *B* and *V* filters, respectively. The time of the secondary minimum at HJD 2 451 843.418 was simply derived from the previous epoch using the phase of 0.55 given in the paper of Bulut & Demircan (2001). Moreover, three new times were derived directly from the Hipparcos photometry data using the light-curve profile fitting method. Nevertheless, only 11 times of minimum light were collected in our preliminary analysis, with 4 secondary eclipses among them. The resulting apsidal motion parameters are again given in Table 2. The O–C residuals for all times of minimum with respect to the linear part of the apsidal motion equation are shown in Fig. 3 as explained above.

Table 3. Eclipsing binaries with orbital eccentricities higher than 0.4.

System	Spectral type	e	P [days]	Source reference
V883 Cen	B5	0.7	35.45	Hensberge et al. (2004)
LV Her	F9	0.61	18.44	Torres et al. (2001)
V1143 Cyg	F5+F5	0.540	7.641	Giménez & Margrave (1985)
DI Her	B4+B4	0.489	10.55	Martynov & Lavrov (1987)
V2283 Sgr	A0+A7	0.488	3.471	Wolf (2000)
V541 Cyg	B9+B9	0.474	15.33	Khaliullin (1985)
V345 Lac	B8	0.455	7.492	this paper
MZ Lac	A0+A0	0.421	3.158	Wolf et al. (1998)
V1647 Sgr	A1+A2	0.414	3.283	Wolf (2000)
FT Ori	A0+A3	0.405	3.150	Wolf & Šarounová (1995)

**Fig. 4.** O–C residuals for the times of minimum of CR Sct. See legend for Fig. 1.

3.4. CR Sct

The detached eclipsing binary CR Scutum (=GSC 5697.2825 = FL 2561; $\alpha_{2000} = 18^{\text{h}}48^{\text{m}}48^{\text{s}}$, $\delta_{2000} = -9^{\circ}30'11''$, $V_{\text{max}} = 10.9$ mag; Sp. A3) has also attracted little attention, being a rather faint early-type binary with a small orbital eccentricity ($e = 0.09$) and a period of about 4.2 days. Its variability was discovered photographically by Oosterhoff (1943), who obtained also the first light elements

Pri. Min. = HJD 2 428 069.626 + 4^d192325 · E.

As far as we know, this star has not been studied since then. All times of minimum light published by Oosterhoff (1943) and our new minima obtained at Tien-Shan and Ondřejov observatories were included, with the weighting scheme mentioned in Sect. 3. A total of 19 times of primary and secondary minima were used. Preliminary apical-motion elements are also given in Table 2, the corresponding O–C diagram is plotted in Fig. 4.

4. Conclusions

We discovered apical motion and derived new apical motion elements for four eccentric eclipsing binaries by means of an O–C diagram analysis. The shortest period of apical motion among all discussed binaries was found for V401 Lac; the longest period was derived for V345 Lac.

Concerning the orbital parameters, V401 Lac seems to be very similar to HS Her, which is another eclipsing binary with a relatively short orbital period of 1.64 days and a similar apical-motion period of about 78 years (Wolf et al. 2002). For V401 Lac, it is also highly desirable to obtain new, high-dispersion and high-S/N spectroscopic observations and apply modern disentangling methods to them to obtain the radial-velocity curves of both binary components and, therefore, derive accurate masses for this important system.

Table 3 summarizes ten known eccentric eclipsing binaries with the eccentricity $e > 0.4$. There is a small subgroup of EEBs of similar spectral types (about A0–A3), similar periods (between 3.0 and 3.5 days) and also values of the orbital eccentricity ($e \approx 0.4$): MZ Lac, RU Mon, FT Ori, V1647 Sgr and V2283 Sgr.

Only a small part of the apical motion period for all these binaries is well covered by precise photoelectric and CCD observations. Therefore, new high-accuracy timings of these eclipsing systems are necessary to enlarge the timespan for a better analysis of the apical motion and to confirm the parameters given above. A spectroscopic orbit, allowing a precise mass determination, should also be obtained for a more thorough study and a more definitive determination of orbital and physical properties of all systems discussed in this paper.

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